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MATHEMATICS AND GEOMETRY TOWARDS IDEALITY IN «DOMUS»'S IDEAL HOUSES

1. The philosophical foundations: Plato and Kant

When Plato reasons on dialectics, which is the highest form of knowledge, since it moves from particularity to universality, he argues that the first two steps towards dialectics are arithmetic and geometry. The question is: «What sort of knowledge is there which would draw the soul from becoming to being?» (*Resp.* 521d), namely, from the disordered instability of what varies to the ordered stability of what does not vary? And the answer is that the first step is what moves to «What is absolute unity» (*Resp.* 525a), namely, «the way in which the study of the one has a power of drawing and converting the mind to the contemplation of true being» (*Resp.* 525a). More precisely, «all arithmetic and calculation [...] appear to lead the mind towards truth [...] in a very remarkable manner» (*Resp.* 525a-b): «arithmetic has a very great and elevating effect, compelling the soul to reason about abstract number, and rebelling against the introduction of visible or tangible objects into the argument» (*Resp.* 525d), since in «those numbers which can only be realized in thought» (*Resp.* 526a) «there is a unity such as you demand, and each unit is equal, invariable, indivisible» (*Resp.* 526a). Thus, «arithmetic is a kind of knowledge in which the best natures should be trained» (*Resp.* 526c) and «which legislation may fitly prescribe» (*Resp.* 525b): «we must endeavour to persuade those who are to be the principal men of our State to go and learn arithmetic, not as amateurs, but they must carry on the study until they see the nature of numbers with the mind only» (*Resp.* 525 b-c), and «the philosopher also, because he has to rise out of the sea of change and lay hold of true being, and therefore he must be an arithmetician» (*Resp.* 525b). And geometry is the second step towards dialectics, since «the knowledge at which geometry aims is knowledge of the eternal, and not of aught perishing and transient» (*Resp.* 527b). At last, the dialectic-

tician, «as one who attains a conception of the essence of each thing» (*Resp.* 534b), can be formed.

Plato's arguments found the nature of Western knowledge: knowing means passing from particularity to universality – knowing means passing from reality (where we can find, for instance, the irreducible variety of one hundred particular houses) to ideality (where we can find, for instance, the reduced unity of one house as we understand that the one hundred particular houses share the one essence of being a house). Arithmetic and geometry are the first two steps towards ideality as they make use of numbers, namely, of the capacity of passing from variety to unity – arithmetic and geometry make use of what may be thought of as the first cornerstone of Western knowledge: *abstraction*. Firstly, we identify one hundred different real houses and, secondly, we identify the one reason why we use the word 'house' for the one hundred different real houses, namely, we distinguish what does not vary (and we retain it) from what varies (and we do not retain it). The former (what does not vary) does not coincide with a real house, since it can be identified within our thought, and not within reality, but it is essential for the latter (for what varies) for two reasons: it gives the latter the possibility, firstly, of being identified and, secondly, of evolving from a more imperfect real status (for instance, the real house X characterised by the flaw $F_{(X)}$) to a less imperfect real status (for instance, the real house Y characterised by the flaw $F_{(Y)}$, being $F_{(Y)}$ the evolution of $F_{(X)}$). Thus, abstraction is a most powerful tool for making reality evolve.

Plato's ideality gives Western knowledge another most powerful tool, which may be thought of as its second cornerstone: *idealisation*. Once we have identified what does not vary in one hundred different real houses, we can use it as the basis of a sort of perfect model (perfected by us by adding something or by subtracting something), which is ideal, and not real. Starting from Plato's notion of idea, Kant works on a notion of ideal which is fundamental to understand the use of the ideal model in Western thought, from sciences to arts. According to Kant, the ideal results, firstly, from what we may call an abstraction on the basis of the «aesthetic normal idea» (Kant 1790, 5: 233), where our imagination can «superimpose one image on another and by means of the congruence of several of the same kind [...] arrive at a mean that can serve them all as a common measure» (Kant 1790, 5: 234), and, secondly, from, what we may call an idealisation on the basis

of the «idea of reason» (Kant 1790, 5: 233), where our reason «determines a priori the end on which the internal possibility of the object rests» (Kant 1790, 5: 234) through a sort of perfecting. At last, the resulting ideal works as the perfect model: the ideal is «the original image (*prototypon*) of all things, which all together, as defective copies (*ectypa*), take from it the matter for their possibility» (Kant 1781, A 578/B 606). More precisely, the ideal is our best standard: «we have in us no other standard for our actions than the conduct of this divine human being, with which we can compare ourselves, judging ourselves and thereby improving ourselves» (Kant 1781, A 569/B 597), since the «ideals, even though one may never concede them objective reality (existence), are nevertheless not to be regarded as mere figments of the brain; rather, they provide an indispensable standard for reason, which needs the concept of that which is entirely complete in its kind, in order to assess and measure the degree and the defects of what is incomplete» (Kant 1781, A 569/B 597-A 570/B 598). Again, what the ideal is for is the real – the ideal works to make the real evolve.

2. The ideal house: the fifteen projects in «Domus»

The use of the ideal for the evolution of the real is precisely the objective of the fifteen projects published in «Domus» between 1942 and 1943. The editor of the journal invited the most important Italian architects «to design the impossible, to push [...] the vision till when, as it is seen, the nostalgia of it as an already missed thing arises, irremediably» («Domus» 1942, 312)¹.

We should start by highlighting two aspects. The first is that, indeed, the seventeen architects who designed the fifteen projects (two projects are co-authored) had quite a strong philosophical background (in particular, the Milanese schools of architecture and philosophy were strictly intertwined), which strengthens the meaning of a philosophical reading of their projects. The second is that, from a rigorously philosophical perspective, they do not design ideal houses, but real houses methodologically founded on the thought of the ideal house, since, as we learn from both Plato and Kant, what is ideal cannot be made real – ideality is a standard, and not an objective, for reality.

Now, let us consider an overview of the fifteen projects before focusing on the mathematical and geometric tools chosen as

¹ The translations from «Domus» are mine.

the most promising to design houses methodologically founded on the thought of the ideal house. We should start by arguing that there is no thought of the ideal house without the thought of the ideal human being for whom to design the house. Thus, there is a sort of double abstraction: one which addresses the question of what any possible house shares with the others as an invariant and one which addresses the question of what any possible human being shares with the others as an invariant, since the latter founds the former, as it is best exemplified by the words of Peressutti (who thinks of «every man's life», 1942, 313), of Belgioioso (who wants «to express much more general problems concerning living architecture: I would like to have contributed to the generality of cases», 1942, 327), of Rogers (who works on the «house for everyone, for you, for me, for the Anonymous», 1942, 333) and of Cattaneo (who asks himself what the house should be «considering man's life», 1942, 500). Indeed, the overview of the fifteen projects may result from the *ideal human being* the seventeen architects think of:

1. the first thing they share is that the ideal human being is a part, and not a counterpart, of nature, and has to follow its, and not autonomous, rules. This is represented by the ideal house in two ways in particular: by designing it within nature and by designing it in continuity with nature. In the first case, we have Peressutti's house, which is within «the horizon, the sky, the sea, the earth, the greenery, some rocks» (1942, 313), Banfi's house, which is a sort of camping van, Belgioioso's house, which is a sort of tent in the mountains, Zanuso's house, which is in a meadow, Rogers' house, which is within «multi-coloured spots among the winding greenery» (1942, 333), De Luca's house, which is «open to the sun, the sea, the light» (1942, 372), Aloisio's house, which is in «a flowered greenery with endless yellow flowers» (1942, 416), Cocchia's house, which is on a hill between the gulf of Naples, the Posillipo hill and the mount Vesuvius, Cattaneo's house, which has a garden and an orchard, Pica's house, which is in the Roman countryside, Mollino's house, which is on a hill, Romano's house, which is on the sea, Latini's house, which is «on the sea, on the submerged green of marine meadows» (1943, 150) and Bianchetti and Pea's house, which is «in front of an open landscape» (1943, 198). In the second case, we have Peressutti's house, which has a rock inside, Banfi's house, which can be shaped according to the landscape, Belgioioso's house, which is «permeated by nature» (1942, 324),

Zanuso's house, which has terraces which «continue outside, in the garden, the life inside» (1942, 329), Rogers' house, which «grows from the soil like a plant» (1942, 333), De Luca's house, which «will be a garden» (1942, 372), Diotallevi and Marescotti's house, which has «movable glasses between the rooms inside and the spaces outside» (1942, 419), Cocchia's house, which has an «oasis inside» (1942, 461), Cattaneo's house, which has a veranda, Pica's house, which has a «most extended pensile impluvium» (1942, 2) inside, Romano's house, which is on the sea, Latini's house, which is a «scale of the rock itself» (1943, 152) and Bianchetti and Pea's house, which has a «continuous contact with the landscape outside» (1943, 198).

