A Closed-Form Optimal Tuning of Mass Dampers for One Degree-of-Freedom Systems Under Rotating Unbalance Forcing

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1 Introduction

An efficient way to introduce additional damping into structures and machines is the application to these systems of a TMD, also known as dynamic vibration absorber. The classical design procedure for a TMD coupled to an undamped primary system forced harmonically was setup by Den Hartog [1] and Brock [2]. They derived the optimal damping and frequency parameters of the dynamic absorber, as a function of the damper mass and on the characteristics of the primary system.

Since then, several researchers thoroughly studied the design and the dynamic properties of the TMDs. A summary of the optimal parameters following different optimization criteria is given in Refs. [3] and [4]. The criteria for the optimization of the steady-state response mainly consist in H_2 or H_∞ optimizations, while those for the transient response aim at maximize the real part of system eigenvalues [5]. Krenk [6] proposed a practical approach to determine the optimal damping ratio, and he analyzed the root locus of the coupled system to assess its free vibration response.

Most absorber tuning procedures consider an undamped primary system. For damped primary system, several attempts have been made to extend the analytical approach [4,7], and for lightly damped systems approximate analytical solutions are available in Ref. [8]. A number of studies have focused on the approximate and numerical solutions. These include, but are not limited to, numerical optimization schemes [9,10], frequency locus method [11], and min–max criteria [12,13].

The design procedure for TMDs has also been extended to multiple degrees-of-freedom absorbers (e.g., see Ref. [14]) or to TMDs with different configurations such as sky- or ground-hook [15,10]. The case of TMD coupled to a flexible system is discussed in Ref. [16]. Moreover, the design procedure has been recently extended in Ref. [17] to the case of an energy harvesting TMD, where the damper is replaced by a piezo-electric system for energy scavenging.

In this paper, a simple explicit design procedure for a TMD applied to an undamped system forced by a rotating unbalance is presented. In such systems, the magnitude of forcing frequency depends quadratically on the forcing frequency, and this feature leads to optimal parameters that differ from classical values, as it is shown through the paper.

The optimization procedure follows Brock's approach for TMD tuning [2,15], and leads to the optimal tuning parameters that were already obtained by Puksand [18], using the frequency locus method [11,19], for the same problem. The optimal tuning parameters are then analyzed for the case of damped primary system, using a numerical analysis, and a procedure for the selection of optimal parameters in this case is provided.

The rotating unbalance forcing is very common in rotating machinery, and a practical application of a TMD based on the results achieved in this paper has been presented in Ref. [20].

2 Equations of Motion

The basic model of a damped TMD, connected to a primary system and forced by a rotating unbalance, is illustrated in Fig. 1. The figure shows the primary structure with total mass m_0 (inclusive of unbalance), stiffness k_0, and damping c_0. An unbalanced mass m_u with eccentricity r rotates at constant speed Ω. The secondary system has a mass m and it is connected to the primary structure with a spring of stiffness k and a damper with viscosity c.

It is convenient to describe the motion of the system in terms of the absolute motion of the primary system x and the relative motion y of the mass of the TMD with respect to the structural mass. With these two independent variables, the equation of motion of the system can be written as

$$\begin{bmatrix} 1+\mu & \mu \\ \mu & \mu \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} 2\zeta\omega & 0 \\ 0 & 2\zeta_\mathrm{d}\omega_\mathrm{d}\mu \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} \omega_0^2 & 0 \\ 0 & \omega_\mathrm{d}^2\mu \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} \dfrac{m_\mathrm{u}}{m_0} r\Omega^2 e^{i\Omega t} \\ 0 \end{Bmatrix} \tag{1}$$

where some characteristic parameters of the coupled system are introduced, as shown below:

$$\mu = \frac{m}{m_0} \quad \omega_0^2 = \frac{k_0}{m_0} \quad \omega_\mathrm{d}^2 = \frac{k}{m} \quad \zeta = \frac{c_0}{2\sqrt{k_0 m_0}} \quad \zeta_\mathrm{d} = \frac{c}{2\sqrt{km}} \tag{2}$$

