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Research Article

A Parameter-Dependent Approach to Observer-Based H_∞ Control for Networked Control LPV Systems

Yanhui Li,[1] Xiujie Zhou,[1] Chang Zhang,[1] and Hamid Reza Karimi[2]

[1] *College of Electrical and Information Engineering, Northeast Petroleum University, Daqing, Heilongjiang 163318, China*
[2] *Department of Engineering, Faculty of Engineering and Science, The University of Agder, 4898 Grimstad, Norway*

Correspondence should be addressed to Yanhui Li; ly_hui@hotmail.com

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We address the observer-based H_∞ controller design problem for networked control LPV (NC LPV) systems, which are network-based systems that depend on unknown but measurable time-varying parameters. According to the analysis of the special issues brought by introducing network into LPV systems and the state reconstruction based on the observer, a new augmented model is established with two independent time-varying delays, which can carry out the controller and observer collaborative design effectively. Based on the parameter-dependent Lyapunov stability theory, a sufficient condition is proposed to ensure that the closed-loop system is asymptotically stable with a guaranteed H_∞ performance level γ, in which the coupling between Lyapunov function matrices and the system matrices existed. By using the Projection Lemma and introducing a slack matrix, the decoupling is achieved successfully, which refers to reducing conservatism. In the present study, the condition for stability analysis and control synthesis is formulated in terms of the parameterized linear matrix inequality (PLMI), which is infinite-dimensional and can be transformed into finite by using the basis function method and gridding technique. A numerical example is given to demonstrate the high validity and merit of the proposed approach.

1. Introduction

The control systems in which the control loops are closed over a real-time network are called the networked control systems (NCSs). It is well known that NCSs have been finding applications in a broad range of areas such as automotive, aerospace, mobile sensor networks, and industrial manufacture [1–3]. The existing researches on NCSs are mainly focused on linear systems [4], uncertain systems [5], time-delay systems [6, 7], stochastic systems [8], and so forth. Among these works, the H_∞ model reference tracking control problem for linear NCSs with a constant or time-varying sampling period was discussed in [4], where the LMI-based method and a multiobjective optimization methodology were used. The LMI method was also adopted to stabilize a class of delay plants based on NCSs [6] and further extended as recursive LMIs in the literature [9]. More specifically, [6] introduced some free matrices to obtain the stability criteria. Different from the previous method, [7] presented a new approach on stability and H_∞ performance for systems with two successive delay components in the state by exploiting a new Lyapunov-Krasovskii functional. Moreover, another control scheme was also developed by using the stochastic ways to describe the variations of the delays for NCSs [8]. However, to the best of the authors' knowledge, the study on the linear parameter-varying plant under the network environment is uncommon in recent years.

The traditional LPV systems constitute a class of linear systems whose dynamics depends on time-varying parameters that are real-time measurable, which have attracted lots of attention [10–13]. To mention a few, a delay-independent analysis and synthesis for LPV systems were discussed by using parameter-dependent Lyapunov-Krasovskii functionals in [10]. The result is easy to check while the absence of information on the delay may cause conservativeness from another perspective. Then the delay-dependent conditions were provided to reduce it [11] and further highlighted by the recent researches [12, 13]. It is worth pointing out that the conventional analysis and synthesis theories are not suitable when considering LPV systems under the network

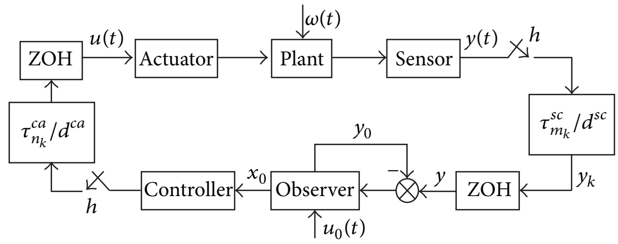

Figure 1: A new structure of the NC LPV systems.

environment, since it is required to deal with not only the parameter varying problems but also the issues such as delays, packet dropouts, and time sequence confusion that are caused by the insertion of the communication network. Therefore, it is significant to revalue these previous results and develop some new proper methods that can be applied to the NC LPV systems.

In the current researches, there exist different strategies for systems analysis, such as state feedback control [14], output feedback control [15, 16], sliding mode control [17, 18], and filtering [19]. However, it is impossible or prohibitively expensive to measure all of the state variables in many practical engineering systems. This is an essential issue to be considered for NC LPV systems. For the purpose of stability analysis and control synthesis, the state estimation is an important task, which can be achieved by designing an observer. There have been some nice results available dealing with observer-based control problem [20, 21]. The work [20] discussed the observer-based H-infinity controller design problem for NCSs with packet dropouts in the multiple channels case, and [21] was concerned with the continuous-time networked control system with random sensor delays. From these results, it is noted that very few efforts have been paid on the study of the NC LPV systems with the states that cannot be measured completely, which motivates this present study.

In this present work, our attention is focused on looking forward to designing an observer-based H_∞ controller for NC LPV systems. The first concern of this paper is to construct a new network-based structure by taking full consideration of the sensor-to-controller channel and controller-to-actuator channel, in which the network-induced delays and packet dropouts are treated to be equivalent to two time-varying delays. It is shown that a new closed-loop system model is established by augmenting the original system with an observer-based controller. Afterwards, a parameter-dependent Lyapunov-Krasovskii functional is used for stability analysis and H_∞ controller design, which can avoid the conservatism introduced by the bounding of cross-terms. The process is accomplished by using the Projection Lemma and introducing slack matrix. The important point is that the problem of stability analysis and control synthesis is formulated in the form of the PLMI, which is infinite-dimensional and can be transformed into finite by using the basis function method and gridding technique.

The notation used in this paper is standard. $\mathbb{R}^n$ denotes the n-dimensional real Euclidean space. $\mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices. $\mathbb{N}$ denotes the natural numbers set. The notations X^T and X^{-1} denote its transpose and inverse when it exists, respectively. Given a symmetric matrix $X = X^T$, the notation $X > 0$ ($X \geq 0$) means that the matrix X is real positive definiteness (semidefiniteness). By diag we denote a block diagonal matrix with its input arguments on the diagonal. I denotes the identity matrix. The symbol $*$ within a matrix represents the symmetric entries. $\|\cdot\|$ stands for either the Euclidean vector norm or its induced matrix 2-norm.

