

# Dealing with a multiple criteria environmental problem with interaction effects between criteria through an extension of the ELECTRE III method

M. Bottero <sup>a,\*</sup>, V. Ferretti <sup>a</sup>, J. R. Figueira <sup>d</sup>, S. Greco <sup>b,e</sup>, B. Roy <sup>c</sup>

<sup>a</sup> Department of Regional and Urban Studies and Planning, Politecnico di Torino, Italy

<sup>b</sup> Department of Economics and Business, University of Catania, Sicily, Italy

<sup>c</sup> LAMSADE, Université Paris-Dauphine, Paris, France

<sup>d</sup> CEG-IST, Instituto Superior Técnico, Universidade de Lisboa, Portugal

<sup>e</sup> Portsmouth Business School, Centre of Operations Research and Logistics (CORL), University of Portsmouth, Portsmouth, United Kingdom

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## 1. Introduction

The starting point of the work presented in this article comes from a request made by a decision maker. It was not for help in making a decision as such, but rather for a comparison of five requalification actions or projects for an abandoned quarry. In addressing the problem for study, the comparison has to take into account several stake-holders' different points of view. To do so, an adequate and coherent family of criteria has to be built. The authors of the current study had good reasons to think that in a context of sustainability assessment they should not discard *a priori* the possibility of interactions between some pairs of criteria. Indeed, in the context of sustainability assessment, economic sustainability has an ecological cost and ecological sustainability has an economic cost (Munda, 2005). Following this reasoning, it is possible to state that in the particular context of sustainability assessment the different aspects (required for the construction of criteria) usually interact with each other, reflecting the natural dynamics of environmental and land-use territorial systems. Consequently, it seemed justified to try to highlight potential

synergies, redundancies, or other phenomena among coalitions of criteria.

Given the possible existence of interaction between some pairs of criteria, we undertook a detailed overview of the main approaches used in the literature to consider the evaluation of interaction effects. This analysis led to the choice of an extension of ELECTRE III with interactions (Figueira, Greco, & Roy, 2009), which has proved to be an appropriate method (*cf.* Section 2, below). The comparison of five alternative requalification projects for an abandoned quarry (*cf.* Section 3) seemed to the authors a very good opportunity to test this extension of ELECTRE III. Of particular interest was the question of whether with this method it was possible to highlight and build robust conclusions, taking into account the existence of some arbitrariness when assigning values to the main parameters (weights of criteria and interaction coefficients). To assign numerical values to these parameters and to implement the method in general, it was necessary to form a focus group (*cf.* Section 3). Section 4 reports the way the focus group worked. Section 5 provides the results of the application and discusses the findings through a sensitivity analysis, which enabled us to highlight and build robust conclusions. Finally, Section 6 contains the main conclusions that can be drawn from the research.

\* Corresponding author. Tel.: +39 011 090 6464; fax: +39 011 090 7499.  
E-mail address: marta.bottero@polito.it (M. Bottero).

## 2. Adopting an MCDA method to handle interaction between criteria

When the analyst was confronted with this case study, he had good reasons to believe that he should take into account the interactions between criteria (this was later confirmed, see Section 3). Under these conditions, the choice of a multiple criteria approach was examined. Thus, the multiple criteria methods taking into account interaction between criteria were reviewed to adopt a suitable one.

There is currently a great variety of multiple criteria decision aiding (MCDA) methods and this means that the task of adopting the appropriate method for a certain decision-aiding situation is not an easy one (see Roy & Słowiński, 2013). There are also a certain number of methods considering the interaction between criteria. It should be noted that there is no interaction between criteria in the case of preference independence (see, e.g., Keeney and Raiffa, 1976). The possible weakening of preference independence, implying some form of interaction between criteria, has been under discussion for some time (see, e.g., Fishburn and Keeney, 1975; Keeney, 1981). Probably the most well-known method emerging from this literature is the multilinear utility functions (Keeney & Raiffa, 1976). This method aggregates values of the considered criteria through a weighted-sum of products of the marginal utilities corresponding to the single criterion over all the subsets of criteria. The limitations of this aggregation procedure are the difficulty in defining the many, and to some extent heterogeneous, weights (one for each subset of criteria) and the marginal utility functions themselves. Another methodology to deal with interaction among and between criteria considers non-additive integrals, such as the Choquet integral (Choquet, 1953; Grabisch, 1996) and the Sugeno integral (Sugeno, 1974) and their generalization, such as the bipolar Choquet integral (Grabisch & Labreuche, 2005) or the level dependent Choquet integral (Greco, Matarazzo, & Giove, 2011). The basic idea of this approach is that the interaction between criteria can be modeled through the non-additive importance of criteria represented by the value assigned to each subset of criteria by a capacity, called also a fuzzy measure. This is valid at least in the basic version of the Choquet integral and Sugeno integral. This approach has several drawbacks (see Roy, 2009), the most important of which is that they require evaluations on criteria expressed on the same scale (for a proposal to determine the common scale that is necessary in order to apply the Choquet integral, see Angilella, Corrente, and Greco, 2015). Recently, the interaction between criteria through a weakening of the preference independence condition has been given some consideration in the domain of Artificial Intelligence through GAI-networks (Gonzales & Perny, 2004), as well as UCP-networks (Boutillier, Bacchus, & Brafman, 2001), which are based on the idea of Generalized Additive Independence (GAI) decomposition introduced by Fishburn (1970). They allow for aggregating performances on the considered criteria through the sum of marginal utilities related to subsets of criteria. The main problem with these methods is the difficulty in eliciting the marginal utilities from preference information given by the decision maker. Another approach, recently proposed to deal with the interaction of criteria, is the use of enriched additive value functions. Besides the usual sum of marginal utility functions related to each of the considered criteria, these have some further terms representing interaction between a small number of couples of criteria in terms of bonus, in case of synergy between criteria, or penalization, in case of redundancy between criteria (Greco, Mousseau, & Słowiński, 2014). Since the decision maker could have some difficulties in defining for which couples there is synergy or redundancy, the couples of interacting criteria are singled out with an ordinal regression approach on the basis of some preference information expressed by the decision-maker in terms of pairwise preference comparison of alternatives. Another possibility, recently proposed in the literature (Corrente, Figueira, & Greco, 2014), is to apply a Choquet-like aggregation method in aggregation of the preference functions out-

ranking methods apart from ELECTRE methods, such as the PROMETHEE methods.

Taking into account all the above aggregation procedures, the choice of an extension of ELECTRE III taking into account interactions between criteria (Figueira et al., 2009) was judged to be completely adequate for dealing with the case study presented in the next section, for the following reasons:

- (i) ELECTRE methods allow for dealing with heterogeneous scales. In the present study the performances of the actions were expressed on ordinal scales for four criteria, while for the other two criteria the scales were quantitative.
- (ii) ELECTRE methods are able to take into account purely ordinal scales, thus maintaining their original concrete verbal meaning. In other words, there is no need to convert the original scales into abstract ones with an arbitrary imposed unit and range.
- (iii) ELECTRE methods also allow for taking into account indifference and preference thresholds when modeling imperfect knowledge of data. In our study it was necessary to take imperfect knowledge into account; for such a purpose the definition of indifference and preference thresholds seemed perfectly adequate.
- (iv) The generalization of ELECTRE methods allows for consideration of the interaction between some couples of criteria, which seemed to be present in our study. In addition, it was considered to be the right opportunity for testing the applicability of ELECTRE III with interactions.

## 3. Case study: the requalification of an abandoned quarry

This study deals with the characterization and comparison of alternative projects for the requalification of an abandoned quarry located in Northern Italy. In particular, this study concerns the analysis and the comparison of five projects in order to rank them from the best to the worst one. Details about the case study are provided in what follows.

### 3.1. A brief description of the context

The application performed in the present research is based on the results coming from a previous study where the alternative options have been identified and investigated (Bottero, Ferretti, & Pomarico, 2014; Brunetti, 2007). The area under analysis refers to a quarry that has been abandoned since 1975 and covers a total surface of 65,000 square meters, with a depth of approximately 25 meters from the ground level. Due to its abandoned state the area is now characterized by uncontrolled vegetation growth and by water-filled pits. Furthermore, the area under analysis is part of the Provincial ecological system of environmentally valuable sites.

For the reclamation of the area five alternative projects have been considered, that can be described as follows: (1) basic reclamation, (2) to plant a forest, (3) development of a wetland, (4) implementation of the ecological network, and (5) construction of a recreational structure. It is worth mentioning that the projects represent real projects, which are now under investigation from the Municipal Authority for the transformation of the area. The five alternative options that were proposed for the requalification of the abandoned quarry can be described in a more detailed form as follows:

1. *Basic reclamation*: This alternative involves the filling of the quarry, the implementation of security measures on the quarry's slope characterized by landslide risk, the laying of the topsoil, the natural evolution of the vegetation, and the accelerated growth of the autochthonous black locust wood.
2. *Valuable forest*: This alternative involves the filling of the quarry, the implementation of security measures on the quarry's slope characterized by landslide risk, the laying of the topsoil, the

**Table 1**  
Survey of the most relevant stakeholder groups.

Stakeholders	Level	Description
Forestry Corp	National	The Forestry Corp is a National Police Force in charge for the defense of natural heritage and landscape. In case of deep transformation of the area under investigation, a delegate from the Forestry Corp will take place in the Environmental Impact Assessment procedure.
Regional Authority	Regional	The Regional Authority is in charge for the territorial and landscape planning and for the environmental management. In this case, if the project will require a modification of the Municipal Plan, the Regional Authority will have to approve the change. Furthermore, the Landscape Regional Plan identifies the area as valuable from a landscape point of view.
Regional Environmental Authority	Provincial	The Provincial Authority is responsible for the territorial and landscape planning and for the environmental management at the provincial level. The interests in the case under examination are related to the fact that the area is part of the Provincial Ecological Network that links many territorial areas of particular importance from a naturalistic point of view. Moreover, the Provincial Authority is in charge for controlling all the operations related to mining activities (opening of new activities, exercise, closure and environmental rehabilitation).
Municipal Technical Office	Local	The Municipal Technical Office is in charge for controlling all the construction activities. In this case, it will evaluate the transformation project in order to verify if it complies with the legislation.
Mayor	Local	The Mayor is the chief of the Municipality and has the responsibility of approving or rejecting the transformation project. The interests are related above all to ensure the quality of life of the local population and to grant the financial-economic stability of the Municipal Authority.
Local practitioners	Local	They represent the practitioners having a bureau in the zone under analysis and working in the field of architecture, urban planning and agronomy. They could be involved in the transformation project for the area.
Inhabitants	Local	The local population could be affected by the transformation project. Their interests are related to preserve the environmental system and to increase the level of services in the area.
Private entrepreneurs	Local	They represent the private bodies that might be interested in investing money in the transformation project that considers the construction of a multi-functional area.

cover with drainage material, and the establishment of an oak-hornbeam wood.

