

## Research Article

# Fault Detection for Network Control Systems with Multiple Communication Delays and Stochastic Missing Measurements

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This paper is concerned with fault detection problem for a class of network control systems (NCSs) with multiple communication delays and stochastic missing measurements. The missing measurement phenomenon occurs in a random way and the occurrence probability for each measurement output is governed by an individual random variable. Besides, the multiple communication delay phenomenon reflects that networked control systems have different communication delays when the signals are transferred via different channels. We aim to design a fault detection filter so that the overall fault detection dynamics is exponentially stable in the mean square. By constructing proper Lyapunov-Krasovskii functional, we acquire sufficient conditions to guarantee the stability of the fault detection filter for the discrete systems, and the filter parameters are also derived by solving linear matrix inequality. Finally, an illustrative example is provided to show the usefulness and effectiveness of the proposed design method.

## 1. Introduction

Over the past few decades, the fault detection problem has been attracting extensive research attention from scholars [1–6]. In a fault detection progress, we aim to construct a residual signal and compute a residual evaluation function which can then be compared with a predefined threshold; if the residual exceeds the threshold we set, then the fault is detected and an alarm of fault is generated. In many industrial applications, various filter schemes have been proposed recently for systems which assume that the measurements always contain a true signal. However, systems may exhibit process and measurement noise inputs. One approach to this problem is  $\mathcal{H}_\infty$  filtering, and the advantage of this approach is that the noise signals in the  $\mathcal{H}_\infty$  filtering setting are arbitrary signals with bounded energy, and no exact statistics are required to be known. So,  $\mathcal{H}_\infty$  filter has been widely applied to many actual systems due to its high accuracy and robustness [7–13].

Considering the fault detection problem in a class of networked control systems, some new problems have merged

out [14–16]. Communication delays and missing measurements are important issues in NCSs. Some existing literatures assume that measurement signal is completely lost. In fact, there often may be a part of the measurement information loss, and each individual sensor may also have different data loss probability [17–24]. For example, in [21], a model of multiple missing measurements has been presented by using a diagonal matrix to account for the different missing probability for individual sensors. The finite-horizon robust filtering problem has been considered in [22] for discrete-time stochastic systems with probabilistic missing measurements subject to norm-bounded parameter uncertainties. A Markovian jumping process has been employed in [23] to reflect the measurement missing problem. Moreover, the optimal filter design problem has been tackled in [24] for systems with multiple packet dropouts by solving a recursive difference equation (RDE). In addition, the presence of communication delays not only reduces relative stability and robustness but also degrades the performance. So far, many researchers have studied the stability and controller design

problems for networked systems in the presence of communication delays [25–33].

Summarizing the above discussion, in this paper, we are motivated to study the fault detection problem for a class of network control systems with multiple communication delays and stochastic missing measurements. A fault detection filter is constructed through the establishment of the existing model; then the addressed fault detection problem is converted into an auxiliary  $\mathcal{H}_\infty$  filtering problem. Sufficient conditions are established for the existence of the fault detection filter, and then the corresponding solvability conditions for the desired filter gains are established. In the end, a practical simulation example is given to show the effectiveness of the proposed method. The main contributions of this paper can be listed as follows. (1) A model is proposed to describe multiple communication delays, and randomly occurring packet dropout phenomenon is also considered. (2) The  $\mathcal{H}_\infty$  performance requirement and the fault detection specification can be obtained by employing stochastic analysis technique. (3) Sufficient conditions are established under which the augmented system is exponentially mean-square stable and satisfies the performance constraint for all nonzero exogenous disturbances under zero-initial condition.

*Notation.* The notation used in the paper is fairly standard.  $\mathbb{R}^n$ ,  $\mathbb{R}^{n \times m}$ , and  $\mathbb{Z}$  ( $\mathbb{Z}^+$ ,  $\mathbb{Z}^-$ ) denote, respectively, the  $n$ -dimensional Euclidean space, the set of all  $n \times m$  real matrices, and the set of integers (nonnegative integers, negative integers). The notation  $\|A\|$  refers to the norm of a matrix  $A$  defined by  $\|A\| = \sqrt{\text{tr}(A^T A)}$ . 0 represents zero matrix of compatible dimensions. The  $n$ -dimensional identity matrix is denoted as  $I_n$  or simply  $I$ , if no confusion is caused. The notation  $P > 0$  means that  $P$  is real symmetric and positive definite.  $M^T$  represents the transpose of the matrix  $M$ .  $\text{diag}\{\dots\}$  stands for a block-diagonal matrix.  $\mathbb{E}\{x\}$  and  $\mathbb{E}\{x | y\}$  will, respectively, mean expectation of the stochastic variable  $x$  and expectation of  $x$  conditional on  $y$ . \* is used as an ellipsis for terms induced by symmetry in symmetric block matrices. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