2. Problem Statement

Consider a LPV system described as follows:

$$\begin{aligned}\dot{x}(t) &= A(\rho(t))\,x(t) + B(\rho(t))\,u(t) + B_1(\rho(t))\,\omega(t),\\ y(t) &= C_1(\rho(t))\,x(t),\\ z(t) &= C_2(\rho(t))\,x(t) + D(\rho(t))\,\omega(t),\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the system state vector and control input vector, respectively. $y(t) \in \mathbb{R}^r$ is the measured output of the plant, $z(t)$ is the controlled output, and $\omega(t) \in L_2[0,\infty)$ is the disturbance input with finite energy. The system matrices $A(\rho(t))$, $B(\rho(t))$, $B_1(\rho(t))$, $C_1(\rho(t))$, $C_2(\rho(t))$, and $D(\rho(t))$ are assumed to be known continuous functions of a time-varying parameter vector $\rho(t)$. It is noted that the parameter vector $\rho(t) = [\rho_1(t), \rho_2(t), \ldots, \rho_s(t)]^T$, $\rho_i(t)$ is assumed to be measurable in real-time and $\dot{\rho}_i(t) = v_i(t) \in [\underline{v}_i, \overline{v}_i]$, $i = 1, 2, \ldots, s$. For convenient description, we write $\rho(t)$, $\dot{\rho}(t)$ as ρ, $\dot{\rho}$ in the following context.

Under the network environment, both of the sensor-to-controller channel and controller-to-actuator channel are taken into account. A new structure of the NC LPV system with an observer-based controller is constructed in Figure 1. Taking into consideration the network-induced delays and making the estimation error as small as possible, we design a dynamic observer with the following construction to estimate the states of the plant:

$$\begin{aligned}\dot{x}_o(t) &= A(\rho)\,x_o(t) + B(\rho)\,u_o(t) + L(\rho)\,(y(t) - y_o(t)),\\ y_o(t) &= C_1(\rho)\,x_o(t),\end{aligned} \tag{2}$$

where $x_o(t)$ is the state of the observer, $u_o(t)$ is the input of the observer, and $L(\rho)$ is the observer gain.

As shown in Figure 1, the measured output $y(t)$ is sampled with periodic sampling interval h and transferred over the sensor-to-controller channel. In the case of no dropouts, the sampling signals arrive the controller at the instant $m_k h + \tau^{sc}_{m_k}$ where $\tau^{sc}_{m_k}$ is the sensor-to-controller delay and m_k ($k = 1, 2, \ldots$) are time-stamps of packets that successfully reach the controller.

Thus, for $t \in [m_k h + \tau^{sc}_{m_k}, m_{k+1} h + \tau^{sc}_{m_{k+1}})$, we have

$$\begin{aligned} \dot{x}_o(t) &= A(\rho)x_o(t) + B(\rho)u_o(t) \\ &\quad + L(\rho)\left(y(m_k h) - y_o(m_k h)\right), \\ y_o(m_k h) &= C_1(\rho)x_o(m_k h). \end{aligned} \tag{3}$$

The control signals are sent at times $n_k h$ and arrive at the plant at times $n_k h + \tau^{ca}_{n_k}$, where $n_k \in \mathbb{N}$ are time-stamps of control signals received by the actuator. Then, for $t \in [n_k h + \tau^{ca}_{n_k}, n_{k+1} h + \tau^{ca}_{n_{k+1}})$, we have

$$u(t) = u_o(t) = K(\rho)x_o(n_k h), \tag{4}$$

where $K(\rho)$ is the controller gain.

Remark 1. Assume that the delays $\tau^{sc}_{m_k}$ and $\tau^{ca}_{n_k}$ are both less than one sampling period. For the control input used by the plant $u(t)$ and the control input used by the observer $u_o(t)$, since $u(t)$ is constructed from data sent by the controller, in general, $u(t) = u_o(t)$.

Remark 2. Since delays and packet dropouts may occur in both the sensor-to-controller channel and controller-to-actuator channel, we have $\{n_k\}_{k=1}^{\infty} \subseteq \{m_k\}_{k=1}^{\infty} \subseteq \mathbb{N}$.

Defining $\eta_1(t) = t - m_k h$, $\eta_2(t) = t - n_k h$, then $m_k h = t - \eta_1(t)$, $n_k h = t - \eta_2(t)$. Thus, (3) and (4) can be rewritten as

$$\begin{aligned} \dot{x}_o(t) &= A(\rho)x_o(t) + B(\rho)u(t) \\ &\quad + L(\rho)\left(y(t - \eta_1(t)) - y_o(t - \eta_1(t))\right), \\ y_o(t - \eta_1(t)) &= C_1(\rho)x_o(t - \eta_1(t)), \\ u(t) &= K(\rho)x_o(t - \eta_2(t)), \end{aligned} \tag{5}$$

where

$$\begin{aligned} \eta_1(t) &\in \left[\tau^{sc}_{m_k}, (m_{k+1} - m_k)h + \tau^{sc}_{m_{k+1}}\right], \\ \eta_2(t) &\in \left[\tau^{ca}_{n_k}, (n_{k+1} - n_k)h + \tau^{ca}_{n_{k+1}}\right], \quad \forall k \in \mathbb{N}. \end{aligned} \tag{6}$$

Remark 3. By the above definition, $\eta_1(t)$ is a piecewise continuous function, which changes whenever the sensor signal reaches the controller. The derivative of $\eta_1(t)$ is always equal to one, except at the transition point. In this paper, the delays are assumed to be bounded; that is, $\eta_j(t) \in [0, \overline{\eta}_j]$ and $\dot{\eta}_j(t) \le \mu_j$, $j = 1, 2$.

In this paper, the packet dropouts are treated as a delay which grow beyond the defined bounds. Then the original system with delays and packet dropouts is equivalent to a system with time-varying delays which satisfy

$$\begin{aligned} \eta_1(t) &\in \left[\min_k \tau^{sc}_{m_k}, (m_{k+1} - m_k)h + \max_k \tau^{sc}_{m_{k+1}}\right], \\ \eta_2(t) &\in \left[\min_k \tau^{ca}_{n_k}, (n_{k+1} - n_k)h + \max_k \tau^{ca}_{n_{k+1}}\right], \quad \forall k \in \mathbb{N}. \end{aligned} \tag{7}$$

By introducing the estimation error $e(t) = x(t) - x_o(t)$, we get the following augmented system:

$$\begin{aligned} \dot{\xi}(t) &= \overline{A}(\rho)\xi(t) + \overline{A}_1(\rho)\xi(t - \eta_1(t)) \\ &\quad + \overline{B}(\rho)\xi(t - \eta_2(t)) + \overline{B}_1(\rho)\omega(t), \\ z(t) &= \overline{C}(\rho)\xi(t) + \overline{D}(\rho)\omega(t), \end{aligned} \tag{8}$$

where $\xi^T(t) = [x^T(t) \quad e^T(t)]$ and the system matrices are

$$\begin{aligned} &\overline{A}(\rho) = \begin{bmatrix} A(\rho) & 0 \\ 0 & A(\rho) \end{bmatrix}, \qquad \overline{A}_1(\rho) = \begin{bmatrix} 0 & 0 \\ 0 & -L(\rho)C_1(\rho) \end{bmatrix}, \\ &\overline{B}(\rho) = \begin{bmatrix} B(\rho)K(\rho) & -B(\rho)K(\rho) \\ 0 & 0 \end{bmatrix}, \qquad \overline{B}_1(\rho) = \begin{bmatrix} B_1(\rho) \\ B_1(\rho) \end{bmatrix}, \\ &\overline{C}(\rho) = [C_2(\rho) \quad 0], \qquad \overline{D}(\rho) = D(\rho). \end{aligned} \tag{9}$$

This paper aims to design an observer-based H_∞ controller such that the closed-loop system (8) satisfies the following properties simultaneously.

