Finally making sense of the double-slit experiment: A quantum particle is never a wave

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Feynman stated that the double-slit experiment "...has in it the heart of quantum mechanics. In reality, it contains the only mystery," and "nobody can give you a deeper explanation of this phenomenon than I have given; that is, a description of it." We rise to the challenge with a novel alternative to the wavefunction-centered interpretations: instead of a quantum wave passing through both slits, we have a localized particle with non-local interactions with the other slit. Key to this explanation is dynamical nonlocality, which naturally appears in the Heisenberg picture as nonlocal equations of motion. This insight led us to develop a new approach to quantum mechanics using pre- and post-selection, weak measurements, deterministic and modular variables. We consider those properties of a single particle which are deterministic to be primal. The Heisenberg picture allows us to specify the most complete enumeration of such deterministic properties in contrast to the Schrödinger wavefunction which remains an ensemble property. We exercise this approach by analyzing a version of the double-slit experiment augmented with post-selection, showing only it, and not the wavefunction approach, can be accommodated within a time-symmetric interpretation, where interference appears even when the particle is localized. While the Heisenberg and Schrödinger pictures are equivalent formulations, nevertheless, the framework presented here has led to new insights, new intuitions and new experiments that were missed from the old perspective.

Heisenberg picture | modular momentum | double-slit experiment

Beginning with de Broglie [1], the physics community embraced the idea of particle-wave duality expressed e.g. in the double-slit experiment. The wave-like nature of elementary particles was further enshrined in the Schrödinger equation which describes the time evolution of quantum wave-packets.

It is often pointed out that the formal analogy between Schrödinger wave interference and classical wave interference allows us to interpret quantum phenomena in terms of the familiar classical notion of a wave. Indeed, wave-particle duality was construed by Bohr and others as the essence of the theory, and in fact its main novelty. Even so, the foundations of quantum mechanics community have consistently raised many questions (see [2–5]) centered on the *physical* meaning of the wavefunction.

From our perspective, and consistent with ideas first expressed by Born [6] and thereafter extensively developed by Ballentine [7, 8], a wavefunction represents an *ensemble* property as opposed to a property of an individual system.

What then is the most thorough approach to ontological questions concerning single particles (using standard non-relativistic quantum mechanics)? We propose an alternative interpretation for quantum mechanics relying on the Heisenberg picture, which, though mathematically equivalent to the Schrödinger picture, is very different both conceptually. For example, within the Heisenberg picture, the primitive physical properties will be represented by *deterministic operators*, which are operators whose measurements: 1) do not disturb individual particles and 2) have deterministic outcomes [9]. By way of example, the modular momentum operator will arise as particularly significant in explaining interference phenomena. This approach along with the use of Heisenberg's unitary equations of motion, introduce a notion of dynamical nonlocality. Dynamical nonlocality should be distinguished from the more familiar kinematical nonlocality (implicit in entangled states [10] and previously analyzed in the Heisenberg picture by Deutsch and Hayden [11]), because dynamical nonlocality has observable effects on probability distributions (unlike e.g. measurements of one out of two spins in Bell states which does not change their probability distribution). Within the Schrödinger picture, dynamical nonlocality is manifest in the unique role of *phases*, which, while unobservable locally, may subsequently influence interference patterns. Finally, in addition to the initial state of the particle, we will also need to take into account a final state in order to form a two-fold set of deterministic properties (one deterministic set based on the initial state and a second based on the final state). The above amounts to a time-symmetric Heisenberg-based interpretation of non-relativistic quantum mechanics.

Significance Statement

We put forth a *time-symmetric interpretation* of quantum mechanics which does not stem from the wave properties of the particle. Rather, it posits corpuscular properties along with *nonlocal* properties, all of which are *deterministic*. This suggests a new approach, which takes deterministic properties in the Heisenberg picture as primitive, instead of the wavefunction, which remains an ensemble property. This way, within a double-slit experiment, the particle goes only through one of the slits. In addition, a nonlocal property originating from the other, distant slit, has been affected through the Heisenberg equations of motion. Under the assumption of nonlocality, *uncertainty* turns out to be crucial in order to preserve causality. Hence, a (qualitative) uncertainty principle can be derived, rather than assumed.