## 2. Problem Formulation

In this paper, we consider the fault detection problem for a class of network control systems with multiple communication delays and stochastic missing measurements; then a NCS model can be represented by the following dynamic model:

$$\begin{aligned} x(k+1) &= (A + \Delta A)x(k) \\ &+ \sum_{m=1}^N (A_d + \Delta A_d)x(k-m) \\ &+ D_1\omega(k) + E_f f(k), \\ \bar{y}(k) &= Cx(k) + D_2\omega(k), \\ x(k) &= \psi(k) \quad \forall k \in \mathbb{Z}^-, \end{aligned} \quad (1)$$

where  $x(k) \in \mathbb{R}^{n_x}$  represents the state vector,  $x(k-m) \in \mathbb{R}^{n_x}$  denotes the state delay of the system, and  $\omega(k) \in \mathbb{R}^{n_\omega}$  denotes the unknown disturbance, while  $f(k) \in \mathbb{R}^{n_f}$  is the fault of the system.  $\bar{y}(k) \in \mathbb{R}^{n_y}$  denotes the system progress output.  $\psi(k)$  ( $k \in \mathbb{Z}^-$ ) is given random initial conditions satisfying  $\sup_{k \in \mathbb{Z}^-} \mathbb{E}\{\|\psi(k)\|^2\} < \infty$ .  $\Delta A$  and  $\Delta A_d$  are internal perturbation arising from uncertain factors  $A$  and  $A_d$ .  $A$ ,  $A_d$ ,  $D_1$ ,  $E_f$ ,  $C$ , and  $D_2$  are known real matrices with appropriate dimensions. Besides, the real-valued matrices  $\Delta A$  and  $\Delta A_d$  represent the norm-bounded parameter uncertainties of the following structure:

$$[\Delta A \quad \Delta A_d] = H_a F(k) [E_a \quad E_d], \quad (2)$$

where  $H_a$ ,  $E_a$ , and  $E_d$  are known constant matrices with appropriate dimensions, and  $F(k)$  is an unknown matrix function satisfying

$$F^T(k) F(k) \leq I. \quad (3)$$

Then, we model the missing measurements via a diagonal matrix consisting of a series of mutually independent random variables satisfying certain probabilistic distributions on the interval  $[0 \ 1]$ . So, the multiple missing measurements are described by

$$\begin{aligned} y(k) &= \Xi Cx(k) + D_2\omega(k) \\ &= \sum_{j=1}^r \beta_j C_j x(k) + D_2\omega(k), \end{aligned} \quad (4)$$

where  $y(k) \in \mathbb{R}^{n_y}$  is the actual measurement signal output, and  $\Xi = \text{diag}\{\beta_1, \dots, \beta_r\}$ ,  $\beta_j$  ( $j = 1, \dots, r$ ) are a series of mutually independent random variables; it is assumed that  $\beta_j$  has the probabilistic density function  $q_j(S)$  ( $j = 1, \dots, r$ ) on the interval  $[0 \ 1]$  with mathematical expectation  $u_j$  and variance  $\sigma_j^2$ .  $C_j$  is defined by

$$C_j = \text{diag} \left\{ \underbrace{0, \dots, 0}_{j-1}, 1, \underbrace{0, \dots, 0}_{r-j} \right\} C. \quad (5)$$

For presentation convenience, we denote

$$\bar{\Xi} = \mathbb{E}\{\Xi\}, \quad \tilde{\Xi} = \Xi - \bar{\Xi}. \quad (6)$$

*Remark 1.* Because missing measurements in system occur in a stochastic way,  $\beta_j$  is the probabilistic missing statue of the  $j$ th sensor and can take value on the interval  $[0 \ 1]$ . In addition, for every sensor, their probability to take different values may differ from each other, which reflects stochastic character of the multiple missing measurements in system model.

Then, we can easily describe a NCS model with multiple communication delays and stochastic missing measurements as

$$\begin{aligned} x(k+1) &= (A + \Delta A)x(k) \\ &+ \sum_{m=1}^N (A_d + \Delta A_d)x(k-m) \\ &+ D_1\omega(k) + E_f f(k), \\ y(k) &= \Xi Cx(k) + D_2\omega(k), \\ x(k) &= \psi(k) \quad \forall k \in Z^-. \end{aligned} \quad (7)$$

The key step of fault detection schemes is the construction of a dynamic system called a fault detection observer or filter, in which the residual signal is generated in order to decide whether a fault has occurred or not. In this paper, according to the above formula, we build a fault detection filter whose model can be described as follows:

$$\begin{aligned} \hat{x}(k+1) &= A_f \hat{x}(k) + B_f y(k), \\ r(k) &= C_f \hat{x}(k) + D_f y(k), \end{aligned} \quad (8)$$

where  $\hat{x}(k) \in \mathbb{R}^{n_x}$  represents the filter state vector,  $r(k) \in \mathbb{R}^{n_r}$  is the so-called residual that is compatible with the fault vector  $f(k)$ , and  $A_f$ ,  $B_f$ ,  $C_f$ , and  $D_f$  are appropriately dimensioned filter matrices to be determined. We set the following variables:

$$\begin{aligned} e(k) &= r(k) - f(k), \\ \eta(k) &= [x^T(k) \quad \hat{x}^T(k)]^T, \\ \nu(k) &= [\omega^T(k) \quad f^T(k)]^T. \end{aligned} \quad (9)$$

