Orthometric correction and normal heights for Italian levelling network: a case study

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Introduction

The official height datum in Italy is defined by the high-precision levelling network established by Istituto Geografico Militare (IGM; Italian Geographic Military Institution), whose total length exceeds 20,000 km. The heights assigned to levelling benchmarks, whose mutual distance is about 1 km, are obtained from the adjustment of the levelling measurements carried out along the whole network and periodically repeated. Furthermore, in the 1970s of the twentieth century, IGM performed gravity measurements on most survey points of the levelling lines already established at that time, according to the bylaws defined in the first version of United European Levelling Network (UELN; see for example Ihde et al. 2006). They prescribed the determination of geopotential numbers adjusted on the network crossings in order to establish a unique levelling network over the whole west Europe (Sacher et al. 1998; Ihde et al. 2006). Subsequently, in the 1990s, new UELN solutions were produced, taking into account new data coming from eastern Europe countries and from the densification of the levelling networks of some central and northern Europe countries. In the same years, with the establishment of the European Reference Frame (EUREF) network of GPS permanent stations, the EUVN (European Vertical Network, Ihde et al. 1998; Ihde et al. 2000; Kenyeres et al. 2010) project was started in order to link the European levelling network to the reference system defined by EUREF. Finally, the European Vertical Reference System (EVRS) has been established, whose following realizations take into account the updates of the height data provided by the various countries (Sacher et al. 2009). Italy too has recently contributed providing data from about 80 stations distributed all over the country. In the framework of this relevant European project, it is advisable to provide gravity along the entire Italian levelling network. This is in order to compute the proper corrections to spirit levelling increments. As already mentioned, gravity along the Italian levelling lines has not been

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measured extensively and only part of the lines has been surveyed with gravity. Thus, corrections can be evaluated only along this subset of the Italian levelling network. However, the Italian gravity database used for the estimation of the Italian gravimetric geoid (Barzaghi et al. 2007) is quite dense, and gravity points are homogeneously distributed over the entire Italian area (but for the Alpine region where the gravity data coverage is poor). Based on this data set, interpolation of gravity along the levelling lines could be performed and corrections to levelling increments computed on the entire Italian levelling network. The aim of this paper is to set up a test to prove that this procedure is feasible and that reliable corrections can be evaluated. Also, comparisons are set up with gravity predictions based on the EGM2008 geopotential model in order to test for its effectiveness in the same application. This is a remarkable test due to the high-frequency pattern of the gravity field of the chosen test area which, besides the strong gravity signal implied by topography, is also characterized by a relevant geophysical component related to the Ivrea body. The poor gravity coverage of the area makes this test even more significant. If the interpolation procedure is effective with a poor gravity distribution, better results will be obtained in the rest of Italy where a dense gravity coverage exists.

The area under study

The test field chosen to check the influence of orthometric and normal corrections is a region located between north-western Piedmont and Aosta Valley, which is the central area of the so-called Ivrea body. This area shows two different critical aspects: significant height variations in short distances and relevant variations of geoid undulation values due to strong inhomogeneities in the mass densities. In the Ivrea area, a closed levelling line has been established (represented in Fig. 1), including line 155 (117.8 km length) and part of the line AF (85.1 km out of 130.2 km of AF line total length) of the Italian high-precision levelling network, entirely inside Italy, whose height varies from about 250 m to more than 2,600 m. Unfortunately, along part of this line, no gravity measurements are available (the one marked with a blue ellipse in Fig. 1). On the other hand, the existing measurements have been carried out partly some decades ago, partly recently, with different instruments and procedures. The problem is now to evaluate if data gaps can be filled, predicting the missing values with the gravity database available for the Italian quasi-geoid computation. As already mentioned, this database in the Alps region is by far less dense than in the remaining part of the Italian peninsula; this matter will be faced in the next section.