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The wavefunction represents an ensemble property

The question of the meaning of the wavefunction is central to many controversies concerning the interpretation of quantum mechanics. We adopt neither the standard ontic nor the epistemic approaches to the meaning of the wavefunction. Rather, we consider the wavefunction to represent an *ensemble* property as opposed to a property of an individual system. This resonates with the *ensemble interpretation* of the wavefunction which was initiated by Born [6] and extensively developed by Ballentine [7, 8]. According to this interpretation, the wavefunction is a statistical description of a hypothetical ensemble, from which the probabilistic nature of quantum mechanics stems directly. It does not apply to individual systems. Ballentine justified an adherence to this interpretation by observing that it overcomes the measurement problem - by not pretending to describe individual systems, it avoids having to account for *state reduction* (collapse). We concur with Ballentine's conclusion but not with his reasoning. Instead, we contend that the wavefunction is appropriate as an ontology for an ensemble rather than an ontology for an individual system. Our principle justification for this is because the wavefunction can only be *directly verified* at the ensemble level. By "directly verified" we mean measured to an arbitrary accuracy in an arbitrarily short time (excluding practical and relativistic constraints).

Indeed, we only regard directly verifiable properties to be intrinsic. Consider for instance how probability distributions relate to single particles in statistical mechanics. We can measure, e.g., the Bolztmann distribution, in two ways - either instantaneously on thermodynamic systems or using prolonged measurements on a single particle coupled to a heat bath. We do not attribute the distribution to single particles because instantaneous measurements performed on single particles yield a large error. Conversely, when the system is large, containing $N \gg 1$ particles (the thermodynamic limit), the size of the error, which scales like $\sqrt{N}$, is relatively very small. In other words, the verification procedure transitions into the category of being "directly verified" only as the system grows. Because of this, the distribution function is best viewed as a property of the entire thermodynamic system. On the single particle level, it manifests itself as probabilities for the particle to be found in certain states. However, the intrinsic properties of the individual particle are those which can be verified directly, namely position and momentum, and only they constitute its real properties.

Similarly as to how distributions in statistical mechanics can be directly verified only on a thermodynamic system, the wavefunction can be directly verified only on quantum ensembles. Continuing the analogy, on a single particle level, the wavefunction can only be measured by performing a prolonged measurement. This prolonged measurement is a *protective* measurement [12]. Protective measurements can be implemented in two different ways: the first is applicable for measuring discrete non-degenerate energy eigenstates and is based on the adiabatic theorem [13]; the second, more general way, requires an external protection in the form of the quantum Zeno effect [14]. In either of these two ways, a large number of identical measurements is required in order to approximate the wavefunction of a single particle. We conclude that analogous to how statistical mechanical distributions become properties for thermodynamic systems, the wavefunction is a property of a quantum *ensemble*.

Unlike Born, we do not wish to imply that the wavefunction description is somehow incomplete (and could become 'complete' with the addition of a classical-like reality, such as with a hidden variable theory). Nor do we oppose the consequence of the PBR theorem [15] which states that the wavefunction is determined uniquely by the physical state of the system. We only mean to suggest that the wavefunction cannot constitute the primitive ontology of a single quantum particle/system. That being said, and contrary to ensemble interpretation advocates, we will not duck out of proposing a single-particle ontology. In what follows, we expound such an ontology based on *deterministic operators*, which are unique operators whose measurement can be carried out on a single particle without disturbing it and with predictable, definite, outcomes. Since properties corresponding to these operators can be directly verified at the single particle level, they constitute the *real* properties of the particle. In order to derive this ontology, we turn the spotlight to the Heisenberg representation.

Formalism and ontology

In the Schrödinger picture, a system is fully described by a continuous wavefunction ψ. Its evolution is dictated by the Hamiltonian and calculated according to Schrödinger's equation. As will be shown below, in the Heisenberg picture, a physical system can be described by a set of Hermitian *deterministic operators*, evolving according to Heisenberg's equation while the wavefunction remains constant.

