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# EFFICIENT MODELLING OF FORCES AND LOCAL STRAIN EVOLUTION DURING DELAMINATION OF COMPOSITE LAMINATES

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## Abstract

*FEM analyses based on cohesive zone models are a well-assessed methodology to predict onset and propagation of delamination in composites. In this work, a specific modelling technique based on a cohesive zone model is applied to analyse Double Cantilever Beam (DCB) and 4-point bending End Notched Flexure (4-ENF) tests, focusing on the evolution of forces as well as of internal local strains, which have been monitored by Fibre Bragg Grating sensors embedded in the specimens. The numerical approach is based on explicit FEM computations and presents some appealing advantages with respect to conventional models, since it does not use zero-thickness cohesive elements and does not require a non-physical penalty stiffness to be introduced between adjacent plies. In the cases presented, such approach is applied to model both force response and local strain evolution during the stable propagation of delamination in mode I and mode II, in the presence of fibre bridging phenomena and taking into account frictional effects between crack faces. The paper presents the experimental results and analyses the data acquired by the sensors embedded in the specimens. Then, the general accuracy and the computational advantages of the numerical approach proposed are evaluated considering numerical benchmarks. Models of the tests are developed at different levels of through-the-thickness mesh refinement and sensitivity analyses are performed to point out the effects on the overall and local response of significant model parameters, such as the length attributed to the process zone in the cohesive zone model and the friction coefficient in the contact interaction between crack faces. Numerical results and numerical-experimental correlation prove that the modelling technique and the methodologies applied to represent fibre bridging and frictional effects represent efficient tools to reliably model complex delamination processes.*

## Keywords:

B. Delamination B. Fracture Toughness C. Finite Element Analyses D. Mechanical Testing

## 1 - Introduction

The development of approaches to the analysis of delamination in composite laminates is of a paramount importance for application of composite materials. Delamination can be promoted by technological processes, foreign object impact or by highly concentrated loads and can be regarded as one of the major threats to structural integrity of a composite structure [1]. Numerical analyses of delamination may reduce the large amount of testing required in design methodologies based on damage tolerance, which take into account the existence of a certain amount of damage in a composite structure, often represented by delamination scenarios. Nowadays, the prediction of the effects of delamination on the structural response is also important for the development of Structural Health Monitoring (SHM) systems. In particular, the design of systems based on the application of strain sensors, such as embedded Fibre Bragg Grating (FBG) sensors, requires an accurate prediction of the local strain field close to delaminated zones [2, 3].

The introduction of cohesive zone models in finite element analyses has proved to be a successful approach to model delamination. A large number of results [4] proves that cohesive zone approach is particularly suited to model fracture in the interfaces between adjacent plies of a laminates, where the plane of crack propagation is known in advance. In Finite Element (FE) analyses, the cohesive zone model is implemented in a traction-displacement constitutive law, which is typically attributed to a layer of zero-thickness or infinitesimal thickness interface elements, introduced between the elements modelling single plies or sub-laminates. Cohesive elements have been also successfully applied to model the local strain evolution during crack propagation, which was acquired by FBG sensors embedded in the structure for the design of SHM systems [2,3] and for the identification of traction-displacement laws in the presence of fibre bridging phenomena [5,6].

Indeed, one of the major drawbacks of cohesive elements derives from the representation of fracture processes by means of displacement discontinuities, which requires, in a FE model, the introduction of degrees of freedom that are inactive until the onset of a crack. Hence, before delamination, relative displacements at the interface must be inhibited to avoid the introduction of undesirable compliance. This is done by using high penalty stiffness levels in cohesive elements, but such solution can introduce several numerical problems [7, 8, 9]. Therefore, calibration of penalty stiffness is an important issue in cohesive zone approach and guidelines have been provided to obtain the best trade-off between computation effectiveness and accuracy [9]. Another issue for the effective application of cohesive zone models to design and analysis of real-world structural components is represented by convergence problems, which often arise in the presence of unstable crack propagations and involve

difficulties in the accomplishment of analyses. Such problems complicates the application of cohesive zone models, in particular when the location of interlaminar damage is not known a-priori and multiple damage scenarios have to be taken into account.

The modelling technique proposed in [10], which was applied to model complex delamination scenarios in different significant cases [10,11,12], overcomes the typical difficulties involved in the adoption of zero-thickness interface elements. Such technique was developed for quasi-static analyses performed by explicit FE codes, which eliminate **convergence** problems in dealing with delamination **phenomena**, but require a very small stable time-step, which depends on the stiffness of the elements [10, 13]. Therefore, high penalty stiffness levels lead to additional problems in quasi-static explicit computations, due to the severe reduction of the stable time step required for the analysis. In the alternative method proposed in [10] composite laminates are represented by the superposition of a stack of bi-dimensional elements, which carry membrane stress components, and of finite-thickness three-dimensional connection elements, which model the average out-of-plane stress-strain state between the adjacent layers in the stack. No penalty stiffness is required and the stiffness parameters of three-dimensional connection elements are the physical transverse shear and thickness moduli of the composite material. Consequently, stable time step can be significantly higher than in explicit analyses carried out by using traditional cohesive elements.

In this work, such approach is applied to model the force response as well as the local strain evolution during crack propagation, acquired by FBG sensors carried by optical fibres embedded in glass-fibre reinforced composite specimens. Double Cantilever Beam tests (DCB) and four-point bending tests on End Notched Flexure specimens (4-ENF) are considered. In the tests performed, the overall and local responses of the laminates are influenced by fibre bridging and by the presence of friction between the crack faces. Hence, the development of a reliable model must also represent such phenomena.

A first objective of the paper is a quantitative assessment of accuracy and computational advantages of the modelling technique proposed by comparing the numerical results with reference models. Then, the overall response and the local strain evolution acquired by FBG sensors are considered to experimentally validate the predictions of numerical models and to discuss the influence of the model parameters used to represent fibre bridging in DCB tests and the friction between the crack faces in 4-ENF tests. Moreover, numerical correlation are carried

out by using different through-the-thickness mesh refinements in order to provide some guidelines for the development of computationally effective and accurate FE models for the prediction of the force required to drive delamination and of the local strain states during delamination propagation.

The paper is organised into five sections, including this introduction. In the next section, the experimental activity is presented and the evolution of local strain field during delimitation propagation is analysed. In a third section, the modelling technique is discussed and numerical benchmarks for validation are introduced. The application of the modelling technique to the experimental tests is described and in a fourth section. The main findings are finally summarised in a conclusive section.

