The Pillars of Metrology

Alessandro Ferrero

The more I study metrology, the more I get persuaded that the measuring activity is an implicit part of our lives, something we are not really aware of, though we do or rely on measurements several times a day. When we check time, put fuel in our cars, buy food, just to mention some everyday activity, either we measure something or we trust measurements done by somebody else.

It is quite immediate to conclude that, nowadays, everything is measured and measurement results are the basis of many important decisions. Interestingly enough, measurement has always played an important role in mankind's evolution and I fully agree with Bryan Kibble's statement that the measuring stick came before the wheel, otherwise the wheel could not have been built [1].

The measuring stick is also one of the most ancient instruments, and we find it together with time measuring instruments and weighs in almost every civilization of the past, proving that measurement is one of the most important branches of science, and there is no civilization without measurement. It proves also the intimate connection existing between instrumentation and measurement, being the two sides of a single medal: the measurement science, or metrology.

Solid Pillars and Useful Tools

This leads many people to think that instruments are the pillars of metrology, something without which metrology cannot exist. Although I do not deny the importance of instruments, I prefer to consider them as important tools, instead of pillars.

Instruments extend our senses. We can see, but without the telescope, we could have never seen Jupiter's satellites as Galileo did. We sense pressure, but without the barometer we could never have realized that the atmosphere acts as a weight. We cannot sense electricity in the same way as we sense other physical quantities, since it may interfere with our vital functions in a destructive way. So, we need a voltmeter to measure an electric voltage and, in this respect, instruments are good and necessary tools.

So, why cannot we consider them also pillars, especially if they are strictly needed, as with all electromagnetic quantities but light? Because instruments are wrong! I am aware that this is a strong statement, and many people may find this incorrect, since we can realize very accurate instruments. But this is not the point. Even the most accurate measurement systems, even the standards do not realize the measurement unit exactly. They are maybe wrong for just one part in 10^{15}, as the standard of time [2], the most accurate one we have ever built, but they can never provide the "true" value of the measurand, that is the quantity we want to measure [3].

From an epistemological point of view, this small inaccuracy has an extremely important meaning. Metrology is a science, and, as such, a way to knowledge. We cannot base knowledge on something that we are aware is wrong, no matter how wrong it is. It is like building a cathedral on pillars grounded on sand, instead of solid rock.

This means that we have to look for the true pillars of metrology not in the instruments, but in those methods that allow us to realize that our measurement results can only provide incomplete information about the measurand and quantify how incomplete this information is. This is the only way we can turn incomplete and therefore useless information into useful knowledge. By doing this we discover that the most important pillars of metrology, as I will try to explain in the following sections, are three: uncertainty, calibration, and traceability.

Uncertainty

Uncertainty has probably become the best-known concept in metrology after the Guide to the Expression of Uncertainty in Measurement (GUM) [4] was published in 1995. According to the GUM we know that

> "*when all of the known or suspected components of error have been evaluated and the appropriate corrections have been applied, there still remains an uncertainty about the correctness of the stated result, that is, a doubt about how well the result*

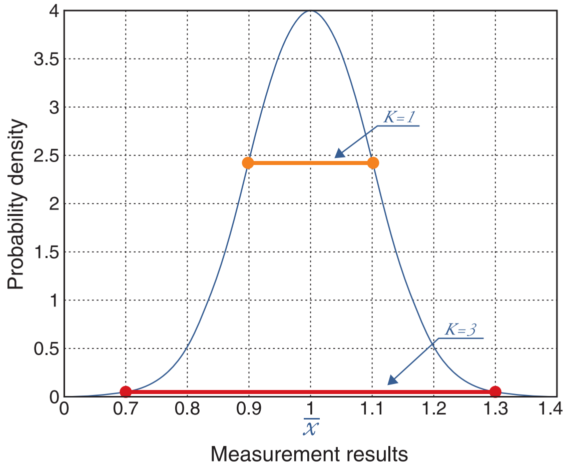

Fig. 1. Distribution of the values that can be attributed to the measurand, according to a normal probability distribution. The two intervals represent confidence intervals with coverage probability of 68.27% (*K*=1) and 99.73% (*K*=3).