2. The second thing they share is that the ideal human being has two essential dimensions, namely, that of what we may call needs and that of what we may call aspirations, the former being physical (for instance, the need of being protected by a roof) and the latter being mental (for instance, the aspiration of feeling protected by a roof which represents the dweller's identity, being made by «green enamelled shingles», Aloisio 1942, 416). This is represented by the ideal house in one way in particular: by dividing an area which is little, dark and hidden (which is the area which answers human needs, such as feeding and cleaning oneself) from an area which is big, illuminated and evident (which is the area which answers human aspirations, such as sharing a conversation with others in a beautiful living room and reading a book alone in a beautiful study). Peressutti divides a crystal cube (the living area) from an aluminium cube (from the utility area). Banfi divides the living area from the utility area as «two well distinct parts» (1942, 318), Belgioioso divides «well defined areas on the basis of their different functions, and well spaced-out» (1942, 324), Zanuso divides an area which is «bigger, more illuminated, richer in elements and colours» (1942, 329) from an area which is «smaller, snugger, darker» (1942, 329), De Luca divides the living area from «a services cell never visible from inside the house» (1942, 373), Cattaneo divides the living area from the bedroom («If from the dining room it will be necessary to walk a lot to go to the bedroom, even that little effort will highlight the definitely separated functions of the two rooms», 1942, 508) and Romano divides three autonomous blocks, namely, the living area, the sleeping area and the utility area.

3. The third thing they share is that the ideal human being's aspirations are even stronger than needs. This is represented by the ideal house in one way in particular: by carefully focusing on details (which is outwardly paradoxical, since the fifteen projects are extremely synthetically presented. But, meaningfully, the extreme synthesis does not mean the absence of details, even extremely carefully focused). Banfi and Belgioioso work on structural details. De Luca chooses a «Vietri ceramic dining table» (1942, 372), Diotallevi and Marescotti design projecting roofs «shaped on the basis of the solar cycle» (1942, 419), Cocchia obsessively works on the colours of mosaics, walls, ceilings and frames, Cattaneo equally obsessively works on religious symbols in the «family room» (1942, 505), Pica imagines a ceiling «frescoed by an abstruse abstractionist» (1943, 4) and gates «minutely historiated» (1943, 4), Mollino chooses «a contoured pivoted turning table» (1943, 54), Latini designs glasses which «provide yellow or blue light by filtering it from outside» (1943, 153) and Bianchetti chooses «big gneisses with grass and flowers within the interstices» (1943, 202).

4. And the fourth thing they share is that the ideal human being has two other essential dimensions, namely, that of what we may call request for isolation and that of what we may call request for relationship. This is represented by the ideal house in one way in particular: by dividing an area whose composition gives the human being a space for facilitating isolation from the others from an area whose composition gives the human being a space for facilitating relationship with the others. Zanuso designs both «sufficiently high walls» (1942, 329) for the garden and «white and transparent» walls for the house (1942, 329), Rogers thinks of a house which is both «far from yours, enough to sing out of tune without being heard, and yet near, so that I could greet you by moving my hands and you could reply» (1942, 333), and whose «walls are boundaries of the outside world, not obstacles, by opening them all, closing, half-closing» (1942, 333), De Luca imagines a house which is both «open to the sun, the sea, the light» and «isolated [...] from outside» (1942, 372), being «a solitary corner» (1942, 372) where «to be in the company of oneself» (1942, 372), Diotallevi and Marescotti want a house which is both isolated and in a city, Cattaneo designs «a well-built wall» (1942, 501) for the garden and «bedrooms which serve the individual's isolation within the family» (1942, 506), and yet «the family room» as the

cornerstone of the house, Pica wants a house which is both «not too far from downtown» (1943, 2) and a space where «to cocoon myself [...] in the company of my studies and dreams» (1943, 2), Mollino thinks of a part of the house entirely made of glass, and yet this has to be «obscured and concealable» (1943, 54), and Latini imagines a house which is both near the town and «a sort of tower, a sort of hermitage» (1943, 152) which encloses «my little absolute and remote world» (1943, 152).

3. The golden ratio

Now, we are ready to focus on the precise mathematical and geometric tools used by the seventeen architects to work on ideality, both to represent what the ideal house may be and to represent who the ideal human being at its foundation may be.

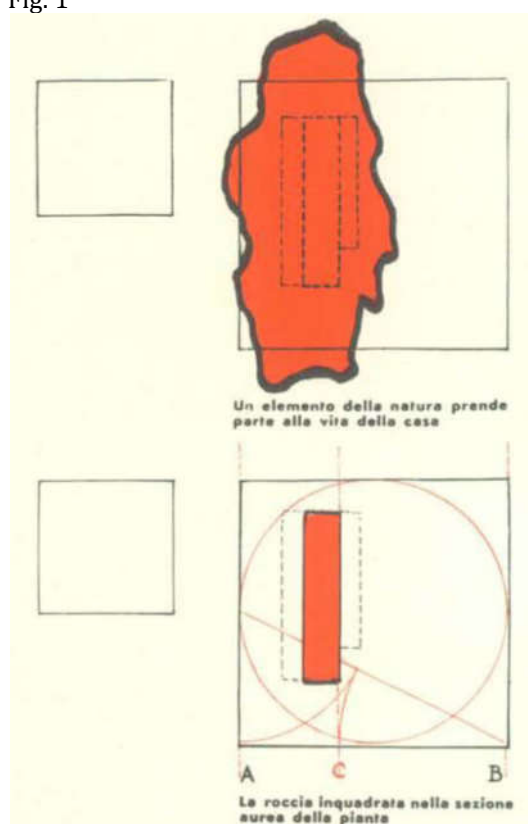
The first tool is the golden ratio. Two cases are particularly meaningful: Peressutti's and Pica's. Peressutti uses the golden ratio in four cases: to frame the rock within the first floor, to relate the second floor and the third floor to the first floor, to divide the flooring of the first floor into two areas and to set the heights. More precisely, in the first case, as we can see in Fig. 1², Peressutti, after having identified a rock as a foundation of the possibility of designing the house in continuity with nature, frames the rock within the first floor by neglecting the aluminium cube (the utility area, which is the smallest cube), by focusing on the crystal cube (on the living area, which is the biggest cube) and by calculating the golden ratio of its side. The rock is framed as follows. Firstly, Peressutti geometrically identifies the point C, which divides the segment AB into two segments: the shortest extreme AC and, at last, the mean proportional CB, which is the golden ratio of AB and AC. Secondly, Peressutti geometrically identifies the point M (Fig. 2³), which divides the segment AC into two segments: the mean proportional AM, which is the golden ratio of AC and MC, and shortest extreme MC. Thus, Peressutti projects M and C, obtaining the shortest side of the rock, and uses the measure of AM to identify its beginning and the measure of MC to identify its end. In-

² The English translations of Peressutti's captions (Fig. 1 is a drawing of his) are the following: «An element of nature takes part in the life of the house» and «The rock framed within the golden ratio of the plan». All the figures are by the architects themselves, if not differently specified.

³ The English translation of Peressutti's caption (Fig. 2 is a drawing of his) is the following: «The relationship between the upper floors and the square of the base is given by the golden ratio».

deed, the choice of giving the relationship between house and nature the form of the golden ratio is meaningful: it is a powerful symbol of a vision according to which the house is ideal if the first thing it takes care of is its relationship with nature (relationship which requires nothing less than the most perfect number⁴) – we may say that the house is ideal if it represents that the first thing the ideal human being takes care of is his relationship with nature (relationship which requires nothing less than the most perfect number).

Fig. 1

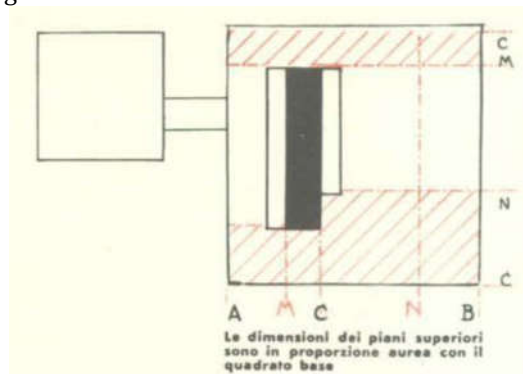


In the second case, as we can see in Fig. 2, firstly, Peressutti designs the second floor by using the golden ratio as the relationship between this and the first floor: the second floor is made by two rectangles, the first being the primary one (it is the bedroom), identified by the letters CB as the longest side and CN as the shortest side (given that CB coincides with the mean proportional

⁴ On the characteristics of the golden ratio see the end of paragraph 4 in part 2.

of the side of the first floor AB and CN coincides with the mean proportional of the side of the second floor CB), and the second being the secondary one (it is the passage towards the utility area), identified by the letters AC as the longest side and C up to the rock as the shortest side (given that AC coincides with the shortest extreme of the side of the first floor AB and C up to the rock coincides with the mean proportional of the side of the second floor AC). Secondly, Peressutti designs the third floor by using the golden ratio as the relationship between this and both the first floor and the second floor: the third floor is made by one rectangle whose longest side coincides with the side of the first floor AB and whose shortest side MC coincides both with the shortest extreme of the side of the second floor AC and with the shortest side of the rock. Indeed, there are two meaningful symbolic choices: the first is the total absence of the golden ratio from the aluminium cube (from the utility area), whose side is half of the side of the crystal cube, and the second is the total presence of the golden ratio in the crystal cube (in the living area), from its first floor to its second floor and to its third floor. These choices work as a powerful symbols of a vision according to which the house is ideal if it is the space which serves human aspirations even more than human needs – we may say that the house is ideal if it represents that the ideal human being is the one who is distinguished by having aspirations even more than by having needs.