Comparison between predicted and observed gravity data

The computation of the orthometric heights requires the availability of gravity values along the spirit levelling lines. When the gravity has not been observed with an adequate density along levelling lines, it is necessary to predict the gravity

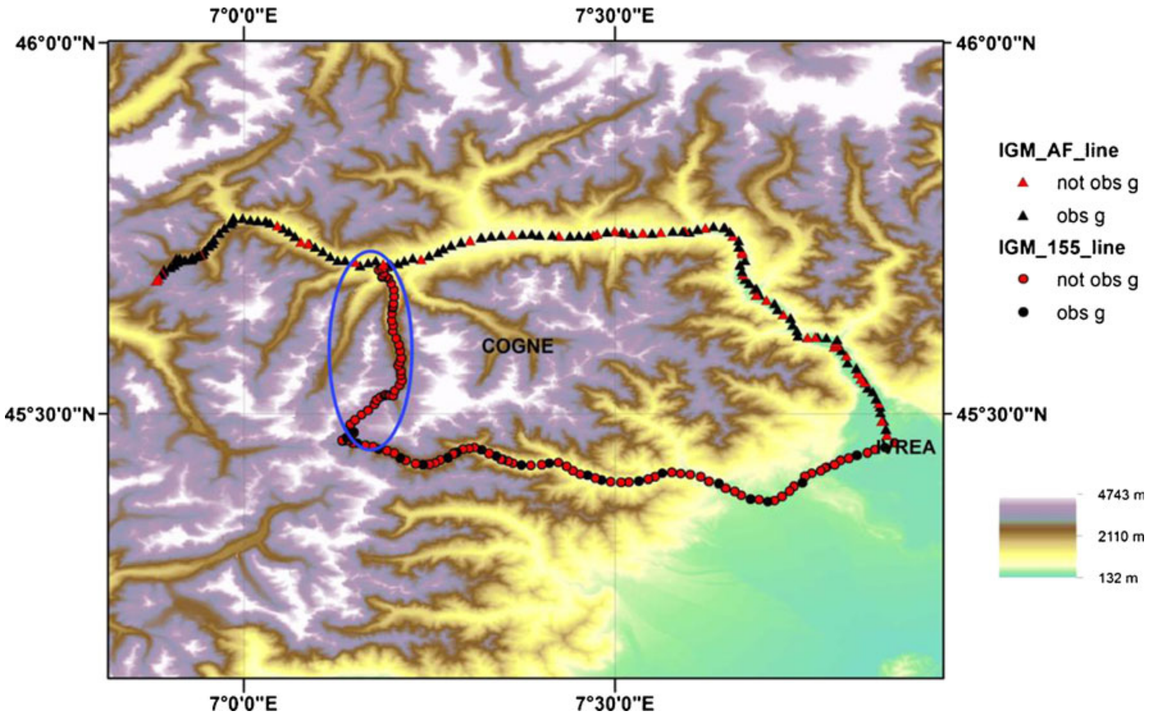

Fig. 1 The two IGM levelling lines (*AF* and *155*), where the presence or the absence of gravity data is shown, in particular, in blue, a full segment without gravity observations is highlighted

values from existing gravity data or from gravity models. In the area of interest, both the Italgeo05 gravity data base (the one used for computing the last Italian geoid estimate, Barzaghi et al. 2007; Albertella et al. 2008) and the EGM08 global geopotential model (Earth Gravitational Model, Holmes and Pavlis 2008) are available, allowing gravity data prediction on the whole territory (Barzaghi and Carrion 2009). The key question is to assess the accuracy and the precision that can be obtained in the predicted gravity values and further on how gravity-based corrections to the levelling increments are influenced. As for an example, one can check for the influence of the predicted data uncertainties on the computation of geopotential differences ΔW. To this aim, the discretised formula $\Delta W = -\sum\overline{g}_i\Delta n_i$ (Heiskanen and Moritz 1967) can be used, where $\overline{g}_i = \frac{g_i+g_{i+1}}{2}$ and Δn_i are the observed levelling increments. The expression of the error is

$$\delta\Delta W = -\sum\delta\overline{g}_i\Delta n_i - \sum\overline{g}_i\delta\Delta n_i \quad (1)$$

The first term in the right-hand side represents the contribution of gravity measurement errors. Its variance is given by (Gentile et al. 2011)

$$E\left\{\left(\sum\delta\overline{g}_i\Delta n_i\right)^2\right\} = \sum\Delta n_i\Delta n_j E\left\{\delta\overline{g}_i\delta\overline{g}_j\right\} \quad (2)$$

Assuming for simplicity that gravity measurements are statistically independent with the same std $\sigma(g)$, owing to the expression of $\overline{g}_i$, only consecutive terms of the sum are correlated (these simplifications are not so relevant since we are only looking for an approximated value of the variance).