> *of the measurement represents the value of the quantity being measured."*

This is one of the first sentences in the Introduction (section 0) of the GUM, and its meaning goes often unnoticed, despite the importance of its implications: the need for calibration and traceability.

Without entering into details, for which the reader is referred to [5], assuming that "*all of the known or suspected components of error have been evaluated and the appropriate corrections have been applied*" implies that the residual reasons for the "*doubt about how well the result of the measurement represents the value of the quantity being measured*" are the effects of random phenomena acting on the measuring system when measurement is performed.

Under this assumption, each indication provided by the measuring system, that is the measured value x_m, can be considered as a single realization of a random variable X, whose realizations depend on the values taken by the random phenomena affecting the measurement process. A random variable can be fully described by the associated probability density function (pdf), and given a pdf, it can be represented by its first two moments: mean value $\overline{x}$ and standard deviation σ.

Therefore, a measurement result can be mathematically described by a random variable and, if suitable assumptions can be made about the pdf, based on repeated measurement or *a priori* knowledge [4], [5], it can be given in terms of the pdf mean value – representing the maximum likelihood value – and standard deviation – quantifying the dispersion of values about the mean value.

The considered standard deviation is called, in metrology, *standard uncertainty* [4]. It is noted with small letter u, and is employed to quantify the "*doubt about how well the result of the measurement represents the value of the quantity being measured.*" It is well known that, having assumed a given pdf, it is possible to define intervals about the measured value (mean value of the distribution) whose half width is given by the standard uncertainty or its multiple as shown in Fig. 1 for the case of a normal distribution.

Those intervals represent confidence intervals in which the unknown and unknowable [5] measurand value is expected to lie with specified coverage probability, whose value depends on the considered pdf and multiple K of the standard uncertainty [4]. K is called coverage factor and the half width $U = Ku$ of the considered confidence interval is called *expanded uncertainty*.

According to the above considerations that have synthetically summarized the fundamental concept given by the GUM, standard uncertainty is the pillar that carries our capability of quantifying the doubt about how well the measured value represents the measurand. Through standard uncertainty, we can quantify the lack of complete knowledge about the measurand and, hence, we can change the obtained measurement result from a useless number into an useful interval of confidence.

However, there is still one critical question to be answered: How can we be sure that the measurand value lies inside the obtained confidence interval (as it is in the case of the black dot in Fig. 2)? How can we exclude that our evaluation of measurement uncertainty is incorrect, so that the measurand value does not lie inside the obtained interval (as it is the case of the red dot in Fig. 2)?

We must consider again the statement in Section 0 of the GUM; If "*all of the known or suspected components of error have been evaluated and the appropriate corrections have been applied,*" (and of course, we have not made any gross mistake in evaluating u) then, we can be reasonably sure that the obtained interval encompasses "*a large fraction of the distribution of values that could reasonably be attributed to the measurand*" [4].

Here is the key point that the GUM takes for granted and does not cover: How can we ensure that all of the known or suspected components of error have been evaluated and the appropriate corrections have been applied? We need a second pillar!

Calibration

An old, very simplistic definition stated that measurement is the comparison of the measurand with a proper standard. The more modern approach to measurement, based on a strict model, involves many more processes [6], but standards still have a fundamental role in determining whether all of the known or suspected components of errors have been correctly evaluated and compensated for. This can be achieved by

Fig. 2. Correct uncertainty estimation when the measurand value (black dot) falls inside the obtained interval, and incorrect uncertainty estimation when the measurand value (red dot) falls outside the obtained interval.