Fig. 2



This powerful symbolic vision is highlighted by the third case. As we can see in Fig. 3⁵, Peressutti divides the flooring of the first floor into two areas by using the golden ratio. More precisely, the parabola which divides the living room (which is closer to the rock) from the dining room (which is closer to the glasses) is inscribed into golden proportions: its focus is identified by the horizontal and vertical projections of CB, which coincides with the mean proportional of the side of the first floor AB, and its tangents converge to the point C, which is the lowest extreme of CM, which coincides both with the shortest extreme of the side of the second floor AC (given by the golden ratio, as we have seen) and with the shortest side of the rock (given by the golden ratio, as we have seen). According to a symbolic perspective, we may add that the choice to set the living room into a more privileged position (it is inside the parabola) and the dining room into a less privileged position (it is outside the parabola) even highlights the vision according to which the ideal house serves human aspirations (for instance, sharing a conversation with others in a beautiful living room) more than human needs (for instance, feeding oneself, even if the beautiful dining room adds an element of aspiration to what is essentially a need).

Fig. 3

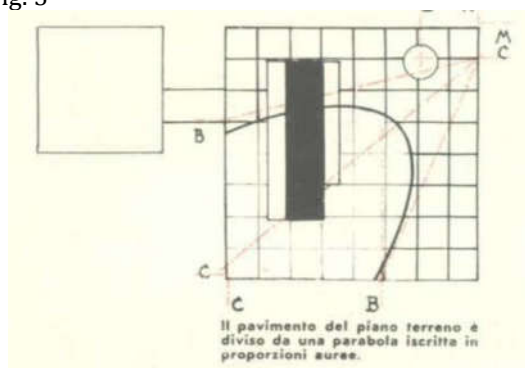
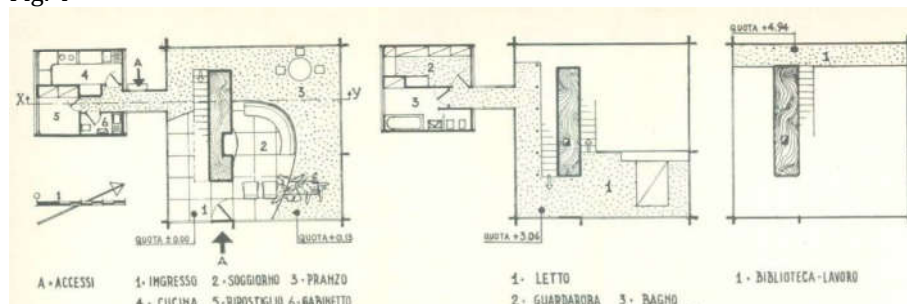


Fig. 4 makes this vision clearer by adding furniture details. In the first floor, the parabola inscribed into golden proportions corresponds to a white flooring, whereas the other area corresponds to a black flooring (as we know from the axonometry): in Western

⁵ The English translation of Peressutti's caption (Fig. 3 is a drawing of his) is the following: «The flooring of the first floor is divided by a parabola which is framed within golden ratios».

thought, the former colour usually symbolises positivity, whereas the latter colour usually symbolises negativity. Thus, and again, the living room is privileged over the dining room, namely, the dimension of aspirations is privileged over the dimension of needs. There is another meaningful furniture detail: the two areas are joined twice, by a part of the rock in the first case and by a plant in the second case, namely, always by nature – we may say that, symbolically, both the house and the human being are ideal if the dimension of needs is naturally joined to, namely, naturally evolves towards, the dimension of aspirations. The position of the entrance is equally meaningful: it is both inside the parabola and in front of the shortest side of the rock, namely, its position is particularly privileged by the golden ratio – we may say that, symbolically, entering the ideal house means entering the existence of a particularly valuable creature, namely, of the human being, who cannot be perfect, but can evolve by aspiring to perfection. And, both in the first floor and in the second floor, it is clear that the aluminium cube (the utility area) has no parts founded on the golden ratio: both the kitchen, the box room and the toilet in the first floor and the wardrobe and the bathroom in the second floor are rectangles not founded on the golden ratio, which, again, symbolically serves the dimension of aspirations (the entrance, the living room and the dining room in the first floor, the bedroom in the second floor and the library in the third floor are rectangles founded on the golden ratio), and not the dimension of needs.

Fig. 4



In the fourth case, as we can see in Fig. 5, Peressutti sets the heights of the crystal cube, and not of the aluminium cube, by using the golden ratio. The height of the house (AB in the first drawing from the left) coincides with the side of the first floor. Thus, the height of the first floor (AC in the first drawing from the left)

coincides with the shortest extreme of the side of the first floor, the total of the heights of the second floor and of the third floor (CB in the first drawing from the left) coincides with the mean proportional of the side of the first floor, the total of the heights of the first floor and of the second floor (AC in the second drawing from the left) coincides with the mean proportional of the side of the first floor and the height of the third floor (CB in the second drawing from the left) coincides with the shortest extreme of the side of the first floor. Thus, the heights of the three floors are different: the heights of the first floor and of the third floor do not vary, coinciding with the shortest extreme of the side of the first floor, whereas the height of the second floor varies, coinciding with the difference between the side of the first floor and twice the shortest extreme of the side of the first floor. The result is that the heights of the first floor and of the third floor are higher than the height of the second floor, namely, the space for the entrance, the living room, the dining room and the library is bigger than the space for the bedroom. The use of the golden ratio to set the heights is, again, meaningful from a symbolic perspective, since it represents a vision according to which the house is ideal if it extremely highlights the space for a totally active work on aspirations (we are particularly lively when we are awake in a living room and in a library) over the space for a not totally active work on aspirations (we are not particularly lively when we are not awake in a bedroom) – we may say that the house is ideal if it represents the ideal human being as the one who is distinguished by the aspiration to evolve, namely, to make both the space of his life and his life evolve. Lastly, the golden ratio serves the roof (Fig. 5, the third drawing from the left), whose front gets its depth from CA, which coincides with the shortest extreme of the side of the first floor, and whose back gets its depth from AM, which coincides with the difference between the side of the first floor and twice the shortest extreme of the side of the first floor. Thus, through the symbolic power of the golden ratio, the roof can be the answer both to the need of being protected and to the aspiration of feeling protected by a roof which represents the dweller's identity.

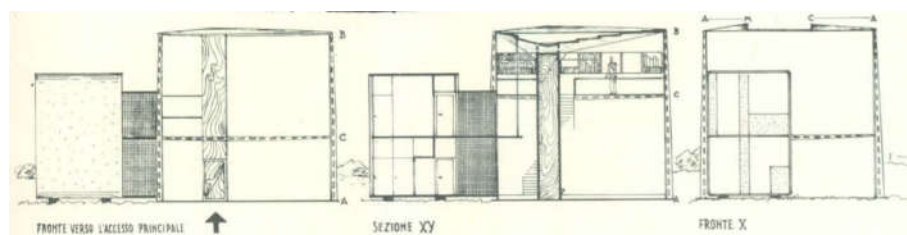


Fig. 5

Pica uses the golden ratio in two cases⁶: to design the porch and to set the statue of Apollo with the Muse. More precisely, in the first case, as we can see in Fig. 6, Pica designs the porch, which corresponds to the area around the pillars, by projecting the longest side of the staircase on the patio. Thus, Pica obtains a porch whose longest side coincides with the golden ratio (with the mean proportional) of the total of its longest and shortest sides and its longest side. In the second case, as we can see both in Fig. 6 and in Fig. 7, Pica sets the statue of Apollo with the Muse by identifying the golden ratio (the mean proportional) of the flight of steps: the former divides the latter into two segments, whose lowest part is the golden ratio (the mean proportional) of the flight of steps and whose upper part is the shortest extreme of the flight of steps. The symbolic use of the golden ratio both for the porch and for the statue of Apollo with the Muse is clearest: the former corresponds to the entrance and the latter corresponds to what lets in the entrance – we may say that any visitor should immediately perceive that the ideal house is founded on the vision given both by the golden ratio and by Apollo with the Muse, namely, on the vision of the ideal human being as the one who lives following the greatest possible harmony (with nature, with the others and with himself).