(I) The closed-loop system (8) is asymptotically stable.

(II) Subjected to the zero initial condition and all nonzero $\omega(t)$, the controlled output $z(t)$ satisfies $\|z(t)\|_2^2 \le \gamma^2 \|\omega(t)\|_2^2$.

Then, the closed-loop system (8) is said to be asymptotically stable with H_∞ performance level γ.

Lemma 4 (Projection Lemma). *Given a symmetric matrix $\Psi \in \mathbb{R}^{m \times m}$ and two matrices $\mathscr{E}$ and $\mathscr{F}$ of column dimension m, there exists an $\mathscr{X}$ such that the following LMI holds:*

$$\Psi + \mathscr{E}^T \mathscr{X}^T \mathscr{F} + \mathscr{F}^T \mathscr{X} \mathscr{E} < 0 \tag{10}$$

if and only if the following projection inequalities with respect to $\mathscr{X}$ are satisfied:

$$\begin{aligned} \mathscr{N}_{\mathscr{E}}^T \Psi \mathscr{N}_{\mathscr{E}} &< 0, \\ \mathscr{N}_{\mathscr{F}}^T \Psi \mathscr{N}_{\mathscr{F}} &< 0, \end{aligned} \tag{11}$$

where $\mathscr{N}_{\mathscr{E}}$ and $\mathscr{N}_{\mathscr{F}}$ denote arbitrary bases of the null space of $\mathscr{E}$ and $\mathscr{F}$, respectively.

Lemma 5. *For any matrices $\mathscr{R} > 0$, $\delta > 0$ and $\mathscr{W} : [-\delta, 0] \to \mathbb{R}^n$, there has*

$$\delta \int_{-\delta}^{0} \mathscr{W}^T(s)\mathscr{R}\mathscr{W}(s)\,ds \geq \left(\int_{-\delta}^{0} \mathscr{W}(s)\,ds\right)^T \mathscr{R}\left(\int_{-\delta}^{0} \mathscr{W}(s)\,ds\right). \tag{12}$$

3. Main Results

In this section, we will first present a sufficient condition for the existance of the observer-based H_∞ controller that guarantees the closed-loop system to be asymptotically stable with H_∞ performance level γ by constructing a parameter-dependent Lyapunov functional. Then, the Projection Lemma is applied to realize the decoupling between the Lyapunov function matrices and system matrices. Lastly, the solution to the observer-based H_∞ controller design problem is obtained by using appropriate basis functions and gridding technique.

Theorem 6. *For the given positive constants $\overline{\eta}_1$, $\overline{\eta}_2$, μ_1, and μ_2, if there exist a continuously differentiable matrix function $P(\rho) > 0$, matrices $Q_j > 0$, $R_j > 0$, $j = 1, 2$, and a scalar $\gamma > 0$ such that the following PLMI holds for all ρ, then the closed-loop system* (8) *is asymptotically stable with H_∞ performance level γ:*

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & P(\rho)\overline{B}_1(\rho) & \overline{C}^T(\rho) & \overline{\eta}_1\overline{A}^T(\rho)R_1 & \overline{\eta}_2\overline{A}^T(\rho)R_2 \\ * & \Pi_{22} & 0 & 0 & 0 & \overline{\eta}_1\overline{A}_1^T(\rho)R_1 & \overline{\eta}_2\overline{A}_1^T(\rho)R_2 \\ * & * & \Pi_{33} & 0 & 0 & \overline{\eta}_1\overline{B}^T(\rho)R_1 & \overline{\eta}_2\overline{B}^T(\rho)R_2 \\ * & * & * & -\gamma^2 I & \overline{D}^T(\rho) & \overline{\eta}_1\overline{B}_1^T(\rho)R_1 & \overline{\eta}_2\overline{B}_1^T(\rho)R_2 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -R_1 & 0 \\ * & * & * & * & * & * & -R_2 \end{bmatrix} < 0, \tag{13}$$

where

$$\begin{aligned} \Pi_{11} &= P(\rho)\overline{A}(\rho) + \overline{A}^T(\rho)P(\rho) + \sum_{i=1}^{s}\left(\upsilon_i\frac{\partial P(\rho)}{\partial \rho_i}\right) + Q_1 + Q_2 - R_1 - R_2, \\ \Pi_{12} &= P(\rho)\overline{A}_1(\rho) + R_1, \\ \Pi_{13} &= P(\rho)\overline{B}(\rho) + R_2, \\ \Pi_{22} &= -(1-\mu_1)Q_1 - R_1, \\ \Pi_{33} &= -(1-\mu_2)Q_2 - R_2. \end{aligned} \tag{14}$$

Proof. Construct a Lyapunov-Krasovskii functional as

$$V(\xi_t, \rho) = V_1(\xi, \rho) + V_2(\xi_t, \rho) + V_3(\xi_t, \rho), \tag{15}$$

where $P(\rho) > 0$, $Q_j > 0$, $R_j > 0$, $j = 1, 2$, and

$$\begin{aligned} V_1(\xi, \rho) &= \xi^T(t)P(\rho)\xi(t), \\ V_2(\xi_t, \rho) &= \sum_{j=1}^{2}\left(\int_{t-\eta_j(t)}^{t}\xi^T(\alpha)Q_j\xi(\alpha)\,d\alpha\right), \\ V_3(\xi_t, \rho) &= \sum_{j=1}^{2}\left(\int_{-\overline{\eta}_j}^{0}\int_{t+\theta}^{t}\dot{\xi}^T(\beta)\overline{\eta}_jR_j\dot{\xi}(\beta)\,d\beta d\theta\right). \end{aligned} \tag{16}$$