In the traditional Hilbert space framework for quantum mechanics along with ideal measurements, the state of a system is a vector $|\psi\rangle$ in a Hilbert space $\mathcal{H}$ and any observable $\hat{A}$ is a Hermitian operator on $\mathcal{H}$. The eigenstates of $\hat{A}$ form a complete orthonormal system for $\mathcal{H}$. When an ideal measurement of $\hat{A}$ is performed, the outcome appears at random (with a probability given by initial $|\psi\rangle$) and corresponds to an eigenvalue within the range of $\hat{A}$'s allowed spectrum. Thereafter, from the perspective of the Schrödinger picture, the ideal measurement leads to the "collapse" (true or effective, depending on one's preferred interpretation) of the wavefunction from $|\psi\rangle$ into an eigenstate corresponding to that eigenvalue. This can be verified by performing subsequent ideal measurements which will yield the same eigenvalue. This "collapse" corresponds to a disturbance of the system.

On the other hand, one could invert the process and consider non-disturbing measurements of the "deterministic subset of operators" (DSO). This set involves measurement of only those observables for which the state of the system under investigation is already an eigenstate. Therefore, no collapse is involved. This set answers the question "what is the set of Hermitian operators $\hat{A}_\psi$ for which ψ is an eigenstate?" for any state ψ:

$$\hat{A}_\psi = \{\hat{A}_i \text{ such that } \hat{A}_i|\psi(t)\rangle = a_i|\psi(t)\rangle\,, a_i \in \mathbb{R}\}. \quad [1]$$

This question is dual to the more familiar question "what are the eigenstates of a given operator?" Clearly, $\hat{A}_\psi$ is a subspace closed under multiplication. Moreover, $[\hat{A}_i, \hat{A}_j] = \hat{A}_k \in \hat{A}_\psi$ is such that $\hat{A}_k|\psi\rangle = 0$.

Theorem [16]: Let $\mathcal{H}$ be a Hilbert space, $\hat{A}$ be an operator acting on it, and $|\psi\rangle \in \mathcal{H}$. Then

$$\hat{A}|\psi\rangle = \langle\hat{A}\rangle|\psi\rangle + \Delta A|\psi_\perp\rangle, \quad [2]$$

where $\langle\hat{A}\rangle = \langle\psi|\hat{A}|\psi\rangle$, $\Delta A^2 = \langle\psi|(\hat{A} - \langle\hat{A}\rangle)^2|\psi\rangle$, and $|\psi_\perp\rangle$ is a vector such that $\langle\psi|\psi_\perp\rangle = 0$.

The physical significance of DSO stems from the possibility to measure them without disturbing the particle, i.e. without inducing collapse. As long as only such eigenoperators are measured, they all evolve unitarily by applying Heisenberg's equation separately to each of them. DSOs (whose measurement outcomes are completely certain) are dual to the "completely uncertain operators," whose measurement outcomes are completely *un*certain. Complete uncertainty means that they satisfy the condition that all their possible measurement outcomes are equiprobable [17]. Thus, no information can be gained by measuring them. Mathematically, the two limiting cases represented by Eq. 2 are given by *deterministic* operators for which $\Delta A|\psi_\perp\rangle = 0$ and *completely uncertain operators* for which $\langle\hat{A}\rangle = 0$ (a necessary but insufficient condition as will be described below).

An important ingredient to consider of our proposed interpretation is a final state of the system. The idea that a complete description of a quantum system at a given time must take into account two boundary conditions rather than one is known from the two-state vector formalism (TSVF). This approach has its roots in the works of Aharonov, Bergman and Lebowitz [18], but it has since been extensively developed [19], and has led to the discovery of numerous interesting phenomena [17].

The TSVF provides an extremely useful platform for analyzing experiments involving pre- and post-selected ensembles. *Weak measurements* enable us to explore the state of the system during intermediate times without disturbing it [20, 21]. The power to explore the pre- and post-selected system by employing weak measurements motivates a literal reading of the formalism, that is, as more than just a mathematical tool of analysis. It motivates a view according to which future and past play equal roles in determining the quantum state at intermediate times, and are hence equally *real*. Accordingly, in order to fully specify a system, one should not only pre-select, but also post-select a certain state using a projective measurement. In the framework we propose within this article, adding a final state is equivalent to adding a second DSO in addition to the one dictated by the initial state. This two-fold set form the basis for the primal ontology of a quantum mechanics for individual particles.