Then, we can get the overall fault detection dynamics governed by the following system:

$$\eta(k+1) = (\bar{A} + \hat{A})\eta(k) + \sum_{m=1}^N \bar{A}_d \eta(k-m) + \bar{B}\nu(k), \quad (10)$$

$$e(k) = r(k) - f(k) = (\bar{C} + \hat{C})\eta(k) + \bar{D}\nu(k),$$

where

$$\begin{aligned} \bar{A} &= \begin{bmatrix} A + \Delta A & 0 \\ B_f \Xi C & A_f \end{bmatrix}, & \hat{A} &= \begin{bmatrix} 0 & 0 \\ B_f \Xi C & 0 \end{bmatrix}, \\ \bar{A}_d &= \text{diag}\{A_d + \Delta A_d, 0\}, \\ \bar{B} &= \begin{bmatrix} D_1 & E_f \\ B_f D_2 & 0 \end{bmatrix}, & \bar{C} &= [D_f \Xi C \quad C_f], \\ \hat{C} &= [D_f \Xi C \quad 0], & \bar{D} &= [D_f D_2 \quad -I]. \end{aligned} \quad (11)$$

*Remark 2.* There is a big probability of the existence of errors between theoretical and practical systems due to unexpected

factors in NCSs. In order to overcome this phenomenon, it is natural to assume system uncertainties. In this paper, we assume that the uncertainties occur on not only regular item of the system but also time-delay item. Therefore, this is a more general description for the NCSs.

The main purpose of this paper is to design a fault detection filter such that the overall fault detection dynamics is exponentially stable in the mean square and, at the same time, the error between the residual signal and the fault signal is made as small as possible. Until now, the fault detection problem to be addressed in this paper can be described by the following two steps.

*Step 1.* For system (7), we construct a fault detection filter as the model of (8); then we can obtain the residual signal  $r(k)$ . Furthermore, the filter is designed so that the overall fault detection system (10) is exponentially mean-square stable with the following  $\mathcal{H}_\infty$  performance constraint under zero-initial condition:

$$\sum_{k=0}^{\infty} \mathbb{E} \{ \|e(k)\|^2 \} \leq \gamma^2 \sum_{k=0}^{\infty} \|v(k)\|^2, \quad (12)$$

where  $v(k) \neq 0$ , and  $\gamma > 0$  is made as small as possible.

*Step 2.* We set up a fault detection measure to judge whether a fault occurs. In this paper, we adopt two variables: an evaluation function  $J(k)$  and a threshold  $J_{\text{th}}$ . The faults can be detected by comparing these two variables:

$$J(k) = \left\{ \sum_{k=s-\mathcal{L}}^{k=s} r^T(k) r(k) \right\}^{1/2}, \quad J_{\text{th}} = \sup_{w \in \ell_2, f=0} \mathbb{E} \{ J(k) \}, \quad (13)$$

where  $\mathcal{L}$  denotes the length of the finite evaluating time horizon. Based on (13), the occurrence of faults can be detected by comparing  $J(k)$  with  $J_{\text{th}}$  according to the following rule:

$$\begin{aligned} J(k) > J_{\text{th}} &\implies \text{with faults} \implies \text{alarm}, \\ J(k) \leq J_{\text{th}} &\implies \text{no faults}. \end{aligned} \quad (14)$$

### 3. Main Results

First of all, let us introduce the following lemmas which will be used in deriving our main results.

**Lemma 3** (Schur complement). *Given constant matrices  $S_1$ ,  $S_2$ ,  $S_3$ , where  $S_1 = S_1^T$  and  $0 < S_2 = S_2^T$ , then  $S_1 + S_3^T S_2^{-1} S_3 < 0$  if and only if*

$$\begin{bmatrix} S_1 & S_3^T \\ S_3 & -S_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -S_2 & S_3 \\ S_3^T & S_1 \end{bmatrix} < 0. \quad (15)$$

**Lemma 4** (S-procedure). *Let  $N = N^T$ ,  $H$  and  $E$  be real matrices with appropriate dimensions, and let  $F^T(k)F(k) \leq I$ . Then, the inequality  $N + HFE + (HFE)^T < 0$  if and only if there*

exists a positive scalar  $\varepsilon$  such that  $N + \varepsilon HH^T + \varepsilon^{-1} E^T E < 0$  or, equivalently,

$$\begin{bmatrix} N & \varepsilon H & E^T \\ \varepsilon H^T & -\varepsilon I & 0 \\ E & 0 & -\varepsilon I \end{bmatrix} < 0. \quad (16)$$

For convenience of presentation, we first discuss the nominal system without parameter uncertainties  $\Delta A$  and  $\Delta A_d$  in Theorems 5 and 6 and will eventually extend our main results to the general case in Theorem 7. Therefore, in Theorems 5 and 6, we redefine  $\bar{A}$  and  $\bar{A}_d$  as

$$\bar{A} = \begin{bmatrix} A & 0 \\ B_f \bar{E} C & A_f \end{bmatrix}, \quad \bar{A}_d = \text{diag}\{A_d, 0\}. \quad (17)$$