⁶ Thanks to Luigi Cocchiarella for our reflections on the golden ratio, and on Pica's use of it in particular.

Fig. 6

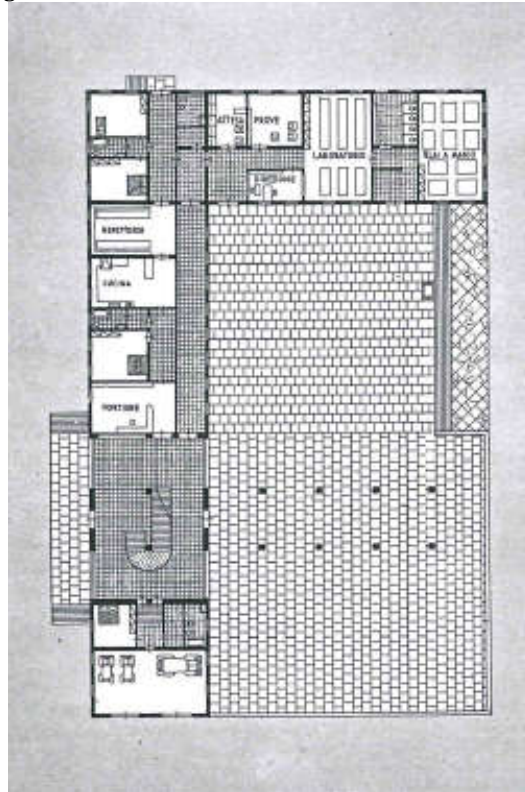
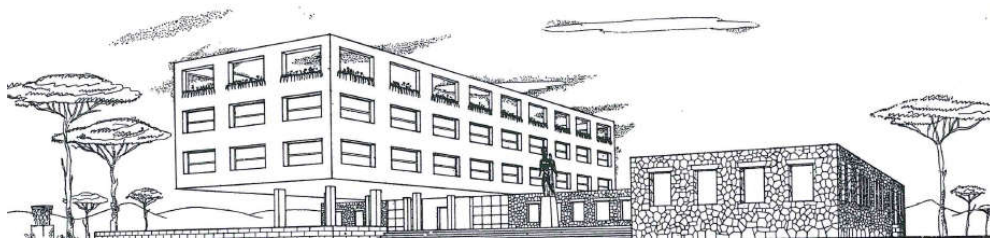


Fig. 7



4. The hidden golden ratio

Interestingly enough, it is possible to find the golden ratio in several of the fifteen projects, even if it is not explicitly mentioned by the architects in their descriptions. The reasons may be two: firstly, they may be consciously using it without mentioning it and, secondly, they may be unconsciously using it, which is even more

interesting, since it means that the golden ratio is an almost automatic tool when it comes to idealisation, at least for Western architects. Zanuso's house is a golden rectangle: its shortest side is the golden ratio of its longest side (Fig. 8 and Fig. 8a, in which I show in light blue the shortest side AE and the longest side AB). De Luca's house and garden form a golden rectangle, whose shortest side is the golden ratio of its longest side (Fig. 9 and Fig. 9a, in which I show in light blue the shortest side AC and the longest side AB). Aloisio's house includes a golden rectangle, which is visible if we take off the upper area of the garage, and whose shortest side is the golden ratio of its longest side (Fig. 10 and Fig. 10a, in which I show in light blue the shortest side AC and the longest side AB. See Fig. 11 for a 3D drawing of his house). Cattaneo's «family room», which is the cornerstone of his house, and garden are almost golden rectangles, whose shortest sides are almost the golden ratios of their longest sides (Fig. 12, Fig. 13, Fig. 12a, in which I show in light blue the shortest side AB and the longest side AC, and Fig. 13a, in which I show in light blue the shortest side AB and the longest side AC. See Fig. 14 for a 3D drawing of his house). Romano's house includes an almost golden rectangle, which is visible, interestingly enough, if we take in the living area and the sleeping area and take off the utility area, and whose shortest side is the golden ratio of its longest side (Fig. 15 and Fig. 15a, in which I show in light blue the shortest side AC and the longest side AB. See Fig. 16 for a 3D drawing of his house). And Bianchetti and Pea's house includes two almost golden rectangles, which, interestingly enough, correspond, in the first case, to the total of the entrance, the living room and the dining room and, in the second case, to the patio (Fig. 17 and Fig. 17a, in which I show in light blue, in the first case, the shortest side AC and the longest side AB, and, in the second case, the shortest side EG and the longest side EF).