Nonlocal dynamics and wave-like behaviour

Interference patterns appear in both classical and quantum grating experiments (most conveniently analyzed in a double-slit setup, which will be referred to hereinafter, even though our results are completely general). We are taught that the explanation for interference phenomena is shared across both domains, the classical and quantum: a spatial wave(function) traverses the grating, one part of which goes through the first slit while the other part goes through the second slit, before the two parts later meet to create the familiar interference pattern. While it is indeed tempting to extend the accepted classical explanation into the quantum domain, nevertheless, there are important breakdowns in the analogy. For example, in classical wave theory, one can predict what will happen when the two parts of the wave finally meet based on entirely *local* information available along the trajectories of the wavepackets going through the two slits. However, in quantum mechanics, what tells us where the maxima and minima of the interference will be located is the *relative* phase of the two wavepackets. While we can measure the local phase in classical mechanics, we cannot in principle measure the individual local phases for a particle since this would violate gauge symmetry [17]. Only the phase *difference* is observable, but it cannot be deduced from measurements performed on the individual wavepackets (until they overlap). The analogy is therefore only partial. For this reason, we contend that the temptation to jump on the wavefunction bandwagon should be resisted. Our goal now is to show how quantum interference can be understood without having to say that each particle passed through both slits at same time as if it were a wave. For this purpose, we examine those operators that are relevant for all interference phenomenon. When we transform back to the Schrödinger picture and apply these operators, we will see that these operators are sensitive to the relative phase, which, again, is the property which determines the subsequent interference pattern.

We therefore consider the state $\psi_\phi(x,t) = \psi_1(x,t) + e^{i\phi}\psi_2(x,t)$ which, in the Schrödinger picture represents the wave at the double-slit. We now ask which operators $\hat{f}(x,p)$ belong to the DSO, $\hat{A}_{\psi_\phi}$. In addition, we ask which operators are sensitive to the relative phase ϕ. It is not difficult to show that if we limit ourselves to simple functions of position and momentum, i.e. any polynomial representation of the form:

$$\hat{f}(x,p) = \sum a_{mn} x^m p^n$$

then any resulting operator is *not* sensitive to the relative phase between different "lumps" of the wavefunction (i.e. lumps centered around each slit). This suggests that simple moments of position and momentum are not the most appropriate dynamical variables to describe quantum interference phenomena. Indeed it is easy to prove the following:

Theorem [16]: Let $\psi_\phi(x,t) = \psi_1(x,t) + e^{i\phi}\psi_2(x,t)$ and assume no overlap of $\psi_1(x,0)$ and $\psi_2(x,0)$ ($t=0$ is when the particle is going through the double-slit). If m,n are integers, then for all values of t, and choices of phases α, β:

$$\int [\psi_\alpha^*(x,t)x^m p^n \psi_\alpha(x,t) - \psi_\beta^*(x,t)x^m p^n \psi_\beta(x,t)]\,dx = 0 \quad [3]$$

Let us now consider operators of the form $\hat{f}(x,p) := e^{ipL/\hbar}$ (where L is the distance between the slits). Evolving this through the Heisenberg equation:

$$i\hbar\frac{\partial\hat{f}(x,p)}{\partial t} = [\hat{f},\hat{H}],$$

where $H = p^2/2m + V(x)$ appropriate for the double-slit. In this particular case, we obtain a *nonlocal* equation of motion:

$$\frac{\partial\hat{f}(x,p)}{\partial t} = [e^{ipL/\hbar}, V(x)] = \frac{1}{\hbar}[V(x+L) - V(x)]e^{ipL/\hbar}, \quad [4]$$

that is, the value of $\dot{\hat{f}}$ depends not only on the potential at x but also at the remote $x+L$. This operator leads us naturally to realize that the variable that accounts for the effect of the double-slit is not p but its modular version. Indeed, since

$$e^{ipL/\hbar} = e^{i(p+2\pi k\hbar/L)L/\hbar}, \qquad k \in \mathbb{Z}$$

the observable of interest is the modular momentum, i.e.