## **2 - Experimental response and detection of local internal strains during stable delamination propagation**

### **2.1 Manufacturing of glass fibre reinforced composite specimens with embedded optical fibres**

The evolution of local strain fields during the development of an interlaminar damage depends on mode propagation of interlaminar fractures and on the overall response of the laminate under the applied loads. Composite laminates with embedded optical fibres (OF) carrying Fibre Bragg Gratings (FGB) were manufactured to carry out an experimental analysis of internal strain field evolution during delamination. The material used was a S2 Glass fibre reinforced composite with CYTEC 5216 epoxy matrix and the following in-plane elastic properties in material axis:  $E_{xx} = 47.5$  GPa,  $E_{yy} = 13.5$  GPa,  $\nu_{xy} = 0.25$ ,  $G_{xy} = 5.896$  GPa [14]. Specimens were cut from laminates with a  $[0]_{48}$  lay-up sequence and an average final cured thickness of 10.39 mm. In all the specimens, a 13  $\mu\text{m}$ -thick film of Polytetrafluoroethylene (PTFE), with a length of 80 mm, was interposed at the mid-plane of each laminate to obtain a pre-crack.

Laminates were produced by means of a vacuum bag process, by using metallic mould and counter-mould and an elastomeric frame made of stiff non-silicon rubber (Airpad® Airtech). Optical fibres carrying 12 mm long FBG's were embedded in some of the specimens. The technology used to embed the fibres is shown in Fig. 1, which is referred to the manufacturing process. Optical fibres were protected by possible damages at the egress of the laminate by using a PTFE tubing, which was partially embedded at the border of the laminate and passed through the stiff elastomeric frame.

Specimens were cut from the laminates and used to perform Double Cantilever Beam (DCB) tests and End Notched Flexure tests in four-point bending configuration (4-ENF), represented in Figs. 2-a and 2-b, respectively. All specimens had a length of 300 mm and a width of 25 mm. In three of the DCB specimens tested, an OF was inserted 2 plies above the pre-cracked central interlaminar layer. In one of these specimens a second OF

was also embedded at the mid-plane of the upper DCB arm (Fig. 2-a). In two specimens used in 4-ENF tests, two OF's were used. In each specimen, the first fibre was inserted two plies above the pre-cracked plane and the other was positioned two plies below the upper surface (Fig. 2-b).

## 2.2 - Interlaminar fracture tests in mode I and mode II

DCB tests were performed following ASTM D5528-01 [15] as a guideline, by using an MTS 858 servo-hydraulic system. Overall, four tests were performed. A pre-opening procedure was carried out to promote a crack advance of few millimetres, so that the subsequent tests started with a crack front representing a typical damage condition, which is different from the defect artificially induced by the PTFE insert. Crosshead speed used during loading and unloading was of 1 mm/min and 5 mm/min, respectively. Crack advancement was monitored by means of pictures taken at a sampling frequency of 5 s by a fixed camera. Fibre bridging phenomena were observed during the opening phase at the crack tip, as it shown in Fig. 3. The resistance curves (*R-curves*), reported in Fig. 3 were obtained by applying all the four data reduction methods reported in [15], namely Beam Theory (BT), Modified Beam Theory (MBT), Compliance Calibration (CC) and Modified Compliance Calibration (MCC).

All methods confirm the presence of a significant *R-curve* effect, as reported in Fig. 3. Indeed, the critical energy release rate ( $G_{IR}$ ) increases with crack length ( $\Delta a$ ), from an initial ( $G_{IRi}$ ) value of about 0.2 kJ/m<sup>2</sup> until a steady state value ( $G_{IRss}$ ) of about 0.8 kJ/m<sup>2</sup>. Such behaviour indicates the development of a fracture process zone during the crack propagation, related to the fibre bridging phenomena. The length required by the crack to reach a steady propagation can be set at about 100 mm. Such length is actually influenced by the type of the test performed [16] and can be referred to as the length of the fracture process zone in the DCB test ( $l_{pz\_DCB}$ ).

Four 4-ENF tests were performed with the layout reported in Fig. 2-b, characterised by  $a = 43.75$  mm,  $2L = 125$  mm and  $L_f = 75$  mm. An MTS 858 system was used and a crosshead speed of 1 mm/min was applied to perform the tests. The stable propagation regime of the crack in 4-ENF tests made possible monitoring the evolution of critical energy release rate with the crack length, by applying a compliance calibration technique (CC) and a linear interpolation of the compliance vs. crack length curve, as suggested in [17]. The data required to evaluate the interpolation coefficients were obtained by positioning the specimen in the 4-point bending fixture, so to evaluate the compliance for different crack lengths. In such tests, load levels were kept below the values required for crack opening. For 4-ENF tests, pre-opening was promoted in unstable mode-II propagation, by performing an End Notched Flexure test in three-point bending configuration. After pre-opening, a further propagation of the

fracture was carried out in 4-ENF configuration, so that the faces of the pre-crack were directly in contact, without being separated by the PTFE insert, in the entire zone internal to the support cylinders. Hence, the friction between the specimen arms, which is known to affect results of 4-ENF tests [17,18], was generated in the final test by the interaction of crack faces developed in a realistic mode II propagation process. In the final test, the same picture acquisition procedure described for the DCB tests was adopted to monitor crack advance.

The trend of mode II toughness values ( $G_{II\bar{R}}$ ) against crack length ( $a$ ) is presented in Fig. 4. Application of data reduction method obtained a non-negligible R-curve effect: the mean value of fracture toughness in Mode II at the beginning of crack propagation is about  $1.8 \text{ kJ/m}^2$  and it raises at  $2.2 \text{ kJ/m}^2$  at  $a = 95 \text{ mm}$ , when the crack has almost reached the internal loading roller on the other side of the specimen pre-cracked end. However, several authors [17, 18, 19] point out that the results obtained in the ENF tests can be affected both by geometrical nonlinearities and by frictional effects. Hence, the development of an accurate model to discuss the origin of the R-effect and to identify the friction coefficient between the faces is one of the objectives of the numerical activity that will be presented in this work.

### 2.3 - Strain field acquisition during stable crack propagations

The FBG sensors embedded in the specimens recorded the evolution of internal strain during the tests. An optical spectrum analyzer was used to acquire the spectra reflected by the sensors and to evaluate an average value of the strain in the 12 mm long FBG. Spectrum acquisitions were performed at discrete instants, at each millimeter of crosshead displacement in both DCB and ENF tests.

The three DCB tests performed with optical fibers embedded in the specimens provided the results presented in Fig. 5. Measures are referred to the final phase of tests only, excluding the pre-opening phase. Crack advance is measured from the initial position of the PTFE inserts. The vertical lines in Fig. 5 show the position of the FBG sensors.