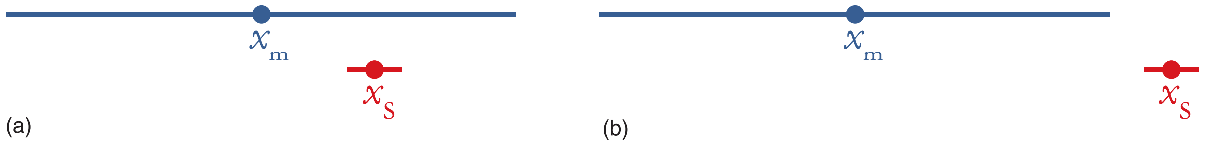

Fig. 3. Measured value x_m together with the interval defined by the expanded uncertainty (blue line), and standard value x_s together with the interval defined by the expanded uncertainty (red line) when all components of errors have been compensated for (a) and when they have not (b).

following one of the key processes in measurement, and our second pillar: *Calibration*. Its definition is given by the International Vocabulary of Metrology (VIM) [7] as:

> *"Operation that, under specified conditions, in a first step, establishes a relation between the quantity values with measurement uncertainties provided by measurement standards and corresponding indications with associated measurement uncertainties and, in a second step, uses this information to establish a relation for obtaining a measurement result from an indication."*

According to this definition, calibration is performed in two steps.

First Calibration Step

In this first step, the measurement system we want to calibrate is employed to measure the standard (or, as we will see in the next section, a device that can be considered as a standard, based on our third pillar).

As also recalled by the VIM definition of calibration, a standard is a practical realization of a measurement unit and features its own, known uncertainty. The measurement system under calibration provides an indication (measured value) that is also characterized by its uncertainty, evaluated according to the GUM. The results of the first calibration step can hence be those very schematically depicted in Fig. 3.

Fig. 3a shows that the value of the measured standard falls inside the interval provided by the expanded uncertainty about the measured value x_m returned by the employed measurement system when measuring the standard. Therefore, it is possible to state that the measurand value does actually lie in the interval $x_m - U$, $x_m + U$ given by the expanded uncertainty. Since the standard value x_s is known with a standard uncertainty $u(x_s)$, the standard uncertainty $u(x_m)$ of the employed measurement system should be properly corrected to take into account this additional contribution, so that the standard uncertainty of the measurement system after calibration is given by:

$$u'(x_m) = \sqrt{u^2(x_m) + u^2(x_s)} \qquad (1)$$

Fig. 3b shows the opposite and less fortunate situation when the measurand value lies outside the interval provided by the expanded uncertainty about the measured value x_m. Assuming that $u(x_m)$ has been correctly evaluated, the conclusion is that some of the components of error could not be recognized and compensated for. How shall we proceed in such a case?

Second Calibration Step

The VIM definition instructs us on how to proceed, when it defines calibration as the operation that "…*in a second step, uses this information to establish a relation for obtaining a measurement result from an indication*" [7].

In the simple case shown in Fig. 3b, the relation that we have to establish is very simple. The measurement result x can be obtained from indication x_m by simply adding the quantity $\Delta = x_s - x_m$ to x_m, so that the corrected measurement result is given by: $x = x_m + \Delta$.

Once again, since correction Δ depends on x_s, its standard uncertainty must be taken into account again in evaluating the standard uncertainty on x, in a similar way as that shown by (1):

$$u(x) = \sqrt{u^2(x_m) + u^2(x_s)} \qquad (2)$$

The simple example, shown in Fig. 3b, is related to a single measured value. In general, measurement systems have a measurement range, and different situations may occur in the different points of the scale, so that the relation to establish should be capable of obtaining the measurement result from every indication along the scale.

This is well explained by the VIM, in the first note to the definition of calibration that states [7]:

> *"A calibration may be expressed by a statement, calibration function, calibration diagram, calibration curve, or calibration table. In some cases, it may consist of an additive or multiplicative correction of the indication with associated measurement uncertainty."*

An Important Comment

One of the most common mistakes made by people who are not familiar with the fundamentals of metrology is to perceive the first step alone, in the above definition, as being calibration. This is so common that the VIM itself warns against this mistake in its note 3 to the definition of calibration [7].