Fig. 8

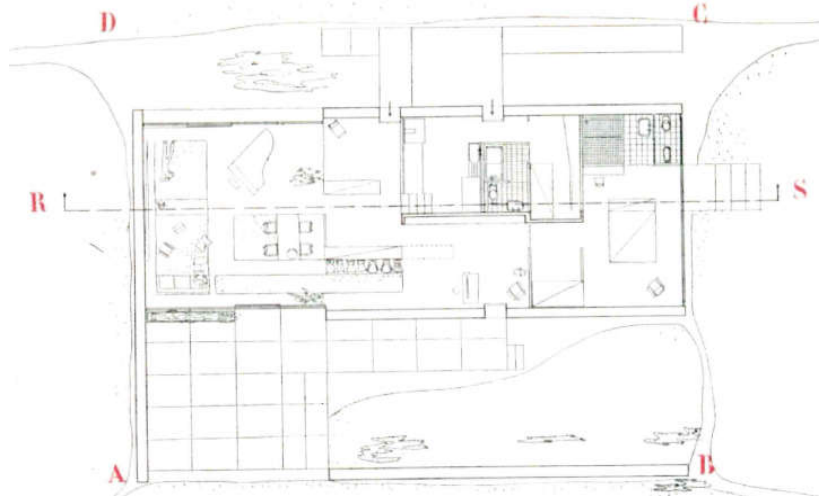


Fig. 8a

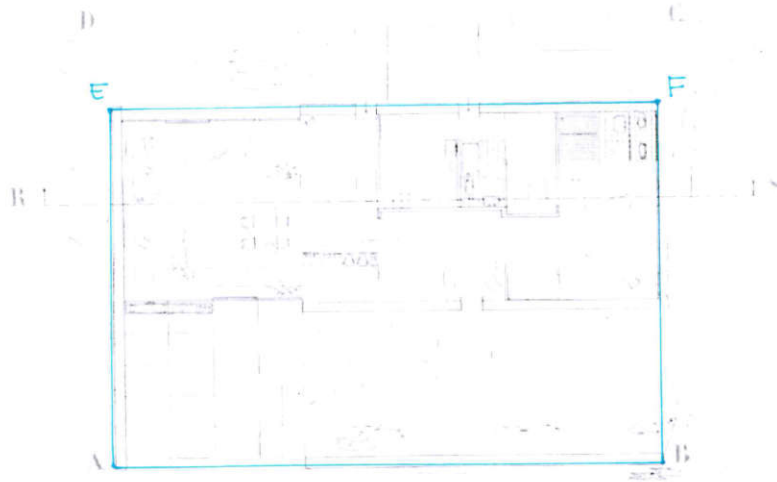


Fig. 9

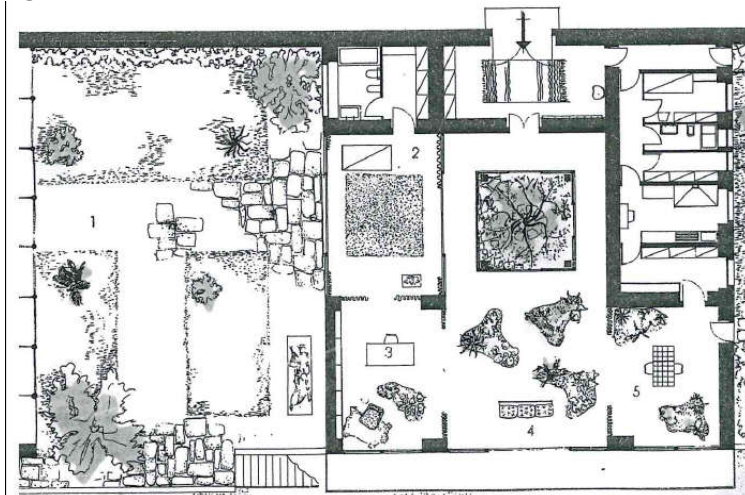


Fig. 9a

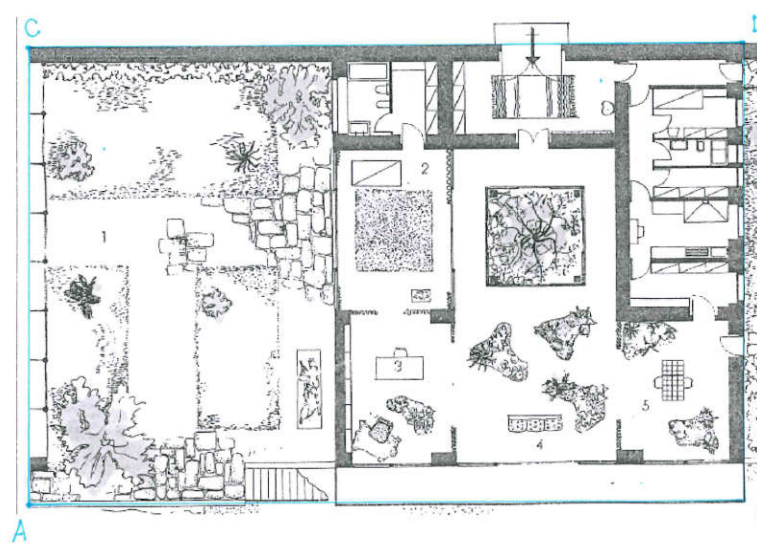


Fig. 10

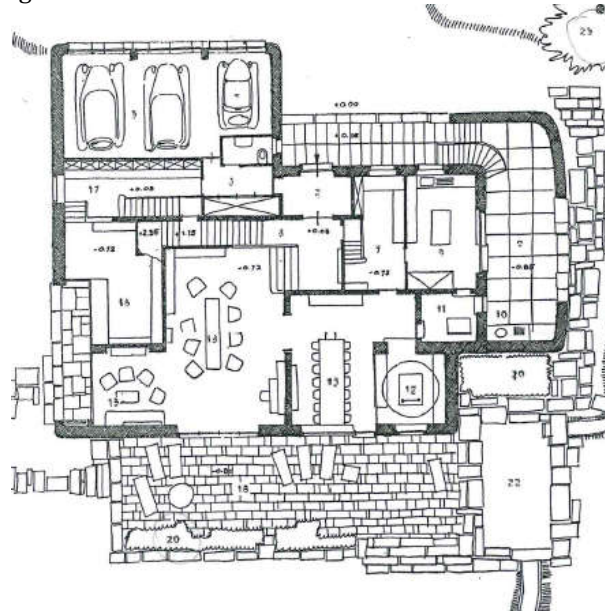


Fig. 10a

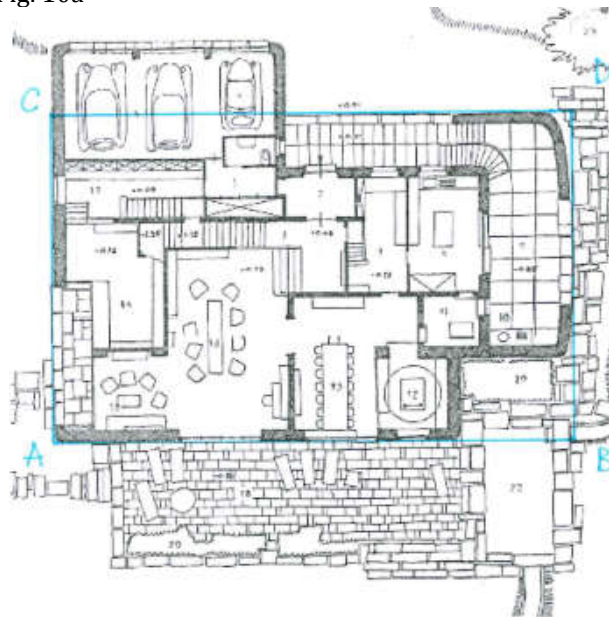


Fig. 11

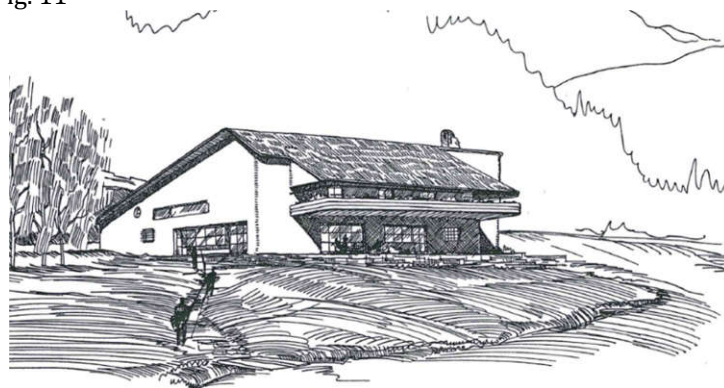


Fig. 12

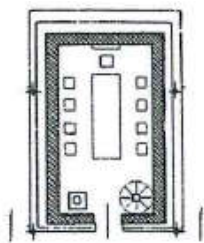


Fig. 12a

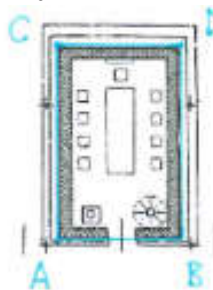


Fig. 13

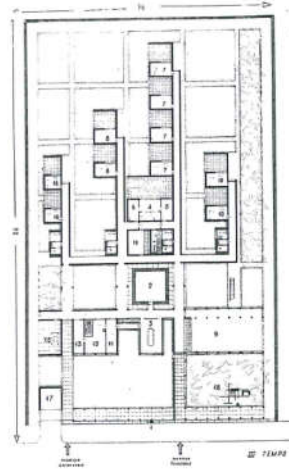


Fig. 13a

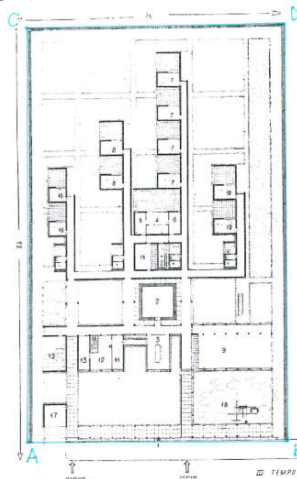


Fig. 14

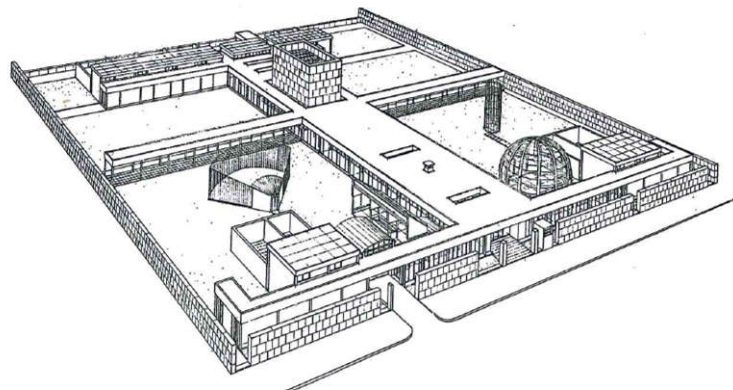


Fig. 15

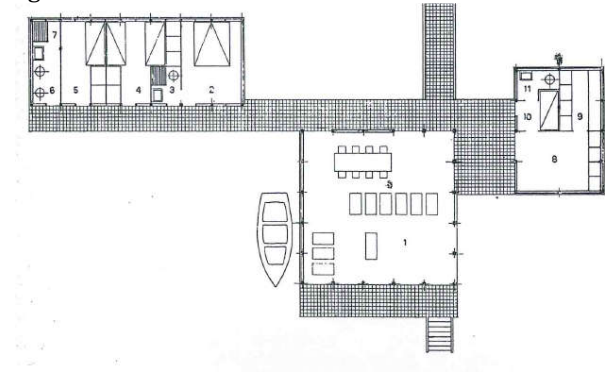


Fig. 15a

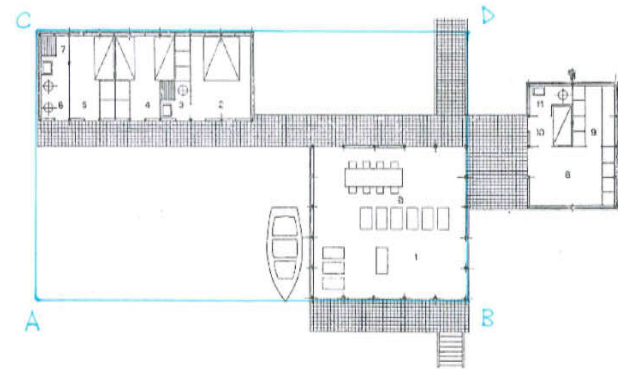


Fig. 16

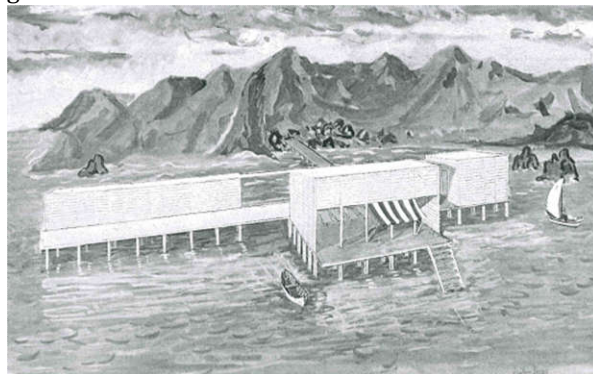
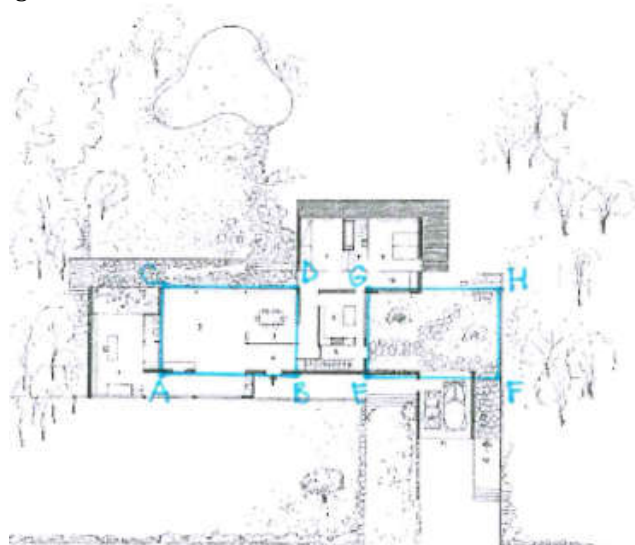


Fig. 17



Fig. 17a



Now, we are ready to highlight the philosophical reason why the golden ratio is a particularly powerful tool to work on ideality, both to represent what the ideal house may be and to represent who the ideal human being at its foundation may be. The golden ratio has always been, since ancient Greek thought, what is quintessentially incommensurable. More precisely, the golden rectangle, which is extensively used by the seventeen architects, is char-

acterised by what follows: once we have identified the sequence of smaller and smaller golden rectangles included into the initial golden rectangle, we can realise that this sequence converges towards a point which is unattainable (called God's eye by the mathematician Pickover). The incommensurability of the golden ratio is highlighted by its irrationality: the golden ratio is considered the most irrational number, since the fraction which represents it cannot be broken correspondingly to a number higher than 1. The philosophical meaning of the incommensurability of the golden ratio may be thought of as what follows: being incommensurable means being ontologically totally different from reality, and being ontologically totally different from reality means being ideal – indeed, ideality is what Western thought invented (contrarily, for instance, to Far Eastern thought) as the ontological counterpart of reality: the latter is imperfect, perishable, changeable and particular and the former is perfect, imperishable, unchangeable and universal, namely, the latter is what there is and the former is the rule of what there is.

Ideality, together with its unchangeability and universality, is the philosophical meaning of the golden ratio for other reasons. As far as unchangeability is concerned, the golden ratio corresponds, for instance, to the non-natural number whose reciprocal and square keep their decimals unchanged. As far as universality is concerned, the golden ratio is, for instance, the possible root of any equation of the following sort: $x^n - x^{n-1} - x^{n-2} = 0$. Indeed, since the nineteenth century in particular, the golden ratio, for its unchangeability and universality, has been extensively used in the arts to represent ideality, following other scholars' reflections (for instance, according to Fechner, any human being is characterised by a natural disposition towards the golden ratio, which frequently occurs in nature). And in the twentieth century its use has been even strengthened, starting from architecture, with Le Corbusier's Modulor, which, on the basis of the golden ratio, identifies the forms of human spaces, namely, the ways in which architecture should be designed. Lastly, several studies have tried to find the golden ratio anywhere, from the human being (from the relationship between stance and swing, see Iosa *et alii* 2013, to the relationship between systole and diastole, see Yetkin *et alii* 2014) to botany (to the forms of the leaves in particular) and to astronomy (to the forms of the galaxies in particular).

5. The modulus