The average strains rapidly **increase** as the crack tip passes below the sensor. In the final part of the test, when  $da > 90 \text{ mm} \div 100 \text{ mm}$ , the trend of the FBG-measured strain resembles the one of the force applied to the DCB specimen. During the development of the process zone, the evolution of the longitudinal strain in the sensor zone is affected by the presence of bridging tractions in the wake of the crack tip, as confirmed by the studies presented in [5,6].

The evolution of internal strains in the two 4-ENF tests endowed with OF's is plotted against the crosshead displacement,  $\delta$ , in Fig. 6. The corresponding force vs. displacement curves are also represented in the same plot.

The deviation from linear trend for the FBG sensors positioned close to the surface of the ENF specimens occurs

at about  $\delta = 1.65$  mm for both the specimens, in correspondence of the initial reduction of tangent modulus of the force vs. displacement curves, which indicates the onset of crack propagation. The FBG sensors near the cracked interface recorded a sudden increment of strain at about  $\delta = 1.8$  mm  $\div$  1.9 mm. Contemporarily, the slope of the strain vs. displacement curves for the FBG close to the surface changes again. Finally, at  $\delta \cong 2.3$  mm, both the strain curves exhibit a final change of slope, which probably indicates the complete development of the crack below the FBG sensors.

### **3 - Efficient hybrid modelling technique for quasi-static explicit analyses**

#### **3.1 - Conventional cohesive zone models and issues related to penalty stiffness**

The tests presented in the previous section can be modelled by using approaches based on cohesive zone models, which are typically implemented by using elements having zero or very small interface thickness, set between plies or sub-laminates. Figure 7 shows an interface element between two sub-laminates during a fracture process. Such elements are characterized by traction-displacement laws that link the stress transmitted through the interface,  $\sigma$ , to the displacement discontinuities between the adjacent surfaces,  $\delta$ .

A damage mechanics approach is applied in the constitutive law, to model the degradation of the interface. The traction-displacement response is linear until damage onset. Then damage will increase according to a prescribed law to represent the degradation of the interface in processes evolving according to one of the three basic opening modes (*I*, *II* and *III*). Several cohesive zone models consider a bi-linear response [20,21,22], with a triangularly shaped traction-displacement curve that is completely characterized by the slope in the elastic range, the maximum stress levels,  $\sigma_{I0}$  and  $\sigma_{II0}$ , and the displacements  $\delta_{IF}$  and  $\delta_{IIF}$  shown in Fig. 7-b.

Mixed mode processes are addressed by introducing a strength criterion and a toughness criterion. Algorithms for the introduction of such criteria into constitutive laws have been presented by several authors, as in [20, 21, 22].

In order to avoid relative motions at the interfaces before the onset of delamination, extremely high values for the interface stiffness element are required, often referred to as penalty stiffness. Such stiffness is the slope of the traction-displacement curve in the linear elastic range, indicated with symbol  $K$  in Fig. 7-b. It is known that penalty stiffness can lead to numerical spurious oscillations [7, 8, 9]. On the contrary, an excessively compliant interface lead to an unrealistic behaviour of the numerical model [9]. The rule that was proposed in [9] is given in Eq.(1), where  $E_{zz}$  is the physical through-thickness stiffness of the material,  $t_s$  is the thickness of the sub-laminates that are connected by the cohesive element, also shown in Fig. 7-a, and  $\alpha$  is a non-dimensional parameter.

$$K_c = \frac{\alpha E_{zz}}{t_s} \quad (1)$$

According to [9], the values of  $\alpha > 50$  are required to carry out a successful computation. Equation (1) indicates that the lower is the thickness of the layers that are connected, the higher is the penalty stiffness that should be introduced to avoid undesired behaviour.

Further complications arise if an explicit time integration scheme is chosen for the computation. Such an approach is typically adopted to model high-velocity transients but it can be considered very effective to perform quasi-static analysis and to model stable and unstable crack propagations, even in approaches dedicated to damage development in fatigue loading [10,11,12,23].

Unfortunately, explicit approaches involve the adoption of a conditionally stable integration algorithm with a time step that is inversely proportional to the stiffness of the elements [13]. In view of that, calibration of penalty stiffness may play a critical role in the effective application of the approach, because the presence of extremely stiff connections, represented by cohesive elements, may easily lead to unacceptable computational time costs.

### **3.2 - 2D/3D Hybrid modelling approach without penalty stiffness**

A particular modelling technique has been presented in [10] to avoid the introduction of zero-thickness cohesive elements and the need of penalty stiffness. Such technique is suited to explicit computations and can be interpreted as a decomposition of composite into two phases, one carrying in-plane stress components acting in the plies (or sub-laminates) and the other one carrying the average out-of-plane stress components between the mid-plane of the plies or sub-laminates. For a lamina or a symmetric sub-laminate, the phase carrying the in-plane stress components is represented by a bi-dimensional element, such as a membrane element, with nodes in the mid-plane of the layer. Bi-dimensional elements are connected by three-dimensional solid elements that represent the phase carrying out-of-plane stress components. In explicit computations, solid elements with a reduced integration scheme are normally adopted and, when they are used as a connection between two bi-dimensional elements, the strain at their single integration point turns out to represent the average strain between the mid-planes of two adjacent layers. Figure 8 shows the scheme of a composite laminate divided into sub-laminates (Fig. 8-a), the structural idealization adopted in a conventional cohesive approach (Fig. 8-b) and the idealization that is proposed in the alternative hybrid modelling technique (Fig. 8-c). It should be observed that the bi-dimensional elements are characterized by the physical stiffness of the ply or sub-laminate that they represent, so that the two phases are actually superposed in the same volume occupied by the composite material.

In a simplified stress condition with no stress gradient in the  $y$ -direction, the translational equilibrium of a single lamina can be formalized as in Eq. (2), considering the membrane force per unit with,  $N_x$ , and the shear stress transferred through the interfaces.

$$\frac{dN_{xx}}{dx} = \tau_{xz}(z + \Delta z) - \tau_{xz}(z) \quad (2)$$

Considering Fig. 8-c, a similar equation, expressed in Eq. (3), governs the equilibrium of the membrane in the hybrid technique.

$$\frac{dN_{xx}}{dx} = \bar{\tau}_{xz}(z + \Delta z) - \bar{\tau}_{xz}(z) = \bar{G}_{xz} \bar{\gamma}_{xz}(z + \Delta z) + \bar{G}_{xz} \bar{\gamma}_{xz}(z) \quad (3)$$

where the out-of-plane shear stress components are computed on the basis of the average strain between the mid-plane of the layers multiplied by the physical shear modulus of the material. If the adjacent layers have different shear moduli, an equivalent stiffness can be computed by considering that the total shear compliance in the volume between the two mid-planes is the sum of the two material compliances weighted by the semi-thickness of each layer.