Taking the time derivative of V_1, V_2, and V_3, respectively, we can obtain

$$\begin{aligned} \dot{V}_1(\xi, \rho) &= \dot{\xi}^T(t)P(\rho)\xi(t) + \xi^T(t)\dot{P}(\rho)\xi(t) + \xi^T(t)P(\rho)\dot{\xi}(t), \\ \dot{V}_2(\xi_t, \rho) &= \xi^T(t)(Q_1 + Q_2)\xi(t) - \sum_{j=1}^{2}\left\{\left[1 - \dot{\eta}_j(t)\right]\xi^T\left(t - \eta_j(t)\right)Q_j\xi\left(t - \eta_j(t)\right)\right\} \\ &\leq \xi^T(t)(Q_1 + Q_2)\xi(t) - \sum_{j=1}^{2}\left\{\left(1 - \mu_j\right)\xi^T\left(t - \eta_j(t)\right)Q_j\xi\left(t - \eta_j(t)\right)\right\}, \\ \dot{V}_3(\xi_t, \rho) &= \sum_{j=1}^{2}\int_{-\overline{\eta}_j}^{0}\left[\dot{\xi}^T(t)\overline{\eta}_jR_j\dot{\xi}(t) - \dot{\xi}^T(t+\theta)\overline{\eta}_jR_j\dot{\xi}(t+\theta)\right]d\theta \\ &\leq \sum_{j=1}^{2}\left\{\overline{\eta}_j^2\dot{\xi}^T(t)R_j\dot{\xi}(t) - \overline{\eta}_j\int_{t-\eta_j(t)}^{t}\dot{\xi}^T(s)R_j\dot{\xi}(s)\,ds\right\}. \end{aligned} \tag{17}$$

Applying Lemma 5 to $\dot{V}_3$,

$$\dot{V}_3(\xi_t, \rho) \leq \sum_{j=1}^{2}\left\{\overline{\eta}_j^2\dot{\xi}^T(t)R_j\dot{\xi}(t) - \frac{\overline{\eta}_j}{\eta_j(t)}\left(\int_{t-\eta_j(t)}^{t}\dot{\xi}(s)\,ds\right)^T \times R_j\left(\int_{t-\eta_j(t)}^{t}\dot{\xi}(s)\,ds\right)\right\}$$

$$\leq \sum_{j=1}^{2} \left\{ \overline{\eta}_j^2 \dot{\xi}^T(t) R_j \dot{\xi}(t) - \left[\xi(t) - \xi\left(t-\eta_j(t)\right)\right]^T \times R_j \left[\xi(t) - \xi\left(t-\eta_j(t)\right)\right] \right\}. \tag{18}$$

For $\omega(t) \equiv 0$, gathering all the derivative terms, we have

$$\dot{V} \leq \varepsilon^T(t) \Upsilon \varepsilon(t) = \varepsilon^T(t) \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} \\ * & \Upsilon_{22} & \Upsilon_{23} \\ * & * & \Upsilon_{33} \end{bmatrix} \varepsilon(t), \tag{19}$$

where

$$\begin{aligned}
\varepsilon^T(t) &= \left[\xi^T(t) \quad \xi^T(t-\eta_1(t)) \quad \xi^T(t-\eta_2(t))\right], \\
\Upsilon_{11} &= P(\rho)\overline{A}(\rho) + \overline{A}^T(\rho)P(\rho) \\
&\quad + \sum_{i=1}^{s}\left(\upsilon_i \frac{\partial P(\rho)}{\partial \rho_i}\right) + Q_1 + Q_2 - R_1 - R_2 \\
&\quad + \overline{\eta}_1^2 \overline{A}^T(\rho) R_1 \overline{A}(\rho) + \overline{\eta}_2^2 \overline{A}^T(\rho) R_2 \overline{A}(\rho), \\
\Upsilon_{12} &= P(\rho)\overline{A}_1(\rho) + R_1 \\
&\quad + \overline{\eta}_1^2 \overline{A}^T(\rho) R_1 \overline{A}_1(\rho) + \overline{\eta}_2^2 \overline{A}^T(\rho) R_2 \overline{A}_1(\rho), \\
\Upsilon_{13} &= P(\rho)\overline{B}(\rho) + R_2 \\
&\quad + \overline{\eta}_1^2 \overline{A}^T(\rho) R_1 \overline{B}(\rho) + \overline{\eta}_2^2 \overline{A}^T(\rho) R_2 \overline{B}(\rho), \\
\Upsilon_{22} &= -(1-\mu_1)Q_1 + \overline{\eta}_1^2 \overline{A}_1^T(\rho) R_1 \overline{A}_1(\rho) \\
&\quad + \overline{\eta}_2^2 \overline{A}_1^T(\rho) R_2 \overline{A}_1(\rho) - R_1, \\
\Upsilon_{23} &= \overline{\eta}_1^2 \overline{A}_1^T(\rho) R_1 \overline{B}(\rho) + \overline{\eta}_2^2 \overline{A}_1^T(\rho) R_2 \overline{B}(\rho), \\
\Upsilon_{33} &= -(1-\mu_2)Q_2 + \overline{\eta}_1^2 \overline{B}^T(\rho) R_1 \overline{B}(\rho) \\
&\quad + \overline{\eta}_2^2 \overline{B}^T(\rho) R_2 \overline{B}(\rho) - R_2.
\end{aligned} \tag{20}$$

By using the Schur Complement Lemma, inequality (13) implies $\Upsilon < 0$ and system (8) is asymptotically stable. And then we will discuss the H_∞ performance of the closed-loop system.

To establish the prescribed H_∞ performance level γ, we define

$$J_{z\omega} = \int_0^\infty \left[z^T(t) z(t) - \gamma^2 \omega^T(t) \omega(t)\right] dt. \tag{21}$$

Under zero initial condition, $V(0) = 0$ and $V(\infty) \geq 0$. Thus,

$$J_{z\omega} \leq \int_0^\infty \left[z^T(t) z(t) - \gamma^2 \omega^T(t) \omega(t) + \dot{V}(t)\right] dt. \tag{22}$$

For any nonzero $\omega(t) \in L_2[0,\infty)$,

$$\begin{aligned}
J_{z\omega} &\leq \int_0^\infty \varsigma^T(t) \Xi \varsigma(t)\, dt \\
&= \int_0^\infty \varsigma^T(t) \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} \\ * & * & \Xi_{33} & \Xi_{34} \\ * & * & * & \Xi_{44} \end{bmatrix} \varsigma(t)\, dt,
\end{aligned} \tag{23}$$