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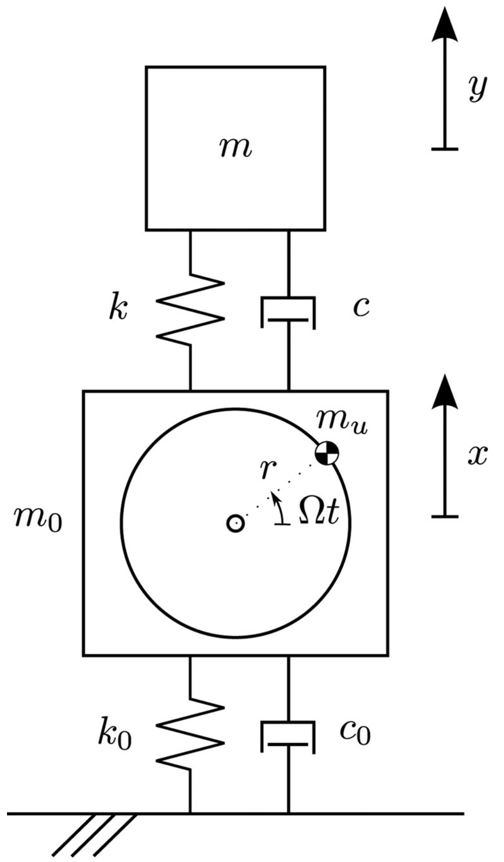

Fig. 1 Damped TMD connected to a primary system forced by a rotating unbalance

Typically, in the design of a TMD, the natural circular frequency ω_0 of the primary system and its damping coefficient ζ are given, while the secondary system is selected choosing a value for the mass ratio μ, the frequency ω_d, and the damping coefficient ζ_d.

The analysis of the system can be further generalized, if the following nondimensional parameters are introduced:

$$\tilde{x} = \frac{x}{\frac{m_u}{m_0} r} \quad \tilde{y} = \frac{y}{\frac{m_u}{m_0} r} \quad \tilde{t} = \Omega t \quad f = \frac{\omega_d}{\omega_0} \quad g = \frac{\Omega}{\omega_0} \tag{3}$$

where $\tilde{x}$ and $\tilde{y}$ are nondimensional expressions of x and y, $\tilde{t}$ is a nondimensional time, f is the frequency ratio, and g is the nondimensional forcing frequency. Introducing these coefficients in Eq. (1), we obtain the nondimensional form of the equation of motion.

$$\begin{bmatrix} (1+\mu)g^2 & \mu g^2 \\ \mu g^2 & \mu g^2 \end{bmatrix} \begin{Bmatrix} \ddot{\tilde{x}} \\ \ddot{\tilde{y}} \end{Bmatrix} + \begin{bmatrix} 2\zeta g & 0 \\ 0 & 2\zeta_d \mu f g \end{bmatrix} \begin{Bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{y}} \end{Bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & f^2\mu \end{bmatrix} \begin{Bmatrix} \tilde{x} \\ \tilde{y} \end{Bmatrix} = \begin{Bmatrix} g^2 \\ 0 \end{Bmatrix} e^{i\tilde{t}} \tag{4}$$

where the dot symbol now represents the derivative with respect to the nondimensional time $\tilde{t}(\partial/\partial t = \Omega(\partial/\partial\tilde{t}))$.