**Theorem 5.** Consider the nominal system model (10) and suppose that the filter parameters are given. The nominal fault detection filter (8) is exponentially mean-square stable with a disturbance attenuation level  $\gamma > 0$ , if there exist matrices  $P > 0$ ,  $Q_k > 0$  ( $k = 1, 2, \dots, N$ ) satisfying

$$\Psi^T \check{P} \Psi + \hat{\Psi}^T \check{P} \hat{\Psi} + \bar{P} < 0, \quad (18)$$

where

$$\begin{aligned} \bar{Z} &= \underbrace{[\bar{A}_d \ \dots \ \bar{A}_d]}_N, & \hat{C}_j &= [D_f C_j \ 0], \\ \bar{\mathcal{A}} &= [\bar{A} \ \bar{Z} \ \bar{B}], & \mathcal{E}_m &= [\bar{C} \ 0 \ \bar{D}], \\ \Psi &= [\bar{\mathcal{A}}^T \ \mathcal{E}_m^T]^T, & \check{\mathcal{A}} &= \begin{bmatrix} \sum_{j=1}^m \sigma_j \hat{A}_j & 0 & 0 \end{bmatrix}, \\ \check{\mathcal{E}} &= \begin{bmatrix} \sum_{j=1}^m \sigma_j \hat{C}_j & 0 & 0 \end{bmatrix}, & \hat{\Psi} &= [\check{\mathcal{A}}^T \ \check{\mathcal{E}}^T]^T, \\ \check{P} &= \text{diag}\{P, I\}, & \mathcal{F} &= \text{diag}\{-Q_1, \dots, -Q_N\}, \\ \bar{Q}_k &= \sum_{k=1}^N Q_k - P, & \hat{P} &= \text{diag}\{\bar{Q}_k, \mathcal{F}\}, \\ \bar{P} &= \text{diag}\{\hat{P}, -\gamma^2 I\}. \end{aligned} \quad (19)$$

*Proof.* Choose a Lyapunov functional for system (10):

$$V(k) = V_1(k) + V_2(k), \quad (20)$$

where

$$\begin{aligned} V_1(k) &= \eta^T(k) P \eta(k), \\ V_2(k) &= \sum_{m=1}^N \sum_{i=k-m}^{k-1} \eta^T(i) Q_m \eta(i). \end{aligned} \quad (21)$$

Then, along the trajectory of augmented system (10) with  $\nu(k) = 0$ , we have

$$\begin{aligned} &\mathbb{E}\{\Delta V_1(k)\} \\ &= \mathbb{E}\left\{\left[\left(\bar{A} + \hat{A}\right)\eta(k) + \sum_{m=1}^N \bar{A}_d \eta(k-m) + \bar{B}v(k)\right]^T P \right. \\ &\quad \times \left[\left(\bar{A} + \hat{A}\right)\eta(k) + \sum_{m=1}^N \bar{A}_d \eta(k-m) + \bar{B}v(k)\right] \\ &\quad \left. - \eta(k)^T P \eta(k)\right\}, \\ &\mathbb{E}\{\Delta V_2(k)\} \\ &= \mathbb{E}\left\{\sum_{m=1}^N \left[\eta^T(k) Q_m \eta(k) - \eta^T(k-m) Q_m \eta(k-m)\right]\right\}. \end{aligned} \quad (22)$$

For notational convenience, we denote

$$\begin{aligned} \eta(k-\tau) &= [\eta^T(k-1) \ \dots \ \eta^T(k-N)]^T, \\ \hat{\xi}(k) &= [\eta^T(k) \ \eta^T(k-\tau)]^T, \quad \xi(k) = [\hat{\xi}^T(k) \ v^T(k)]^T, \\ \mathcal{A} &= [\bar{A} \ \bar{Z}], \quad \hat{\mathcal{A}} = \begin{bmatrix} \sum_{j=1}^m \sigma_j \hat{A}_j & 0 \end{bmatrix}, \quad \hat{A}_j = \begin{bmatrix} 0 & 0 \\ B_f C_j & 0 \end{bmatrix}. \end{aligned} \quad (23)$$

In the following, we first prove the exponential stability of the fault detection dynamics system (10) with  $\nu(k) = 0$ . Therefore, we can easily have

$$\mathbb{E}\{\Delta V(k)\} \leq \mathbb{E}\{\hat{\xi}^T(k) (\mathcal{A}^T P \mathcal{A} + \hat{\mathcal{A}}^T P \hat{\mathcal{A}} + \hat{P}) \hat{\xi}(k)\}. \quad (24)$$

By utilizing Schur complement Lemma 3, we know that  $\mathbb{E}\{\Delta V(k)\} < 0$  if (18) is true. Furthermore, along the same line of the proof for Theorem 1 in [34], it can be concluded that the discrete-time nominal system of (10) with  $\nu(k) = 0$  is exponentially mean-square stable.

