Specific accelerating factor: One more tool in motor sizing projects

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1. Introduction

A classical electrotechnical approach in motor design is finding precise load performance requirements in order to built the most appropriate customized machine for the application. This is the practice for big motors and/or for particularly critical applications, in which development costs are justified. In automation or mechatronic field, instead, developing brand new motors for every different application is basically unsustainable; despite that, high performances are equally requested. Only solution is choosing as best as possible the motor from a commercial list of existing ones. This procedure is called *motor sizing*, and it's absolutely different from the previous approach; electrical machines are seen as ''black-boxes'', and manufacturer's catalogues are generally the only available data.

The criteria for the correct choice of electric brushless motor and gearbox in the automation field have been widely studied since 1980s [1]. Some procedures consider purely inertial loads applied to the motor [2], while others consider a more generic load [3,4]. Generally, as presented in [5], the mechanical efficiency and the inertia of the transmission are not considered until the verification phase, while in [6] such effects are taken into account since the beginning of the machine design.

Whatever method is used, two situations can happen:

– More than one couple motor–gearbox is suitable for the given application;
– None known motor–gearbox is suitable for the given application;

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Considering this, a good sizing method should also answer the following designer's questions:

(a) *"For the many suitable motor–gearboxes I've found, can I calculate, with poor datasheets information, a parameter that makes a performance classification for the given application?"*
(b) *"Can I find an index, theoretic or practical, that gives the idea how a certain motor is far from top quality performances?"*
(c) *"I haven't found a suitable motor in my database, but can I hope to find something good in another database/catalogue or my load requirements are too heavy?"*

Methodologies presented in literature [1–8] are primarily based on the thermal analysis, searching for the maximum torque the motor can exert without overheating. Most of these methods define a parameter starting from the characteristics of the motor, independent by the given application; this quantity can be calculated using only the information collected in the manufacturer catalogs. Several different definitions of this parameter are present in the literature. One of the most common is based on the relationship between the motor rated torque and its moment of inertia.

This parameter is called "accelerating factor" or "continuous duty power rate factor" or simply "power rate". The power rate is a well known concept in literature and different considerations have been derived to handle it as a usefulness motor coefficient in comparison with the torque-to-inertia ratio [9–11].

The accelerating factor is useful for assessing whether a motor is able to perform a given task, but it's not sufficient to answer previous questions; no comparison between suitable motors is available (two brushless motors, very different in terms of construction, size and weight, may have the same accelerating factor), and a superior limit of the accelerating factor does not exist. For this reason the designer of an automatic machine has difficulties in understanding whether the choice done is the best or not.

This work is focused on the analysis of the accelerating factor from a phenomenological point of view, analyzing data available on catalogs of motor manufacturers and trying to find a relationship between this parameter and the construction features of a brushless motor. The purpose is not to find a formula to help the motor manufacturer to improve the performance of their devices, but to help the designer to choose the most valid motor for his application. In particular, through the results of this activity, the designer has a tool to assess the chosen motor and to understand if something better could be available on the market; a benchmark for the accelerating factor would help in selecting the best motor–transmission coupling.

The writ is structured as follows: in the first section a brief compendium of [4] reprises the accelerating factor sizing method. In the second section the performance of some brushless motors are compared. In the third section the investigation on the electromechanical model of a brushless motor is discussed and in the fourth section a new parameter useful to compare the performance of different motors is introduced. Finally conclusions are drawn in the last section.

Symbols used in the paper are in Table 1.

2. Selection criterion

2.1. The model

A complex automatic machine can be divided into simpler subsystems, able to operate one degree of freedom. As shown in Fig. 1 they can be summed up in three key parts: servo-motor, transmission and load. While the load characteristics are completely known as they depend on the machine task, the motor and the transmission are unknown until their selection.

Brushless motors working range consists of a continuous working zone (rated torque zone) and a dynamic zone (related to the maximum motor torque $T_{M,max}$) (Fig. 2).

Knowing only essential data from catalogs, the rated torque is usually considered constant and equal to $T_{M,N}$ up to maximum motor speed $\omega_{M,max}$ [12].