The steady-state response of the system subjected to a rotating unbalance forcing is described by the complex nondimensional amplitudes $\tilde{X}$ and $\tilde{Y}$ that take the following form:

$$\tilde{X} = \frac{[(f^2 - g^2) + i2\zeta_d f g]g^2}{\Delta_R + i\Delta_I} \tag{5}$$

$$\tilde{Y} = \frac{g^4}{\Delta_R + i\Delta_I} \tag{6}$$

with the following denominator terms:

$$\begin{aligned} \Delta_R &= [g^4 - ((1+\mu)f^2 + 4\zeta\zeta_d f + 1)g^2 + f^2] \\ \Delta_I &= 2[-((1+\mu)\zeta_d f - \zeta)g^3 + (\zeta_d f + \zeta f^2)g] \end{aligned} \tag{7}$$

We can notice that the steady-state responses of primary and secondary system depend on the nondimensional parameters g, f, ζ, ζ_d, and μ. In particular, the dependence upon the mass ratio μ always appears in the form of $(1+\mu)$. The significant difference with respect to a constant amplitude harmonic forcing is the presence of a factor g^2 in the numerator of Eqs. (5) and (6), due to the dependence of forcing on the centripetal acceleration.

In the limit of infinite damping ratio ζ_d or infinite stiffness k, no relative motion y is allowed and the effect of a single primary system with mass $m + m_0$ is created. The frequency of the limiting situation of a locked damper will be denoted ω_∞, and its nondimensional form f_∞. They are given by

$$\begin{aligned} \omega_\infty &= \sqrt{\frac{k_0}{m_0 + m}} = \frac{\omega_0}{\sqrt{1+\mu}} \\ f_\infty &= \frac{\omega_\infty}{\omega_0} = \frac{1}{\sqrt{1+\mu}} \end{aligned} \tag{8}$$

The dependence upon the mass ratio can be then expressed with a dependence upon the parameter $f_\infty^2 = (1+\mu)^{-1}$.

In the practical design of a TMD, the mass ratio μ is usually limited and chosen by the designer, considering the overall dimensions of the system. The frequency ratio f and the damping ratio ζ_d are then chosen as a function of μ using optimality criteria. For systems with a rotating unbalance forcing, optimal parameters differ from the classical values obtained for harmonic forcing, as it is demonstrated in Sec. 4 and 5.

3 Dynamic Amplification Function for Undamped Primary System

Since TMDs are in general applied to primary systems with very low damping, in the rest of the analysis the structural damping ratio ζ will be neglected, because this assumption strongly simplifies the analytical optimization of the TMD parameters. The effects of structural damping on the dynamic response of the system will be studied numerically in Sec. 6.

This simplification allows to write the nondimensional dynamic amplification $\tilde{X}$ and the square of its magnitude as

$$\tilde{X} = \frac{A + 2i\zeta_d B}{C + 2i\zeta_d D} \quad |\tilde{X}|^2 = \frac{A^2 + (2\zeta_d)^2 B^2}{C^2 + (2\zeta_d)^2 D^2} \tag{9}$$

where A, B, C, and D are, respectively,

$$A = g^2(f^2 - g^2) \tag{10}$$

$$B = fg^3 \tag{11}$$

$$C = g^4 - (1 + f^2 f_\infty^{-2})g^2 + f^2 \tag{12}$$

$$D = fg(1 - g^2 f_\infty^{-2}) \tag{13}$$

Figure 2 shows the magnitude $|\tilde{X}|$ as a function of the forcing frequency g, for frequency ratio $f = 1$, mass ratio $\mu = 0.05$, and for different values of ζ_d. We notice that, like in the standard procedure of the TMD optimization, there are two neutral frequencies g_a and g_b where the value of $|\tilde{X}|$ is independent of ζ_d (sometimes called "fixed points"). For $\zeta_d \to \infty$, the system response tends to a unique undamped peak of resonance at $g = f_\infty$.