3. *Wetland*: This alternative involves the partial filling of the quarry, the implementation of security measures on the quarry' slope characterized by landslide risk, the surface sealing, the creation of a lake, the planting of wetland vegetation, and the natural evolution of the surrounding wood and of the wetland.
4. *Ecological network*: This alternative consists of the partial filling of the quarry, the implementation of security measures on the quarry's slope characterized by landslide risk, the surface sealing, the realization of lakes, pedestrian and equestrian pathways, and recreational areas, the predisposition of information and educational material, and the natural evolution of the existing wood.
5. *Multi-functional area*: This alternative involves the partial filling of the quarry, the implementation of security measures on the quarry's slope characterized by landslide risk, and the construction of sports and residential structures that are completely self-sufficient in terms of energy and waste water disposal and that are harmoniously integrated with the landscape.

### 3.2. Stakeholders: their concerns, values, and expectations

One crucial point of a decision process consists of the identification and classification of the stakeholder groups (entities), which can be defined as those who can affect the realization of organizational goals or group of individuals affected by the realization of the organizational goals. It has been recognized that mapping the stakeholders allows the comprehension of fundamental issues, such as the level of interest of each stakeholder group to impress its expectations on the project decisions and the power with which each group can affect the project decisions. In the present analysis, the environmental planning and management involve different stakeholders with conflicting objectives and interests. It would thus be necessary to consider the opinions of all the stakeholders for a sound decision aiding process.

Table 1 surveys the relevant stakeholders which can have a role in the process. The stakeholders are all the individuals or entities/institutions which are related to the use and/or the management of the area, including the Regional Authority, the Provincial Authority, the Regional Environmental Authority, the Forestry Corp, the Municipal Technical Office, the Mayor, the local practitioners, the inhabi-

tants, and the private entrepreneurs. This structure comprises all the involved stakeholders.

### 3.3. Building a coherent family of criteria

This subsection is devoted to the construction of the family of criteria, the identification of projects and their performances, as well as the definition of the discriminating thresholds associated with criteria (see Appendix A).

#### 3.3.1. Criteria

Starting from the overall objective of the analysis, which is the identification of the most sustainable ranking of the projects for the reuse of the abandoned quarry, a coherent set or family of criteria that reflects all the concerns relevant to the decision problem has been identified, paying attention to their exhaustiveness, cohesive-ness, and non-redundancy (Roy & Bouyssou, 1993). The criteria considered in the present application were selected based on the relevant international literature (Bascetin, 2007; Golestanifar & Bazzazi, 2010; Rey-Valette, Damart, & Roussel, 2007; Soltanmohammadi, Osanloo, & Bazzazi, 2010) and on the requirements coming from the legislative framework in the context of Environmental Impact Assessment (first of all, the European Directive 2014/52/EU). In order to find the most suitable project for the reuse of the abandoned quarry, a family of six criteria has been built (Table 2). Both quantitative and qualitative criteria have been used for the analysis.

Two of the co-authors of this paper (M. Bottero and V. Ferretti) are very familiar with problems in the domain of sustainability assessments. They had good reasons to think that we should not discard *a priori* the possibility of the existence of interactions between some pairs of criteria (*cf.* Section 1). The work performed in this paper (*cf.* Section 4) shows that they were right.

#### 3.3.2. Performances table

Table 3 presents the performances of the five projects  $\{a_1, a_2, a_3, a_4, a_5\}$  according to the six considered criteria. The criteria "profitability", "new services for the population", and "environmental effects" are expressed on the following seven-level qualitative scale: very bad (1), bad (2), rather bad (3), average (4), rather good (5), good (6), very good (7). This is a numerical but ordinal scale, used for encoding the original verbal scale. It is useful only to work with figures

**Table 2**  
Description of the considered criteria.

Criteria	Unit	Description
Investment costs	Euros	This criterion models the construction costs [min ].
Profitability	Qualitative judgment	This criterion refers to the financial efficacy of the investment and to the consequences that the project may determine on the local economic system in terms of public revenues [max ].
New services for the population	Qualitative judgment	This criterion models the availability of new services for the population, such as green areas, recreational areas, sports structures, etc. [max ].
Landscape ecology	hectares	This criterion models the effects of the project on the landscape quality, on bio-diversity conservation and on the local ecological network, in terms of hectares of naturalized area [max ].
Environmental effects	Qualitative judgment	This criterion models the effects that the project is likely to produce on the physical environment (hydrology, geo-technical conditions, etc.) [max ].
Consistency with the local planning requirements	Yes/no	This criterion models the presence of constraints that could affect the transformation project and to the consistency with the planning instruments in force [max ].

**Table 3**  
Performances table with discriminating threshold values.

	Investment ( $g_1$ )	Profitability ( $g_2$ )	Services ( $g_3$ )	Landscape ( $g_4$ )	Environment ( $g_5$ )	Consistency ( $g_6$ )
$a_1$	30,000	Rather bad (3)	Very bad (1)	2	Average (4)	Yes (1)
$a_2$	45,000	Rather bad (3)	Rather good (5)	5	Rather good (5)	Yes (1)
$a_3$	90,000	Very bad (1)	Good (6)	3.2	Very good (7)	Yes (1)
$a_4$	120,000	Very bad (1)	Very good (7)	3.5	Good (6)	Yes (1)
$a_5$	900,000	Very good (7)	Very good (7)	1	Rather bad (3)	No (0)
$q_j$	15,000	1	1	0.5	1	0
$p_j$	20,000	1	1	1	1	0

instead of the verbal statements; these figures are simply the rank of the verbal statements.

### 3.3.3. Thresholds

To take into account the imperfect character of data, ELECTRE methods make use of discriminating (indifference and preference) thresholds (see Subsection A.2). Table 3 thus presents also the discriminating thresholds (indifference,  $q$ , and preference,  $p$ ) identified for the six criteria. In particular:

- For the qualitative criteria (“profitability”, “new services for the population”, and “environmental effects”) the indifference thresholds and the preference thresholds are both equal to 1; this means that two performances which are put on two consecutive levels on the qualitative scale cannot be considered as significantly different (these values have not a quantitative meaning, for more details see Roy, Figueira, and Almeida-Dias 2014);
- For the criterion “consistency with the local planning requirements”, there are no thresholds (both are equal to 0).
- For the quantitative criteria (“investment costs” and “landscape ecology”) the indifference threshold could not be 0 and the preference threshold had to be strictly higher than the indifference threshold. In particular, for the criterion “investment costs” a difference bigger than 20,000 Euros means that the cheaper alternative is strictly preferred and a difference of 15,000 Euros is compatible with an indifference between the two alternatives. As for the criterion “landscape ecology” the indifference threshold was fixed at 0.5 hectares while the preference threshold at 1 hectare.

### 3.4. Why to constitute a focus group?

The implementation of ELECTRE III with interaction between criteria requires the specification of the role that the different criteria must play as well as the nature of the interactions which may exist between these criteria. This is performed by assigning numerical values and in-determination margins to several parameters. For such a purpose, the analyst considered appropriate to form a panel of experts working together, side-by-side, through the application of a focus group technique (see, for instance, Morgan, 1988; Morgan and Krueger, 1993; Stewart and Shamdasani, 1990). These experts should be able to take

into account the points of view of the different stakeholders. Indeed, in the present application, a close attention was devoted to the formation of a group of experts having a balanced background composition. For this reason, an expert in the field of economic evaluation, an environmental engineer, and an expert in landscape ecology constitute this focus group.

## 4. Presentation of the work with the focus group

The work by the focus group has been organized in three major phases according to the following structure. In a first phase (say, a learning phase), the analyst promoted an individual discussion with each expert for reflecting and thinking about the relative importance of criteria, and then, built a set of weights for each one of the three experts, separately. In a second phase, the main task of the analyst was devoted to promote a discussion about the sets of weights obtained in the previous phase, and then, help in building a consensual set of weights for the group. Finally, in the third phase, the analyst led the experts to discuss and work side-by-side about the nature of the interactions which may exist between criteria and about the way of taking into account such interactions. Each one of these three phases will make the object of one of the following three sub-sections. Let us underline that in what follows it is not a matter of veto thresholds. The analyst explained to the experts what a veto effect consisted in. The experts thought that, considering the nature of the case under study, there was no reason to assign a veto power to any of the six considered criteria.

### 4.1. Phase 1 (learning): construction of a set of individual weights

Indeed, this first phase is a learning phase, which is intended to make the notion of the relative importance of the different criteria understandable for the three experts. These experts worked separately, the object being to give the possibility to each one of the three experts to explain the way they wanted to differentiate the role every criterion must play, according to the opinion of each one of them. With such a purpose in mind, the analyst used the SRF (Simos-Roy-Figueira) method for helping and assisting the experts.

This stage started with a collective presentation of the way this method (Figueira & Roy, 2002) should be able to help the experts to

**Table 4**

Ranking of criteria and blank cards for the three experts (where  $g_1$  = "investment costs",  $g_2$  = "profitability",  $g_3$  = "services",  $g_4$  = "landscape",  $g_5$  = "environmental" effects and  $g_6$  = "consistency";  $r_j$ , represents the position of criterion  $j$ , for  $j = 1, \dots, 6$ ;  $n_j$ , is the number of blank cards between positions  $j$  and  $j + 1$  in the ranking of criteria, for  $j = 1, \dots, 5$ ; and,  $N$  is the overall number of blank cards for each expert).