$$p_{mod} := p \bmod p_0,$$

where $p_0 = 2\pi\hbar/L$. Eq. (4) differs considerably from the classical evolution which is given by the Poisson bracket:

$$\frac{d}{dt}e^{i2\pi p/p_0} = \left\{e^{i2\pi p/p_0}, \hat{H}\right\} = -i\frac{2\pi}{p_0}\frac{dV}{dx}e^{i2\pi p/p_0}, \qquad [5]$$

which involves a *local derivative*, suggesting that the classical modular momentum changes only if a *local* force $\frac{dV}{dx}$ is acting on the particle. We thus understand that even though commutators have a classical limit in terms of Poisson brackets, they are fundamentally different because they entail nonlocal dynamics. The connection between nonlocal dynamics and relative phase via the modular momentum suggests the possibility of the former taking the place of the latter in the Heisenberg picture. The nonlocal equations of motion in the Heisenberg picture thus allow us to consider a particle going through only one of the slits, yet it nevertheless has nonlocal information regarding the other slit.

Unlike ordinary momentum, modular momentum becomes, upon detecting (or failing to detect) the particle at a particular slit, *maximally* uncertain. The effect of introducing a potential at a distance from the particle (i.e. of opening a slit) is equivalent to a nonlocal rotation in the space of the modular variable (see [22]). Denote it by $\theta \in [0, 2\pi)$. Suppose the amount of nonlocal exchange is given by $\delta\theta$ (i.e. $\theta \to \theta + \delta\theta$). Now "maximal uncertainty" means that the probability to find a given value of θ is independent of θ, i.e. $P(\theta) = constant = \frac{1}{2\pi}$. Under these circumstances, the shift in θ to $\theta + \delta\theta$ will introduce no observable effect, since the probability to measure a given value of θ, say θ_1, will be the same before and after the shift, $P(\theta_1) = P(\theta_1 + \delta\theta_1)$. We shall call a variable that satisfies this condition a "completely uncertain variable".

Theorem (Complete uncertainty principle for modular variables) [17]: Let Φ be a periodic function, which is uniformly distributed on the unit circle. If $\langle e^{in\Phi}\rangle = 0$ for any integer $n \neq 0$, then Φ is completely uncertain.

When a particle is localized to within $|x| < L/2$, the expectation value of $e^{ipL/\hbar}$ vanishes. This is obvious since $e^{ipL/\hbar}$ functions as a translation operator, shifting the wavepacket outside $|x| < L/2$, i.e. outside its region of support. Accordingly, when a particle is localized near one of the slits, as in the case of either ψ_1 or ψ_2, then $\langle e^{inpL/\hbar}\rangle = 0$ for every n. It then follows from the complete uncertainty principle that the modular momentum is completely uncertain. Accordingly, all information about the modular momentum is lost once we find the position of the particle. This onset of complete uncertainty is crucial in order to prevent signaling and preserve causality. As an example, suppose we apply a force arbitrarily far away from a localized wavepacket. We thus change operators depending on the modular momentum instantly, since modular momentum relates remote points in space. If we could measure this change on the wavepacket then we could violate causality, but all such measurements are precluded by the complete uncertainty principle.

The fact that the modular momentum becomes uncertain upon localization of the particle also fits well with the fact that interference is lost with localization. In the Schrödinger picture, interference loss is understood as a consequence of wavefunction collapse. Once the superposition is reduced, there is nothing left for the remaining localized wavepacket to interfere with. The Heisenberg picture, however, offers a different explanation for the loss of interference which is not in the language of collapse: if one of the slits is closed by the experimenter, a nonlocal exchange of modular momentum with the particle occurs. Consequently, the modular momentum becomes completely uncertain, thereby erasing interference and destroying the information about the relative phase.