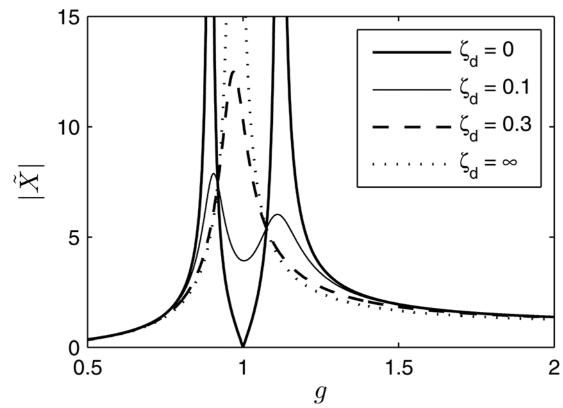

Fig. 2 Magnitude of $\tilde{X}$ versus g with nonoptimal TMD parameters ($f = 1$ and $\mu = 0.05$)

The optimization procedure exploits the existence of the two neutral frequencies and consists of two steps:

(1) The dynamic amplification at the two neutral frequencies is set equal by choosing the appropriate TMD frequency f (as explained in Sec. 4).
(2) Damping ζ_d is then selected in order to have the maxima of the dynamic amplification function at the two neutral frequencies.

4 Optimal Frequency Tuning

Looking at the structure of the dynamic amplification formula in Eq. (9), at the neutral frequencies g_a and g_b the following equation must hold for the magnitude to be independent of ζ_d:

$$\frac{B^2}{A^2} = \frac{D^2}{C^2} \Rightarrow AD = \pm BC \tag{14}$$

The use of the "+" sign leads to the trivial solution $g = 0$. This is the static solution when there is no forcing at all, and the system is at rest. On the contrary, the use of the "−" sign leads to the following quadratic equation in g^2:

$$(1 + f_\infty^{-2})g^4 - 2(1 + f^2 f_\infty^{-2})g^2 + (2f^2) = 0 \tag{15}$$

whose solutions g_a^2 and g_b^2 depend on the choice for the parameters f and f_∞. These roots need not to be computed explicitly at this point, but only in the form of the sum of their reciprocal,[2]

$$\frac{1}{g_a^2} + \frac{1}{g_b^2} = \frac{1}{f^2} + \frac{1}{f_\infty^2} \tag{16}$$

The optimal frequency parameter is determined by specifying the value of the dynamic amplification at the neutral frequencies. At these frequencies, the response magnitude is independent of ζ_d, and therefore it can be evaluated for $\zeta_d \to \infty$

$$|\tilde{X}|_{a,b} = \lim_{\zeta_d \to \infty} |\tilde{X}|_{a,b} = \left|\frac{B}{D}\right| = \left|\frac{g_{a,b}^2}{1 - g_{a,b}^2 f_\infty^{-2}}\right| \tag{17}$$

If we impose the condition $|\tilde{X}|_a = |\tilde{X}|_b$, since $g_a < f_\infty < g_b$, we get the following relationship:

$$\frac{1}{g_a^2} + \frac{1}{g_b^2} = \frac{2}{f_\infty^2} \tag{18}$$

[2]Given a quadratic equation $ax^2 + bx + c = 0$ with solutions x_1 and x_2, then $(1/x_1) + (1/x_2) = -(b/c)$.

The tuning of the frequency parameter f follows from the combination of Eqs. (16) and (18). The result for f is

$$f_{\text{opt}} = f_\infty = \frac{1}{\sqrt{1+\mu}} \tag{19}$$

that is higher than the optimal classical value $f_{\text{classic}} = f_\infty^2$ [2,6]. With this choice for f, which guarantees $|\tilde{X}|_a = |\tilde{X}|_b$, the dynamic amplification at the neutral frequencies has magnitude

$$|\tilde{X}|_{a,b} = \sqrt{\frac{2}{\mu(1+\mu)}} \tag{20}$$

5 Optimal Tuning for Damping

The optimal value for ζ_d is selected in order to have a local maximum at the neutral frequencies g_a and g_b, i.e., $((\partial|\tilde{X}|/\partial g)\big|_{a,b} = 0)$.