where

$$\begin{aligned}
\varsigma^T(t) &= \left[\xi^T(t) \quad \xi^T(t-\eta_1(t)) \quad \xi^T(t-\eta_2(t)) \quad \omega^T(t)\right], \\
\Xi_{11} &= \Pi_{11} + \overline{\eta}_1^2 \overline{A}^T(\rho) R_1 \overline{A}(\rho) \\
&\quad + \overline{\eta}_2^2 \overline{A}^T(\rho) R_2 \overline{A}(\rho) + \overline{C}^T(\rho)\overline{C}(\rho), \\
\Xi_{12} &= \Pi_{12} + \overline{\eta}_1^2 \overline{A}^T(\rho) R_1 \overline{A}_1(\rho) \\
&\quad + \overline{\eta}_2^2 \overline{A}^T(\rho) R_2 \overline{A}_1(\rho), \\
\Xi_{13} &= \Pi_{13} + \overline{\eta}_1^2 \overline{A}^T(\rho) R_1 \overline{B}(\rho) \\
&\quad + \overline{\eta}_2^2 \overline{A}^T(\rho) R_2 \overline{B}(\rho), \\
\Xi_{14} &= \overline{\eta}_1^2 \overline{A}^T(\rho) R_1 \overline{B}_1(\rho) \\
&\quad + \overline{\eta}_2^2 \overline{A}^T(\rho) R_2 \overline{B}_1(\rho) + \overline{C}^T(\rho)\overline{D}(\rho), \\
\Xi_{22} &= -(1-\mu_1)Q_1 + \overline{\eta}_1^2 \overline{A}_1^T(\rho) R_1 \overline{A}_1(\rho) \\
&\quad + \overline{\eta}_2^2 \overline{A}_1^T(\rho) R_2 \overline{A}_1(\rho) - R_1, \\
\Xi_{23} &= \overline{\eta}_1^2 \overline{A}_1^T(\rho) R_1 \overline{B}(\rho) + \overline{\eta}_2^2 \overline{A}_1^T(\rho) R_2 \overline{B}(\rho), \\
\Xi_{24} &= \overline{\eta}_1^2 \overline{A}_1^T(\rho) R_1 \overline{B}_1(\rho) + \overline{\eta}_2^2 \overline{A}_1^T(\rho) R_2 \overline{B}_1(\rho), \\
\Xi_{33} &= -(1-\mu_2)Q_2 + \overline{\eta}_1^2 \overline{B}^T(\rho) R_1 \overline{B}(\rho) \\
&\quad + \overline{\eta}_2^2 \overline{B}^T(\rho) R_2 \overline{B}(\rho) - R_2, \\
\Xi_{34} &= \overline{\eta}_1^2 \overline{B}^T(\rho) R_1 \overline{B}_1(\rho) + \overline{\eta}_2^2 \overline{B}^T(\rho) R_2 \overline{B}_1(\rho), \\
\Xi_{44} &= \overline{\eta}_1^2 \overline{B}_1^T(\rho) R_1 \overline{B}_1(\rho) \\
&\quad + \overline{\eta}_2^2 \overline{B}_1^T(\rho) R_2 \overline{B}_1(\rho) + \overline{D}^T(\rho)\overline{D}(\rho) - \gamma^2 I.
\end{aligned} \tag{24}$$

Applying the Schur Complement Lemma to (13) leads to $\Xi < 0$. Using the zero initial condition,

$$\int_0^\infty z^T(t) z(t)\, dt < \int_0^\infty \gamma^2 \omega^T(t) \omega(t)\, dt. \tag{25}$$

We have $\|z(t)\|_2^2 \leq \gamma^2 \|\omega(t)\|_2^2$; namely, the system has a prescribed H_∞ performance level γ. The proof is completed. □

Remark 7. Traditional Lyapunov functional is generally selected as

$$V(\xi_t,\rho)=V_1(\xi,\rho)+V_2(\xi_t,\rho), \tag{26}$$

where

$$\begin{aligned} V_1(\xi,\rho)&=\xi^T(t)P\xi(t),\\ V_2(\xi_t,\rho)&=\int_{t-\eta(t)}^{t}\xi^T(\alpha)Q\xi(\alpha)\,d\alpha. \end{aligned} \tag{27}$$

The Lyapunov matrix P is invariant for all varying parameters ρ and the obtained results based on this Lyapunov function are delay-independent, which lead to more conservativeness. In this paper, the Lyapunov functional matrix $P(\rho)$ is selected as a parameter-dependent matrix that varies with the varying parameters and the delay-dependent term of $V_3(\xi_t,\rho)$ is also added, which can reduce the conservation of the results. However, the drawback of the condition given by Theorem 6 is that the PLMI (13) involves multiple product terms of the Lyapunov function matrices and system matrices. This indicates that inequality (13) is nonlinear and makes the controller design difficult. Therefore, a slack matrix is introduced to eliminate the coupling between the parameter-dependent Lyapunov function matrices and the system matrices.

Theorem 8. *For the given positive constants $\overline{\eta}_1$, $\overline{\eta}_2$, μ_1, and μ_2, if there exist a continuously differentiable matrix function $P(\rho)>0$, matrices $Q_j>0$, $R_j>0$, $j=1,2$, U, and a scalar $\gamma>0$ such that the following PLMI holds for all ρ, then the closed-loop system* (8) *is asymptotically stable with H_∞ performance level γ:*

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & U^T\overline{A}_1(\rho) & U^T\overline{B}(\rho) & U^T\overline{B}_1(\rho) & 0 & U^T & \overline{\eta}_1R_1 & \overline{\eta}_2R_2\\ * & \Phi_{22} & R_1 & R_2 & 0 & \overline{C}^T(\rho) & 0 & 0 & 0\\ * & * & -(1-\mu_1)Q_1-R_1 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & -(1-\mu_2)Q_2-R_2 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & -\gamma^2I & \overline{D}^T(\rho) & 0 & 0 & 0\\ * & * & * & * & * & -I & 0 & 0 & 0\\ * & * & * & * & * & * & -P(\rho) & 0 & 0\\ * & * & * & * & * & * & * & -R_1 & 0\\ * & * & * & * & * & * & * & * & -R_2 \end{bmatrix}<0, \tag{28}$$

where

$$\begin{aligned} \Phi_{11}&=-\left(U+U^T\right),\\ \Phi_{12}&=P(\rho)+U^T\overline{A}(\rho),\\ \Phi_{22}&=-P(\rho)+Q_1+Q_2-R_1-R_2+\sum_{i=1}^{s}\left(v_i\frac{\partial P(\rho)}{\partial\rho_i}\right). \end{aligned} \tag{29}$$

Proof. The first 7×7 diagonal subparameter linear matrix inequality of (28) can be rewritten as

$$\Psi+E^TUF+F^TU^TE<0 \tag{30}$$

with

$$\Psi=\begin{bmatrix} 0 & P(\rho) & 0 & 0 & 0 & 0 & 0\\ * & \Phi_{22} & R_1 & R_2 & 0 & \overline{C}^T(\rho) & 0\\ * & * & -(1-\mu_1)Q_1-R_1 & 0 & 0 & 0 & 0\\ * & * & * & -(1-\mu_2)Q_2-R_2 & 0 & 0 & 0\\ * & * & * & * & -\gamma^2I & \overline{D}^T(\rho) & 0\\ * & * & * & * & * & -I & 0\\ * & * & * & * & * & * & -P(\rho) \end{bmatrix},$$

$$\begin{aligned} E&=\left[-I\ \ \overline{A}(\rho)\ \ \overline{A}_1(\rho)\ \ \overline{B}(\rho)\ \ \overline{B}_1(\rho)\ \ 0\ \ I\right],\\ F&=[I\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]. \end{aligned} \tag{31}$$