Frequently, in industrial applications, the machine task is periodic with cycle time t_a much smaller than the motor thermal time constant. The motor behavior can therefore be analyzed through the root mean square (rms) value of T_M defined as:

$$T_{M,rms} = \sqrt{\frac{1}{t_a}\int_0^{t_a} T_M^2 \, dt} \quad (1)$$

namely the torque that, acting steadily over the cycle, generates the total energy dissipation.

The selection of the actuator requires to check the following conditions:

– rated motor torque:

$$T_{M,rms} \leqslant T_{M,N}; \quad (2)$$

– maximum motor speed:

$$\omega_M \leqslant \omega_{M,max}; \quad (3)$$

– maximum motor torque:

$$T_M(\omega_M) \leqslant T_{M,max}(\omega_M). \quad (4)$$

Fig. 2 graphically show the meaning of inequalities (2)–(4).

Conditions (2)–(4) are well known in literature and represent the starting point of all the procedures for motor and reducer selection.

2.2. The accelerating factor and the load factor

The motor torque T_M can be written as:

$$T_M = \tau T_L^* + J_M\dot{\omega}_M = \tau T_L^* + J_M\frac{\dot{\omega}_L}{\tau} \quad (5)$$

where:

$$T_L^* = T_L + J_L\dot{\omega}_L \quad (6)$$

is the generalized resistant torque at the load shaft. In Eq. (6) all the terms related to the load are known.

The root mean square value of the torque T_M is computed from Eqs. (1) and (5):

$$T_{M,rms}^2 = \int_0^{t_a} \frac{1}{t_a}\left(\tau T_L^* + J_M\frac{\dot{\omega}_L}{\tau}\right)^2 dt = \tau^2 T_{L,rms}^{*^2} + J_M^2\frac{\dot{\omega}_{L,rms}^2}{\tau^2} + 2J_M(T_L^*\dot{\omega}_L)_{mean} \quad (7)$$

Introducing Eqs. (7) in (2) and dividing by the motor momentum of inertia J_M:

$$\frac{T_{M,N}^2}{J_M} \geqslant \tau^2\frac{T_{L,rms}^{*^2}}{J_M} + J_M\frac{\dot{\omega}_{L,rms}^2}{\tau^2} + 2(T_L^*\dot{\omega}_L)_{mean} \quad (8)$$

Now two parameters can be defined: the motor's accelerating factor α and the load factor β:

$$\alpha = \frac{T_{M,N}^2}{J_M} \quad (9)$$

$$\beta = 2\left[\dot{\omega}_{L,rms}T_{L,rms}^* + (\dot{\omega}_L T_L^*)_{mean}\right] \quad (10)$$

The coefficient α does not depend on the machine task, it's easy to calculate from manufacturer catalogs and can be traced back to the quantities used in [7–9,15].

On the contrary, the coefficient β depends only on the working conditions because it represents the power rate required by the system. The measurement unit of both factors is (W/s). Substituting α and β in Eq. (8):

$$\alpha \geqslant \beta + \left[T_{L,rms}^*\left(\frac{\tau}{\sqrt{J_M}}\right) - \dot{\omega}_{L,rms}\left(\frac{\sqrt{J_M}}{\tau}\right)\right]^2 \quad (11)$$

Since the term in brackets is always positive, or null, the load factor β represents the minimum value of the right hand side of Eq. (11).

2.3. Range of suitable transmission ratio

Solving the biquadratic inequality (11), for each motor, there is a range of acceptable gear ratios:

$$\tau_{min}, \tau_{max} = \frac{\sqrt{J_M}}{2T_{L,rms}^*}\left[\sqrt{\alpha - \beta + 4\dot{\omega}_{L,rms}T_{L,rms}^*} \pm \sqrt{\alpha - \beta}\right] \quad (12)$$

With:

$$\tau_{min} \leqslant \tau \leqslant \tau_{max} \quad (13)$$

the condition expressed in Eq. (2) is satisfied. The range width $\Delta\tau$ is a function of the difference between the two factors α and β:

$$\Delta\tau = \tau_{max} - \tau_{min} = \frac{\sqrt{J_M}}{T_{L,rms}^*}\sqrt{\alpha - \beta} \quad (14)$$

Table 1
Nomenclature.