This approach is quite tricky. As a matter of fact, if we try to compute the optimal value of ζ_d imposing that $(\partial|\tilde{X}|/\partial g)\big|_{a,b} = 0$, we get a very complex expression that even standard symbolic computation software are not able to resolve. A workaround for this problem is to apply a perturbation method, following Brock's approach for the standard TMD optimization [2,15]. Instead of computing $(\partial|\tilde{X}|/\partial g)\big|_{a,b} = 0$, we impose that $|\tilde{X}|^2$ evaluated in $g^2 = g_{a,b}^2 + \delta$ has value equal to $|\tilde{X}|_{a,b}^2$. We then compute the limit for $\delta \to 0$ and we solve for ζ_d^2. Synthetically, the following system has to be solved:

$$\begin{cases} \zeta_d^2 = \dfrac{A^2 - C^2|\tilde{X}|^2}{4(D^2|\tilde{X}|^2 - B^2)} & \text{from Eq. (9)} \\ g^2 = g_{a,b}^2 + \delta \\ |\tilde{X}|^2 = |\tilde{X}|_{a,b}^2 = \dfrac{2}{\mu(1+\mu)} \end{cases} \tag{21}$$

From the previous system of equations, ζ_d can be expressed as a polynomial expression of δ in the form

$$\zeta_d^2 = \frac{P_0 + P_1\delta + P_2\delta^2 + \ldots}{Q_0 + Q_1\delta + Q_2\delta^2 + \ldots} \tag{22}$$

where the ratio P_0/Q_0 is indeterminate in the form 0/0. The limit for $\delta \to 0$ can be solved applying *de l'Hospital* rule

$$\lim_{\delta \to 0} \zeta_d^2 = \frac{P_1}{Q_1} \tag{23}$$

Two different solutions are thus obtained, one for $g = g_a$ and one for $g = g_b$, namely,

$$\zeta_d\big|_{a,b}^2 = \frac{3\mu}{4(\mu+2)} \pm \sqrt{\frac{\mu}{2(\mu+1)}} \tag{24}$$

The optimal value $\zeta_{d,\text{opt}}^2$ can be selected as the mean of these two results,

$$\zeta_{d,\text{opt}}^2 = \frac{\zeta_{d,a}^2 + \zeta_{d,b}^2}{2} = \frac{3\mu}{4(\mu+2)} \tag{25}$$

We notice that this parameter is larger than the standard optimal parameter $\zeta_{d,\text{classic}}^2 = (3\mu/8(\mu+1))$.

6 Analysis and Discussion of Results

The optimal values for the frequency and damping ratios as a function of the mass parameter μ are shown in Fig. 3. The choice