Now, we are in a position to deal with the  $\mathcal{H}_\infty$  performance of the nominal system of (10). Under zero-initial condition,  $J(n)$  can be described as the following forms:

$$\begin{aligned} J(n) &= \mathbb{E} \sum_{k=0}^n \left[ e^T(k) e(k) - \gamma^2 v^T(k) v(k) \right] \\ &\leq \mathbb{E} \sum_{k=0}^n \left[ e^T(k) e(k) - \gamma^2 v^T(k) v(k) + \Delta V(k) \right]. \end{aligned} \quad (25)$$

Then we have

$$J(n) \leq \mathbb{E} \left\{ \sum_{k=0}^n \xi^T(k) (\Psi^T \check{P} \Psi + \hat{\Psi}^T \check{P} \hat{\Psi} + \bar{P}) \xi(k) \right\}. \quad (26)$$

If there exist  $P > 0, Q_k > 0$  ( $k = 1, 2, \dots, N$ ) satisfying

$$\Psi^T \check{P} \Psi + \widehat{\Psi}^T \check{P} \widehat{\Psi} + \bar{P} < 0, \quad (27)$$

then we will have  $J(n) < 0$  by considering Theorem 5.

Letting  $n \rightarrow \infty$ , we can obtain

$$\sum_{k=0}^{\infty} \mathbb{E} \{ \|e(k)\|^2 \} \leq \gamma^2 \sum_{k=0}^{\infty} \|v(k)\|^2 \quad (28)$$

which is equivalent to the inequality in (12). To this end, the proof of Theorem 5 is complete.  $\square$

According to the analysis results established, we will deal with the fault detection filter design problem.

**Theorem 6.** Consider the nominal system model (10), and let  $\gamma > 0$  be a given constant scalar which represents  $\mathcal{H}_\infty$  noise attenuation level bound. The desired full-order fault detection filter of form (8) exists if there exist matrices  $P > 0, Q_k > 0$ , ( $k = 1, 2, \dots, N$ ),  $X$ , and  $K$  satisfying

$$\Omega = \begin{bmatrix} \bar{P} & * \\ \Gamma & -\check{\mathcal{P}} \end{bmatrix} < 0, \quad (29)$$

where

$$\begin{aligned} \Gamma_{11} &= \begin{bmatrix} P\widehat{A}_0 + X\widehat{R}_1 & P\bar{Z} \\ K\widehat{R}_1 & 0 \end{bmatrix}, & \Gamma_{12} &= \begin{bmatrix} P\bar{B}_0 + X\widehat{R}_2 \\ \widehat{E}_0 + K\widehat{R}_2 \end{bmatrix}, \\ \Gamma_{21} &= \begin{bmatrix} X\widehat{R}_4 & 0 \\ K\widehat{R}_4 & 0 \end{bmatrix}, & \widehat{R}_1 &= \begin{bmatrix} 0 & I \\ \bar{\Xi}C & 0 \end{bmatrix}, \\ \Gamma &= \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & 0 \end{bmatrix}, & \widehat{A}_0 &= \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \\ \bar{B}_0 &= \begin{bmatrix} D_1 & E_f \\ 0 & 0 \end{bmatrix}, & \widehat{R}_2 &= \begin{bmatrix} 0 & 0 \\ D_2 & 0 \end{bmatrix}, \\ \widehat{R}_4 &= \begin{bmatrix} 0 & 0 \\ \sum_{j=1}^m \sigma_j C_j & 0 \end{bmatrix}, & \widehat{E}_0 &= [0 \quad -I], \\ \widehat{E} &= [0 \quad I]^T, & \check{\mathcal{P}} &= I_2 \otimes \check{P}, \end{aligned} \quad (30)$$

and  $\bar{P}$  and  $\check{P}$  are defined in Theorem 5. Furthermore, if there exist  $P > 0, Q_k > 0$ , ( $k = 1, 2, \dots, N$ ),  $X$ , and  $K$  satisfying (29), then the fault detection filter parameters in the form of (8) are given as follows:

$$[A_f \quad B_f] = (\widehat{E}^T P \widehat{E})^{-1} \widehat{E}^T X, \quad [C_f \quad D_f] = K. \quad (31)$$

*Proof.* First, let us rewrite the parameters in Theorem 5 in the following form:

$$\begin{aligned} \bar{A} &= \widehat{A}_0 + \widehat{E}L\widehat{R}_1, & \sum_{j=1}^m \sigma_j \widehat{A}_j &= \widehat{E}L\widehat{R}_4, \\ \bar{B} &= \bar{B}_0 + \widehat{E}L\widehat{R}_2, & & \\ \bar{C} &= K\widehat{R}_1, & \sum_{j=1}^m \sigma_j \widehat{C}_j &= K\widehat{R}_4, \\ \bar{D} &= \widehat{E}_0 + K\widehat{R}_2, & L &= [A_f \quad B_f]. \end{aligned} \quad (32)$$