Symbol	Description	Unit
T_M	Motor torque	(Nm)
J_M	Motor momentum of inertia	(kg m^2)
$T_{M,rms}$	Motor root mean square torque	(Nm)
$T_{M,N}$	Motor nominal torque	(Nm)
$T_{M,max}$	Motor maximum torque	(Nm)
T_{M,N_c}	Motor nominal torque at zero speed	(Nm)
ω_M	Motor angular speed	(rad/s)
$\dot{\omega}_M$	Motor angular acceleration	(rad/s^2)
$\omega_{M,max}$	Maximum motor speed	(rad/s)
T_L	Load torque	(Nm)
J_L	Load momentum of inertia	(kg m^2)
J_T	Transmission inertia	(kg m^2)
T_L^*	Generalized load torque	(Nm)
$T_{L,rms}^*$	Generalized load root mean square torque	(Nm)
$T_{L,max}$	Load maximum torque	(Nm)
ω_L	Load angular speed	(rad/s)
$\dot{\omega}_L$	Load angular acceleration	(rad/s^2)
$\dot{\omega}_{L,rms}$	Load root mean square acceleration	(rad/s^2)
$\omega_{L,max}$	Maximum load speed	(rad/s)
$\tau = \omega_L/\omega_M$	Transmission ratio	
τ_{min}	Minimum acceptable transmission ratio	
τ_{max}	Maximum acceptable transmission ratio	
$\Delta\tau = \tau_{max} - \tau_{min}$	Range of acceptable transmission ratio	
η	Transmission mechanical efficiency	
$(\ldots)_{mean}$	Average value of the quantity in brackets (various)	
$[\ldots]_{max}$	Maximization of the quantity in brackets (various)	
α	Accelerating factor	(W/s)
α_p	"proof" Accelerating factor (test motor)	(W/s)
α_{cat}	Catalogue accelerating factor	(W/s)
α_s	Specific accelerating factor	(W/(ms))
$\alpha_{s,max}$	Maximum valued specific accelerating factor	(W/(ms))
β	Load factor	(W/s)
t_a	Cycle time	(s)
$\bar{\varepsilon} = [\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n]$	Motor's internal construction parameters or coefficients (hidden)	(various)
$\bar{\gamma} = [\gamma_1, \gamma_2, \ldots, \gamma_m]$	Motors's known data (available on catalogue)	(various)
P	Motor power	(W)
E	Motor phase voltage	(V)
I	Line current	(A)
k_w	Winding factor	
U	Number of conductors in series per phase	
EMF	Electromotive force	(V)
E_c	Electromotive force of a single conductor	(V)
f	Supply frequency	(Hz)
p	Poles number	
q	Slots number	
k_B	Flux density coefficient at the air gap	
A	Linear current density	(A/m)
l	Active rotor length	(m)
D	Rotor diameter	(m)
τ_p	Pole pitch	(m)
τ_c	Slot pitch	(m)
δ	Air gap	(m)
PM	Permanent magnet	
h_m	Magnet height	(m)
B_δ	Maximum flux density at the air gap	(T)
B_{av}	Average flux density at the air gap	(T)
B_r	Residual flux density	(T)
H_c	Coercivity magnetic force	(A/m)
H_0	Coercivity reverse magnetic force	(A/m)
μ_0	Air magnetic permeability	(H/m)
μ_{rev}	Magnetic reverse permeability	(H/m)
M_0	No load magneto motive force	(A)
θ_δ	Air gap reluctance	(H^{-1})
θ_m	Magnetic internal reluctance	(H^{-1})
θ_{disp}	Leakage reluctance	(H^{-1})
k_{disp}	Magnetic leakage coefficient	
A_m	Effective area of the magnet	(m^2)
A_δ	Effective area of the air gap	(m^2)
ϕ_δ	Maximum magnetic flux at the air gap	(Wb)
ϕ_m	Maximum magnetic flux provided by the magnet	(Wb)
K_c	Carter factor	
ρ_{eq}	Rotor equivalent density	(kg/m^3)
I_t	Total slot current	(A)

Table 1 (*continued*)