Other tools are used by the seventeen architects to represent ideality through unchangeability and universality. One of the most frequent tools is the modulus, which is used as a sort of guarantee: given a modulus, any part of the house is founded on it, namely, any part of the house works to represent an identical ideal of what the house should be – any part of the house works to represent an identical ideal of who the human being should be.

Let us analyse the most meaningful cases. Banfi chooses a square as a modulus to found the living area of his house (Fig. 18 and Fig. 19 show three out of five possible versions of the plan), which is rigorously divided into two areas: the camping van, which corresponds to the utility area and is not founded on the modulus, and the movable structure, which, shaped according to the landscape, corresponds to the living area and is founded on the modulus. As Banfi specifies, «The living area is made by 22 square elements (2m x 2m) for the roof and for the flooring, by 24 elements for the outside wall and by 2 [elements] for the inside wall, all being 2m x 2m long»⁷ (1942, 318). As for the roof, «it is made by curtain elements which are 2m x 2m long [...]; thus, it is possible to give it the desired form, required by the landscape» (1942, 318). As for the flooring, «it is made by plank elements 2m x 2m long, which rest on screwing supports which allow to create the horizontal level without modifying the varied inclination of the soil» (1942, 318). As for the walls, they «are made by supporting elements screwed to the flooring supports and by panels 2m x 1,9m long to be fitted between the supports» (1942, 318). The most interesting thing to highlight is that the modulus founds the possibility of varying the house (according to what nature requests) – it is precisely what does not vary, namely, what is unchangeable and universal, that founds the possibility of varying (according to what nature requests). Let us look at Fig. 18: here we can find the first version of the house proposed by Banfi (which, interestingly enough, is, among the versions, the nearest to the golden rectangle), where the living area (the living room, the dining room, the study and the terrace) is on the right and the sleeping area is on the left. Banfi proposes five versions of the ideal house: two rectangles, two squares and one irregular form. Let us look at Fig. 19: here we can find two other versions of the

⁷ Here «element» means modulus.

house proposed by Banfi. In the first case, the house maximises the indoor dimension: 24 elements correspond to the indoor space, where the living area (the living room, the dining room and the study) is on the right and the sleeping area is on the left. In the second case, the house maximises the outdoor dimension: 17 elements correspond to the indoor space and 7 elements correspond to the outdoor space. The living area (the dining room and the terrace, which, given its extension, works as a living room) is on the left and the sleeping area is on the right. Thus, from a symbolic perspective, we may say that the modulus gives Banfi two meaningful possibilities. The first possibility is to make visible that working on the ideal house means working on the ideal human being, namely, on the result of an abstraction, firstly, and of an idealisation, secondly, which get to a universal model. Thus, Banfi uses a sort of constant: if there is something universal within the human being, then there is something universal within the house (namely, the modulus does not vary, since it is a constant). The second possibility is to make visible that the ideal human being is a part, and not a counterpart, of nature, and has to follow its, and not autonomous, rules. Thus, Banfi uses a sort of variable: if nature changes, then the house changes (namely, the sequence of the modulus varies, since it is a variable).

Fig. 18

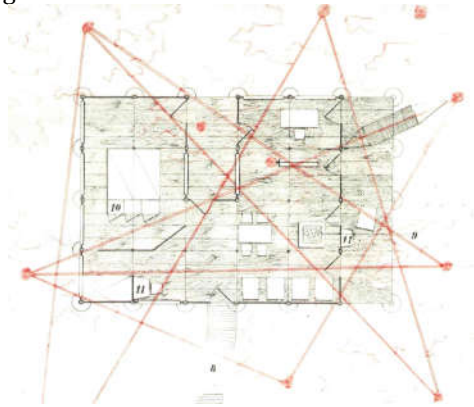
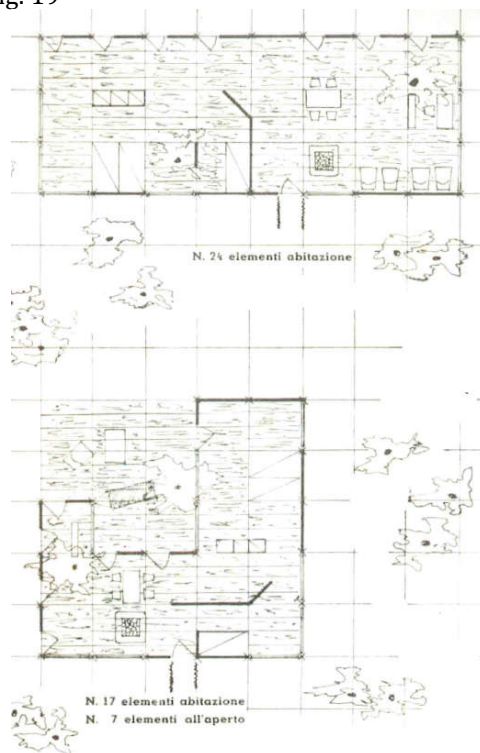