There is a domain, in particular, into which calibration is almost always perceived as *sending the instrument to the calibration lab and use the received calibration certificate only to show the ISO 9000 auditor that we are compliant with article 7.6 of the ISO Std. 9001:2008*: It is the field of quality management system.

Indeed, if we refer to calibration as only the first step in the VIM definition and we neglect the second step, we are simply wasting money to only establish "*a relation between the quantity values with measurement uncertainties provided by measurement standards and corresponding indications with associated measurement uncertainties*" (in simple words, obtain a set of indications as those shown in Fig. 3). We waste money because we do not use the obtained relation to obtain *a measurement result from an indication*. Again, in simple words, if we do not use the data provided by the calibration certificate to correct the measured values, we will always have a useless indication, but not a useful measurement result.

It is immediate to perceive that, if the first calibration step shows that an instrument has a 2% gain error, if we do not correct it in the second calibration step, we obtain exactly the same result as if we did not calibrate the instrument at all! Sometimes this practice may become rather annoying, as with the speed traps on the Italian highways. The devices that measure the car speed are sent to an accredited calibration lab, but if you get a ticket, nowhere can you find evidence that the calibration data have been used to obtain a measurement result showing the speed you were driving at, with associated measurement uncertainty, from a mere indication. In the end, you pay a fine, which does partly cover the cost of the calibration lab, but nothing grants you that your speed has been properly measured. For a metrologist, this is rather disappointing!

It is however also important to outline that step 1 and step 2 must be performed under the same measurement conditions. It is immediate to perceive that we cannot correct an indication obtained at 45 °C with the calibration data obtained at 23 °C. The result would be useless, unless we can ensure that the indications provided by the measurement system under calibration are independent of temperature. Therefore, a good calibration process should be capable of transferring the results obtained during step 1, usually performed in a controlled lab environment, into those calibration functions, calibration diagrams, or calibration curves capable of obtaining a measurement result from an indication ***under the real operating conditions***. This additional step may result in an increase of measurement uncertainty, but it is necessary to fully characterize the measured value from a correct metrological perspective.

Traceability

According to the considerations in the previous sections, one may think that two pillars – uncertainty and calibration – are enough to provide solid and stable bases to metrology. In principle, this is correct. However, there is a practical problem that, if not considered, might undermine our pillars and make the whole construction collapse.

Measurement standards are complex and delicate devices, are kept and maintained in the National Metrology Institutes, and are not easily accessible. On top of this, it makes little sense, from a very practical and economical point of view, to use a primary standard to calibrate a digital voltmeter that costs a few tens of dollars.

On the other hand, the primary standard is the only reference that can be used to obtain "*a measurement result from an indication*." The solution to this puzzle is our third pillar: ***Metrological traceability***.

Once again, let us refer to the VIM for its definition [7]: "*property of a measurement result whereby the result can be related to a reference through a documented unbroken chain of calibrations, each contributing to the measurement uncertainty.*"

To fully understand this definition, we need to refer again to the VIM. Its note 2 to this definition is: "*Metrological traceability requires an established calibration hierarchy,*" where "*calibration hierarchy*" is defined, again by the VIM [7], as: "*sequence of calibrations from a reference to the final measuring system, where the outcome of each calibration depends on the outcome of the previous calibration.*"

Here is the solution to the puzzle. There is no need to use directly the primary standard to calibrate our measuring system if we can use, instead of the standard, a measuring system that has been calibrated using the standard, or, in turn, another measuring system that has been calibrated using the standard. In principle, this chain can be as long as we wish, provided it is *unbroken*, that is we can always find the primary standard at the beginning of the chain. Of course, there is a price to pay: uncertainty necessarily increases along the chain of calibrations. This is the direct consequence of (2), where $u(x)$ becomes $u(x_s)$ in the next calibration along the chain, thus causing the increment of uncertainty.