Finally, one obtains

$$E\left\{\left(\sum\delta\overline{g}_i\Delta n_i\right)^2\right\} = \frac{1}{2}\sigma^2(g)\left(\sum\Delta n_i^2 + \sum\Delta n_i\Delta n_{i+1}\right) \quad (3)$$

Clearly this quantity is strongly dependent on the altimetric profile. For example, for a line with a total height difference of 2,000 m, divided into 20 steps of 100 m each, setting $\sigma(g)$= 10 mgal, one obtains $\left(E\{(\sum\delta\overline{g}_i\Delta n_i)^{2}\}\right)^{1/2}\cong 4\ gal\cdot m$, corresponding to a height difference of about 4 mm. Hence, it is not necessary to use highly accurate gravimetric data for the computation of geopotential differences.

Having this number in mind, we performed the prediction of gravity on the levelling lines. This has been done on the previously described lines and for the benchmarks where observed gravity data are available (27 points on 155 line and 102 points on AF line).

To get the estimates from the Italgeo05 (Barzaghi et al. 2007; Albertella et al. 2008) gravity database (Borghi et al. 2007, see Fig. 2), we started from the Italgeo05 gravity residuals (Δg_{res}), which had been computed on a $2'\times 2'$ grid, since the prediction based on a smoother signal with respect to the whole gravity vector should improve the reliability of the results. The residuals had been computed according to remove-solve-restore technique:

$$\Delta g_{res} = \Delta g_{free\text{-}air} - \Delta g_{model} - \Delta g_{RTC} \quad (4)$$

where

$$\Delta g_{free\_air} = g_P - \gamma_Q \quad (5)$$

γ_Q is the normal gravity in point Q, homologous of P along the vertical line

Δg_{model} is the long wavelength component, obtained from the geopotential model GPM98CR (Wenzel 1998)

Δg_{RTC} corresponds to the residual terrain effect (Forsberg 1994), computed evaluating the terrain volumes with a DTM.

Then the gravity residuals have been predicted on the benchmarks applying the fast collocation algorithm (Bottoni and Barzaghi 1993). At this step, the gravity signal must be pieced together again adding the model component (Δg_{model}), as well as the residual terrain correction component (Δg_{RTC}), which is computed between the Earth's surface and the telluroid.

$$\Delta g_{free\_air} = \Delta g_{res} + \Delta g_{model} + \Delta g_{RTC} \quad (6)$$

Finally, the normal gravity value (γ) has been calculated and added to the Δg_{free_air} to obtain the estimated gravity values ($\widehat{g}$), see Eq. (7).

$$\widehat{g} = \gamma + \Delta g_{free\_air} \quad (7)$$

In Table 1, the statistics of the differences between the estimated and the observed values are shown for the two IGM lines (an outlier rejection has also been made to remove two anomalous gravity observations). The average values underline the presence of a bias: this is well explained if we take into account that the IGM observed values are referred to a gravity reference system, the Potsdam one, prior to IGSN71, introducing a bias of 14 mgal. When this bias is removed (see Table 2), the agreement is satisfactory, especially taking into account that in the area of interest, the gravity database is particularly sparse (see Fig. 3). This positive result, in view of the comments on formula [3], suggests that the use of the