Symbol	Description	Unit
α_r	Slot filling factor	
b_c	Slot width	(m)
h_c	Slot height	(m)
S	Current density	(A/m^2)
ρ_{cu}	Copper resistivity at nominal over temperature	(Ω/m)
V_{cond}	Volume of conductors in a slot	(Ω/m)
λ	Thermal transfer coefficient	(W/m^2)
A_{ex}	Area of thermal transfer	(m^2)
$\theta\prime$	Conductor nominal over temperature	(C)
N	ID number of motors in database	
M	Motor mass	(kg)
x	Width (equal to height) of motors in database	(m)
z	Depth of motors in database	(m)
k_l	Active length factor	(m)

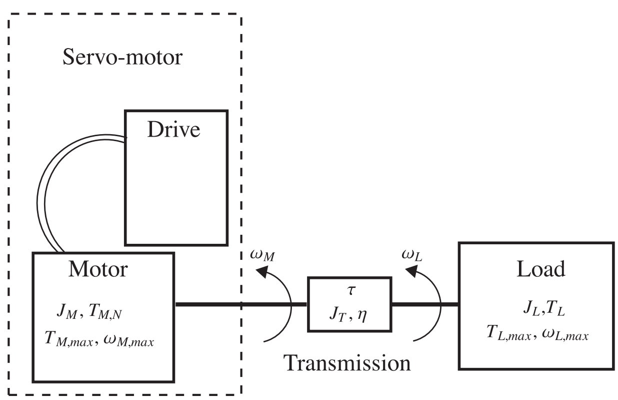

Fig. 1. Model of a generic servo-system.

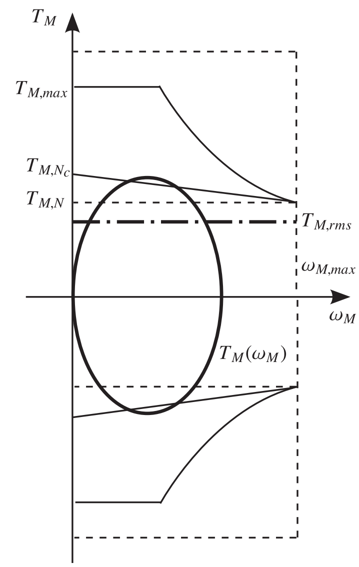

Fig. 2. A simplified speed/torque curve of a common brushless motor. The bold line represents $T_M(\omega_M)$, the torque requested to the motor during a cycle.

3. Brushless motors's performances

Following the sizing procedure shown in [4], and reprised in previous paragraph, α is able to discriminate whether a motor, regardless of type, can fulfill a given task.

The designer who is involved in the choice of the servomotor normally consults the catalogs of commercial devices searching for one with enough accelerating factor.

The process is then based only on the designer experience, instinct and amount of accessible catalogues. Previous questions are still without answer. In this situation the designer is in trouble, because he has no benchmarks; sometimes motors with similar dimensions and power result in extremely different values of α. On the contrary, motors with very different features can have very tight values of accelerating factor. The idea behind this work is providing a tool to help the designer making a choice more rational and systematic.

This tool must be easy to obtain with available catalogues data, easy to use in a sizing procedure and supported by a well-known theoretic base. Extreme accuracy is not necessary and neither possible, because with catalogues data, usually rounded, very precise calculations have no sense. Furthermore, the mechatronics engineer has to manage the behavior of all the servo-system, he cannot focus on the motor part only; he simply needs a rough but efficient benchmark to evaluate and compare the possible choices.

The first approach was statistic: collecting a consistent amount of motor data to have a general overview of the state of the art and better orient the following efforts; a database composed by commercial available motors has been created. It contains about 300 different motors sensible data, as reported in [13,14], and allows to compare the information available from the catalogs. Information collected are: brand, model, type of motor (AC or DC), torque coefficient, winding electrical resistance, number of poles, geometrical dimensions and, naturally, motor nominal torque and the rotor momentum of inertia. The motors are identified by a unique number (N) and they can vary significantly in their constructive features. Such a big quantity of data it's not useful for research only, but it generally increases the possibility to select the better sized motor.

The accelerating factor has been computed considering a rectangular working field, with nominal torque $T_{M,N}$ delivered at maximum speed $\omega_{M,max}$ (Fig. 2).