A fundamental aspect of the technique proposed is that the average out-of-plane strain components in the volume between the two connected layers can be used to describe the separation of the two mid-planes during a fracture process, as it is shown in Fig. 9-a. Indeed, within a small strain assumption, the average strain can be related to the vector of relative displacement between the mid-planes, as indicated in Eq. (4).

$$\begin{aligned} \bar{\boldsymbol{\varepsilon}} &= \left\{ \bar{\varepsilon}_{zz} \quad \bar{\gamma}_{xz} \quad \bar{\gamma}_{yz} \right\}^T = \left\{ U_z^+ - U_z^- \quad U_x^+ - U_x^- \quad U_y^+ - U_y^- \right\}^T / t_k = \\ &= \left\{ \Delta_z \quad \Delta_x \quad \Delta_y \right\}^T / t_k = \boldsymbol{\Delta} / t_k \end{aligned} \quad (4)$$

where  $t_k$  is the distance between the mid-planes.

The relative displacements defined in Eq. (4) are used to represent fracture processes. In particular, relative displacement for the mode I, mode II and mode III opening are expressed as in Eq. (5).

$$\Delta_I = \begin{cases} \Delta_z & \text{if } \Delta_z > 0 \\ 0 & \text{if } \Delta_z \leq 0 \end{cases} ; \quad \Delta_{II} = \Delta_x ; \quad \Delta_{III} = \Delta_y \quad (5)$$

Within the assumption of equivalent properties in mode II and III, the toughness required to create a new surface in pure fracture processes is given in Eq. (6), which expresses the work required to separate the two mid-planes in axial and shear opening mode as a function of the area below the stress-strain response of the connection element.

$$\int_0^{\infty} \sigma_I d\Delta_I = t_k \int_0^{\infty} \sigma_I d\varepsilon_I = G_{Ic} ; \int_0^{\infty} \sigma_{II} d\Delta_{II} = t_k \int_0^{\infty} \sigma_{II} d\varepsilon_{II} = G_{IIc} \quad (6)$$

Accordingly, the traction-separation law presented in Fig. 7 is replaced by the constitutive bi-linear response shown in Fig. 9-b. Such law is attributed to three-dimensional connection elements, which will have a null response in the in-plane stress components. This constitutive response models the transverse shear stiffness and the out-of-plane axial stiffness of the laminate, and has the form reported in Eq. (7).

$$\begin{Bmatrix} \sigma_{zz} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} E_{zz} & 0 & 0 \\ 0 & G_{xz} & 0 \\ 0 & 0 & G_{yz} \end{bmatrix} (1-d) \begin{Bmatrix} \varepsilon_{zz} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \quad (7)$$

where  $d$  is a scalar damage variable that evolves in order to obtain the required bi-linear response so to model the strength and the toughness of the connection between the layers. The components of Green-Lagrange strain tensor were used to implement the constitutive law in a VUMAT Fortran subroutine, used to perform Abaqus/Explicit analyses.

For the cases presented in this paper, the assumption of identical properties in mode II and mode III fracture process is introduced. Moreover, the damage variable evolves according to a bi-linear stress-strain response as the one represented in Fig. 9, so that the main model parameters are the stiffness terms included in Eq. (7), the strengths in axial and shear stress conditions,  $\sigma_{I0}$  and  $\sigma_{II0}$ , and the final strains  $\varepsilon_{IF}$  and  $\varepsilon_{IIF}$ , which can be calculated from Eq. (6). A quadratic strength criterion and a toughness criterion based on B-K formulation [24] were introduced in the constitutive law to model mixed mode processes, although, in the applications presented in this paper, only cracks that nominally propagates in pure mode I or mode II are considered.

### 3.3 - Assessment of efficiency and accuracy of the hybrid technique

Further details and applications of the modelling technique based on 2D/3D elements can be found in [10]. The technique was also successfully applied to model several quasi-static tests involving multiple delaminations, which are reported in [10, 11, 12]. In this section, the computational effectiveness of the modelling approach is assessed by comparing the analysis of a quasi-static DCB experiment on a fabric specimen with an analysis performed using zero-thickness cohesive elements. The test was performed on a specimen made of AS4/8852 plain weave fabric pre-preg, with a length of 200 mm, a width of 25 mm, and a  $[0]_{24}$  lay-up sequence. Experimental procedures, in-plane and out-of-plane material properties are presented in [25]. A conventional model was developed and it is shown in Fig. 10-a. The two arms of the DCB specimen were modelled by using six solid elements

through the thickness of the laminate (element C3D8R [13]) with 1.25 mm x 1.25 mm in-plane dimensions. Cohesive elements (element COH3D8 [13]) with a thickness of 0.02 mm were used to model the single interlaminar layer between the two arms. The same test was modelled by using the hybrid approach with the same in-plane mesh refinement. In the hybrid model, shown in Fig. 10-b, shell elements (element S4R [13]) were adopted to model 12 sub-laminates, each one representing two plies in the laminate. Solid elements (element C3D8R [13]) endowed with the material law previously described were used to connect the shell elements.

Shell elements were used instead of membrane elements to reduce the risk of hourglass phenomena, which can occur when membrane are used as bi-dimensional element in the hybrid technique. Analyses reported in [10] show that the performance of models with shell and membrane element is essentially identical. The properties reported in Table 1 were used for cohesive elements in the two models. In the conventional model, the values of penalty stiffness in the zero-thickness cohesive elements ( $K_{nn}$ ,  $K_{ss}$  and  $K_{tt}$ ) were defined in accordance with Eq.(1).

The analysis of the DCB test was performed by using Abaqus/Explicit. A smoothly increasing velocity was applied to a rigid body that was created on the external surface at the tip of the upper DCB arm. A similar rigid body was created and kept fixed on the lower DCB arm. No mass scaling [13] was applied in both conventional and hybrid model. A density of  $1.2 \times 10^3 \text{ kg/m}^3$  was assigned to zero-thickness cohesive elements, in agreement with the physical value of a resin rich layer, whereas the physical density of the composite material was distributed between the two phases in the hybrid modelling approach, in order to maximize the stable time step. It is worth noting that such mass distribution does not affect the model properties, since Abaqus/Explicit lumps the mass at the nodes of the elements to obtain a diagonal mass matrix [10, 13] and the two phases of the model share the same nodes.