The null-spaces of E and F are

$$\mathcal{N}_E = \begin{bmatrix} \overline{A}(\rho) & \overline{A}_1(\rho) & \overline{B}(\rho) & \overline{B}_1(\rho) & 0 & I \\ I & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix},$$
$$\mathcal{N}_F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}. \quad (32)$$

Applying the Projection Lemma, we obtain the following two inequalities which are equivalent to (30):

$$\mathcal{N}_E^T \Psi \mathcal{N}_E < 0, \quad (33)$$

$$\mathcal{N}_F^T \Psi \mathcal{N}_F < 0. \quad (34)$$

Substituting $\mathcal{N}_E$ and Ψ into (33), we obtain

$$\begin{bmatrix} P(\rho)\overline{A}(\rho) + \overline{A}^T(\rho)P(\rho) + \Phi_{22} & P(\rho)\overline{A}_1(\rho) + R_1 & P(\rho)\overline{B}(\rho) + R_2 & P(\rho)\overline{B}_1(\rho) & \overline{C}^T(\rho) & P(\rho) \\ * & -(1-\mu_1)Q_1 - R_1 & 0 & 0 & 0 & 0 \\ * & * & -(1-\mu_2)Q_2 - R_2 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & \overline{D}^T(\rho) & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -P(\rho) \end{bmatrix} < 0. \quad (35)$$

It is shown that (35) is equivalent to the first 5×5 diagonal subparameter linear matrix inequality of (13). Similarly, (34) leads to an inequality which is included in (35).

Defining $\Delta_1 = \begin{bmatrix} \mathcal{N}_E^T & 0 \\ 0 & I \end{bmatrix}$ and performing congruence transformations to (28), we can obtain (13) in Theorem 6. The proof is completed. □

Remark 9. By applying the Projection Lemma to (13) and introducing slack matrix U, the parameter-dependent Lyapunov functional matrices and the system matrices are decoupled in Theorem 8. This technique can bring additional flexibility in the controller synthesis problem and results in far less conservative conditions than with customary approaches. The further solving process is given as follows.

Theorem 10. *For the given positive constants $\overline{\eta}_1$, $\overline{\eta}_2$, μ_1, and μ_2, if there exist a scalar $\gamma > 0$ and matrices $X(\rho) > 0$, $Y_j > 0$, $Z_j > 0$, G, H, U_{21}, M_1, $M_a(\rho)$, $M_{lc}(\rho)$, $N_b(\rho)$, $S_b(\rho)$, $N_g(\rho)$, $S_g(\rho)$ such that the following PLMI holds for all ρ, then the closed-loop system (8) is asymptotically stable with H_∞ performance level γ:*

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} & \Gamma_{15} & 0 & \Gamma_{17} & \Gamma_{18} & \Gamma_{19} \\ * & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} & 0 & \Gamma_{26} & 0 & 0 & 0 \\ * & * & \Gamma_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Gamma_{44} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & D^T(\rho) & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & \Gamma_{77} & 0 & 0 \\ * & * & * & * & * & * & * & -\Gamma_{23} & 0 \\ * & * & * & * & * & * & * & * & -\Gamma_{24} \end{bmatrix} < 0, \quad (36)$$

where

$$\Gamma_{11} = \begin{bmatrix} -(G + G^T) & -(H + G^T + M_1) \\ * & -(H + H^T) \end{bmatrix},$$

$$\Gamma_{12} = \begin{bmatrix} X_1(\rho) + G^T A(\rho) & X_2(\rho) + G^T A(\rho) + M_a(\rho) \\ X_2^T(\rho) + H^T A(\rho) & X_3(\rho) + H^T A(\rho) \end{bmatrix},$$

$$\Gamma_{13} = \begin{bmatrix} 0 & -M_{lc}(\rho) \\ 0 & 0 \end{bmatrix},$$

$$\Gamma_{14} = \begin{bmatrix} N_b(\rho) & N_b(\rho) - N_g(\rho) \\ S_b(\rho) & S_b(\rho) - S_g(\rho) \end{bmatrix},$$

$$\Gamma_{15} = \begin{bmatrix} G^T B_1(\rho) + U_{21}^T B_1(\rho) \\ H^T B_1(\rho) \end{bmatrix},$$

$$\Gamma_{17} = \begin{bmatrix} G^T & G^T + M_1 \\ H^T & H^T \end{bmatrix}, \quad \Gamma_{18} = \begin{bmatrix} \overline{\eta}_1 Z_{11} & \overline{\eta}_1 Z_{12} \\ \overline{\eta}_1 Z_{12}^T & \overline{\eta}_1 Z_{13} \end{bmatrix},$$

$$\Gamma_{19} = \begin{bmatrix} \overline{\eta}_2 Z_{21} & \overline{\eta}_2 Z_{22} \\ \overline{\eta}_2 Z_{22}^T & \overline{\eta}_2 Z_{23} \end{bmatrix}, \quad \Gamma_{23} = \begin{bmatrix} Z_{11} & Z_{12} \\ * & Z_{13} \end{bmatrix},$$

$$\Gamma_{24} = \begin{bmatrix} Z_{21} & Z_{22} \\ * & Z_{23} \end{bmatrix}, \quad \Gamma_{26} = \begin{bmatrix} C_2^T(\rho) \\ C_2^T(\rho) \end{bmatrix},$$

$$\Gamma_{22} = \left[-X(\rho) + \sum_{j=1}^{2} \left(Y_j - Z_j \right) + \sum_{i=1}^{s} \left(v_i \frac{\partial X}{\partial \rho_i} \right) \right],$$

$$\Gamma_{33} = \begin{bmatrix} -(1-\mu_1) Y_{11} - Z_{11} & -(1-\mu_1) Y_{12} - Z_{12} \\ * & -(1-\mu_1) Y_{13} - Z_{13} \end{bmatrix},$$

$$\Gamma_{44} = \begin{bmatrix} -(1-\mu_2) Y_{21} - Z_{21} & -(1-\mu_2) Y_{22} - Z_{22} \\ * & -(1-\mu_2) Y_{23} - Z_{23} \end{bmatrix},$$

$$X(\rho) = \begin{bmatrix} X_1(\rho) & X_2(\rho) \\ * & X_3(\rho) \end{bmatrix}, \quad Y_j = \begin{bmatrix} Y_{j1} & Y_{j2} \\ * & Y_{j3} \end{bmatrix},$$

$$Z_j = \begin{bmatrix} Z_{j1} & Z_{j2} \\ * & Z_{j3} \end{bmatrix}. \tag{37}$$

Furthermore, the parameters of controller and observer are

$$K(\rho) = B^{-1}(\rho) G^{-T} N_b(\rho),$$
$$L(\rho) = U_{21}^{-T} M_{lc}(\rho) \left(U_{21}^{-T} M_1 H^{-1} \right)^{-1} H^{-1} C_1^{-1}(\rho). \tag{38}$$