In Fig. 3 the motors are sorted by growing moment of inertia; to better display all data, the maximum value was used as reference base to divide the others, obtaining a per-unit (pu) scale; with the same approach, the per-unit nominal torque and accelerating factor are reported too.

Similar α values are obtained through extremely different torques $T_{M,N}$ and inertia J_M;

as example, two motors compared in Table 2 have $\alpha \cong 42 \cdot 10^3$ [W/s] with different nominal torque, moment of inertia and size.

This simple case means there's not a unique correlation between the accelerating factor and the motor dimensions. In [13] a relationship between the external motor dimensions and the accelerating factor has been investigated but no conclusive results have been derived.

Synchronous brushless motors for automation field are extremely heterogeneous in terms of dimensions and performance, so a deeper theoretical analysis is necessary searching for the influence of construction parameters on the accelerating factor.

4. Electromechanical approach

In order to investigate the meaning of the factor α, it should be expressed as a function of a certain number of motor's parameters:

$$\alpha = f(\bar{\varepsilon}, \bar{\gamma}) \tag{15}$$

where $\bar{\varepsilon} = [\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n]$ is a vector of internal construction parameters or coefficients (hidden) and $\bar{\gamma} = [\gamma_1, \gamma_2, \ldots, \gamma_m]$ is a vector of known data available on catalogue.

The target motor considered for the study is a sinusoidal brushless motor, isotropic and with a three phase star connection. This is not true for all database's devices, but it's a good way to start the analysis by the same point of reference.

Once again, purpose of the model is not the synthesis of the best design, but the analysis of several machines searching for the benchmark in sizing process; differences of approach are evident if compared with [19], in which the focus is instead the motor construction using detailed geometrical relations.

Following equations are well known in literature [16]; the intention is insulating in (15) a function g of the kth term of $\bar{\gamma}$, obtaining a definition of the specific accelerating factor:

$$\alpha_s = \frac{\alpha}{g(\gamma_k)} \tag{16}$$

Explicit formulation of Eq. (15) will be later verified on a real target motor; after that, choosing reasonably a maximizing set of $\bar{\varepsilon}, \bar{\gamma}$ parameters, an estimation of $\alpha_{s,max}$ can be performed:

$$\alpha_{s,max} = \frac{[f(\bar{\varepsilon}, \bar{\gamma})]_{max}}{g(\gamma_k)} \tag{17}$$

4.1. The motor torque

Starting from the power P absorbed by the motor and considering all the power to be active power ($\cos\varphi \simeq 1$), the following can be obtained:

$$P = 3EI = 3k_w U E_c I = T_M \omega_M \tag{18}$$

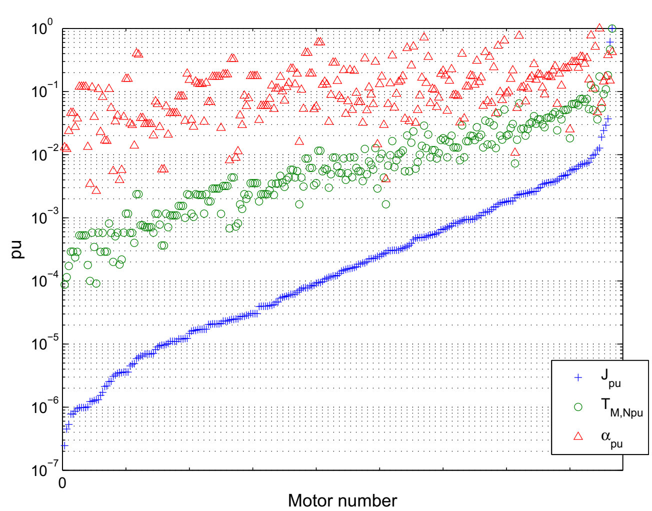

Fig. 3. Motors ordered with increasing values of J_M.

Table 2
Comparison between motors 44 and 230.

N		44	230
$T_{M,N}$	(Nm)	1.3	3.3
J_M	(kg m^2)	4.00E−05	2.60E−04
α	(W/s)	42.25E+03	41.88E+03
x	(m)	120E−03	100E−03
z	(m)	82E−03	158E−03

where E is the motor phase voltage, I the line current and k_w is the winding factor. The term U represents the number of active conductors in series per phase which contribute to generate the EMF. E_c is the EMF for every single active conductor.