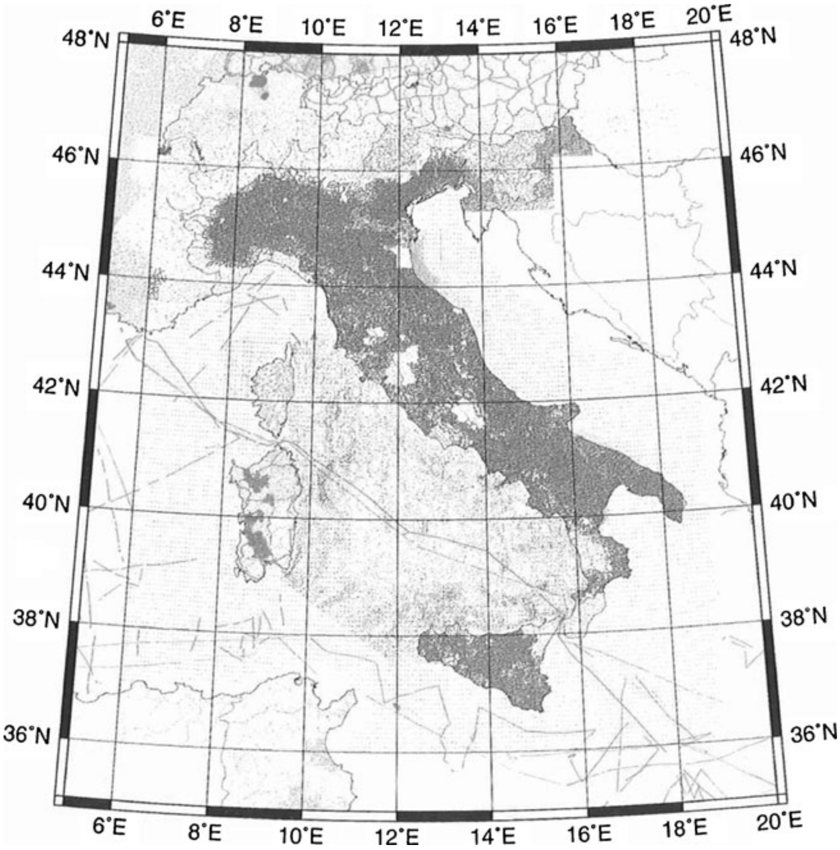

Fig. 2 Italgeo05 gravity database

estimated values could be equivalent to the use of observed values, especially because, as described before, the area of interest presents criticalities both in terms of height and gravimetric signal gradients. Anyway, for further analysis in the following sections, the orthometric correction will be computed both with the estimated and the observed values.

Finally, the gravity values have been predicted using the EGM08 global geopotential model. In this case, the $\Delta g_{\mathrm{model}}$ component has been computed from the model coefficients on the points of interest. Then, the normal gravity value γ has been added to obtain the gravity estimation:

$$\widehat{g}_{\mathrm{EGM08}} = \gamma + \Delta g_{\mathrm{model}} \tag{8}$$

In Table 3, the statistics of the differences between the EGM08 estimated values and the observed ones are shown (in the differences, the Potsdam bias has been consistently removed).

The differences are significant since the geopotential model does not take into account the high-frequency component of the gravity signal coming from the topography. High-frequency gravity components coming from intra-crustal density anomalies are not included in the global geopotential model signal as well. So, it is expected that statistics in Table 3 are worse than those listed in Table 2. It must be also stressed that the standard deviation of the residual is at best around 25 mgal which, according to formula [3], should imply a poor precision in the corrected geopotential differences.

Table 1 Statistics of differences between gravity values estimated with Italgeo05 database and the observed ones before removing the bias with respect to the Potsdam gravity reference

	155 line	AF line
Number of points	27	102
Average	−15.463 mgal	−12.195 mgal
Standard deviation	4.794 mgal	5.914 mgal
Minimum value	−24.870 mgal	−32.730 mgal
Maximum value	−7.520 mgal	−0.950 mgal

Table 2 Statistics of differences between gravity values estimated with Italgeo05 database and the observed ones after the removal of the bias with respect to the Potsdam gravity reference

	155 line	AF line
Number of points	27	102
Average	−1.463 mgal	1.805 mgal
Standard deviation	4.794 mgal	5.914 mgal
Minimum value	−10.870 mgal	−18.730 mgal
Maximum value	6.480 mgal	13.050 mgal