Table 1 - Interlaminar properties for the model of the DCB test used in modelling technique assessment

$K_{nn}$ (GPa/mm)	$K_{ss} = K_{tt}$ (GPa/mm)	$\sigma_{I0}$ (MPa)	$\sigma_{II0}$ (MPa)	$G_{Ic}$ (kJ/m <sup>2</sup> )	$G_{IIc}$ (kJ/m <sup>2</sup> )	$E_{zz}$ (GPa)	$G_{xz} = G_{yz}$ (GPa)
120.0	45.0	20	50	1.064	1.75	7.8	3

The numerical experimental correlation reported in Fig. 10-c shows that both models qualitatively and quantitatively capture the response of the test. However, Table 2 compares the computational cost of the two model obtained by using a workstation endowed with Dual-Quad-Core Intel Xeon processors with 2.66 GHz CPU clock. Computations were parallelized by using 8 CPU's.

The analysis with the hybrid model is about one order of magnitude faster than the conventional one, despite the higher number of interfaces modelled and of total d.o.f.'s, thanks to the noticeable increment of the stable time step. The CPU time can be further reduced in both analyses by artificially increasing the density of material, that is by applying a mass scaling or a selective mass scaling [13]. For instance, quasi-static analyses of a DCB test reported in [26], with a single interface, adopted a density increased of a  $10^5$  factor for cohesive elements and of a  $10^3$  factor for solid bricks in the DCB arms. The non-physical thickness attributed to conventional cohesive elements can also be changed in Abaqus [13], but this directly affect the mass of the model.

Table 2 - Computational performance of conventional and hybrid DCB model

Cohesive Model	d.o.f.	Stable Time Increment	#8 CPU's Tot. time
Proposed	182864	$1.356 \times 10^{-7}$	113 min
Conventional	108876	$7.684 \times 10^{-9}$	1003 min

When mass scaling is applied, the density attributed to the elements is increased, so that larger stable time increments can be used to reduce computational time. In the case of a selective mass scaling, such density increment can be attributed only to the elements modelling the interlaminar layers. However, the increment of mass due to such operation becomes very large when many interlaminar layers are modelled and dynamic oscillations can be more easily excited when density is increased of several orders of magnitude, as in example reported in [26]. Such effect, eventually, forces a reduction of the loading rate required to perform a quasi-static analysis and finally leads to an increment of computational time costs. It should also be observed that, in the conventional approach, penalty stiffness has actually to be increased according to Eq. (1) if many interlaminar layers are modelled, because thinner and therefore stiffer sub-laminates are connected by interface elements. If penalty stiffness is increased, stable time increment is consequently reduced and a higher density is required to restore the original computational time, so that the problems originated by the increment of mass are amplified. Indeed, according to authors' experience, the set-up of an analysis based on a conventional approach, including the same number of interlaminar layers used in the hybrid model presented in this section, turns out to be quite difficult and the final computational cost is much higher with respect to the performances obtained by adopting the hybrid technique. A second benchmark is presented to prove that the modelling technique can accurately represent the local in-plane stress-strain state in the plies even in the case of multi-directional laminates. To accomplish such objective, quasi-static analyses of a carbon-reinforced laminate undergoing three-point bending were performed by using

Abaqus/Explicit. A laminate made of unidirectional carbon plies (IM7/8552) was considered, with quasi-isotropic  $[45/0/-45/90]_3s$  lay-up and in plane dimensions of  $150 \times 25 \text{ mm}^2$ . A reference model at the level of the single ply was defined, by means of a uniform grid made of  $1.25 \text{ mm} \times 1.25 \text{ mm} \times 0.28 \text{ mm}$  solid elements (elements C3D8R [13]) and characterised by the elastic properties reported in [25].

A FE hybrid model was developed with the same mesh refinement level adopted in the reference one. Thus, each ply was modelled by adopting membrane elements (elements M4R [13]), and membranes were connected by means of 23 solid elements in the thickness direction (elements C3D8R [13]). A central rigid cylinder, with a smoothly increasing vertical velocity, was used to apply the load and two lower rigid cylinders, set at a distance of 50 mm from the central one, were used to model lateral supports. The contour of the longitudinal  $\varepsilon_{xx}$  strain in the reference model, in laminate reference frame, is reported in Fig. 11-aA, which is referred to a total deflection of 1 mm. The numerical load exerted by the central roller in such a condition is of 1.584 kN and 1.578 kN for the reference and the proposed hybrid model respectively. Hence, the overall stiffness of the laminate turns out to be correctly captured by the hybrid model.

The hybrid technique accuracy in the prediction of local stress-strains levels is investigated by considering the through-the-thickness distribution of local  $\sigma_{xx}$  stress in each membrane element, expressed in material axes. The result, reported in Fig. 11-b, proves that the proposed hybrid technique can correctly represent the local stress-strain field at the level of the single lamina also in laminates with complex lamination sequences.

## **4 - Numerical analyses of DCB and ENF tests**

### **4.1 - Application of hybrid technique with different through-the-thickness mesh refinements**

The experimental overall responses and local internal strain evolutions obtained in DCB and 4-ENF tests represent a significant benchmark for a numerical approach, since the ability of accurately capturing both forces and local strain fields can be assessed in the presence of complicated effects due to fibre bridging, frictions and geometrical non-linearities.

The hybrid modelling technique was used to perform a sensitivity analysis by varying the parameters of the cohesive zone models that are more difficult to identify from the experimental results, such as process zone length and friction coefficient. Moreover, models of the DCB and of the 4-ENF tests were developed by adopting different type of meshes, with a different level of refinement in the through-the-thickness direction. Figure 12 reports the details of the meshes, in the zones at end of the pre-crack. In all the hybrid meshes single plies or sub-

laminates are represented by bi-dimensional elements (S4R [13]) and are connected by layers of three dimensional connection elements (C3D8R [13]) endowed with the cohesive zone model implemented in the VUMAT subroutine.

In a first type of model (Hybrid Model, Fig. 12-a), all the 48 plies of the specimens are represented by single bi-dimensional elements connected by 47 layers of solid connection elements. In the second type (Coarse Hybrid Model, Fig. 12-b), each bi-dimensional element represents a sub-laminate made of two plies. Finally, in the third type (Local Hybrid Model, Fig. 12-c), only a thickness corresponding to 6 plies, close to the mid-plane of the specimens, is modelled by using an hybrid mesh, with each bi-dimensional element representing a single ply. The remaining thickness of the specimen arms is modelled by using continuum shell elements (SC8R elements [13]). The position of the optical fibre closest to the pre-cracked interfaces, where local strains were measured in the experimental tests, is also shown in Fig. 12.