Note also that since $p = p_{mod} + N\hbar/L$ for some integer N, the uncertainty of p is greater or equal to that of p_{mod} (the integer part can be uncertain as well). For this reason, a complete uncertainty of the modular momentum p_{mod} (which means its distribution function is uniform in the interval $[0, \hbar/L)$) sets $\hbar/L$ as a lower bound for the uncertainty in p, i.e. $\Delta p \geq \hbar/L$. This inequality parallels the Heisenberg uncertainty principle, equating it in the case of $\Delta x = L$.

At first blush, it appears that, as axioms, dynamical nonlocality and relativistic causality nearly contradict each other. Nevertheless, by prohibiting the detection of nonlocal action, complete uncertainty enables one to reconcile nonlocality with relativistic causality so that they may "peacefully co-exist." This is why we regard this principle as very fundamental.

Measuring nonlocal operators

Consider a system described at time $t = 0$ by a vector $|\psi\rangle$ in a Hilbert space. Fundamental properties of operator valued functions allow us to reconstruct $|\psi\rangle$ using weak measurements of the position of the particle at various instants t. Indeed, if we call $\rho(x,t)$ the density of $\psi(x,t)$, namely

$$\rho(x,t) = \psi^*(x,t)\psi(x,t)$$

then we can calculate its Fourier transform

$$\mathcal{F}\rho(k,t) = \int_{\mathbb{R}} \psi^*(x,t)\psi(x,t)e^{ikx}\,dx. \qquad [6]$$

For a given operator $\hat{A}$ we can write its expectation value as

$$\overline{\hat{A}_x(t)} = \langle\psi(x,t)|\hat{A}|\psi(x,t)\rangle \qquad [7]$$

therefore Eq. (6) is nothing but the expectation value of e^{ikx}. Note that in Eq. (7) we have been using the Schrödinger picture with a time-evolving state $\psi(x,t)$. Re-writing Eq. (7) in the Heisenberg picture:

$$\langle\psi(x,t)|\hat{A}|\psi(x,t)\rangle = \langle\psi(x,0)|\hat{A}(t)|\psi(x,0)\rangle$$

We know the two pictures are equivalent: the time evolution has simply been moved from the vector in the Hilbert space to the operator. Given that $x(t) = x(0) + p(0)\frac{t}{m}$ we have

$$\overline{e^{ikx(t)}} = \overline{e^{ik(x(0)+p(0)\frac{t}{m})}}.$$

If we set $\alpha = k$, $\beta = k\frac{t}{m}$ we see that, as time t changes,

$$\overline{e^{i(\alpha x(0)+\beta p(0))}}$$

assumes all the possible values. Hence, nonlocal operators at $t = 0$ can be measured locally at some later time. The following theorem shows that this description is exhaustive.

Theorem: The collection, for all $(\alpha, \beta) \in \mathbb{R}^2$,

$$f(\alpha,\beta) = \int_{\mathbb{R}} \psi^*(x)e^{i(\alpha x+\beta p)}\psi(x)\,dx,$$

uniquely determines the state ψ.

Proof: Integration with respect to α lets us find $\psi^*(0)\psi(\beta)$ for all β. This amounts to finding $\psi(x)$ when setting $\psi(0) = 1$.

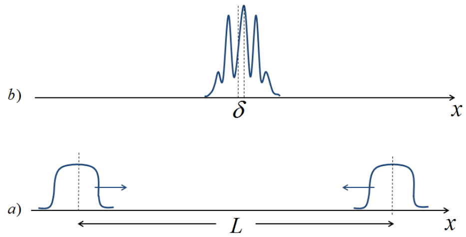

Fig. 1. Interference of two wavepackets. (a) The density of the initial superposition (8) of the two wavepackets. (b) The interference pattern at the time T when the wavepackets completely overlap. The shift δ of the interference pattern is proportional to the relative phase ϕ.

The double-slit experiment revisited

Performing certain experiments involving post-selection allows us both to measure interference and deduce which-path information. But the Schrödinger picture is very awkward with such experiments which posit both wave and particle properties *at the same time*. Alternatively, in the Heisenberg picture, the particle has both a definite location and a nonlocal modular momentum which can "sense" the presence of the other slit and therefore create interference. This description thus evades difficulties present in the Schrödinger picture.