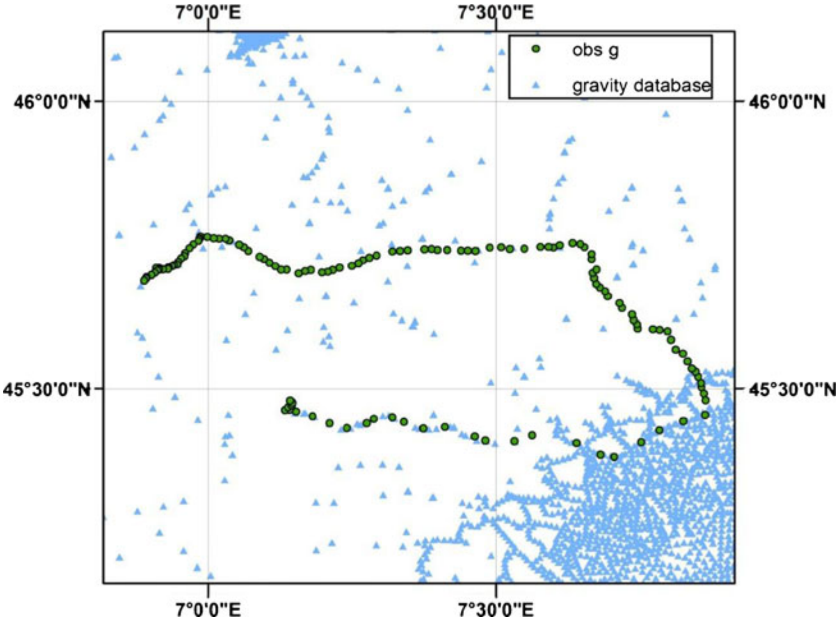

Fig. 3 Observed gravity data on lines AF and 155 and gravity database distribution in the area of interest

The orthometric correction computation for the two levelling lines

As well known, the orthometric correction is needed to take into account the non-parallelism among the level surfaces (the equipotential surfaces of the gravity field). In our computations, the standard formula given in Heiskanen and Moritz (1967) has been considered

$$\mathrm{OC}_{AB} = \sum_{A}^{B} \frac{g-\gamma_0}{\gamma_0} \times \Delta n + \frac{\overline{g}_A-\gamma_0}{\gamma_0} \times H_A - \frac{\overline{g}_B-\gamma_0}{\gamma_0} \times H_B \quad (9)$$

where:

OC_{AB} is the orthometric correction on the AB interval

Δn is the levelling increment

g is the average gravity value along the levelling line

γ_0 is normal gravity for an arbitrary standard latitude, in this case φ=45°:γ_0= 980.6294 gal

$\overline{g}_A$ and $\overline{g}_B$ are the mean values of the gravity along the plumb line between the ground points, A and B respectively, and the corresponding point on the geoid. The $\overline{g}$ value has been computed considering the normal density ρ=2.67g/cm^3, according to the simplified Prey reduction, with the formula:

$$\overline{g} = g + 0.0424 \times H \quad (10)$$

where g is measured in gals and H in kilometres

Table 3 Statistics of differences between the gravity data estimated with EGM08 and the observed ones after the Potsdam bias removal

	155 line	AF line
Number of points	27	102
Average	58.120 mgal	55.370 mgal
Standard deviation	63.260 mgal	24.330 mgal
Minimum value	−38.820 mgal	−11.160 mgal
Maximum value	150.05 mgal	119.540 mgal

H_A and H_B are the orthometric heights of two benchmarks of a line.

To take into account the non-availability of orthometric heights in the computation (H_A and H_B in Eq. (9), the orthometric correction along the levelling lines should be computed iteratively. Nevertheless, it has been verified that the use of non-corrected heights in Eq. (9) does not affect the results at the sub-millimetre level. This can also be evaluated observing that a variation of a few centimetres in H_A or H_B (which are expressed in kilometres) has an impact on the second and third term of Eq. (9): when these variations are multiplied by $\frac{\overline{g}_A-\gamma_0}{\gamma_0}$, which is in the order of magnitude of about $10^{-4} \div 10^{-5}$, they give very small contributes, of the order of $10^{-3} \div 10^{-4}$ mm. Adding these contributions over the whole line, the effect on the orthometric correction computations is at most about $10^{-1} \div 10^{-2}$ mm. Despite the fact that the iteration could be neglected, it has been taken into account in the computation, for the sake of completeness.