#### 4.2 - Development and calibration of DCB numerical model

The model of the DCB tests performed on the glass-fibre reinforced specimens with embedded optical fibres was developed by adopting the hybrid modelling technique, in the three versions presented in the previous section. The in-plane dimensions of the elements were fixed to 0.5 mm in the crack propagation direction and 1 mm along the width of the specimen. The elastic properties of the glass fibre reinforced plies, reported in section 2.1, were used to characterize the orthotropic model attributed to shell elements. For the out-of plane properties that characterize the stiffness of the solid connection element, a transverse isotropy assumption was introduced, so that  $E_{zz} = E_{yy} = 13.5$  GPa and  $G_{xz} = G_{xy} = 5.896$  GPa. Moreover, since the cohesive law implemented is based on the assumption of identical properties in mode II and mode III, all the shear moduli were set equal to the same value:  $G_{xy} = G_{xz} = G_{yz} = 5.896$  GPa.

A bi-linear cohesive response cannot represent the traction-displacement law in the case of fiber bridging that originates a non-linear softening response in mode I opening [5, 6, 16, 27, 28]. A tri-linear traction-displacement law is more appropriate and can be obtained by superimposing two elements with a bi-linear response [27], as it is shown in Fig. 13. Such approach requires the identification of only two superposition parameters to model fibre bridging,  $n$  and  $m$ , with  $0 \leq n, m \leq 1$ , which define the strength and the toughness attributed to the two laws.

Strength is distributed according to the parameter  $n$ , so that  $\sigma_{c1} = n\sigma_c$  and  $\sigma_{c2} = (1-n)\sigma_c$ , where  $\sigma_c$  is the overall axial interlaminar strength, whereas the fracture toughness levels attributed to the cohesive responses are  $G_1 = mG_c$  and  $G_2 = (1-m)G_c$ , where  $G_c$  is the overall toughness of the interlaminar layer. Techniques for the

identification of superposition parameters are described in [28]. In particular, a semi-analytical procedure is defined, requiring the knowledge of the  $R$ - $a$  curve, and, alternatively, a numerical identification procedure is presented, which only requires the knowledge of the experimental force vs. displacement curve.

The method based on the superimposition of bi-linear laws was adopted to model the cohesive zone in the analyses of the DCB tests. Therefore, in the pre-cracked layer, two three-dimensional connection elements were superimposed. Such elements were characterized with the material parameters derived from the identification of the superposition parameters,  $n$  and  $m$ , which was carried out by applying the semi-analytical approach proposed in [28].

The strength  $\sigma_c$  does not influence the response in a DCB test and was fixed to 20 MPa. The critical energy release rate  $G_c$  of the material was set equal to the steady state value of the energy release rate,  $G_{IRss}$ , evaluated in the tests. The toughness attributed to the first cohesive element represents the initial toughness  $G_{IRi}$  in the  $R$ -curve, so that  $G_1 = mG_c = G_{IRi}$ . Therefore, the  $m$  parameter is identified by setting  $m = G_{IRi}/G_{IRss}$  and parameter  $n$  is the only remaining unknown in the material model. Once the  $m$  and  $n$  parameters are chosen, the numerical process zone length obtained by the superimposition of the two laws is determined [28]. Hence, if the parameter  $m$  is fixed, the experimental value of the process zone length in the DCB test,  $l_{pz\_DCB}$ , makes possible the identification of  $n$ . Such procedure, which involves the application of non-linear equations, is described in details in [28].

Actually, the tri-linear cohesive law obtained by superimposing the two cohesive laws is a simplification of the real traction-displacement law that characterises the fracture process in the interlaminar layer. **Moreover, there is significant uncertainty in the experimental measurement of the crack length, which is required to obtain the steady state energy release rate, especially for materials that develop long process zones.** For such reasons, the numerical-experimental correlation was carried out considering three sets of identification parameters, which are reported in Table 3. The values of parameter  $n$  reported in Table 3 are the solutions of the non-linear problem described in [28] for different values of  $l_{pz\_DCB}$ .

Table 3 - Parameter sets used in the analyses of DCB tests

	$G_c$ (kJ/m <sup>2</sup> )	$m$	$l_{pz\_DCB}$ (mm)	$n$
#1	0.8	0.250	90	0.9900
#2	0.8	0.250	100	0.9917
#3	0.8	0.250	110	0.9930

The values of  $n$  are close to unit and this indicates that the strength attributed to the second connection element is very low, although the selected value of  $m$  leads to attribute 75% of the toughness  $G_c$  to such element. Consequently, the final strain corresponding to a unit damage is very high and the small strain assumption, which was used to develop the relations in Eq. 5, is no more valid.

This requires a modification in the constitutive law attributed to the second connection element. The Green-Lagrange strain component  $\varepsilon_{zz}$  is no more adequate to evaluate the relative displacement between the mid-planes of the layers in the case of large strains and a component of the deformation strain gradient,  $\mathbf{F}$ , provided in the local reference frame of the element, is more conveniently used. Since the formulation of the solid element used to connect bi-dimensional elements involves a constant strain state in the element, the component  $F_{zz}$  is linked to displacement  $\Delta_z$  as it is expressed in Eq. (9).

$$\Delta_z = \frac{\partial u_{zz}}{\partial Z} t_k = (F_{zz} - 1)t_k \quad (9)$$

Therefore, the evolution of the damage variable in Eq (8), for the cohesive element that models fibre bridging effects, is given as a function of the nominal strain  $F_{zz}-1$ . Such quantity, for small strains, has a negligible difference with  $\varepsilon_{zz}$ . The final value of the nominal strain can be obtained by applying Eq. (10), which is a modification of Eq. (7).

$$\int_0^{\infty} \sigma_I d\Delta_I = t_k \int_0^{\infty} \sigma_I dF_{zz} = G_2 = (m-1)G_c \quad (10)$$

### 4.3 - Numerical-experimental correlation in DCB tests

The curves shown in Fig. 14-a are referred to the correlation between experimental tests and force responses in numerical analyses performed by using Hybrid Models, with all the 48 layers separately modelled, and the 3 sets of material parameters reported in Table 3.

The experimental curves also include the pre-opening phase and it can be observed that all the numerical results tend to overestimate the force during such preliminary test. However, in the subsequent DCB test, an acceptable correlation is obtained for all the parameter sets. **It must be remarked that very large errors would be obtained if a simple bi-linear cohesive law were adopted, as it is exemplified in the cases presented in [28].**

The adoption of different process zones originates variations in the central part force response until a displacement of about 13.5 mm, corresponding to force levels of about 110 N. Considering the graphs reported in Fig. 5, the crack length in correspondence of such force levels is about 100 mm. Hence, it appears that the differences in the force responses obtained by assuming different values of the process zone length are difficult to be identified once that the process zone is completely developed.