To emphasize this point, let us consider a simple one-dimensional Gedanken experiment to mimic the double-slit experiment. In the Schrödinger picture, a particle is prepared in a superposition of two identical spatially separated wavepackets moving toward one another with equal velocity (Fig. 1):

$$\Psi_i(x, t=0) = \frac{1}{\sqrt{2}}[e^{ip_0x/\hbar}\Psi(x+L/2)+e^{i\phi}e^{-ip_0x/\hbar}\Psi(x-L/2)], \quad [8]$$

where $\Psi(x)$ is a Gaussian wavefunction. To simplify, we assume the spread Δx obeys $\hbar/p_0 \ll \Delta x \ll L$, hence, the wavepacket approximately maintains its shape up to the time of encounter (our results are general however). The relative phase ϕ has no effect on the local density $\rho(x)$ or any other local feature until the two wavepackets overlap. The phase ϕ manifests itself by shifting the interference pattern by $\delta = \frac{\hbar\phi}{p_0}$.

This initial configuration is identical to that of the standard double-slit setup, but instead of letting the two wavepackets propagate away from the grating to hit a photographic plate, we confine ourselves to one dimension and let them meet at time T on the plane of the grating. Upon meeting, the density of the two wavepackets becomes

$$\rho(x, T) \approx 4|\Psi_i(x)|^2\cos^2(p_0x/\hbar - \phi/2), \quad [9]$$

which displays interference, similar to that of a standard double-slit experiment.

We now augment the experiment with a post-selection procedure, where we place a detector on the path of the wavepacket moving to the right $\Psi_f(x) = e^{ip_0x/\hbar}\Psi(x-L/2)$. While the probability to find the particle there is $\frac{1}{2}$, let us consider an ensemble of such pre- and post-selected experiments which realizes the rare case where *all* the particles are found by this detector (that is, we determine the position operator for the entire ensemble by a post-selection). The two-state, which constitutes the full description of pre- and post-selected systems at any intermediate time t, is given by $\langle\Psi_f(t)|\ |\Psi_i(t)\rangle$.

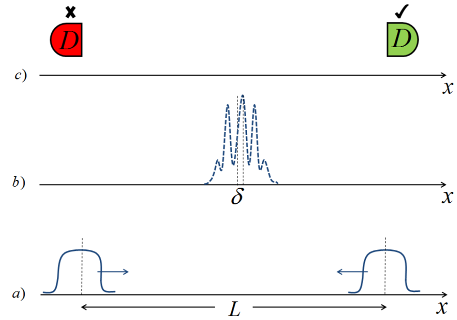

Fig. 2. Weak measurement of the interference pattern. The two wavepackets are pre-selected in (a) and post-selected in b. Weak measurements in (b) performed at $t = T$ show the usual interference pattern in spite of the fact that detector D detects all particles as belonging to just one (moving to the right) wavepacket.

Within the TSVF, we can define a two-times generalization of the pure-state density:

$$\rho_{two-time}(x, T) = \frac{\langle x|\Psi_f\rangle\langle\Psi_i|x\rangle}{\langle\Psi_f|\Psi_i\rangle} = 2|\Psi(x)|^2e^{i(p_0x/\hbar-\phi/2)}\cos(p_0x/\hbar - \phi/2). \quad [10]$$

To measure this density, during intermediate times we perform a weak measurement using $M \gg 1$ projections $\Pi_i(x)$ with the interaction Hamiltonian $H_{int} = g(t)q\sum_i^M \Pi_i(x)$, where q is the pointer of the measuring device, i sums over an ensemble of particles, and $\int_0^\tau g(t)dt = g$ is sufficiently small during the measurement duration τ. For a large enough ensemble, these measurements allow us to observe the two-time density while introducing almost no disturbance to the state of the particle. If we perform many such measurements in different locations within the overlap region, they will add up to a histogram tracing the two-time density in that region (Fig. 2) from which we find the parameter δ which depends on the relative phase ϕ. This gedanken experiment demonstrates a perplexing situation from the point of view of the Schrödinger picture. The real part of this density, which describes the evolution of the two-state, exhibits an interference pattern when weakly measured. However, by virtue of the post-selection, we know that the particle has a determinate position, described by a right-moving wavepacket which went through the left slit. Interference is thus still present despite the fact that the particle is localized around one of the slits. Recall that the interpretation of the particle as having a wave-like nature was originally devised in order to account for interference phenomena, and here we have shown that this is not necessary and in fact inconsistent with a time-symmetric view.