In the previous paragraph, it was pointed out that predicted values should be precise enough to allow a reliable orthometric correction (OC) estimation. To confirm the feasibility of using predicted gravity data instead of the observed ones, the orthometric correction has been computed for the two levelling lines both with observed and predicted gravity. As we can see in Table 4, the orthometric correction along the

Table 4 Orthometric correction computed both with observed and predicted gravity data for the two considered levelling lines

	OC with observed gravity	OC with predicted gravity
Line 155	0.459 m	0.473 m
Line AF	0.295 m	0.292 m

lines has significant values and the results obtained with observed and predicted values are comparable (1.4 cm difference for line 155 and 0.3 cm difference for line AF). The variation between the OC values evaluated for the two lines (around 46 cm for line 155 and around 30 cm for line AF) can be explained considering that line 155 presents 2,400 m height difference , while line AF presents 1,900 m height difference. In this case, the whole AF line is considered, not only the portion involved in the closed loop (see Fig. 1), to consider all available gravity measures.

These results prove that, at least in the area under analysis, the OC based on predicted gravity is substantially equivalent with the one computed with observed data.

This also strengthens what has been previously discussed: it is reasonable to fill the gaps in the observed gravity data (see Fig. 1) for the closed loop formed by 155 and AF lines with the gravity predictions. In this way, it is possible to assess the improvement on the levelling misclosure applying the orthometric correction to the measured height differences. To evaluate the misclosure, only the portion of line AF involved in the closed levelling loop is considered (see Fig. 1).

From Table 5, the significant reduction in the levelling misclosure is evident: it becomes less than half of the error obtained without the orthometric correction. Besides, the reduced value is of the same order of magnitude as the error tolerance for high-precision levelling, which is, according to the $\Delta h = 2.5 \times \sqrt{l}$ formula (Blachut et al. 1979; Intesa Stato and Gruppo di lavoro Reti plano-altimetriche 1998) equal to ~0.035 m (for ~200 km closed line length).

Finally, also the possibility to compute the orthometric correction using predicted values only has been considered. This could be useful in particular when, for the area under study, a gravity database is available, but no gravity observations are given along levelling lines.

Table 5 Levelling misclosure without and with the orthometric correction: gravity observation gaps filled with predicted data

Gravity observation gaps filled with predicted data	Without the OC	With the OC
Levelling misclosure	0.070 m	0.027 m

Also in this case, see Table 6, the misclosure has been significantly reduced, even though it is slightly above the tolerance value.

The gravity data have been estimated considering the EGM08 (Holmes and Pavlis 2008) global model as well. The use of a global model could be necessary when a dense gravity database is not available on the area of interest since the EGM08 proved to be very reliable even locally (Barzaghi and Carrion 2009; Claessens et al. 2009; Kotsakis et al. 2009; Roman et al. 2009). However, worse results are expected since the EGM08 predicted gravity values are in a poor agreement with observed data (see comments below Table 3). The estimated OCs listed in Table 7 seem to confirm this statements: the EGM08 based OC is remarkably different from the one obtained from observed gravity.

To confirm this result, we computed the height values obtained after the orthometric correction computed, considering observed and estimated values, both with Italgeo05 gravity database and EGM08 models (see Table 8). As expected, the corrected heights derived from observed gravity are in a good agreement with those estimated using the Italgeo05 database. On the contrary, the EGM08 corrected heights display significant differences. Also, as it is reasonable, it has been found that major discrepancies among the different estimates are obtained where point heights correspond to their maximum values.

Finally, for the sake of completeness, the levelling misclosure has been computed with gravity values estimated with EGM08 as well (Table 9).

The estimated misclosure is even lower than the one obtained with the Italgeo05 gravity data. However, due to the results of Tables 5, 6 and 8, which prove that EGM08-derived predictions are less accurate than those based on observed gravity, it can be concluded that this is only one admissible random number close to zero.