Experts	$r_1$	$n_1$	$r_2$	$n_2$	$r_3$	$n_3$	$r_4$	$n_4$	$r_5$	$n_5$	$r_6$	$N$
$E_1$ : economic evaluation	$g_6$	2	$g_4$	0	$g_3$	0	$g_1$	0	$g_2$	0	$g_5$	2
$E_2$ : environmental engineering	$g_6$	2	$g_3$	2	$g_2$	1	$g_4$	3	$g_1$	2	$g_5$	10
$E_3$ : landscape ecology	$g_6$	1	$g_2$	1	$g_1$	3	$g_3$	2	$g_4$	2	$g_5$	9

**Table 5**

Normalized weights for each criterion and for each expert.

Experts	$Z$	$w_1$	$w_2$	$w_3$	$w_4$	$w_5$	$w_6$
$E_1$ : economic evaluation	7	19.3	22.4	16.2	13.0	25.5	3.6
$E_2$ : environmental engineering	15	25.9	14.0	8.1	18.0	31.9	2.1
$E_3$ : landscape ecology	14	11.2	6.8	20.0	26.5	33.1	2.4

express their judgments with respect to the relative importance of criteria.

1. The analyst gave to every expert a deck (pack, or set) containing six cards with the front of each card carrying the name (or a label) of every criterion that distinguishes it from the other cards in the deck; the analyst also gave them a big enough number of blank cards; the purpose of such blank cards being explained to the experts by the analyst slightly later on in the interaction protocol.
2. The analyst asked every expert to regroup cards corresponding to criteria of the same weight in order to constitute, if necessary, packets of ties (the analyst said to them that these packets will be, most often, reduced to card, which was definitely the case here for every one of the three experts and for all the six criteria).
3. Then, the analyst asked every expert to rank (or line up) the tied packets by an increasing order of their weights, by explaining them that the least important packet will be assigned to the rank 1, the second least important to the rank 2, and so on.
4. The analyst also called the attention of every expert to think about the fact that two successive tidying up packets of criteria in the ranking can have, according to their opinion, a more or less close importance; after the expert has been reflecting about this more or less close importance, the analyst asked him to materialize it by inserting blank cards in between the successive packets of criteria; the analyst finally explained to each expert that no blank card means that both packets will not have the same weights, but the difference in the weights would be minimal; only one blank card means twice the minimal difference when compared to the absence of blank cards; two blank cards correspond to three times the minimal difference, and so on.

Table 4 contains the information obtained from every expert when applying the procedure above.

The analyst was not surprised to obtain different rankings since the three experts have a very different background and, as a result, their approach in the analysis of the problem is also very different.

At this point, the analyst explained to the experts that in order to assign the numerical values to the weights, which must reflect the relative importance of criteria according to the preference information they provided (*cf.* Table 4), they need to answer an additional question. The analyst, therefore, asked every expert to that he should tell how many times the last packet of criteria (that is to say, the most important) is more important than the first one (this ratio will be de-noted by  $Z$ ). Finally, the analyst specified to every expert that he has three possible alternatives to define this value: a single very definite value, a range, or three distinct values (a minimum, a maximum, and a central value).

**Table 6**

Normalized weights for the expert in economic evaluation according to different values for the ratio  $Z$  (the original value of  $Z$  established by the expert is highlighted in bold).

$E_1$ : expert in economic evaluation	$w_1$	$w_2$	$w_3$	$w_4$	$w_5$	$w_6$
$Z = 7$	<b>19.3</b>	<b>22.4</b>	<b>16.2</b>	<b>13.0</b>	<b>25.5</b>	<b>3.6</b>
$Z = 10$	19.5	22.9	16.1	12.7	26.2	2.6
$Z = 14$	19.6	23.2	16.1	12.5	26.7	1.9
$Z = 15$	19.6	23.2	16.1	12.5	26.8	1.8

The obtained answers, as well as the weights that result from such answers by applying SRF, are provided in Table 5.

The reader will note that one of the experts gave to  $Z$  a very different value (7) of those given by the two other experts (15 and 14). The analyst wanted to know the impact on the set of weights provided by SRF of these differences. For this purpose, the analyst applied SRF to the preference information provided by the first expert, but with different values for the ratio  $Z$  (see Table 6).

#### 4.2. Phase 2: construction of a common set of weights

The analyst began by calling the attention of the experts to the convergence or agreement points. This is essentially related to the position (see Table 4) of criterion  $g_6$  (services), as being the least important, and the position of criterion  $g_5$  (effects), as being the most important one. Then, the analyst especially stressed the divergences. First, the analyst pointed out that the expert in economic evaluation inserted very few blank cards (only 2, while the other experts inserted 9 or 10). This led to a more narrowed set of weights with the value 7 he assigned to the ratio  $Z$ , instead of 14 and 15 (*i.e.*, the values given by the two other experts). Then, the analyst called the attention of the experts to the very major divergence (*cf.* again Table 4). This disagreement is related to the relative position in the ranking of criteria  $g_1$  (investment costs) and  $g_2$  (profitability). The expert in economic evaluation assigned the two criteria, respectively, to ranks 4 and 5, while the two other experts reversed their respective ranks: the expert in environmental engineering put them, respectively, in ranks 5 and 3, and the expert in landscape ecology gave them, respectively, ranks 3 and 2. Besides, if they take into account the place of blank cards, it clearly appears that these two experts wanted to assign a distinctly less important role to criterion  $g_2$ , than the role the expert on economic evaluation wanted to give to this same criterion (this is what Table 5 clearly shows).

These divergences led to some exchanges between the expert in economic evaluation and the two others. By means of several explanations, the three experts shared their opinions. These explanations led them to re-examine the role which they agreed certain criteria

**Table 7**  
Common ranking of criteria and blank cards for the group of experts.

	$r_1$	$n_1$	$r_2$	$n_2$	$r_3$	$n_3$	$r_4$	$n_4$	$r_5$	$n_5$	$r_6$	$N$
Group of the three experts	$g_6$	2	$g_3$	3	$g_1$	1	$g_4$	3	$g_2$	2	$g_5$	11

should play, especially concerning the ranking of the most distant criteria from their domains of expertise.

For instance, the economic evaluation expert succeeded in making understandable for the two other experts the fact that profitability ( $g_2$ ) is much more important than investment costs ( $g_1$ ). She explained that the investment costs in a new project are not very important; indeed, what is really important is the fact that this project can generate important incomes (therefore, a high profitability) in order to remunerate the costs of an intervention (that is, the investment costs). The experts in landscape ecology and in environmental engineering, therefore, saw again how they ranked the criteria and they decided to put profitability ( $g_2$ ) with a higher importance than the investment costs ( $g_1$ ) (while in the first phase of the interaction process the situation was the opposite). Finally, the debate converged to a new ranking:

$$g_6 < g_3 < g_1 < g_4 < g_2 < g_5,$$

where  $<$  means “strictly less important than”. See also Table 7.

Concerning the value of  $Z$ , an agreement was achieved around the value 16. It appeared interesting to the analyst to see also the impact that the ratios 14 and 15 would have (cf. Table 8 below).

The participants found that the work done by the focus group was a very interesting task. The SRF methodology for the determination of weights was very well accepted and it has been considered useful for reflecting the respective role played by the different criteria involved in this study. The discussion raised by the comparison of the three rankings of the cards, proposed in the first phase by the experts working separately, made it possible to realize that the interpretation of the meaning of the criteria was not the same for the three experts. Once this meaning was clarified and unified, the manipulation of the cards-criteria and the visualization of the way they were ranked led quite quickly to an agreement about the respective role the different criteria should play in the decision aiding process. Finally, the assignment of a value for the ratio  $Z$  gave rise to some debates, but the experts easily came to an agreement on the value  $Z = 16$ . As for the choice of the previous value, the agreement about the choice of the values  $Z = 14$  or  $Z = 15$  was also easy to reach, leading to three sets of nonsignificantly different weights (cf. Table 8).

#### 4.3. Phase 3: details of the implementation on how to take into account the interaction between criteria

The analyst organized this phase in three steps: the first one consisted of an explanation about what mechanism every type of interaction effect could take into account; the second was devoted to an inventory of the type of interactions to be taken into account; and the third dealt with the manner of taking the interactions into account.

**Table 8**  
Normalized weights for the group of three experts according to different values for the ratio  $Z$  (the original value of  $Z$  established by the group of experts is highlighted in bold).

Group of the three experts	$w_1$	$w_2$	$w_3$	$w_4$	$w_5$	$w_6$
$Z = 14$	14.9	25.7	7.6	18.5	31.1	2.2
$Z = 15$	14.8	25.8	7.5	18.5	31.3	2.1
$Z = 16$	<b>14.8</b>	<b>25.9</b>	<b>7.5</b>	<b>18.5</b>	<b>31.3</b>	<b>2.0</b>

##### 4.3.1. First step

The analyst explained to the experts the nature of the interactions between criteria that ELECTRE method allows to take into account. For such a purpose, the analyst made use of two examples, the first related to the project of building a new hotel, and the second concerning the purchase of a new digital camera, such as they were introduced in Figueira et al. (2009). On the one hand, the three experts had no difficulty to understand the effects of mutual-strengthening and mutual-weakening. On the other hand, additional explanations were necessary to make the antagonistic effect easier to understand.

The experts raised the following question: “by analyzing the interactions, should we think about what is going on in the general case of a problem of land-use planning, or must we consider directly the particular case in which we are interested in here?” The analyst recommended the experts to begin by considering the general case, and then examined if their conclusions remained valid for the particular case. In land-use planning problems, it is actually possible that certain interaction effects will depend on the particular case under analysis. The experts presented, as an example, an environmental noisy pollution case. The importance of this impact can strongly depend on the morpho-geological characteristics of the territory where the impact will take place. If the impact is produced by a road crossing a village, this impact will be considered in a negative manner, when compared with a case where the road passes in a place where nobody lives in. In a more general way, it is therefore necessary to consider that the decision-aiding problems in an environmental domain must be dealt with by taking into account all the particular features of the site being analyzed.