Proof. Introduce a partition of U and its inverse $W = U^{-1}$ in the following form:

$$U = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix}, \quad W = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix}. \tag{39}$$

Without loss of generality, we assume that U_{21} and W_{21} are invertible and define

$$F_u = \begin{bmatrix} U_{11} & I \\ U_{21} & 0 \end{bmatrix}, \quad F_w = \begin{bmatrix} I & W_{11} \\ 0 & W_{21} \end{bmatrix}. \tag{40}$$

Thus, we can obtain

$$W F_u = F_w, \quad U F_w = F_u. \tag{41}$$

Defining $\Delta_2 = \operatorname{diag}\{F_w, F_w, F_w, F_w, I, I, F_w, F_w, F_w\}$ and applying the congruence transformation by matrix Δ_2 to LMI (28), we can easily infer the following inequality:

$$\begin{bmatrix} \Omega_{11} & \overline{P}(\rho) + \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & 0 & \Omega_{17} & \overline{\eta}_1 \overline{R}_1 & \overline{\eta}_2 \overline{R}_2 \\ * & \Omega_{22} & \overline{R}_1 & \overline{R}_2 & 0 & \Omega_{26} & 0 & 0 & 0 \\ * & * & -(1-\mu_1)\overline{Q}_1 - \overline{R}_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -(1-\mu_2)\overline{Q}_2 - \overline{R}_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & \overline{D}^T(\rho) & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & -\overline{P}(\rho) & 0 & 0 \\ * & * & * & * & * & * & * & -\overline{R}_1 & 0 \\ * & * & * & * & * & * & * & * & -\overline{R}_2 \end{bmatrix} < 0, \tag{42}$$

where

$$\Omega_{11} = \begin{bmatrix} U_{11} + U_{11}^T & U_{21} + U_{11}^T W_{11} + U_{21}^T W_{21} \\ * & W_{11} + W_{11}^T \end{bmatrix},$$

$$\Omega_{12} = \begin{bmatrix} U_{11}^T A(\rho) & U_{11}^T A(\rho) W_{11} + U_{21}^T A(\rho) W_{21} \\ A(\rho) & A(\rho) W_{11} \end{bmatrix},$$

$$\Omega_{13} = \begin{bmatrix} 0 & -U_{21}^T L(\rho) C_1(\rho) W_{21} \\ 0 & 0 \end{bmatrix},$$

$$\Omega_{14} = \begin{bmatrix} U_{11}^T B(\rho) K(\rho) & U_{11}^T B(\rho) K(\rho) W_{11} - U_{11}^T B(\rho) K(\rho) W_{21} \\ B(\rho) K(\rho) & B(\rho) K(\rho) W_{11} - B(\rho) K(\rho) W_{21} \end{bmatrix},$$

$$\Omega_{15} = \begin{bmatrix} U_{11}^T B_1(\rho) + U_{21}^T B_1(\rho) \\ B_1(\rho) \end{bmatrix},$$

$$\Omega_{17} = \begin{bmatrix} U_{11}^T & U_{11}^T W_{11} + U_{21}^T W_{21} \\ I & W_{11} \end{bmatrix},$$

$$\Omega_{22} = -\overline{P}(\rho) + \overline{Q}_1 + \overline{Q}_2 - \overline{R}_1 - \overline{R}_2 + \sum_{i=1}^{s} \left(v_i \frac{\partial \overline{P}(\rho)}{\partial \rho_i} \right),$$

$$\Omega_{26} = \begin{bmatrix} C_2^T(\rho) \\ W_{11}^T C_2^T(\rho) \end{bmatrix},$$

$$\overline{P}(\rho) = F_w^T P(\rho) F_w,$$

$$\overline{Q}_j = F_w^T Q_j F_w, \quad \overline{R}_j = F_w^T R_j F_w. \tag{43}$$

Defining $V = \begin{bmatrix} I & 0 \\ 0 & W_{11}^{-1} \end{bmatrix}$ and performing congruence transformations to (42) by $\Delta_3 = \operatorname{diag}\{V, V, V, V, I, I, V, V, V\}$, we obtain the result of Theorem 10 with

$$X(\rho) = V^T \overline{P}(\rho) V, \qquad Y_j = V^T \overline{Q}_j V,$$

$$Z_j = V^T \overline{R}_j V, \qquad G = U_{11}, \qquad H = W_{11}^{-1},$$

$$M_1 = U_{21}^T W_{21} W_{11}^{-1},$$
$$M_a(\rho) = U_{21}^T A(\rho) W_{21} W_{11}^{-1}, \qquad N_b(\rho) = U_{11}^T B(\rho) K(\rho),$$
$$M_{lc}(\rho) = U_{21}^T L(\rho) C_1(\rho) W_{21} W_{11}^{-1},$$
$$S_b(\rho) = W_{11}^{-T} B(\rho) K(\rho),$$
$$N_g(\rho) = U_{11}^T B(\rho) K(\rho) W_{21} W_{11}^{-1},$$
$$S_g(\rho) = W_{11}^{-T} B(\rho) K(\rho) W_{21} W_{11}^{-1}. \tag{44}$$

Thus, the controller and observer parameters can be achieved. The proof is completed. □

Remark 11. The parameter dependence refers to that (36) is an infinite-dimensional PLMI. In order to solve this problem, the appropriate basis functions and gridding technique are used to transform it into finite-dimensional PLMI. Selecting the appropriate basis functions as $\{f_i(\rho)\}_{i=1}^s$, we have

$$\mathscr{Y}(\rho) = \sum_{i=1}^{s} f_i(\rho) \mathscr{Y}_i > 0. \tag{45}$$

Remark 12. According to Theorem 10, if there exists feasible solution of (36), the observer-based controller can be designed in the form of (5). In addition, we can also treat γ as optimization variable to obtain the optimal disturbance attenuation level, that is, solving the following convex optimization problem:

$$\min \gamma \quad \text{subject to} \ (36), (45). \tag{46}$$

Remark 13. In reality, almost all real-time systems have time-varying parameters, and this paper gives an approach to the observer-based H_∞ controller design problem for LPV systems under network environment. Recently, because of the rapid development of network technologies, investigations on the NCSs have been studied well and sensor networks are proved to be a new research hot spot. In the context of sensor networks, the network-induced phenomena become even severe due primarily to the network size, communication constraints, limited battery storage, strong coupling, and spatial deployment [9, 22]. Therefore, the proposed results of this paper can be also extended to the sensor networks, which will be one of our future research directions.