The geopotential numbers and the normal heights

The normal height H^* is defined as the separation between the ellipsoid and the telluroid, which is the surface of all points $\overline{P}$, lying on the ellipsoid normal through a point P on the Earth's surface, for which $U(\overline{P}) = W(P)$, where W is the physical gravity potential and U is the normal potential. The normal height can be computed from the geopotential number

$$C(P) = W_0 - W(P) \tag{11}$$

Table 6 Levelling misclosure without and with the orthometric correction: predicted gravity data only

Predicted gravity data only	Without the OC	With the OC
Levelling misclosure	0.070 m	0.037 m

Table 7 Orthometric correction computed with observed and gravity data predicted by means of the EGM08 model for the two considered levelling lines

	OC with observed gravity	OC with gravity predicted with EGM08
Line 155	0.459 m	0.662 m
Line AF	0.295 m	0.385 m

(where W_0 is the geoid gravity potential) using the formula

$$H^* = C/\overline{\gamma} \tag{12}$$

where $\overline{\gamma}$ is the average of the normal gravity γ along the ellipsoid normal from the ellipsoid to the telluroid.

$C(P)$ has been computed along the levelling line starting from the value extracted from IGM tables for the initial crossing point of the line and subtracting the increments computed from the measured levelling differences and gravity values:

$$C(P) = C(P_0) - W(P) + W(P_0) \tag{13}$$

where the increment between two successive benchmarks has been computed multiplying the height difference by the average value of the gravity. Thus, it must be underlined that these are correct geopotential increments computed properly taking into account gravity.

The procedure adopted for the computation of $\overline{\gamma}$ is the following: the value $\gamma_{\text{ell}}(\varphi)$ of the normal gravity on the ellipsoid is obtained from (Heiskanen and Moritz 1967) and a constant vertical gradient is assumed, with value 0.3086 mgal/m. Hence,

$$\gamma(\varphi, h) = \gamma_{\text{ell}}(\varphi) - \partial_h\gamma \times h \tag{14}$$

$$\Rightarrow \overline{\gamma}(\varphi, H^*) = \gamma_{\text{ell}}(\varphi) - \frac{1}{2}\partial_h\gamma \times H^* \tag{15}$$

Table 8 Height differences corrected with the OC, computed using observed and estimated gravity values, both with Italgeo05 gravity database and EGM08 model

	Italgeo05 gravity database		EGM08 model	
	155 line	AF line	155 line	AF line
Number of points	27	102	27	102
Average	0.000 m	0.000 m	−0.005 m	−0.012 m
Standard deviation	0.006 m	0.004 m	0.081 m	0.028 m
Minimum value	−0.006 m	−0.006 m	−0.095 m	−0.068 m
Maximum value	0.014 m	0.019 m	0.202 m	0.090 m

In Eq. (15), the unknown value of H^* can be replaced by the value of H obtained from adjusted levelling, which differs by not more than 20 cm. Consequently, the error introduced for $\overline{\gamma}$ does not exceed 0.03 mgal. On the other hand, a 1 % error in $\partial_h\gamma$ sums up to an error on $\overline{\gamma}$ of about 4 mgal at a height of 2,500 m. In order to evaluate the effect of these errors on the value of H^*, one can write

$$H^* + \delta H^* = \frac{C}{\overline{\gamma} + \delta\overline{\gamma}} \cong H^*\left(1 - \frac{\delta\overline{\gamma}}{\overline{\gamma}}\right) \Rightarrow \frac{\delta H^*}{H^*} = -\frac{\delta\overline{\gamma}}{\overline{\gamma}} \tag{16}$$

For example, if $\delta\overline{\gamma}$ =5 mgal, at a height of 2,500 m, the corresponding variation of H^* is 1.25 cm.

Once orthometric and normal heights have been computed, an interesting check of the results is given by the formula (Heiskanen and Moritz 1967)

$$H - H^* = \zeta - N \doteq -\frac{\Delta g_{Bouguer}}{\overline{\gamma}}\Bigg) \times H \tag{17}$$

The normal heights have been computed as described above, using formula [12]. The value 245.601 m at the initial crossing point, $H^*_{P_0}$, obtained from the geopotential number reported in IGM tables, is about 18 cm above the 245.418 m height provided by IGM for two-line crossing point. By the way, the misclosure obtained for normal heights is about 0.024 m.

The differences H-H^* are initially negative and grow with height, reaching the maximum value of about +23 cm at the maximum height of 2,616 m, with an increase of more than 40 cm, as illustrated in Table 10.