##### 4.3.2. Second step

To prepare the experts for this systematic exam, the analyst wished to begin by making them reflect on three different cases of interaction between criteria (one with all the types of effects), which were judged (considering their knowledge on the concrete case) relevant for analysis. Indeed, it is important that the reader can understand in which purpose the three cases were proposed to begin the discussion with the experts.

A possible strengthening effect between criteria  $g_1$  (investment costs) and  $g_2$  (profitability). “Should not we consider that from the very moment a project  $a$  brings more benefits than a project  $a'$ , being project  $a$  less costly, the way these two criteria give their contribution to the credibility of the outranking  $aSa'$  must be more significant than the way we obtain by the simple addition of the two impacts related to each one of these criteria when only one validates the assertion ‘ $a$  is at least as good as  $a'$ ’?” The analyst justified the *raison d’être* of such a strengthening effect by emphasizing that a project of high cost will normally have a high profitability too. The analyst gave the following example: luxurious houses which are very expensive to construct (due to the high quality of materials, sophistication of the thermal and electrical installations, and so on) are normally sold at a very high price, but with a very high profitability too. This argument did not persuade the experts. It does not seem for them to be relevant and appropriate in the considered case, which is related to a project of public interest. In this type of projects, the very large expenses are accompanied, in general, by a rather low profitability (it is, for example, the case of the construction of a public park). It does not seem adequate to the experts, therefore, to take into account a form of

synergy (or mutual-strengthening) between the two criteria, investment costs ( $g_1$ ) and profitability ( $g_2$ ).

A possible weakening effect between criteria  $g_4$  (landscape ecology) and  $g_5$  (environmental effects). “Should not we consider that from the very moment a project  $a$  is at least as good as a project  $a'$ , on each one of the two criteria, the way these two criteria give their contribution to the credibility of the outranking  $aSa'$  must be less significant than the way we obtain by the simple addition of the two impacts related to each one of these criteria when only one validates the assertion ‘ $a$  is at least as good as  $a'$ ’?” The experts recognized that this weakening effect was worth being kept. Indeed, it seems to them that it is very probable that if a project is characterized by a good performance in terms of landscape ecology it will also have a good performance in terms of environmental effects. Consequently, the joint impact of these two criteria must be less than the sum of the impacts which they have when they intervene separately.

Possible antagonism of criterion  $g_5$  (environmental effects) over criterion  $g_2$  (profitability). “Should not we consider that from the very moment a project  $a$  is at least as good as a project  $a'$ , on criterion  $g_2$  (profitability), but  $a'$  is significantly preferred to  $a$  on criterion  $g_5$  (environmental effects), the way criterion  $g_2$  gives its contribution to the credibility of the outranking  $aSa'$  must be less significant than the way we obtain when criterion  $g_5$  does not validate the assertion ‘ $a'$  is significantly preferred to  $a$ ’?” The experts have considered that this antagonism was worth being kept. According to them, if a project (for instance,  $a_1$ , see Table 3) has a so good profitability when compared to another project (for instance,  $a_3$ , see again Table 3), while the respective performances of these two projects on criterion  $g_5$  (environmental effects) lead to make clear an opposition to the outranking of the second project by the first ( $a_1Sa_3$ ), the contribution of criterion  $g_2$  to the credibility of this outranking must really be less than the weight of this criterion,  $w_2$ . To justify this position, the experts made reference to real-world cases as well as to the scientific literature about studies on environmental impacts. Indeed, these studies showed that when a project has less environmental benefits than another one (as  $a_3$  with respect to  $a_1$ ), the benefits which come from the profitability are partly hidden by the least good environmental performance. The reduction on the weights of the criterion profitability, in the computation of the credibility of the considered outranking (for instance,  $a_1Sa_3$ ), appeared to them as an adequate way of taking into account the effect of which it has just been a matter.

The examination of the 12 other cases of possible interaction led the experts to keep a case of mutual-strengthening and a second case of antagonism, as illustrated in the following paragraphs.

Strengthening effect between criteria  $g_1$  (investment costs) and  $g_5$  (environmental effects). The experts justified this interaction effect as follows. A project which is characterized by weak environmental effects has all the chances to be also characterized by low intervention costs (investment costs). This leads to consider that a project where the investment costs are low, but that, however, has good environmental effects is worth being very well appreciated. This effect can be taken into account by assigning to criteria  $g_1$  and  $g_5$ , when they contribute conjointly to validate an outranking, an overall weight greater than the algebraic addition of the weights  $w_1$  and  $w_5$ , which they have when they intervene separately to validate this outranking.

Antagonism of criterion  $g_4$  (ecology) over criterion  $g_2$  (profitability). The arguments to keep this interaction effect are similar to the ones leading to keep the antagonism suggested by the analyst (antagonism of  $g_5$  against or over  $g_2$ ).

#### 4.3.3. Third step

Having identified the four cases of interaction, the analyst must now get the experts to work together about the way of taking into account these four cases. For such a purpose, the analyst must ask the experts to assign a numerical value to the interaction coefficients  $k_{ij}$  and  $k'_{ij}$  as they were defined in Subsection A.3.2. This was made

through a dialogue between the analyst and the experts as we shortly present in what follows.

- (a) “You have considered that it was necessary to take into account a strengthening effect between criteria  $g_1$  (investment costs, weight  $w_1 = 14.8$ ) and  $g_5$  (environmental effects, weight  $w_5 = 31.3$ ). This strengthening effect intervenes when both criteria  $g_1$  and  $g_5$  conjointly contribute to validate the assertion ‘ $a$  outranks  $a'$ ’. In order to take into account this strengthening effect, it is needed, under these conditions, to assign to the coalition of both criteria (investment costs, environmental effects) a weight greater than the sum  $w_1 + w_5 = 14.8 + 31.3 = 46.1$ . Under these conditions, according to your opinion, what is the value which it is necessary to assign to the weight of this coalition?” The experts felt difficulties to answer this question. They understood perfectly the sense of the question, but they did not know how to formulate a meaningful numerical answer. For this reason, they asked if the analyst could provide an interval (a minimum and a maximum value) in which they should place the asked value. First of all, the analyst pointed out that, in the case of a strengthening effect considered extremely weak (in other words, negligible), the minimum value would be 46.1. Secondly, the analyst suggested that in the case of a strengthening effect judged extremely strong, the maximum could be the double, that is, 92.2. On these foundations, the experts agreed to assign a weight of 60 to the coalition of both criteria  $g_1$  and  $g_5$ , when they intervene conjointly. It follows that the value of the strengthening coefficient  $k_{15}$  is:  $60 - 46.1 = 13.9$ , rounded up to 14.
- (b) “You have considered that it was necessary to take into account a weakening effect between criteria  $g_4$  (ecology, weight  $w_4 = 18.5$ ) and  $g_5$  (environmental effects, weight  $w_5 = 31.3$ ). This weakening effect intervenes when both criteria  $g_4$  and  $g_5$  jointly contribute to validate the assertion ‘ $a$  outranks  $a'$ ’. In order to take into account this effect, it is needed, under these conditions, to assign to the coalition of both criteria (ecology and environmental effects) a weight lower than the sum  $w_4 + w_5 = 18.5 + 31.3 = 49.8$ . Under these conditions, according to your opinion, what is the value which it is necessary to assign to the weight of this coalition?” Again, the experts asked the analyst to propose them an interval in which they should place the asked value. First of all, the analyst pointed out that a weakening effect could at most lead to consider that only the most important criterion of the coalition can contribute to the credibility of the outranking (the other criterion bringing no additional information). It follows that an adequate minimum value is 31.3 (maximum weakening effect). Secondly, in the case of a weakening effect considered extremely weak (in other words, considered negligible), the weight of the coalition could remain, equal to  $w_4 + w_5 = 49.8$ . On these foundations, the experts agreed to assign a weight slightly greater than 40 (40.8) to the coalition of both criteria,  $g_4$  and  $g_5$ , when they intervene conjointly. It follows that the value of weakening coefficient  $k_{45}$  is  $40.8 - 49.8 = -9$ .
- (c) “You have considered that it was necessary to take into account an antagonism of criterion  $g_4$  (ecology) over criterion  $g_2$  (profitability, weight  $w_2 = 25.9$ ). This antagonism intervenes when a project ‘ $a$  is at least as good as a project  $a'$ ’ on criterion  $g_2$ , while  $a'$  is significantly preferred to  $a$  on criterion  $g_4$ . In order to take into account this antagonism, it is needed, under these conditions, to assign to the criterion  $g_2$  (profitability) a weight lower than or equal to  $w_2 = 25.9$ . Under these conditions, according to your opinion, what is the value which it is necessary to assign to the weight of this criterion?” The analyst still offered an interval. Again, he firstly started to point out that if the antagonism is extremely weak (in other words, negligible), an adequate

**Table 9**

Interaction coefficients (this table contains all necessary unambiguously information for interactions: the absence of figures characterizes the absence of interactions; the presence of a figure repeated in a symmetrical manner with respect to the main diagonal characterizes a strengthening effect if the figure is positive and a weakening effect if the figure is negative; the presence of a figure appearing only above or under the main diagonal, in other words not repeated in a symmetrical manner, characterizes an antagonism).

	Investment ( $g_1$ )	Profitability ( $g_2$ )	Services ( $g_3$ )	Landscape ( $g_4$ )	Environment ( $g_5$ )	Consistency ( $g_6$ )
Investment ( $g_1$ )					$k_{15} = 14$	
Profitability ( $g_2$ )				$k'_{24} = 6$	$k'_{25} = 6$	
Services ( $g_3$ )						
Landscape ( $g_4$ )					$k_{45} = -9$	
Environment ( $g_5$ )	$k_{51} = 14$			$k_{54} = -9$		
Consistency ( $g_6$ )						

maximum value is  $w_2 = 25.9$ . Secondly, the analyst suggested that in the case of an antagonism judged extremely strong, the minimum value could be half of the weight  $w_2$ , that is to say, 13. The experts had little more difficulties here than in both of the precedent cases to agree about a value. Finally, they decided to assign the value 20. It follows that the value of the antagonism coefficient  $k'_{24}$  is  $25.9 - 20 = 5.9$ , rounded up to 6.