The numerical curves reported in Fig. 14-b are obtained by using different through-the-thickness mesh refinement and different material parameters. Considering Fig. 14-a and Fig. 14-b it can be observed that variations originated by the adoption of different meshes are negligible with respect to the ones induced by a relatively small change in the process zone length.

The numerical-experimental correlation of the local strain evolution is reported in Fig. 15-a, for the Hybrid Models, with all the plies modelled by single shell elements, and the same sets of parameters given in Table 3. Numerical strains were evaluated by averaging the value at the integration points of the connection elements in the same 12 mm-long zones occupied by the FBG sensors in the experimental specimens. Numerical analyses capture with an appreciable accuracy the evolution of the local strains. Indeed, the gradient during the initial raising of the strains is correctly modelled, although numerical values tend to anticipate the experimental trend with all the material parameter sets. Such anticipation could also depend on the uncertainty of the exact position of the FBG sensor. The subsequent diminution of the gradients in the strain vs. displacement curves is also correctly represented, but the transition between the two gradients occurs more smoothly as the length of the process zone is decreased. The presence of a gradient discontinuity in the strain-softening regime of the three-linear laws, which is highlighted in Fig. 13, can explain such tendency. Indeed, if a longer process zone length is used, the discontinuity becomes more severe and the effect on the evolution of local strain is more apparent.

The comparison between local strain evolutions obtained by using different type of models, with different through-the-thickness mesh refinement is presented in Fig. 15-b, considering the same set of material parameters corresponding to  $l_{pz\_DCB} = 100$  mm (Set #2 in Tab. 3). The Hybrid and Local Hybrid Models, with a more refined hybrid mesh, obtain identical responses. The Coarse Hybrid Model, which is characterized by a uniformly refined but coarser hybrid mesh, present some minor discrepancies.

It should also be observed that the position of the integration points used to evaluate the numerical strains exactly corresponds to the physical through-the-thickness position of the optical fibre. The importance of a correct position is evidenced by the numerical curves reported in Fig. 15-b, which reports the numerical strains evaluated in the interlaminar layer immediately above (Upper Interface) and immediately below (Lower Interface) the one where the optical fibre was placed. Such strains are significantly different from the experimental ones.

Finally, results reported in Fig. 15-a indicate that the variations in the strain responses due to the choice of different process zone lengths extent until a displacement of more than 20 mm. The analysis confirms that the acquisi-

tion of internal strains can provide accurate and meaningful information to identify the traction-displacement behaviour at the interface, as it was done in [5]. However, results also confirm that the **three-linear** cohesive law identified by the procedure presented in [28] can adequately model the evolution of the local strain field.

Overall, the results validate the capability of the modelling technique of capturing with an adequate accuracy the evolution of the local longitudinal strains between the plies, even if the technique consider average out-of-plane states of strain between plies (or sub-laminates) instead of the strain at a zero-thickness interface. Moreover, it is shown that application of the hybrid technique can be limited to the few interfaces close to the cracks, as in the Local Hybrid Model, with a significant increase in the efficiency of the model when the location of interlaminar damage propagation is known in advance.

#### **4.4 - Development and numerical results of 4-ENF test model**

The FE models of the specimen used in the 4-ENF tests were developed by using the three types of meshes shown in Fig. 12 with an in-plane size of the elements set to 1 mm x 1 mm. A rigid body was defined to model the upper part of the test fixture (Fig. 2-b), which includes the two internal load application cylinders. Such rigid body can freely rotate along a central axis parallel to the axes of the cylinders. The supporting lower external cylinders were also modelled as rigid surfaces. All the rigid cylinders were set in contact with the external surfaces of the specimen. The pre-crack was modelled by attributing an initial unit damage condition to a set of connection elements. A detail of the FE models, including the rigid elements, is shown in Fig. 16-a, which reports the contour of interlaminar damage variable in correspondence of a deflection of 1.7 mm for one of the developed models.

The modelling technique turned out to be well suited for the introduction of friction between crack faces. In the typical applications of cohesive zone models, axial stiffness is not degraded for compressive strain states in order to model the contact between the delaminated plies [20, 21, 22]. Such choice was also adopted in previous applications of the constitutive law attributed to the three-dimensional connection elements [10,11,12], but, in the present case, the damage variable introduced in Eq. (8) was activated to degrade the modulus  $E_{zz}$  also in case of negative values of the strain  $\varepsilon_{zz}$ . A contact interaction was set to avoid interpenetration between the bi-dimensional elements representing the adjacent plies. **Two surfaces were defined by considering the mid-planes of the bi-dimensional elements and the contact thickness attributed to the contact interaction between these surfaces was set to provide a small clearance in the undamaged state between the contact planes. Such clearance was calibrated to avoid the activation of contact when the connection elements are undamaged, as it is sketched in Fig. 17. Indeed, no contact is required in undamaged conditions, because the elastic response of the connection elements in the**

out-of-plane directions avoid interpenetration and the interaction between the connected layer is completely modelled by such response. For the present case, the out-of-plane material properties reported in section 4.2 were used to characterize the stiffness of connection elements ( $E_{zz} = 13.5$  GPa,  $G_{xz} = G_{yz} = 5.896$  GPa). According to such properties, out-of-plane stress levels of the order of several hundreds of MPa's are required to reduce the distance between the mid-planes of the bi-dimensional elements to 0.9 times the original distance. Based on this consideration, the minimum distance to activate the contact algorithm between the faces of the bi-dimensional elements was set to 0.9 times the initial thickness. Hence contact is not activated if the element is undamaged, as it is shown in Fig. 17-a, unless in the presence of severe compressive loads, which is not the case for the ENF analyses that are presented in this paper. Moreover, it can be reasonably assumed that the activation distance could be further reduced without significant effects on the overall and local numerical response. On the contrary, once the connection element is damaged and can no more oppose to a relative movement of the adjacent layers, the contact interaction is established when the distance between the faces is reduced to 0.9 times the initial thickness, as indicated in Fig. 17-b. In such conditions, contact is required to avoid interpenetration and, in the presence of relative tangential motion, a friction coefficient,  $\mu$ , attributed to such interaction, models the transmission of frictional forces between the two layers.