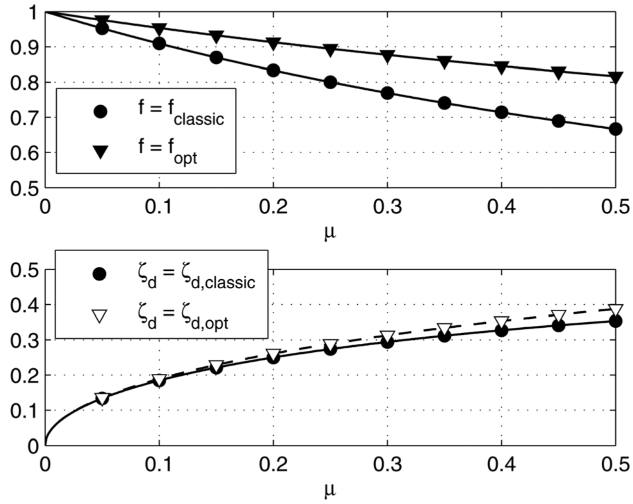

Fig. 3 Optimal TMD parameters as a function of mass parameter μ

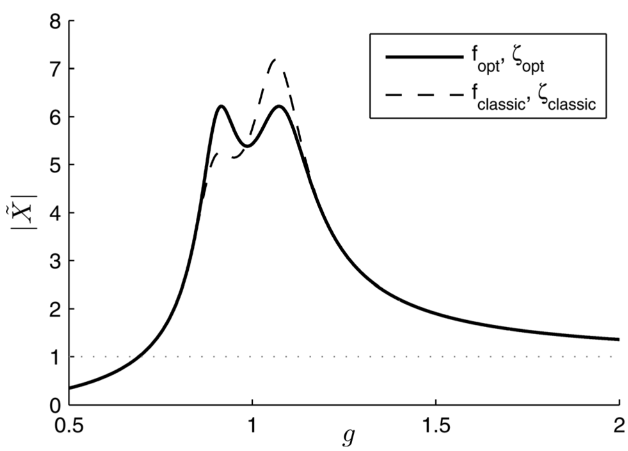

Fig. 4 Magnitude of $\tilde{X}$ versus g with optimal TMD parameters, using $\mu = 0.05$

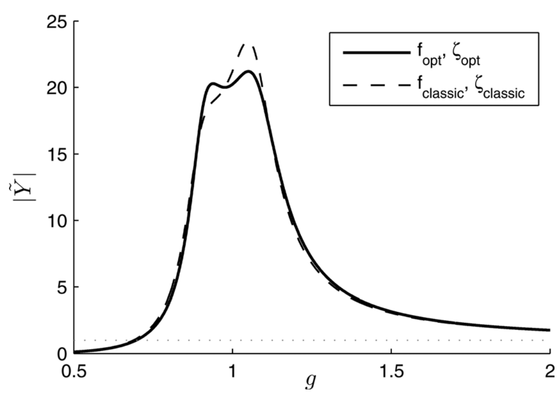

Fig. 5 Magnitude of $\tilde{Y}$ versus g with optimal TMD parameters, using $\mu = 0.05$

of such optimized parameters leads to an optimal frequency response function of the system, as it is shown in Figs. 4 and 5. These results report, respectively, the value of the magnitude of $\tilde{X}$ and $\tilde{Y}$ as a function of the nondimensional forcing frequency g, using differently optimized TMD parameters, when the mass ratio $\mu = 0.05$ is selected.

We notice that a proper design of the TMD parameters allows for a significant reduction of the amplitude of oscillation for the primary system forced by rotating unbalance. This reduction is a nonlinear function of the mass parameter and is represented in Fig. 6, where we plot the ratio between the maxima of the frequency response functions (infinity norm) obtained with classical and optimal parameters as a function of μ. Considering that practical mass ratios are about 0.05–0.10, the reduction can be about 15–18%.

These results hold if we consider a negligible structural damping ζ (see also Ref. [8] for standard TMD). If this parameter is included in the analysis, a practical analytical discussion is not possible and numerical simulations are necessary. As a matter of fact, the two neutral frequencies g_a and g_b disappear and the whole frequency response function depends on ζ and μ, making the presented optimization procedure not applicable. However, it is possible to numerically evaluate the effect of ζ on the dynamic amplitudes $\tilde{X}$ and $\tilde{Y}$.

Considering a damping ratio for the primary system ranging from 0% to 5%, we can numerically compute the optimal ζ_d and f for a given μ. We use the subscript num for the numerically optimized values.

The optimization, in case of $0 < \zeta < \zeta_{\lim}$, leads to a solution in which the frequency and damping parameters are larger than $\zeta_{d,opt}$ and f_{opt}, and the both show a positive trend with ζ.