- (d) The experts considered that the antagonism of criterion  $g_5$  (environmental effects) over criterion  $g_2$  (profitability) was of the same nature and also of the same importance as the antagonism of  $g_4$  (ecology) over criterion  $g_2$ . It led to put  $k'_{25} = 6$ .

Table 9 sums up the results gathered in this third step.

The analyst checked if the positive net balance condition (now with the normalized weights and interaction coefficients) was fulfilled for the common set of weights. It is related to the criteria  $g_2$ ,  $g_4$  and  $g_5$ :

$$w_2 - k'_{24} - k'_{25} = 25.9 - 6 - 6 = 13.9 > 0$$

$$w_4 + k_{45} = 18.5 - 9 = 9.5 > 0$$

$$w_5 + k_{54} = 31.3 - 9 = 22.3 > 0$$

At the end of the meeting, the analyst discussed with the experts about the difficulties they felt when assigning numerical values to the interaction coefficients. The analyst asked the experts the following question: "It would have been easier for you, rather than to answer by assigning a numerical value, to make the evaluation in qualitative terms by appreciating the level of interaction on a semantic scale, such as the following one: (*negligible, weak, medium, strong, extremely strong*)?". Without hesi-

tancy the experts answered in the affirmative. If they like to adopt this mode of questioning, it is naturally necessary to define rules intended to assign numerical values to each one of the levels of interaction characterized in a semantic way. The precedent considerations showed that the rules related to the extreme levels (negligible, extremely strong) depend on the type of the considered interaction effect. Once these minimums and maxima were fixed, we could associate the medium level with the middle of the interval defined by the extreme values, the weak level with the quarter, and the strong level with the three quarters. The resulting values should act then as a basis for a debate with the experts to fix the final numerical value, which it is necessary to adopt.

Despite the difficulties found to assign numerical values to the interaction coefficients, the experts finally agreed to recognize that the way the interactions are taken into account in ELECTRE III was natural and easy to understand. They also understood how the numerical

values they had to assign to the interaction coefficients, were used to change the relative importance of the criteria affected by the interaction effects. This seems us to be very positive conclusions in favor of the method within the context of this concrete application. We do not think that similar conclusions would be obtained with methods based on the Choquet integral. The way in which interactions operate in these methods, through the definition of capacities, seems to us to be much more opaque and, as a result, more difficult to be understandable to the members of a focus group. Finally, let us point out that the antagonistic effect, that turned out to be very appropriate to the land-use planning application considered here, cannot be taken into account with Choquet integral based methods.

## 5. Sensitivity analysis and robustness concerns

The work with the focus group allows to define a consensual set of weights (*cf.* Table 8, for  $Z = 16$ ) as well as a set of values for the inter-action coefficients (*cf.* Table 9). The values of the parameters, which appear in these two tables, determine the role of the different criteria the members of the focus group used for ranking the five projects under analysis. These values were decided following arbitrage and hesitations in order to remove ambiguities which must be taken into account to obtain robust conclusions (*cf.* Section 6). The biggest difficulties which were felt by the members of the focus group concerned the assignment of values to the interaction coefficients. This is why the analyst was interested in an extremely vast set of possible values for such coefficients in order to examine the impact the choice of such a set of values could have on the ranking under analysis. This examination was conducted, in a first phase, with the common set of weights (*cf.* Subsection 5.1, below). Then, the analyst tried to verify if the obtained results remained valid with some sets of weights "close" to the common set of weights (*cf.* Subsection 5.2, below). Finally, the analyst performed the test with a set of rather different weights (*cf.* Subsection 5.3, below). The computational results and experiments presented in this section were performed with a new Q-BASIC implementation<sup>1</sup> of ELECTRE III with interactions between criteria.

### 5.1. Results with the common set of weights: analysis of the interaction effects

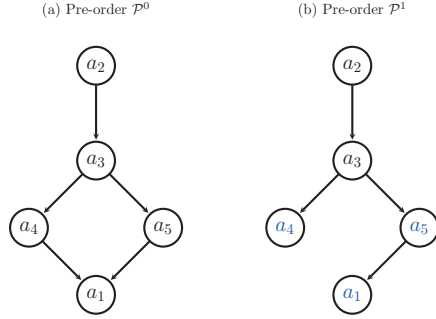
With the common set of weights and in the absence of any interaction between criteria, the application of ELECTRE III leads to the partial pre-order  $\mathcal{P}^0$  (see Fig. 1a). When taking into account the inter-

<sup>1</sup> For more details about this software, please ask Salvatore Greco (salgreco@unict.it).



**Table 10**  
Ranking criteria and different dispositions of the blank cards for the group of experts.

Dispositions	$r_1$	$n_1$	$r_2$	$n_2$	$r_3$	$n_3$	$r_4$	$n_4$	$r_5$	$n_5$	$r_6$	$N$
1	$g_6$	2	$g_3$	2	$g_1$	2	$g_4$	2	$g_2$	2	$g_5$	10
2	$g_6$	3	$g_3$	2	$g_1$	1	$g_4$	2	$g_2$	3	$g_5$	11
3	$g_6$	3	$g_3$	1	$g_1$	3	$g_4$	1	$g_2$	3	$g_5$	11
4	$g_6$	1	$g_3$	3	$g_1$	3	$g_4$	3	$g_2$	1	$g_5$	11



**Fig. 1.** The two obtained pre-orders (rankings).

action effects with their original values (cf. Table 9) the application of ELECTRE III leads to the partial pre-order  $\mathcal{P}^1$  (see Fig. 1b), which only differs from  $\mathcal{P}^0$  for the fact that project  $a_4$  is not any more ranked before project  $a_1$ , but it becomes incomparable to the latter.

In a preliminary analysis, the analyst tried to know the partial pre-order to which the application of ELECTRE III leads when only one of the four interaction effects is taken into account. This was done by making the coefficient varying within a range as wide as possible, by taking into account its meaningfulness. Thus, four cases were successively studied.

*Case 1: Varying  $k_{15}$  within the range  $[0, 45]$  (values bigger than 45 are considered by the analyst completely unrealistic).*

*Case 2: Varying  $k_{45}$  within the range  $[-15, 0]$  (the positive net balance condition allows to go until  $-18.5$ , but this value was judged by analyst not very realistic since it leads to exclude criterion  $g_4$ ; this is why the analyst did not considered useful to include values strictly lower than  $-15$ ).*

*Case 3: Varying  $k'_{24}$  within the range  $[0, 20]$  (the net positive balance condition allows to go until 25.9, but with this value the antagonistic effect cancels absolutely the role of criterion  $g_2$ . The analyst has considered 6 as the minimum weight necessary to keep for criterion  $g_2$  and consequently the limit was 20. This is why 5.9 is the minimum value of the weight, which the analyst considered useful to keep for further analysis).*

*Case 4: Varying  $k'_{25}$  within the range  $[0, 20]$  (the justification is the same as in the previous Case 3).*

In Cases 1, 3, and 4, that is to say, when only, either the mutual-strengthening effect or one of the two antagonistic effects is taken into account, the result is  $\mathcal{P}^0$  (see Fig. 1a); this occurs for whatever the value of the interaction coefficient within the range under analysis. It highlights that, with the considered set of weights, none of the interaction effects taken separately has an impact on the resulting pre-order.

In Case 2, that is to say, when the mutual-weakening effect is taken into account separately, we find:

- $\mathcal{P}^0$ , if the value assigned to the interaction coefficient remains very weak:  $|k_{45}| \leq 1.333$ .
- $\mathcal{P}^1$ , for whatever the value of  $k_{45}$  within the range  $[-15, -1.334]$  (this is the same as in the presence of all interaction effects, with the original values of the interaction coefficients, especially with  $k_{45} = -9$ ).

**Table 11**  
Normalized weights for the group of experts with different dispositions of blank cards and  $Z = 16$ .

Dispositions	$w_1$	$w_2$	$w_3$	$w_4$	$w_5$	$w_6$
1	13.7	25.5	7.8	19.6	31.4	2.0
2	14.8	24.0	9.3	18.5	31.4	2.0
3	13.0	24.0	9.3	20.3	31.4	2.0
4	13.0	27.7	5.6	20.3	31.4	2.0

We shall see in Subsection 5.2 the reasons that justify the presence of a critical threshold which leads to switch from  $\mathcal{P}^0$  to  $\mathcal{P}^1$ .

The analyst then wished to see what would happen, on the left and on the right of the critical value  $-1.333$ , when the three other interaction effects were active, according to the ranges defined in Cases 1, 3, and 4. The analyst verified that the final result is always  $\mathcal{P}^0$  if  $|k_{45}| \leq 1.333$  and  $\mathcal{P}^1$  if  $|k_{45}| \geq 1.334$ .

## 5.2. Analysis with some sets of weights "close" to the common set of weights

As it has been highlighted in Subsection 4.2, after explaining the meaning of each criterion, it was comparatively easy to reach a general consensus about the ranking of criteria by an increasing order of relative importance (cf. Table 7). The number of blank cards in each of the inter-criteria spaces previously defined could have led to numbers of blank cards slightly different from those showed in Table 7. That is why the analyst has chosen four other possible dispositions for the blank cards, strongly contrasting with each other, as shown in Table 10. This choice has been done in order to see if with the set of weights obtained from SRF for each of the new dispositions of blank cards the final results would have been different (see Table 11). Therefore, the analyst followed again the same procedure as the one introduced in Subsection 5.1 by successively replacing the common set of weights by each one of the four new sets of weights presented in Table 11.