The analyses of ENF tests were performed with the use of Abaqus/Explicit, by applying a smoothly increasing downward velocity to the upper reference node. Overall, the same material properties used in the analyses of DCB test were used. The shear strength of the interlaminar layer,  $\sigma_{II0}$ , which does not affect the fracture test response, was set to 50 MPa. The toughness attributed to the connection element in mode II fracture process was varied in the range  $1.5 \text{ kJ/m}^2 \div 2.1 \text{ kJ/m}^2$  and the friction coefficient  $\mu$  was varied in the range  $0.0 \div 0.6$ .

The contours reported in Fig. 16 are referred to a Hybrid Model, with one bi-dimensional element per ply, a toughness of  $1.5 \text{ kJ/m}^2$  and a friction coefficient set to 0.6. The numerical process zone is highlighted in Fig. 16-a. The contour of  $\sigma_{zz}$  stress, shown in Fig. 16-b, indicates the zones along the crack faces with negative  $\sigma_{zz}$  values, which are transmitted through the cracked interface by the numerical contact interaction. Such zones, which are located in correspondence of the support and loading cylinders on the pre-cracked side of the specimens, are the ones where frictional forces develop in the numerical analysis.

Numerical-experimental correlations of force vs. displacement curves is presented in Fig. 18 by using the Hybrid Model, first considering different toughness levels and a null friction coefficient (Fig. 18-a), and then a fixed toughness level of  $1.5 \text{ kJ/m}^2$  with increasing values of  $\mu$  (Fig. 18-b).

The slope of the force vs. displacement curves in the elastic range is appreciably captured by all the numerical analyses, including the geometrical non-linear effect that determines a slight progressive increase of the stiffness. In the analyses performed with a toughness level of 2.1 kJ/m<sup>2</sup> and a coefficient of friction set to zero, the force levels at the beginning of crack propagations are close to the experimental ones. Nevertheless, the numerical slope of the curve during the phase of stable crack propagation underestimates the experimental value. Moreover, such slope remains constant if the toughness is varied. On the contrary, results reported in Fig. 18-b indicate that if the friction coefficient is increased both the force levels at the onset crack propagation and the slope of the curve during the propagation are increased. It can be observed that the analysis performed by using  $G_{IIc} = 1.5$  kJ/m<sup>2</sup> and a coefficient of friction set to 0.6 obtains an appreciable correlation with the experimental curve.

The analysis of the correlation of local strain levels is reported in Fig. 19-a, considering two analysis performed with the Hybrid Model, by using  $G_{IIc} = 1.5$  kJ/m<sup>2</sup> and  $\mu=0.6$  and  $G_{IIc} = 2.1$  kJ/m<sup>2</sup> and  $\mu=0.0$ , respectively. Both the numerical models qualitatively model the trend of local strain close to the cracked interface. Models also capture the non-linear evolution of strain on the outer surface of the specimen. Results indicate that the effects of the crack advancement on the local strain states are delayed in the model with  $G_{IIc} = 2.1$  kJ/m<sup>2</sup> and  $\mu=0.0$  with respect to the case of the model with lower toughness and higher friction. Hence the crack advancement predicted for a given displacement  $\delta$  by the model without friction is lower than the one provided by the model with  $G_{IIc} = 1.5$  kJ/m<sup>2</sup> and  $\mu=0.6$ , which gives results that are in better agreement with experimental data. The delay obtained by the model without friction is confirmed by the final change of slope of the force vs. displacement curves presented in Fig. 18, which occurs both in experiments and in numerical models when the crack passes beyond the right internal cylinder. In the analysis with  $G_{IIc} = 1.5$  kJ/m<sup>2</sup> and  $\mu=0.6$ , the displacement  $\delta$  in correspondence of this phenomenon is in appreciable agreement with the experimental data, whereas the analysis  $G_{IIc} = 2.1$  kJ/m<sup>2</sup> and  $\mu=0.0$  clearly overestimates the deflections required to propagate the crack until the right cylinder.

From a numerical standpoint, the result reported indicates that the models, with the proper choice of material parameters and the introduction of friction, can quite reliably model the overall response and the evolution of local strains in stable mode II propagation. Finally, the results referred to the local strain evolution obtained with all the three type of meshes described in Fig. 12 are reported in Fig. 19-b for the same material parameters  $G_{IIc} = 1.5$  kJ/m<sup>2</sup> and  $\mu=0.6$ . Results confirm that the modelling technique can be applied at different levels of through-the-thickness mesh refinement without significantly affecting the accuracy in the prediction of the local strain state.

## 5 - Concluding remarks

The paper presented the evolution of local strains fields during mode I and mode II stable propagation of delamination in composite laminates and modelled the global and local response of the specimens. Numerical activities were carried out by using a modelling technique based on the decomposition of the laminate in a stack of bi-dimensional elements, carrying in-plane stress components, connected by three-dimensional elements endowed with a cohesive zone model that degrades the out-of-plane physical stiffness properties of composite material. The results reported in the paper quantified the computational advantages in explicit computations provided by such technique and validated its application in the case of multi-directional laminates. The technique was used to model the local strains evolution during delamination processes affected by fibre bridging and frictional effects. The correlation with the internal strains acquired by FBG sensors embedded in laminates provided appreciable results. Numerical results suggest that a **tri-linear** cohesive response is adequate to achieve an acceptable correlation in the evolution of local strain during mode I delamination propagation in the presence of significant fibre bridging. Moreover, numerical-experimental correlation confirmed the possibility of a rapid identification of the cohesive response at an appreciable level of accuracy by means of a semi-analytical procedure specifically developed for **tri-linear** softening laws. The effects of friction between the faces in the stable mode II propagation of delamination **were** also investigated, thanks to the characteristic of the numerical approach, which proved to be well suited to model contact interaction after the failure of interfacial bonds. Results indicate a strong influence of friction on mode II stable propagation of interlaminar damage. Overall, numerical results show that the technique proposed can be used to model all interlaminar layers of a laminate in quasi-static explicit analyses with a reduction of computational cost and of the effort required to set up the analysis. Moreover, analyses performed with different levels of through-the-thickness mesh refinements indicates that models developed according to the proposed technique can reliably approximate both global response and local strain states by using different levels of through-the-thickness mesh refinement. In particular, an accurate modelling of both global and local responses can be achieved by modelling only few interlaminar layers, if numerical analysis are referred to a scenario characterized by the propagation of a single delamination, developing in an interlaminar interface that is known in advance.

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## Figure Captions

*Figure 1: Manufacturing of a laminate endowed with optical sensors*

*Figure 2: Layout and positioning of optical fibres and sensors for DCB (a) and 4-ENF (b) tests*

*Figure 3: R-a curves obtained in DCB tests and fibre bridging phenomena*

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*Figure 5: Internal strains and applied forces vs. crack length in DCB tests*

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