In contrast, the Heisenberg picture tells us that each particle has both a definite position, and, at the same time, it also has nonlocal information in the form of DSOs which are simple functions of the modular momentum [9].

Discussion

After the Schrödinger picture has dominated for many years, we have elaborated a new Heisenberg-based interpretation for quantum mechanics. In this interpretation, individual particles possess deterministic yet nonlocal properties which have no classical analog, whereas the Schrödinger wave can only describe an ensemble. An uncertainty principle appears not as a mathematical consequence, but as a reconciler between metaphysical desiderata - causality and the nonlocality of the dynamics. This complete uncertainty principle (qualitatively) implies the Heisenberg uncertainty principle, but not the other way around. For this reason, we regard it as more fundamental. In turn, uncertainty combined with the empirical demand for definite measurement outcomes, necessitates a mechanism for choosing those outcomes. This demand is met by the inclusion of a final state. It was shown elsewhere [23], that by considering a special final state of the kind we had introduced, but for the entire Universe, the outcomes of specific measurements can be accounted for. This cosmological generalization thereby solves the *measurement problem*. We now understand this final state to constitute a DSO, which may be regarded as hidden variables due to their epistemic inaccessibility in earlier times.

We contend that this interpretation conveys a powerful physical intuition. Internalizing it, one is no longer restricted to thinking in terms of the Schrödinger picture, which is a convenient tool for mathematical analysis, but inconsistent with the pre and post-selection experiments. The wavefunction is an efficient mathematical tool for calculations of experimental statistics. But the use of potential functions is also mathematically efficient even though it is only the fields derived from potentials which are physically real. Hence, mathematical usefulness is not a sufficient criterion by which to fix an ontology. Indeed, while useful for calculating the dynamics of DSOs, wavefunctions are not the real physical objects - only DSOs themselves are. Importantly, considerations pertaining to this ontology have led Aharonov to discover the Aharonov-Bohm effect. The stimulation of new discoveries is the ultimate metric to judge an interpretation.

Intriguingly, the Heisenberg representation which was discussed here from a foundational point of view, is also a very helpful framework for discussing quantum computation [24]. Moreover, in several cases [25], it has a computational advantage over the Schrödinger representation.

For the sake of completeness, it might be interesting to briefly address the notion of kinematic nonlocality arising from entanglement. As noted in Sec. III, a quantum system in two-dimensional Hilbert space, e.g. a spin-1/2 particle, is described within our formalism using two DSOs. For describing a system of two entangled spin-1/2 particles (in a four-dimensional Hilbert space), we would utilize a set of 10 DSOs. It is important to note that the measurements of such operators are nonlocal [26], possibly carried out in space-like separated points. Most of these operators involve simultaneous measurements of the two particles. A (non-deterministic) measurement of one particle would change the combined DSOs, thus instantaneously affecting also the ontological description of the second particle. In [11]. There it was claimed that the information flow in the Heisenberg representation is local, however, in light of the above analysis, this only refers to certain kinds of operators.

We believe that if quantum mechanics were discovered before relativity theory, then our proposed ontology could have been the commonplace one. Before the 20th century, physicists and mathematicians were interested in studying various Hamiltonians having an arbitrary dependence on the momentum, such as $\cos(p)$. In quantum mechanics, these Hamiltonians lead to nonlocal effects as discussed above. The probability current is not continuous under the resulting time-evolution, which makes the wavefunction description less intuitive. However, those Hamiltonians were dismissed as non-physical in the wake of relativity theory, allowing the wavefunction ontology to prosper. We hope that our endorsement of the Heisenberg-based ontology will promote a discussion of this somewhat neglected approach.

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