In the absence of any interaction, we find the pre-order  $\mathcal{P}^0$  for each of the four considered sets of weights (as in the case of the common set of weights). When the initial values of the interaction coefficients are kept, especially  $k_{45} = -9$ , we find (as in the case of the common set of weights) the pre-order  $\mathcal{P}^1$  with the sets of weights from dispositions 1 and 2 (cf. Table 11). With the sets of weights from dispositions 3 and 4, the resulting pre-order is not any more  $\mathcal{P}^1$  but  $\mathcal{P}^0$ , this shows that the mutual-weakening effect is of no impact with these two new sets of weights. The study of Cases 1, 2, 3, and 4 defined in Subsection 5.1 will provide the explanation of this modification.

In Cases 1, 3, and 4, defined, in Subsection 5.1, the obtained results with the four new sets of weights are identical to those obtained with the common set of weights. Moreover, in Case 2 (which takes into account separately the mutual-weakening effect) the critical value, which allows to switch from  $\mathcal{P}^0$  to  $\mathcal{P}^1$ , is  $-1.333$  only when considering the set of weights of Disposition 2: in such a set of weights  $w_4$  has the same value (18.5) as in the common set of weights. With the sets of weights from Disposition 1,  $w_4$  becomes equal to 19.6 and the critical threshold gets the value  $-8.66$ ; this value characterizes a smaller weakening effect than the one that was characterized by  $-9$ . That is why with this new set of weights we still obtain pre-order

$\mathcal{P}^1$  when we take into account all the interaction effects with the initial values of the interaction coefficients, especially  $k_{45} = -9$ . On the contrary, with both sets of weights from Dispositions 3 and 4,  $w_4$  becomes equal to 20.3 and we observe that the critical threshold is at  $-13.33$ ; a value which characterizes a stronger weakening effect than that characterized by  $-9$ . This is why with these two sets of weights we find pre-order  $\mathcal{P}^0$  (and not  $\mathcal{P}^1$ ) when all interaction effects intervene with the initial values of the interaction coefficients, especially  $k_{45} = -9$ .

The previous considerations highlight the coherence of the obtained results. In particular, they make appear the following phenomenon: the higher  $w_4$  the bigger  $|k_{45}|$  must be, so that the mutual-weakening effect leads to rank no more project  $a_4$  in a better position than project  $a_1$ , but shows instead incomparability between these two projects. Such a phenomenon requires an explanation.

This explanation comes from the fact that the mutual-weakening effect affects the way projects  $a_4$  and  $a_1$  compare to each other (since  $a_4$  is strictly preferred to  $a_1$  according to both criteria  $g_4$  and  $g_5$ ). We noticed that in the absence of a mutual-weakening effect ( $k_{45} = 0$ )  $a_4$  is ranked in a better position than  $a_1$ . It is not surprising that this ranking disappears to give place to an incomparability when the mutual-weakening effect becomes strong enough to reduce in a significant manner the power criteria  $g_4$  and  $g_5$  have to validate the outranking of  $a_1$  by  $a_4$ . This explains the presence of a critical threshold with a value of  $k_{45}$ . We must expect that this critical threshold becomes closer to 0 as  $w_4$  decreases, since the credibility of the outranking of project  $a_1$  by  $a_4$  increases when  $w_4$  increases. It is really what we have observed and what it was a matter of an explanation.

### 5.3. Analysis with the set of weights of expert $E_1$

The analyst wished to confront the previous results with those that would have been obtained if the set of weights finally kept had been that one of the experts in economic evaluation,  $E_1$ . Indeed, it was the expert  $E_1$  (as we saw in Subsection 4.2) who contributed to dissipate the poor interpretation of the respective role which it was necessary to attribute to criteria  $g_1$  and  $g_5$ . Nevertheless, in the *consensus* resulting from her intervention, it was assigned to criterion  $g_4$  (landscape ecology) a bigger relative importance than that assigned to criteria  $g_3$  (new services) and  $g_1$  (investment costs), while initially, for the expert  $E_1$ , it was the opposite situation. Moreover, this expert put very few blank cards between the criteria ranks and assigned the value 7 to the Z-ratio. The resulting set of weights from  $E_1$  (cf. Table 6) is much more narrowed and rather different from those considered before (e.g.,  $w_4 = 13$  while previously was  $w_4 \geq 18.5$ ).

The analyst, therefore, performed with this new set of weights the same type of computations as those performed with the sets of weights taken in consideration in the two previous subsections. The obtained results are identical, with two very little exceptions. It is worthwhile to present these two exceptions here, even if they will not affect the general conclusions presented in Section 6. It is, finally, also an agreement aspect, which deserves an explanation.

#### (a) On the mutual-strengthening effect

With all the sets of weights studied in Subsections 5.1 and 5.2, the result is always  $\mathcal{P}^0$  for all the values of  $k_{15}$  within the range  $[0, 45]$ . The mutual-weakening effect thus does not have any impact. With the sets of weights of expert  $E_1$ , the result becomes  $\mathcal{P}^1$  as soon as  $k_{15}$  exceeds 7.99. In other words, from the value 8 and beyond, the result is not any more  $\mathcal{P}^0$  but  $\mathcal{P}^1$ , which means that it is not any more justified to have  $a_4$  better ranked than  $a_1$  since these two projects become incomparable. The reason is the following. First of all, let us point out that the mutual-strengthening effect has no direct impact on the way projects  $a_4$  and  $a_1$  must be compared, since  $a_4$  is strictly preferred to  $a_1$  according to criterion  $g_5$  while it is the opposite with respect to criterion  $g_1$ . There is, therefore, an indirect effect, making

an influence on the way  $a_4$  compares itself with other projects, that explains the fact that  $a_4$  does not rank in a better position than  $a_1$ , when the value of  $k_{15}$  exceeds a critical threshold.

For an explanation of this result, let us firstly highlight that in the ranking provided by the descending distillation,  $a_4$  is always placed in a strictly better position than  $a_1$ , while in the ranking provided by the ascending distillation, it depends on the considered case, either projects  $a_1$  and  $a_4$  are in the same position, or  $a_1$  is in a better position than  $a_4$ . It is this second situation which leads to the incomparability between  $a_1$  and  $a_4$ . We saw that this incomparability appeared only with the set of weights of expert  $E_1$  (in which  $w_4$  is weaker than in the common set of weights) and with a strong enough mutual-strengthening effect:  $k_{15} \geq 0.8$ . This is due to the fact that, in the latter case,  $a_4$  is significantly outranked by two other projects,  $a_2$  and  $a_3$ , while  $a_1$  is only outranked by one,  $a_2$ . In all the other cases, projects  $a_1$  and  $a_4$  are outranked by another single project:  $a_2$  for  $a_1$  and according to the considered case,  $a_2$  or  $a_3$  for  $a_4$ .

#### (b) On the mutual-weakening effect

With the considered sets of weights, the mutual-weakening effect has no impact: either when this effect intervenes separately or when other interaction effects are considered conjointly, the result is still  $\mathcal{P}^0$  and never  $\mathcal{P}^1$ , contrary to the fact that we had observed in Subsection 5.2. This change has nothing of surprising. With the sets of weights previously considered, the critical value of  $|k_{45}|$  which when exceeded leads to  $\mathcal{P}^1$ , it was all the more weak than the weight of criterion  $g_4$  was itself weak. With  $w_4 = 18.5$  this critical value was at 1.333. In the set of weights considered here, we have  $w_4 = 13$ . This explains the fact we observe no critical value at all.

#### (c) On the antagonistic effect

In all the studied cases (including those in Subsection 5.3) this effect has no impact. We will explain why this effect has no impact, especially in the case of expert  $E_1$ , but the argument is the same for all the cases. To do so it is necessary to identify all the ordered pairs  $(a, a')$  where at least one of the two antagonistic effects could have an impact. It is thus the case if and only if " $a$  outranks  $a'$ " on criterion  $g_2$  and " $a'$  is strictly preferred to  $a$ " either on criterion  $g_4$  or on criterion  $g_5$ . The analysis of Table 3 leads to the following ordered pairs:

- Antagonism due to criterion  $g_4$ :  $(a_1, a_2), (a_1, a_3), (a_1, a_4), (a_5, a_2), (a_5, a_3), (a_5, a_4)$ .
- Antagonism due to criterion  $g_5$ :  $(a_1, a_3), (a_1, a_4), (a_2, a_3), (a_5, a_2), (a_5, a_3), (a_5, a_4)$ .

For each of these ordered pairs, the credibility of the outranking of  $a'$  by  $a$  takes into account the weight of criterion  $g_2$ . Indeed, the effect of an antagonism contributes to diminish this weight. When in  $\mathcal{P}^0$ , as well as in  $\mathcal{P}^1$ ,  $a'$  is ranked in a better position than  $a$ , this reduction of the weight of  $g_2$  (and consequently of the antagonistic effects) cannot produce an impact. We still have to explain why the antagonistic effect has no impact when considering the other three ordered pairs, i.e.,  $(a_1, a_4), (a_5, a_4)$ , and  $(a_2, a_3)$ .

1. *Ordered pair  $(a_1, a_4)$* : The incomparability between these two projects with the set of weights of the expert  $E_1$  does not come from a direct comparison of  $a_1$  against  $a_4$  (the outranking credibility is too weak); it comes instead from indirect effects which take into consideration the way  $a_4$  compares itself with other projects. There is, therefore, no reason why the reduction of the credibility degree of the outranking of  $a_4$  by  $a_1$ , following from an antagonistic effect, can lead to a ranking with  $a_1$  in a worst position than  $a_4$  (the only effect which could have an antagonism due to criterion  $g_4$ ).
2. *Ordered pair  $(a_5, a_4)$* : In this case an indirect effect influences the way  $a_4$  compares itself with other projects and explains the fact that  $a_4$  is incomparable to  $a_5$ , for whatever the considered set of

weights. There is, therefore, no reason why the reduction of the credibility degree of the outranking of  $a_4$  by  $a_5$ , following from one any of the two antagonistic effects, can lead to a ranking with  $a_5$  in a worst position than  $a_4$ .