Now, we can rewrite (18) by using Lemma 3 (Schur complement lemma) as follows:

$$\begin{bmatrix} \bar{P} & * \\ \widehat{\Gamma} & -\check{\mathcal{P}}^{-1} \end{bmatrix} < 0, \quad (33)$$

where

$$\begin{aligned} \widehat{\Gamma} &= \begin{bmatrix} \widehat{\Gamma}_{11} & \widehat{\Gamma}_{12} \\ \widehat{\Gamma}_{21} & 0 \end{bmatrix}, & \widehat{\Gamma}_{11} &= \begin{bmatrix} \widehat{A}_0 + \widehat{E}L\widehat{R}_1 & \bar{Z} \\ K\widehat{R}_1 & 0 \end{bmatrix}, \\ \widehat{\Gamma}_{12} &= \begin{bmatrix} \bar{B}_0 + \widehat{E}L\widehat{R}_2 \\ \widehat{E}_0 + K\widehat{R}_2 \end{bmatrix}, & \widehat{\Gamma}_{21} &= \begin{bmatrix} \widehat{E}L\widehat{R}_4 & 0 \\ K\widehat{R}_4 & 0 \end{bmatrix}. \end{aligned} \quad (34)$$

Pre- and postmultiplying inequalities (33) by  $\text{diag}\{I, \check{\mathcal{P}}\}$  and letting  $X = P\widehat{E}L$ , then we can obtain (29) readily. The proof of this theorem is complete.  $\square$

Now, according to previous Theorems 5 and 6, we can do further research about the system with uncertainties described in (7).

**Theorem 7.** Consider the uncertain fault detection system (10) with parameter uncertainties  $\Delta A$  and  $\Delta A_d$ , and let  $\gamma > 0$  be a given constant scalar which represents  $\mathcal{H}_\infty$  noise attenuation level bound. The desired full-order fault detection filter of form (8) exists if there exist matrices  $P > 0, Q_k > 0$ , ( $k = 1, 2, \dots, N$ ),  $X, K$ , and positive  $\varepsilon > 0$  satisfying

$$\begin{bmatrix} \Omega & * & * \\ \widehat{H}_a^T & -\varepsilon I & * \\ \varepsilon \widehat{E}_a & 0 & -\varepsilon I \end{bmatrix} < 0, \quad (35)$$

where

$$\begin{aligned} \bar{H}_a &= [0 \quad 0 \quad 0 \mid 0 \quad |\widehat{H}_a^T \quad 0 \mid 0 \quad 0]^T, \\ \bar{E}_a &= [\widehat{E}_a \quad \widehat{E}_d \quad 0 \mid 0 \quad |0 \quad 0 \mid 0 \quad 0], \\ \widehat{H}_a &= [H_a^T \quad 0]^T, & \widehat{E}_a &= [E_a \quad 0], \\ \widehat{E}_d &= \underbrace{[E_d \quad 0 \quad \dots \quad E_d \quad 0]}_N. \end{aligned} \quad (36)$$

Furthermore, if there exist appropriate matrices  $P > 0, Q_k > 0$  ( $k = 1, 2, \dots, N$ ),  $X$ , and  $K$  satisfying (35), then the fault

detection filter parameters in the form of (8) are given as follows:

$$[A_f \ B_f] = (\widehat{E}^T P \widehat{E})^{-1} \widehat{E}^T X, \quad [C_f \ D_f] = K. \quad (37)$$

*Proof.* According to result (29) in Theorem 6, we can replace  $A$  and  $A_d$  in (29) with  $A + H_a F(k) E_a$  and  $A_d + H_a F(k) E_d$ ; then we can obtain the following form:

$$\Omega + \overline{H}_a F(k) \overline{E}_a + \overline{E}_a^T F^T(k) \overline{H}_a^T < 0, \quad (38)$$

where the corresponding parameters have been defined in (36). According to Lemma 4, we can easily obtain (35), and the proof is then complete.  $\square$

*Remark 8.* From Theorem 7, we can know that the fault detection filter is designed such that the overall fault detection dynamics is exponentially stable in the mean square and, at the same time, the error between the residual signal and the fault signal is made as small as possible.

*Remark 9.* The main results in Theorems 5–7 can be applied to a wide class of network control systems that involve uncertainties, multiple communication delays, and stochastic missing measurements that result typically from networked environments. Sufficient conditions are established for the existence of the desired fault detection filters. The corresponding solvability conditions for the desired filter gains are established, and the explicit expression of such filter matrices is characterized in terms of the solution to a LMI that can be effectively solved.

#### 4. Numerical Example

In this section, we present an illustrative example to demonstrate the effectiveness of the proposed algorithm. Consider the following networked system with multiple communication delays and stochastic missing measurements:

$$\begin{aligned} x(k+1) &= (A + \Delta A) x(k) \\ &\quad + \sum_{m=1}^N (A_d + \Delta A_d) x(k-m) \\ &\quad + D_1 \omega(k) + E_f f(k), \\ y(k) &= \Xi C x(k) + D_2 \omega(k), \\ x(k) &= \psi(k) \quad k \in Z^-. \end{aligned} \quad (39)$$