Bouguer anomalies have been computed along the levelling line using measured or interpolated gravity values. Their variation is very large, from about +60 mgal to about −190 mgal. The Eq. (17)is not satisfied, but the difference between the left- and the right-hand side has a very small variation, between about −17 and −19 cm (see Table 10), with an average value of −17 cm and a standard deviation of 1.3 cm (Table 11). This bias is clearly related to the initial difference between orthometric and normal height mentioned above, which has not been directly checked and has to be investigated more deeply. Some anomalous values can be found corresponding to high altitudes in Table 10, e.g. see points 081

Table 9 Levelling misclosure without and with the orthometric correction: gravity data predicted from EGM08 global model only

EGM08 predicted data only	Without the OC	With the OC
Levelling misclosure	0.070 m	−0.027 m

Table 10 Examples of $\boldsymbol{H - H^* - \Delta g_{\text{Bouguer}}/\gamma \times H}$ bias along 155 line, considering about one point out of ten and including points with maximum height

	$\widetilde{H}$ (without OC) [m]	H (with OC) [m]	H^* [m]	$-\Delta g_{\text{Bouguer}}/\gamma \times H$ [m]	$H - H^*$ [m]	$H - H^* + -\Delta g_{\text{Bouguer}}/\gamma \times H$ [m]
001	235.900	235.908	236.082	0.007	−0.174	−0.167
011	332.886	332.894	333.075	0.013	−0.181	−0.169
021	357.851	357.860	358.042	0.017	−0.182	−0.165
031	484.583	484.625	484.777	−0.016	−0.151	−0.167
041	614.975	615.053	615.167	−0.058	−0.114	−0.172
051	888.885	889.027	889.060	−0.140	−0.032	−0.172
061	1,461.852	1,462.057	1,462.007	−0.225	0.050	−0.175
071	1,754.530	1,754.813	1,754.687	−0.302	0.126	−0.176
081	2,418.217	2,418.623	2,418.421	−0.335	0.203	−0.132
082	2,488.666	2,489.085	2,488.878	−0.388	0.207	−0.181
084	2,616.255	2,616.709	2,616.484	−0.354	0.226	−0.128
091	2,313.048	2,313.423	2,313.227	−0.379	0.196	−0.184
101	1,726.267	1,726.524	1,726.405	−0.305	0.119	−0.186
111	1,292.231	1,292.406	1,292.360	−0.235	0.046	−0.189
121	994.780	994.896	994.918	−0.169	−0.022	−0.191
127	688.356	688.446	688.517	−0.122	−0.070	−0.193

and 084: this behaviour will also be further investigated. Anyway, the results seem to be quite interesting due to the large variations of both constituents of Eq. (17).

Conclusions

The gravity corrections to levelling line increments should be always taken into account for theoretical reasons. However, in many practical applications, they are disregarded as they are considered smaller than the random error associated to spirit levelling. In this paper, we proved that, at least in some particular areas, they have a relevant impact. If they are applied, they can strongly reduce the misclosure on closed levelling loops. In our test area, the western Alps region, we proved that an out-of-tolerance misclosure is substantially reduced to a value which satisfies the tolerance condition for high-precision levelling. The new and interesting point is that the results have been obtained filling the gravity data gaps using interpolated gravity values (gravity observations were not available for the whole loop). It must be further stressed that this was an important test due to the strong topographic variations and the roughness of the gravity field in the considered area. The best results were obtained using the Italgeo05 gravity database which allowed the computation of reliable gravity values. This has been proved by comparing the predicted and the observed available gravity data along the two levelling lines in the area under investigation. The discrepancies have a standard deviation which is in a range of precision allowing the computation of sufficiently reliable corrections. The same computations were also carried out using EGM2008 predicted gravity values obtaining, however, poorer results. Finally, the effectiveness of gravity corrections was also tested by comparing normal and orthometric heights which has been found to be in a reasonable agreement. So, we can state that predicted gravity, based on a reliable gravity database, can be used in computing the corrections to spirit levelling. This opens a new perspective for the computation of these corrections to the whole Italian levelling line data set.

Table 11 Statistics of $\boldsymbol{H - H^* - \Delta g_{\text{Bouguer}}/\gamma \times H}$ along 155 line in metres

Points	130
Average	−0.174
Standard deviation	0.013
Min	−0.193
Max	−0.128

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