3. *Ordered pair* ( $a_2, a_3$ ): Here it is necessary to explain why the antagonistic effect which comes only from  $g_5$  remains compatible with a ranking where  $a_2$  is in a better position than  $a_3$ , even with a maximum antagonistic coefficient  $k'_{25} = 20$ . The value of the credibility degree of the outranking of  $a_3$  by  $a_2$  is equal to:
  - (for the common set of weight): 0.687 in the absence of antagonism and 0.609 with a maximum antagonistic effect.
  - (for the set of weights of expert  $E_1$ ): 0.745 in the absence of antagonism and 0.681 with maximum antagonistic effect.

These reductions of the credibility degree of the outranking  $a_3$  by  $a_2$  were not sufficient to change the way  $a_2$  and  $a_3$  are ranked.

## 6. Conclusions

The results presented in Section 5 lead to the following conclusions:

- (a) In the real case considered in this article the analysis of the results allows us to formulate the following robust conclusions (this term having the sense defined in Roy 2010a; 2010b):
  1. ELECTRE III with interactions between criteria leads to rank  $a_2$  in a better position than  $a_3$ , and these two projects in better positions than the remaining three others; this is valid for whatever the considered sets of weights and interaction coefficients. In the same conditions  $a_5$  is ranked in a better position than  $a_1$ , and  $a_4$  is incomparable to  $a_5$ .
  2. Project  $a_4$  is in general ranked in a better position than  $a_1$  by ELECTRE III with interaction between criteria, except when the values for the interaction coefficients  $k_{15}$  (mutual-strengthening effect) and/or  $k_{45}$  (mutual-weakening effect) exceed a certain critical threshold; in these conditions  $a_4$  becomes incomparable to  $a_1$ .
- (b) These conclusions were obtained following an interactive approach requiring the intervention and interaction with the members of a focus group, that worked together to assign the values of the first sets of weights and interaction coefficients to be introduced in ELECTRE III with interactions for producing the first result. The approach followed to assign such values was easily understood and accepted by the members of the focus group. Uncertainties and ill-determinations which resulted from this approach could be taken into consideration by the method (in particular by using sensitivity analysis) so as to obtain the above introduced conclusions.
- (c) The results of the previous sections have been introduced to the members of the focus. Their reactions were the following. The first important observation that they made concerned the obtained results. In particular, all the participants confirmed that the two best performing alternatives are coherent with their expectations. A second observation concerned the result of the sensitivity analysis with particular reference to the interaction coefficients. All the participants agreed on the importance of taking into account such interaction effects for environmental decision making processes but suggested that further research should be carried out in order to develop a user friendly proto-col for the elicitation of the coefficients.
- (d) The study of this real case allowed us to test the ELECTRE III method with interactions between criteria to support a public decision related with territorial planning processes. The way the whole work has been developed and conducted, the nature of the obtained results, as well as the way the results were accepted constitutes, in our opinion, a validation of this method for helping to make better decisions in this type of contexts.

## Appendix A. Theoretical and methodological background

This appendix is devoted to present the fundamentals of ELECTRE III with interaction between criteria. We shall avoid to present some aspects of ELECTRE III, as for instance, the direct and inverse variable thresholds (Roy et al., 2014), and the details of the distillation procedures. For a more complete description of this particular method the reader can refer to Roy and Bouyssou (1993). The family of ELECTRE methods was designed into two main phases. The first one consists of the construction of one or more outranking relations, while the second is related to the exploitation of these relations (Figueira, Greco, Roy, & Słowiński, 2013; Figueira, Mousseau, & Roy, 2005; Roy, 1985; 1991; 1996). One of the crucial steps of the methodology applied the present work is described in Figueira et al. (2009).

### A.1. Basic data

In what follows  $A$  denotes a set of *potential actions* or projects, as in our case study. In our settings, each action,  $a \in A$ , is defined by a brief label, corresponding to an extensive description. In such a case,  $A$  can be defined as follows,  $A = \{a_1, \dots, a_i, \dots, a_m\}$ . Let  $g$  denote a given *criterion*, built for characterizing and comparing potential actions according to a considered point of view. The characterization of an action  $a \in A$ , denoted by  $g(a)$ , usually represents the *performance* of action  $a$  according to the considered criterion. Let  $F = \{g_1, \dots, g_j, \dots, g_n\}$  denote a *coherent family of criteria* (Roy, 1985; 1996). The sets  $A$  and  $F$  contain our basic data. In what follows we shall also use  $F$  as the set of criteria subscripts.

### A.2. Preference modeling through a pseudo-criterion model

Thresholds are built to take into account the imperfect character of the data from the computation of the performances  $g_j(a)$ , for all  $a \in A$  and  $g_j \in F$ , as well as the arbitrariness that affects the definition of the criteria.

**Definition 1** (Preference threshold). The preference threshold between two performances, denoted by  $p$ , is the smallest performance difference that when exceeded is judged significant of a strict preference in favor of the action having the best performance.

**Definition 2** (Indifference threshold). The indifference threshold between two performances, denoted by  $q$ , is the largest performance difference that is judged compatible with an indifference situation between two actions having different performances.

The definition of the thresholds allows to define a non-classical model for taking into account the decision-makers preferences.

**Definition 3** (Pseudo-criterion with constant thresholds). A criterion  $g_j$  is called a *pseudo-criterion* when two thresholds are associated with  $g_j$ : the *indifference threshold*,  $q_j$ , and the *preference threshold*,  $p_j$ , such that  $p_j \geq q_j \geq 0$ .

From the above definitions, the following binary relations can be derived, for each criterion and considering two actions  $a$  and  $a'$ , where  $g_j(a) \geq g_j(a')$ , for a given criterion  $g_j$  to be maximized.

1.  $|g_j(a) - g_j(a')| \leq q_j$  represents a non-significant advantage of one of the two actions over the other, meaning that  $a$  is *indifferent* to  $a'$  according to  $g_j$ , denoted  $I_{g_j} a'$ .
2.  $g_j(a) - g_j(a') > p_j$  represents a significant advantage of  $a$  over  $a'$ , meaning that  $a$  is *strictly preferred* to  $a'$  according to  $g_j$ , denoted  $a P_{g_j} a'$ .
3.  $q_j < g_j(a) - g_j(a') \leq p_j$  represents an ambiguity zone. The advantage of  $a$  over  $a'$  is a little large to conclude about an indifference between  $a$  and  $a'$ , but this advantage is not enough to conclude about a strict preference in favor of  $a$ . This means that there is a

hesitation between indifference and strict preference. In such a case,  $a$  is *weakly preferred* to  $a'$ , denoted  $aQ_2a'$ .

The following notation about coalitions of criteria will be needed in the remaining of this paper. Let

- $C(ala')$  denote the subset of criteria such that  $aI_ja'$ ;
- $C(aQa')$  denote the subset of criteria such that  $aQ_ja'$ ;
- $C(aPa')$  denote the subset of criteria such that  $aP_ja'$ ;
- $\bar{C}(a'Pa)$  denote the complements of  $C(aPa')$ .

### A.3. Building an outranking relation

Three concepts are needed to the construction of an outranking relation, namely, concordance, non-discordance, and a degree of credibility. These three concepts will be reviewed in this subsection. The extension of the comprehensive concordance index to incorporate three types of interactions between criteria will be presented in this subsection too.

#### A.3.1. Concordance, discordance, and credibility

The following three paragraphs will deal with the three main concepts, needed for the construction of a fuzzy outranking relation (Roy, 1991).

**Concordance index.** For using ELECTRE III it is necessary to associate a set of intrinsic weights with the family of criteria. This set of weights, each one denoted by  $w_j$ , is such that  $w_j > 0$ , for  $j = 1, \dots, n$ , and  $\sum_{j=1}^n w_j = 1$  (assumption). The overall concordance with the assertion of “ $a$  outranks  $a'$ ” is modeled through a *comprehensive concordance index*, denoted  $c(a, a')$ , and defined as follows:

$$c(a, a') = \sum_{j \in C(aPa')} w_j + \sum_{j \in C(aQa')} w_j + \sum_{j \in C(ala')} w_j + \sum_{j \in C(a'Qa)} w_j \varphi_j, \quad (\text{A.1})$$

where

$$\varphi_j = \frac{p_j - (g_j(a') - g_j(a))}{p_j - q_j} \in [0, 1[. \quad (\text{A.2})$$

Let us recall that  $c(a, a')$  (roughly meaning a degree of outranking of  $a$  over  $a'$ ) takes into account the weights of criteria which contribute to validate the assertion, “ $a$  is at least as good as  $a'$ ” denoted by  $aSa'$ . Every criterion leading to  $aPa'$ ,  $aQa'$ , and  $ala'$  is taken into account with its overall weight. It is obvious that a criterion leading to  $a'Pa$  must not be taken into account for validating such an assertion. On the contrary, a criterion leading to  $a'Qa$  must not be completely discarded with respect to its contribution to the assertion  $aSa'$ . This weak preference situation represents a hesitation between  $a'la$  and  $a'Pa$ . The criterion is thus taken into account by a fraction,  $\varphi$ , of its weight. This fraction can be interpreted as the proportion of voters (the weight corresponds to the voting power of the criterion) in favor of the assertion  $aSa'$ . This proportion should be as close as possible to 1 when the hesitation is more in favor of the indifference. It should be zero when we reach the strict preference situation in favor of  $a'$ .

There is a difference that should be pointed out when scales are continuous or when they are discrete (Roy et al., 2014) (for the sake of the simplicity consider the criterion  $g$  and the same two actions  $a$  and  $a'$ ):

1. A *continuous scale* leads to the following formula:

$$\varphi = \frac{p - (g(a') - g(a))}{p - q}, \quad \text{with } q < g(a') - g(a) \leq p, \quad (\text{A.3})$$

for  $p \neq q$ .

This relation leads effectively to:

- (a)  $\varphi = 1$  iff  $g(a') = g(a) + q$ : the only situation that validates  $a'la$  without hesitation.
- (b)  $\varphi = 0$  iff  $g(a') = g(a) + p$ : situation that, due to the continuous nature of the scale, only leads to the absence of the hesitation between  $a'la$  and  $a'Pa$ ; the latter imposes thus its power.

2. When in presence of a *discrete scale* the formula becomes as follows:

$$\varphi = \frac{(p+1) - (g(a') - g(a))}{(p+1) - q}, \quad (\text{A.4})$$

with  $q \leq g(a') - g(a) \leq p$ , for  $p \neq q$ .