The model parameters are given as follows:

$$\begin{aligned} A &= \begin{bmatrix} 0.6 & 0.2 \\ 0 & 0.7 \end{bmatrix}, & A_d &= \begin{bmatrix} 0.03 & 0 \\ 0.02 & 0.03 \end{bmatrix}, \\ D_1 &= \begin{bmatrix} 0.8 \\ 0.3 \end{bmatrix}, & E_f &= \begin{bmatrix} -1 \\ 0.6 \end{bmatrix}, & H_a &= \begin{bmatrix} 0.2 \\ 0.01 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} C &= \begin{bmatrix} 0.2 & -0.1 \\ 0.3 & -0.2 \end{bmatrix}, & D_2 &= \begin{bmatrix} 0.6 \\ 0.7 \end{bmatrix}, & \Xi &= \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix}, \\ E_a &= [0 \ 2], & E_d &= [0.1 \ 2], & F(k) &= \sin(k); \end{aligned} \quad (40)$$

$\beta_1$  and  $\beta_2$  are independent random variables whose probability density functions  $q(\beta_1)$  and  $q(\beta_2)$  satisfy

$$q(\beta_1) = \begin{cases} 0 & \beta_1 = 0 \\ 0.1 & \beta_1 = 0.5 \\ 0.9 & \beta_1 = 1, \end{cases} \quad q(\beta_2) = \begin{cases} 0 & \beta_2 = 0 \\ 0.2 & \beta_2 = 0.5 \\ 0.8 & \beta_2 = 1. \end{cases} \quad (41)$$

We can easily get the mathematical expectation and variance of  $\beta_1$  and  $\beta_2$ :  $u_1 = 0.95$ ,  $u_2 = 0.9$ ,  $\sigma_1 = 0.15$ , and  $\sigma_2 = 0.2$ .

By applying Theorem 7, we can obtain the desired  $\mathcal{H}_\infty$  filter parameters as follows:

$$\begin{aligned} A_f &= \begin{bmatrix} -0.2854 & -0.2854 \\ -0.2854 & -0.2854 \end{bmatrix}, & B_f &= \begin{bmatrix} -0.4828 & -0.1696 \\ -0.4828 & -0.1696 \end{bmatrix}, \\ C_f &= [5.2489 \ 5.2489], & D_f &= [-18.6850 \ 4.9809] \end{aligned} \quad (42)$$

with the optimized performance index  $\gamma^* = 16.61$ .

To further illustrate the effectiveness of the designed fault detection filter, we give a fault signal; for  $k = 0, 1, \dots, 150$ , let the fault signal  $f(k)$  be given as

$$f(k) = \begin{cases} 1, & 50 \leq k \leq 100 \\ 0, & \text{else.} \end{cases} \quad (44)$$

First, we assume our initial conditions as  $x(0) = [\pi/8 \ 0]^T$ ,  $\widehat{x}(0) = [0 \ 0]^T$ , and the external disturbance is  $w(k) = 0$ . The residual signal  $r(k)$  and evolution of residual evaluation function  $J(k)$  are shown in Figures 1 and 2, respectively, which indicate that the designed filter can detect the fault effectively when it occurs.

Next, we consider that the disturbance is given by

$$w(k) = \begin{cases} 0.5 \times \text{rand}[0 \ 1], & 30 \leq k \leq 130 \\ 0, & \text{else,} \end{cases} \quad (45)$$

where the rand function generates arrays of random numbers whose elements are uniformly distributed in the interval  $[0 \ 1]$ . Then, the residual signal  $r(k)$  and evolution of residual evaluation function  $J(k)$  are shown in Figures 3 and 4. Respectively, it can be seen that the residual can not only reflect the fault in time but also detect the fault without confusing it with the disturbance  $w(k)$ .

Selecting a threshold as  $J_{\text{th}} = \sup_{f(k)=0} \mathbb{E}\{\sum_{k=0}^{200} r^T(k)r(k)\}^{1/2}$  and accordingly obtaining that  $J_{\text{th}} = 36.9234$  in Figure 4 represented the Dotted curve after 200 Monte Carlo simulations with no faults. Solid curve represents the residual evaluation of the system. From Figure 4, it can be seen that  $36.6700 = J(76) < J_{\text{th}} < J(77) = 37.1069$ , which means that the fault can be detected in 27 time steps after its occurrence. From simulation results, it can be clearly observed that the smaller the threshold we obtain, the faster the fault detection will take.

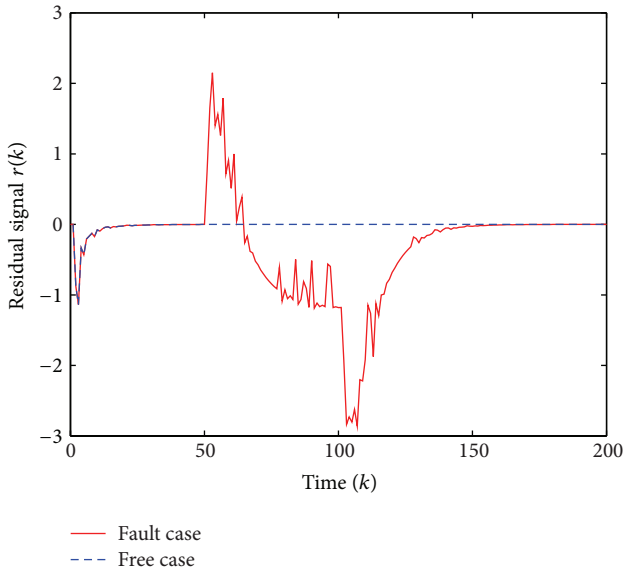


FIGURE 1: Residual signal  $r(k)$  without  $w(k)$ .