4. Numerical Example

In this part, we will use a numerical example to demonstrate the validity of the proposed approach. Consider system (1) with

$$A = \begin{bmatrix} 0 & 1 + 0.2\rho(t) \\ -2 & -3 + 0.1\rho(t) \end{bmatrix},$$
$$B = \begin{bmatrix} 0.2\rho(t) \\ 0.1 + 0.1\rho(t) \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}, \tag{47}$$
$$C_1 = [0 \ \ 0.1], \qquad C_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

where $\rho(t) = \sin(t)$ and $\rho(t) \in [-1, 1]$, $\dot{\rho}(t) \in [-1, 1]$.

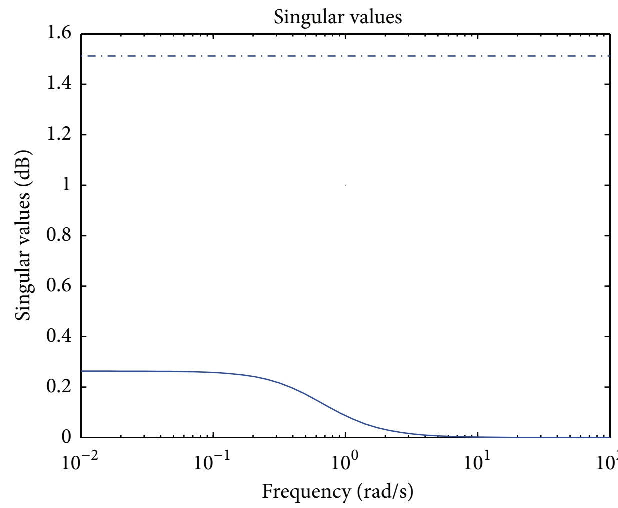

FIGURE 2: The maximum singular value of the closed-loop system.

Selecting the appropriate basis functions as $f_1(\rho) = 1$, $f_2(\rho) = \rho(t)$, then

$$\mathscr{Y}(\rho) = \mathscr{Y}_1 + \rho(t) \mathscr{Y}_2. \tag{48}$$

The maximum allowable delay is $\overline{\eta}_1 = 1.0$, $\overline{\eta}_2 = 0.5$. Solving the PLMI problem in Theorem 10 we obtain an H_∞ performance level $\gamma = 1.5123$ with the parameter matrices of controller and observer as follows:

$$\begin{aligned} K(\rho) &= [2.1460 \ \ -0.5151] \\ &\quad + \rho(t) [-0.1367 \ \ -0.1205], \\ L(\rho) &= \begin{bmatrix} -0.9208 \\ -0.9081 \end{bmatrix} + \rho(t) \begin{bmatrix} -0.0362 \\ -0.0363 \end{bmatrix}. \end{aligned} \tag{49}$$

Given the initial condition $x(0) = [0.1 \ \ 0.25]^T$ and the disturbance signal is

$$\omega(t) = 4e^{(-0.01t)} \sin(0.02\pi t). \tag{50}$$

Figure 2 shows the maximum singular value of the closed-loop system by the obtained controller and observer (49) and the simulation results of the state response and estimation error are shown in Figures 3 and 4, which indicate that the designed observer-based controller is well performed.

5. Conclusions

In this paper, a new network-based structure of LPV systems is concerned by taking full consideration of the sensor-to-controller channel and controller-to-actuator channel, in which the network-induced delays and packet dropouts are treated to be equivalent to two time-varying delays. The design problem of an observer-based H_∞ controller for NC LPV systems is investigated by augmenting the system dynamics and transforming it to a time-varying delayed controller and observer collaborative design problem. Key ideas consist in the parameter-dependent Lyapunov-Krasovskii

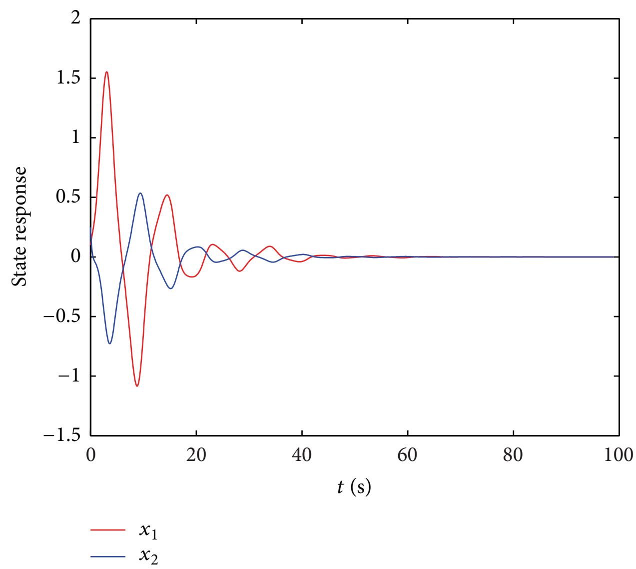

FIGURE 3: The state response of the closed-loop system.

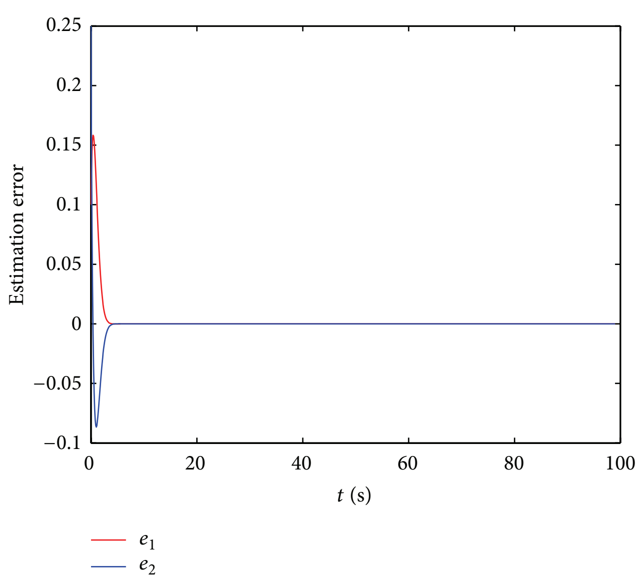

FIGURE 4: The estimation error of the state observer.

functional and the Projection Lemma as well as the introduction of slack matrix which permit eliminating the coupling between the Lyapunov function matrices and the system matrices. Thus, the corresponding analysis and synthesis condition for stabilization and H_∞ performance is expressed in terms of the PLMI, which is infinite-dimensional and can be transformed into finite by using the basis function method and gridding technique. The delay-dependent result refers to reduced conservatism in the synthesis condition with the presented method. Simulation results are presented to demonstrate the high validity and merit of the proposed approach.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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