It means that in this case we can keep the previous formula (3) by replacing  $p$  by  $(p+1)$ . Let us observe that this formula is still valid when  $p = q$ , which corresponds to a situation of absence of weak preference. When  $p = q + 1$ , which corresponds to a unique situation of real hesitation ( $g(a') = g(a) + p$ ), this formula leads to  $\varphi = 1/2$  (which seems a very adequate value). Similarly, if  $p = q + 2$ , each one of the two hesitation situations leads to  $\varphi = 2/3$  and  $\varphi = 1/3$ , respectively.

**Discordance index.** ELECTRE III gives the possibility to introduce a *veto power* to certain criteria by associating with each one of these criteria a *veto threshold*, denoted  $v_j$ , such that  $v_j \geq p_j$ . The *discordance index* is used to take into account such a veto power. The veto power of each criterion is modeled through a *partial discordance index*, denoted by  $d_j(a, a')$ ,  $j = 1, \dots, n$ , and defined as follows:

$$d_j(a, a') = \begin{cases} 1 & \text{if } g_j(a) - g_j(a') < -v_j, \\ \frac{g_j(a) - g_j(a') + p_j}{p_j - v_j} & \text{if } -v_j \leq g_j(a) - g_j(a') < -p_j, \\ 0 & \text{if } g_j(a) - g_j(a') \geq -p_j. \end{cases} \quad (\text{A.5})$$

**Credibility index.** The credibility index is defined as follows:

$$\sigma(a, a') = c(a, a') \prod_{j=1}^n T_j(a, a'), \quad (\text{A.6})$$

where

$$T_j(a, a') = \begin{cases} \frac{1 - d_j(a, a')}{1 - c(a, a')} & \text{if } d_j(a, a') > c(a, a'), \\ 1 & \text{otherwise.} \end{cases} \quad (\text{A.7})$$

This index reflects the way the assertion “ $a$  outranks  $a'$ ” is more or less well justified or founded when taking into account all the criteria from  $F$ .

#### A.3.2. Interactions between criteria

This subsection provides the definitions of the three interaction types as they were defined in Figueira et al. (2009).

- (a) *Mutual-strengthening effect between criteria  $g_j$  and  $g_i$* :

**Definition 4** (Mutual-strengthening effect). If criteria  $g_j$  and  $g_i$  both strongly, or even weakly, support the assertion  $aSa'$  (more precisely,  $g_j, g_i \in \bar{C}(a'Pa)$ ), we consider that their contribution to the concordance index must be larger than the sum of  $w_j + w_i$ , because these two weights represent the contribution of each of the two criteria to the concordance index when the other criterion does not support  $aSa'$ .

We suppose that the effect of the combined presence of  $g_j, g_i \in \bar{C}(a'Pa)$  among the criteria supporting the assertion  $aSa'$  can be modeled by a mutual-strengthening coefficient,  $k_{ji} > 0$ , which intervenes algebraically in  $c(a, a')$ .

- (b) *Mutual-weakening effect between criteria  $g_j$  and  $g_i$* :

**Definition 5** (Mutual-weakening effect). If criteria  $g_j$  and  $g_i$  both strongly, or even weakly, support the assertion  $aSa'$  (more precisely,  $g_j, g_i \in \bar{C}(a'Pa)$ ), we consider that their contribution to the

concordance index must be smaller than the sum of  $w_j + w_i$ , because these two weights represent the contribution of each of the two criteria to the concordance index when the other criterion does not support  $aSa'$ .

We suppose that the effect of the combined presence of  $g_j, g_i \in \bar{C}(a'Pa)$  among the criteria supporting the assertion  $aSa'$  can be modeled using a mutual-weakening coefficient,  $k_{ji} < 0$ , which intervenes algebraically in  $c(a, a')$ , such that  $w_j + k_{ji} > 0$  and  $w_i + k_{ji} > 0$ .

(c) *Antagonism of criterion  $g_h$  over criterion  $g_j$ :*

**Definition 6** (Antagonistic effect). If criterion  $g_j$  strongly, or weakly, supports the assertion  $aSa'$  and criterion  $g_h$  strongly opposes this assertion, we consider that the contribution of the criterion  $g_j$  to the concordance index must be smaller than the weight  $w_j$  that was considered in cases in which  $g_h$  does not belong to  $\bar{C}(a'Pa)$ .

We suppose that this effect can be modeled by introducing an antagonism coefficient  $k'_{jh} > 0$ , which intervenes negatively in  $c(a, a')$ , such that  $w_j - k'_{jh} > 0$ .

**Remark 1.** Let us notice that,

- Cases  $a$  and  $b$  are mutually exclusive, but cases  $a$  and  $c$  and cases  $b$  and  $c$  are not.
- For cases  $a$  and  $b$ ,  $k_{ji} = k_{ij}$ .
- The presence of an antagonism coefficient  $k'_{jh} > 0$  is compatible with both the absence of antagonism in the reverse direction ( $k'_{ij} = 0$ ) and the presence of a reverse antagonism ( $k'_{ij} > 0$ ).

An additional coherency condition is needed.

**Condition 1** (Positive net balance).

$$w_j - \left( \sum_{\{j,i\}: k_{ji}<0} |k_{ji}| + \sum_h k'_{jh} \right) > 0, \quad \text{for all } g_j \in F.$$

This condition is necessary to avoid reducing the weights to zero or negative values.

### A.3.3. An extension of the concordance index

The extension we consider in this paper is the one proposed by Figueira et al. (2009). It takes into account the three interactions effects of the previous sections. Some additional notation is needed. Let

- $L(a, a')$  denote the set of all pairs  $\{j, i\}$  such that  $j, i \in \bar{C}(a'Pa)$ ;
- $O(a, a')$  denote the set of all ordered pairs  $(j, h)$  such that  $j \in \bar{C}(a'Pa)$  and  $h \in C(a'Pa)$ .

The new formula of the concordance index is as follows.

$$c(a, a') = \frac{1}{K(a, a')} \left( \sum_{j \in \bar{C}(a'Pa)} c_j(a, a') w_j + \sum_{\{j,i\} \in L(a, a')} \mathcal{Z}(c_j(a, a'), c_i(a, a')) k_{ji} - \sum_{(j,h) \in O(a, a')} \mathcal{Z}(c_j(a, a'), c_h(a', a)) k'_{jh} \right) \quad (\text{A.8})$$

where

$$K(a, a') = \sum_{j \in F} w_j + \sum_{\{j,i\} \in L(a, a')} \mathcal{Z}(c_j(a, a'), c_i(a, a')) k_{ji} - \sum_{(j,h) \in O(a, a')} \mathcal{Z}(c_j(a, a'), c_h(a', a)) k'_{jh} \quad (\text{A.9})$$

(It should be remarked that in the third summation  $c_h(a', a)$  is always equal to 1.)

**Remark 2.** For the current application we defined the  $\mathcal{Z}$ -function as follows:  $\mathcal{Z}(x, y) = xy$ . An explanation about this choice of this function can be found in Figueira et al. (2009).

### A.4. Exploiting the outranking relation

The exploitation procedure starts by deriving from the credibility degrees two complete pre-orders,  $\mathcal{P}_\delta$  and  $\mathcal{P}_\alpha$ . A final partial pre-order  $\mathcal{P}$  is built as the intersection of the two complete pre-orders. Pre-orders  $\mathcal{P}_\delta$  and  $\mathcal{P}_\alpha$  are obtained according to two variants of the same principle, both acting in an antagonistic way on the floating actions (Figueira et al., 2005).

**Definition 7** (Descending pre-order). The complete pre-order  $\mathcal{P}_\delta$  is defined as a partition of the set  $A$  into  $r$  ordered classes,  $\bar{B}_1, \dots, \bar{B}_\ell, \dots, \bar{B}_r$ , where  $\bar{B}_1$  is the head-class in  $\mathcal{P}_\delta$ . Each class  $\bar{B}_\ell$  is composed of tied actions according to  $\mathcal{P}_\delta$ . The actions in class  $\bar{B}_\ell$  are preferred to those in class  $\bar{B}_{\ell+1}$ . For this reason,  $\mathcal{P}_\delta$  called a *descending* or *to-down* complete pre-order.

**Definition 8** (Ascending pre-order). The complete pre-order  $\mathcal{P}_\alpha$  is defined as a partition of the set  $A$  into  $s$  ordered classes,  $B_1, \dots, B_\ell, \dots, B_s$ , where  $B_s$  is the head-class in  $\mathcal{P}_\alpha$ . Each class  $B_\ell$  is composed of tied actions according to  $\mathcal{P}_\alpha$ . The actions in class  $B_{\ell+1}$  are preferred to those in class  $B_\ell$ . For this reason,  $\mathcal{P}_\alpha$  called a *ascending* or *bottom-up* complete pre-order.

The overall algorithm, composed by the procedures (called distillations) for determining  $\mathcal{P}_\delta, \mathcal{P}_\alpha$ , and then  $\mathcal{P}$  can be succinctly outlined as follows.

1. Determine  $\mathcal{P}_\delta$ , starting the first distillation by defining an initial set  $D_0 := A$ . It leads to the first distilled  $\bar{B}_1$ . After getting  $\bar{B}_\ell$ , at the distillation  $\ell + 1$ , set  $D_0 := A \setminus \{\bar{B}_1 \cup \dots \cup \bar{B}_\ell\}$ . Continue until all the actions in  $A$  are processed.
2. Determine  $\mathcal{P}_\alpha$  by using a similar algorithm. But, now remember that the actions in  $B_{\ell+1}$  are preferred to those in class  $B_\ell$ .
3. The partial pre-order  $\mathcal{P}$  will be computed as the intersection of  $\mathcal{P}_\delta$  and  $\mathcal{P}_\alpha$ .

In the intersection of Step 3 there is incomparability when  $\mathcal{P}_\delta$  and  $\mathcal{P}_\alpha$  provide contradictory results and there is comparability when the results provided by these two pre-orders are compatible.

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