There is still one important point to consider in the definition. The chain of calibrations that implements the metrological traceability should be not only *unbroken*, but also *documented*. This means that every step of the calibration chain has to be properly and adequately documented, so that it is possible to trace back every element that contributed to the declared calibration uncertainty.

It implies also the existence of regulatory authorities and accreditation bodies that assess the competence of the laboratories that implement the chain of calibrations, and issue the necessary accreditation. The general requirements for the competence of testing and calibration laboratories are specified by the ISO/IEC Std 17025:2005 [8], to which the reader is referred for more details on these requirements.

Conclusions

I hope that these brief notes succeeded in clarifying the connection between measurement uncertainty, calibration and metrological traceability. These three concepts are the three pillars that hold metrology, since, as explained in the previous sections, the only way to obtain a useful *measurement result* from a useless *indication* is to evaluate and express the uncertainty associated to that indication through a proper and metrologically traceable calibration. This is the only way to obtain confidence intervals, about the measured value, and ensure that they contain the measurand value with the specified coverage probability. In simple words, only the application of these three concepts allows us to trust as an acceptable measurement result the *wrong* indication of an instrument.

Fig. 4. The nave of the Oviedo Cathedral in Spain and the impressive pillars that hold it. Photo courtesy of Alessandro Ferrero (© 2015, Alessandro Ferrero, used with permission).

I hope I also succeeded in explaining why these three concepts are the most important pillars, although they may not be fully perceived when we approach metrology for the first time. I called them pillars because I like to picture them in my mind as the pillars that hold the awesome gothic cathedrals. When you look at these cathedrals from a distance, the façade and the impressive rampant arches attract you. You may even think that the arches carry the weight. If you remain outside, you cannot understand what is holding up the building.

You must enter the cathedral to discover the pillars that hold the cross vault and, consequently, the entire weight of the building as shown by the recent picture I took inside the Oviedo Cathedral in Spain (Fig. 4). You cannot perceive the columns if you stay outside. Similarly, you cannot understand metrology if you do not enter into the basic concepts of uncertainty, calibration, and traceability. Most importantly, you cannot teach it without paying enough attention to these concepts.

References

[1] B. Kibble, "The measuring stick came before the wheel," *IEEE Instrum. Meas. Mag. IEEE*, vol. 17, no. 3, pp. 28-29, Jun. 2014.

[2] F. Cabiati, "The system of units and the measurement standards," *Modern Measurements: Fundamentals and Applications*, A. Ferrero, D. Petri, P. Carbone, and M. Catelani, Eds., Hoboken, NJ, USA: John Wiley & Sons, Inc., 2015.

[3] D. Petri, L. Mari, and P. Carbone, "A Structured Methodology for Measurement Development," in *IEEE Trans. Instrum. Meas.*, vol. 64, no. 9, pp. 2367–2379, Sep. 2015.

[4] JCGM 100:2008, *Evaluation of Measurement Data – Guide to the Expression of Uncertainty in Measurement, (GUM 1995 with minor corrections)*, Joint Committee for Guides in Metrology, 2008. [Online]. Available: http://www.bipm.org/en/publications/guides/gum.html.

[5] A. Ferrero and S. Salicone, "Measurement uncertainty," *IEEE Instrum. Meas. Mag.*, vol. 9, no. 3, pp.44–51, Jun. 2006.

[6] A. Ferrero and D. Petri, "Measurement models and uncertainty," *Modern Measurements: Fundamentals and Applications*, A. Ferrero, D. Petri, P. Carbone, and M. Catelani, Eds., Hoboken, NJ, USA: John Wiley & Sons, Inc., 2015.

[7] JCGM 200:2012, *International Vocabulary of Metrology – Basic and General Concepts and Associated Terms (VIM 2008 with minor corrections)*, Joint Committee for Guides in Metrology, 2012. [Online]. Available: http://www.bipm.org/en/publications/guides/vim.html.

[8] ISO/IEC Std. 17025:2005, *General requirements for the competence of testing and calibration laboratories*, IEC, Second edition, 2005.