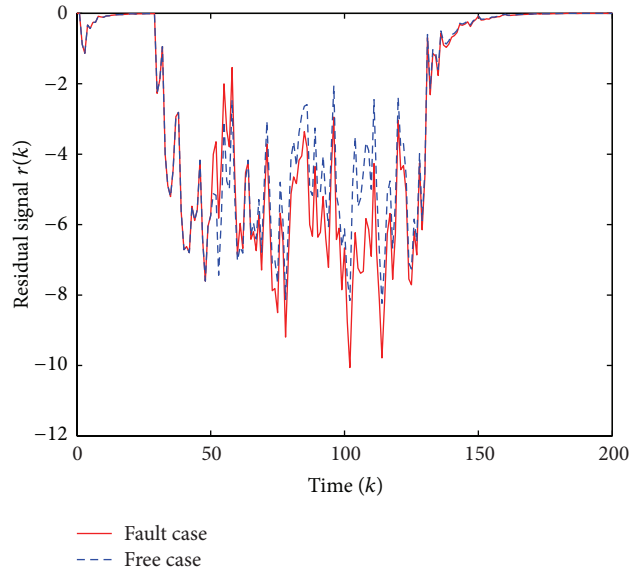


FIGURE 3: Residual signal  $r(k)$  with  $w(k) = 0.5 \times \text{rand}[0, 1]$ ,  $30 \leq k \leq 130$ .

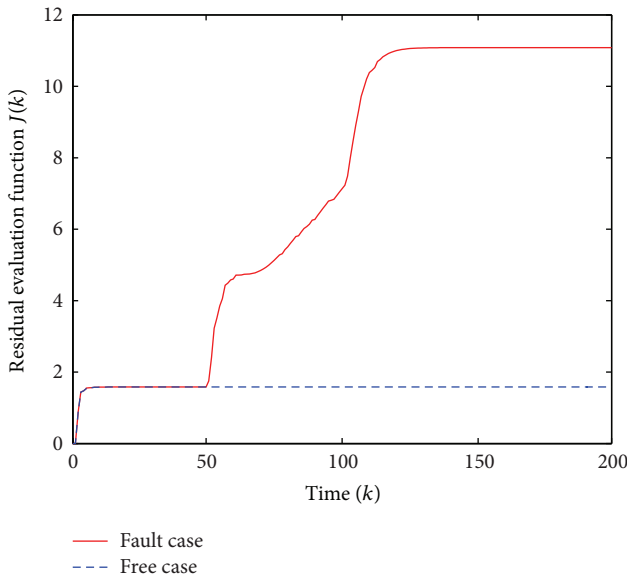


FIGURE 2: Evolution of residual evaluation function  $J(k)$  without  $w(k)$ .

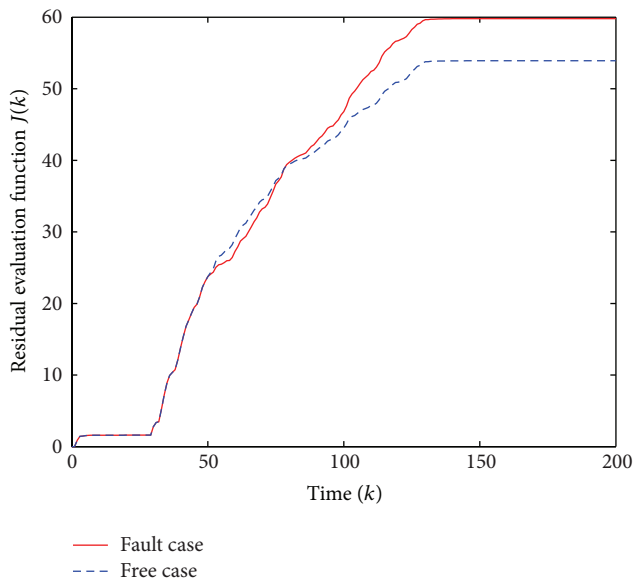


FIGURE 4: Evolution of residual evaluation function  $J(k)$  with  $w(k) = 0.5 \times \text{rand}[0, 1]$ ,  $30 \leq k \leq 130$ .

### 5. Conclusions

In this paper, we have addressed the fault detection problem for a class of network control systems comprising multiple communication delays and stochastic missing measurements. Our purpose is to build up a fault detection filter through an existing model of NCSs such that the overall fault detection dynamics is exponentially stable while preserving a guaranteed performance; at the same time, the error between the residual signal and the fault signal is made as small as possible. At the end, an illustrative simulation example has been given to demonstrate the effectiveness of the fault detection techniques presented in this paper.

### Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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