Fig. 19



Belgioioso chooses an analogous strategy, namely, a square as a modulus to found his house (Fig. 20. See Fig. 21 for a 3D drawing of his house), which is a sort of tent:

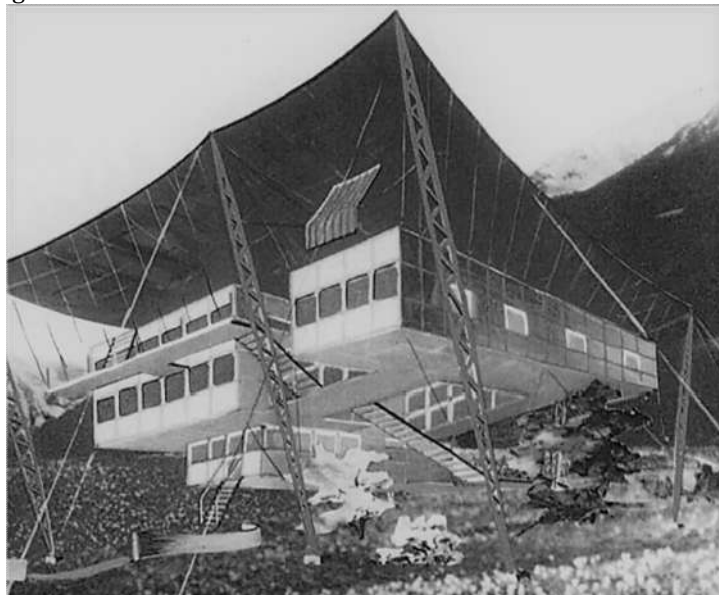
It is evident in this sort of building the necessity of unifying the measures of its constitutive elements. Thus, once 1,2m has been determined as the most suitable interaxis 'modulus', all the capacious elements, both vertical and horizontal, are consequently dimensioned, and the measures of the rooms, both in regard with their lengths and in regard with their heights, result from the multiples (1942, 327).

Again, universality is essential: «This 'house for my family' represents from a spiritual point of view a solution which is quite near to my current ideal life, and yet it wants to express much more general problems concerning living architecture: I would like to have contributed to the generality of cases» (1942, 327). And, again, the reason why universality is essential is that Belgioioso thinks of the ideal human being (of what any human being essentially is) as the foundation of the ideal house (of what any house essentially is).

Fig. 20



Fig. 21



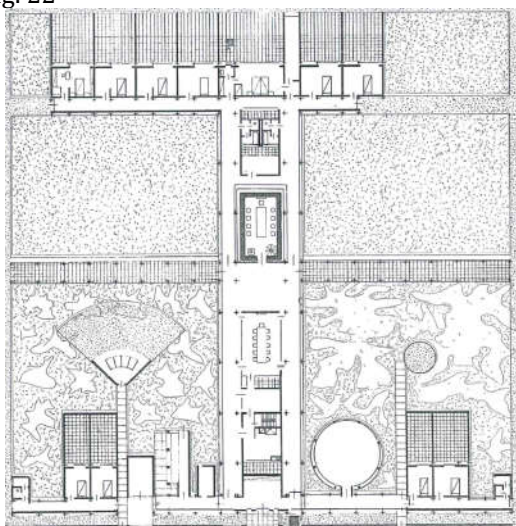
Cattaneo chooses another sort of modulus: since his house is a Christian family house, the modulus is the tool through which it is possible to make the house bigger as the Christian family becomes bigger and bigger. Let us look at Fig. 22: the upper part of the house is the sleeping area, where there are the parents' bedroom (in the middle), the sons' bedrooms (on the right, two in this case) and the daughters' bedrooms (on the left, four in this case). Cattaneo explains:

When the first son is born, his bedroom is built next to his parents' one, and, after the period spent into the cradle, it gets occupied. After a second son of the same gender is born, he moves to the first son's bedroom,

for whom another bedroom is built, and so forth, so that the bedrooms furthest from the parents' one are the elder sons' (1942, 506).

Thus, the modulus, which corresponds to the son's bedroom (identically repeated in regard with any feature, from the indoor structure to the outdoor structure and to the furniture), serves as a sort of symbol of the Christian family: the parents should be the nucleus and the sons and the daughters should gradually distance themselves from the nucleus to start their own families (Zanuso works on an analogous idea, but without specifying the character of the modulus. His house is not «big, but able to become big. A nucleus, like the cell, able to become bigger as the family becomes bigger [...]. On its southern side its extension is expected to be made by the completion of the level of the terraces according to the dimensions of prior elements», 1942, 329).

Fig. 22

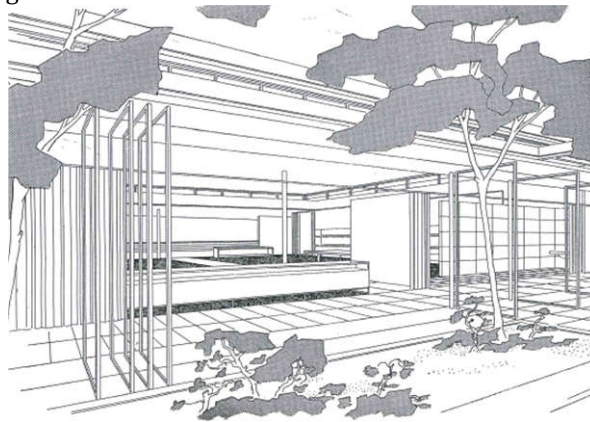


Pica (Fig. 6 and Fig. 7, part 1), when it comes to explain that «all the plan is built on an orthogonal square matrix characterised by a constant modulus 6m long [...]. The height respects the fundamental modular measure too» (1943, 4), adds:

I wanted, as it was right, to postulate those correspondences of rhythmic geometry and numerical abstraction for which we, who are architects, have always had a most special inclination of the heart. [...] Thus, this autobiographical house would like to be spatially defined according to the Renaissance and the mysteriously musical rhythm of a sort of 'paradise of geometry' (1943, 4).

There is a last remark to make on the modulus. As we have already seen, it outwardly serves changeability and particularity and inwardly serves unchangeability and universality. Now, we may add something else: surprisingly enough, flexibility, in its possible articulations, is almost absent from the fifteen projects. Flexibility is present in four projects: Banfi's, Zanuso's, Diotallevi and Marescotti's and Cattaneo's. As for Banfi, Zanuso and Cattaneo, flexibility, as we have already seen, inwardly means universality. As for Diotallevi and Marescotti (see Fig. 23 for a 3D drawing of the interior of their house), their house is characterised by an inward flexibility: the structure of the living room «allow[s] all the dispositions wanted up to the total disappearance into specific side compartments» (1942, 419). But Diotallevi and Marescotti's inward flexibility is a unique case: no other house is characterised by something contrary to a distinctive search for universality – again, almost the totality of the houses is thought of as something that, to be ideal, has to be founded on a distinctive search for something universal, namely, for the answer to the question on what any human being essentially is.

Fig. 23



6. The number 2

There is a last mathematical and geometric tool used by the seventeen architects to work on ideality: the obsessive presence of the number 2, which is used in some cases to divide something explicitly related to the house and in some cases to divide something explicitly related to the human being (more precisely, both to the relationship between human being and nature and to the relationship between a first human being's ontological dimension

and a second human being's ontological dimension). As for the first case, Peressutti's number 2 is given by dividing the crystal cube (the living area) from the aluminium cube (from the utility area): the two areas are connected by a narrow hallway. Banfi's number 2 is given by dividing the living area from the utility area: the house shaped according to the landscape corresponds to the former and the camping van corresponds to the latter, and there is no architectural connection between them. Belgioioso's number 2 is given by dividing the living area from both the sleeping area and the utility area: the former is «included in more capacious dimensions» (1942, 327) and the latter are «dimensioned according to concepts of minimum, since life is spent within them according to functions of purely biological and mechanical necessities» (1942, 327). Zanuso's number 2 is given by dividing the bigger and more illuminated area from the smaller and darker area: the former includes «The dining table, the couch, the armchairs, the raised carpet where to read, to chat, to stay» (1942, 329) and the latter includes «The working armchair, the working table, the arrangements for the wardrobe work» (1942, 329). De Luca's number 2 is given by dividing the living area from the services cell which includes «all the things related to the trivial everyday life, all the necessary, but annoying, things» (1942, 372-373). Aloisio's number 2 is given by dividing the men's area from the women's area: «In the living room, the wives are in a separated area, since, poor them, they cannot stand conversations on fishes anymore» (1942, 417). Diotallevi and Marescotti's number 2 is given by dividing both any indoor block from the totality of the indoor blocks and the totality of the indoor blocks from the outdoor blocks: «Distinct blocks interposed by outdoor spaces, each one with a function related to the overlooking room: living room, dining room, bedrooms, kitchen services, toilets and washhouse» (1942, 419). Cattaneo's number 2 is given by dividing both «The essential elements» (1942, 505), which are the dining room, the kitchen, the parents' bedroom, the children's bedrooms, the study, the wardrobe and the toilets, from «The accessory elements» (1942, 506), which are the servants' bedroom, the guest bedrooms, the living room, the woodshed, the laundry, the green house, the henhouse and the garage, and the living area from the sleeping area, which are distant. Romano's number 2 is given by dividing the part of the house oriented towards the sea from the part of the house oriented towards the land: the former corresponds to the block of

the living area (which includes the living room and the dining room) and the latter corresponds to the blocks of the sleeping area (which includes the parents' bedroom, the children's bedroom, the guest bedroom and the toilets) and of the utility area (which includes the services room, the kitchen, the servants' bedroom and the servants' toilet).