$\zeta_{\lim}$ is a threshold value, and if $\zeta > \zeta_{\lim}$ two different possible TMD tuning strategies, namely, solutions 1 and 2, are possible. Solution 1 follows the same trend of the solutions for $\zeta < \zeta_{\lim}$,

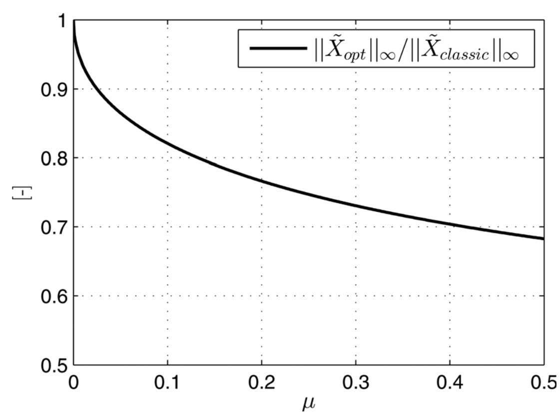

Fig. 6 Ratio between the maxima of the frequency response functions (infinity norm) obtained with classical and optimal parameters as a function of the mass ratio

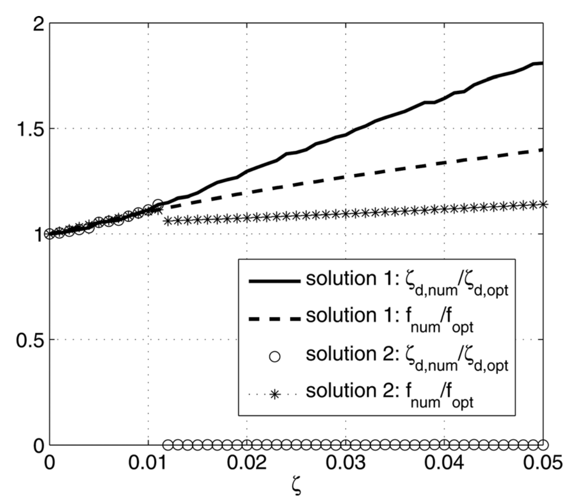

Fig. 7 Numerically optimized values of ζ_d and f as a function of ζ for $\mu = 0.05$: solution 1 and solution 2

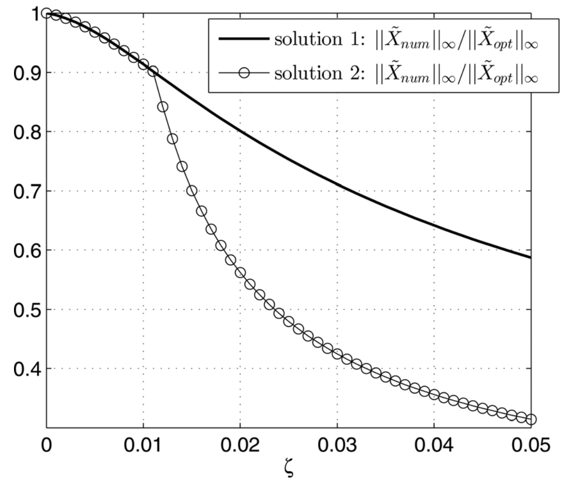

Fig. 8 Ratio of maxima $||\tilde{X}_{\mathrm{num}}||_\infty$ as a function of ζ for $\mu = 0.05$: solution 1 and solution 2

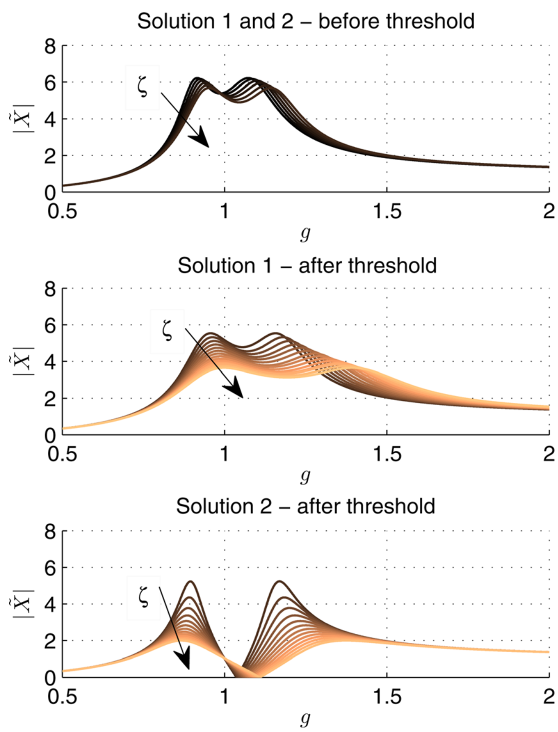

Fig. 9 Optimized frequency response functions $|\tilde{X}_{\mathrm{num}}|$ for solutions 1 and 2, for increasing values on ζ. For this example, with $\mu = 0.05$, the threshold is $\zeta = 0.012$. Solutions 1 and 2—before threshold, solution 1—after threshold, and solution 2—after threshold.

while solution 2 requires a $\zeta_{\mathrm{d,num}} = 0$ and frequency ratios f_{num} with still a positive trend with damping, but with lower values.

For the case of $\mu = 0.05$, $\zeta_{\mathrm{lim}} = 0.012$, the optimized damping and frequency ratio values, with respect to the solution with the undamped primary system $\zeta = 0$ are reported in Fig. 7.

The correspondent maximum amplitudes of vibration, for $\mu = 0.05$, $||\tilde{X}_{\mathrm{num}}||_\infty$ are shown in Fig. 8. After the threshold, the vibration values of solution 2 are significantly lower.

We can analyze the optimized frequency response functions $|\tilde{X}|$ for solutions 1 and 2, for increasing values on ζ: these functions are reported in Fig. 9. We see that solution 1 is the evolution of the standard solution before the threshold, while solution 2 is a solution with an antiresonance and limited resonance peaks. The

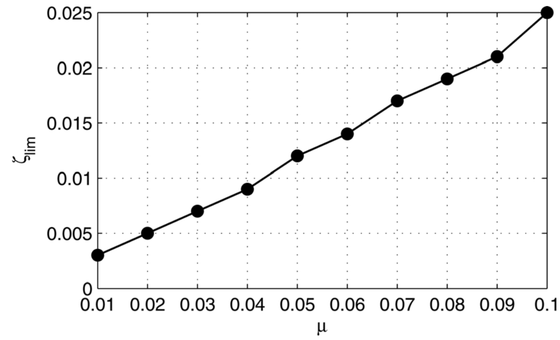

Fig. 10 ζ_{lim} as a function of the mass ratio μ

existence of such antiresonance can be foreseen in Eq. (5), where for $\zeta_{\mathrm{d}} = 0$ we have that $\tilde{X} = 0$ for $g = f$. Of course, this is a case for which the primary source of energy dissipation is given by the primary system, and it should not be classified as a standard "primary system plus TMD"; however, this solution turns out to be more efficient in terms of vibration mitigation.

Finally, we should highlight that the threshold (ζ_{lim}) is also a function of the mass ratio μ. As a final result, we present the trend of this threshold in Fig. 10: It is clear that the presented optimization procedure is valid also for lightly damped primary systems, and in case of damped systems numerical investigations are necessary. The existence of this second solution does not invalidate the optimization procedure presented that still holds for practical values of ζ.

7 Conclusions

A closed-form solution for the optimal values of the parameters of a TMD, coupled to a primary system and forced by a rotating unbalance, has been presented.

The inertial forcing due to the rotating unbalance depends quadratically on the forcing frequency and it leads to optimal tuning parameters that differ from classical values obtained for constant harmonic forcing.

Analytical results demonstrate that frequency and damping ratios, as a function of the mass ratio, should be higher than classical optimal parameters. Indeed, for practical applications, a proper TMD tuning allows to achieve a reduction in the steady-state response of about 20% with respect to the response achieved with a classically tuned damper.

The presented analytical solution, valid for an undamped primary system, has been investigated numerically for the case of damped primary system. If the structural damping is larger than a given threshold two solutions are possible, their performances should be investigated numerically.

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