As for the second case, let us consider, firstly, when the number 2 has to do with the relationship between human being and nature. As we have already seen, the totality of the fifteen projects represents that the ideal human being is a part, and not a counterpart, of nature, and has to follow its, and not autonomous, rules. This is represented by the ideal house by designing it within nature and by designing it in continuity with nature, but in both cases the condition of possibility is realising that the two parts are considered the greatest division to be overcome by the ideal architect. More precisely, according to Rogers, the house has to be thought of as something natural: «My house changes its aspect as the seasons go by; every spring it changes its fronds renewing itself; in summer it has the cool of the woods; in autumn it is coloured, and in winter it lets itself be covered by the snow, and my family germinates underneath waiting for the sun» (1942, 333). Let us consider, secondly, when the number 2 has to do with the relationship between a first human being's ontological dimension and a second human being's ontological dimension. As we have already seen, several of the seventeen architects work both on the relationship between needs and aspirations and on the relationship between request for isolation and request for relationship. More precisely, the ideal human being they think of is characterised by both a sort of constant, which means what joins him with the totality of the others (his commonality), and a sort of variable, which means what divides him from the totality of the others (his uniqueness): according to a mathematical image, the former is «the denominator, the communal factor, equal» (Peressutti 1942, 314), and the latter is «the numerator, the personal fact, always different» (Peressutti 1942, 314). Thus, the human being has to be considered both by making reference to «his vegetative biological entity, as an animal isolated or simply 'put near' the others» (Cattaneo 1942, 500), and by making reference to «his effort to join the others at levels greater than that of his physical individuality» (Cattaneo 1942, 500).

Now, we can try to reason on the symbolical meaning of the number 2. As we have already seen, the number 2 is used to highlight several things: that the ideal human being has a relationship with nature, that the ideal human being has both needs and aspirations and that the ideal human being has both a request for isolation and a request for relationship. If we ask what we can learn from this representation, then we can answer that we can learn two essential things: firstly, that any human being is essentially characterised by complexity and, secondly, that any human being is essentially characterised by relationship. The first word, 'complexity', results from the fact that the totality of the things highlighted has to do with an opposition: a human being acts as a counterpart of nature but is a part of nature, a human being starts from his needs but his aspirations are even stronger and a human being buys a house for himself but furnishes a living room for others. Thus, a human being (any human being, namely, the ideal human being) is a complex creature – and a complex creature requires to be thought of carefully, and a house which is equally carefully thought of. The second word, 'relationship', results from the fact that the totality of the things highlighted has to do with a search for otherness: a human being searches for nature as otherness (namely, as a counterpart), a human being searches for aspirations as otherness (namely, as what he has not) and a human being searches for others as otherness (namely, as what he is not). Thus, a human being (any human being, namely, the ideal human being) is a relating creature – and a relating creature requires possibilities of being both 'concave', to receive, and 'convex', to be received, and a house which is equally both receiving (namely, introverted and, thus, private, for oneself) and received (namely, extroverted and, thus, public, for the city).

We may think that this is the reason why the house is the architect's greatest challenge: it is

the painful problem of man's house. Years, centuries, millennia go by, gods rise and die, their pompous abodes, the powerful men's manors, fountains, theatres and markets set themselves up, but the man still has not learnt to build an abode for himself: a house for everyone, for you, for me, for the Anonymous. [...] Go away from the rubble, and build your house, it is time: you have fiddled with the pyramids, the Coliseums and the big domes enough, show who you are and how much you are worth. I want to be true, to live: I want houses where this is possible (Rogers 1942, 333),

but, «Among the many architectural topics, that of the house, despite being the richest as far as the examples are concerned, is the most roughly defined. The house is studied in its smallest particulars of practical operation, but it is not asked what it really is or should be, considering man's life» (Cattaneo 1942, 500). Thus, the ideal house has a paradigmatic meaning: among the architectural exercises on what is ideal, it is the essential architectural exercise on what is ideal, since working on the ideal house quintessentially means working on «be[ing] true, [...] [on] liv[ing]», on «man's life» – again, working on the ideal house quintessentially means working on what a human being (any human being, namely, the ideal human being) essentially is: the former is the best architecture to answer the question on the latter.

7. Conclusion

The architectural exercise we have been focusing on is instructive to understand what may be thought of as the most distinctive cornerstone of Western science: the mathematisation of empirical knowledge. Since ancient Greek thought, Western science has been characterised by a mathematisation unknown elsewhere, from physics to medicine, to astronomy and to architecture – and mathematisation means universalisation. This is the reason why, for instance, terrestrial magnetism, which has been discovered by the Chinese several centuries before the Europeans, started being used as a compass to orient oneself around the world by the latter, and not by the former: it is precisely a process of mathematisation that is required to make a phenomenon pass from particularity to universality, namely, from a particular token to a universal type – from what really happens here and now to what ideally happens wherever and whenever, even in the furthest point of the biggest ocean.

And what founds the condition of possibility of mathematising, namely, the passage from particularity to universality, is ideality, which is what may be thought of as the most distinctive cornerstone of Western philosophy. But, more precisely, why is ideality crucial for mathematising? The reason of its cruciality can be understood through the example of terrestrial magnetism. Let us suppose we have a dimension, namely, that of reality, where we observe the magnetic phenomenon A, the magnetic phenomenon B and the magnetic phenomenon C, which occur in very different circumstances. Can we hope to orient ourselves if we get to the

furthest point of the biggest ocean? The answer is no, since what A, B and C mean to us has to do with contingency: given A, B and C, we do not have what proves that the phenomenon we will find in the furthest point of the biggest ocean will be identical and, thus, predictable. Now, let us suppose we have a dimension, namely, that of reality, where we observe the magnetic phenomenon A, the magnetic phenomenon B and the magnetic phenomenon C, which occur in very different circumstances, and another dimension, that of ideality, which is invented by us as the dimension where we imagine to retain what does not vary from A to B and to C: we explicitly ask what does not vary from A to B and to C and, after having carefully both observed and reasoned, we explicitly answer that what does not vary from A to B and to C is X, which is ontologically ideal, and not ontologically real, since it exists in our imagination, and not in our reality. Can we hope to orient ourselves if we get to the furthest point of the biggest ocean? The answer is yes, since what X means to us has to do with necessity: given X, we have what proves that the phenomenon we will find in the furthest point of the biggest ocean will be identical and, thus, predictable. Actually, Hume, the greatest philosopher of empiricism, teaches us that there is no empirical necessity: we may get lost in the furthest point of the biggest ocean anyway. But, actually, Hume believes that we have a crucial reason to get the courage, at least, to get to the furthest point of the biggest ocean: we have abstracted, we have idealised and we have found our most powerful tool to orient ourselves, which is X as what can be imagined as the constant of the variants A, B, C and the phenomenon we will find in the furthest point of the biggest ocean – and being imagined as constant of variants means being mathematised.

Now, let us go back to architecture. Mathematising and idealising architecture mean something analogous to mathematising and idealising science. Paradoxically enough, the more we idealise the more science develops, as we have experienced particularly since the seventeenth century, when Galileo Galilei started strengthening idealisation by choosing Plato, and not Aristotle, as the reference point of the scientific method. Art in general and architecture in particular work equally: the more we idealise the more art and architecture develop. We can clearly understand why through the ideal houses: if we pass from the case of terrestrial magnetism to the case of the ideal houses, we may say that, in the first case, idealising means giving the condition of possibility

of the compass, namely, of the tool which, by resulting from a universalisation, can orient us in relation to a brand new reality (in relation to a brand new point to get to) to manage as well as possible, and, in the second case, idealising means giving the condition of possibility of the ideal human being, namely, of the tool which, by resulting from a universalisation, can orient us in relation to a brand new reality (in relation to a brand new house to build) to manage as well as possible – again, idealising means making reality evolve at its best, as philosophers and mathematicians, and more precisely those who were both philosophers and mathematicians, have taught us since the beginning of Western thought.