Defining the linear current density A the current distribution on stator's circumference, I is obtained regardless the particular shape of the stator windings:

$$A = \frac{3UI}{p\tau_p} \tag{19}$$

Expressing also E_c as a function of the mechanical and the electro-magnetic parameters,

$$E_c = \frac{\pi}{\sqrt{2}} f k_B B_\delta \tau_p l = \frac{\pi}{\sqrt{2}} f B_{av} \tau_p l \tag{20}$$

the following is derived:

$$P = k_w \frac{\pi}{\sqrt{2}} f k_B B_\delta \tau_p^2 l A p = k_w \omega_M \frac{\pi^2}{4\sqrt{2}} D^2 k_B B_\delta l A \tag{21}$$

being $\omega_M = 4\pi f/p$, p the poles number, B_{av} the average flux density at the air gap and τ_p the pole pitch. Remembering the (18) the nominal torque is given by:

$$T_{M,N} = \frac{\pi^2}{4\sqrt{2}} k_w k_B B_\delta D^2 l A \tag{22}$$

In this way, the nominal toque is expressed as function of geometric dimensions (rotor diameter D and active rotor length l) and electro-magnetic quantities (maximum flux density at the air gap B_δ and A).

4.2. Flux density at the air gap

The magnetic equivalent circuit of an electric motor per pole can be described as in Fig. 4 [17,18]. The network represents a general disposition of a PM in a magnetic core circuit. Leakage flux, as well as air-gap flux, are determined by the leakage and the air-gap reluctances respectively, whereas PM's determine the flux injected in the circuit. The considerations derived from the network are valid for every motor in the database, even if they have clearly constructive technology differences.

The reluctance of the core is considered to be negligible with respect to the one of the air gap. The no load magneto motive force M_0 is defined as:

$$M_0 = H_0 h_m \tag{23}$$

being H_0 the coercivity reverse magnetic force of the magnet and h_m the magnet height in the direction of magnetization.

Considering the equivalent circuit of Fig. 4, the flux at the air gap can be computed as:

$$\phi_\delta = \frac{\theta_{disp}}{\theta_{disp} + \theta_\delta} \phi_m = k_{disp} \phi_m \tag{24}$$

where k_{disp} takes into account the magnetic flux division between the different paths. The maximum magnetic flux ϕ_m is obtained as:

$$\phi_m = \frac{M_0}{\theta_{tot}} \tag{25}$$

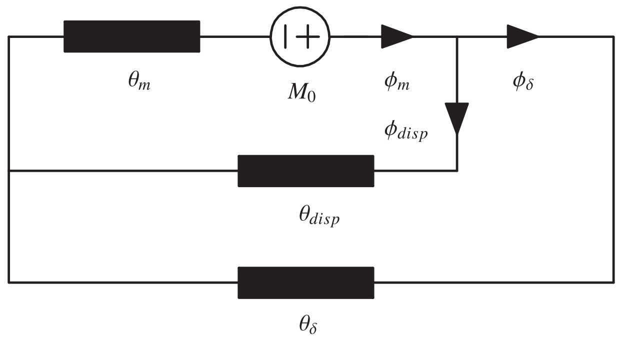

Fig. 4. The magnetic equivalent circuit of an electric motor pole.

where:

$$\theta_{tot} = (\theta_{disp} \| \theta_\delta) + \theta_m = k_{disp} \frac{1}{\mu_0} \frac{\delta K_c}{l \tau_p} + \frac{1}{\mu_{rev}} \frac{h_m}{A_m} \tag{26}$$

in which the Carter factor K_c takes into account the stator saturation ($K_c \cong 1.1$), μ_0 is the magnetic permeability of free space, μ_{rev} is the PM magnetic permeability and A_m is the effective area of the magnet.

The maximum flux density at the air gap B_δ can be computed remembering the (24)–(26):

$$B_\delta = \frac{\phi_\delta}{A_\delta} = k_{disp} \frac{H_0 h_m}{k_{disp} \frac{1}{\mu_0} \delta K_c + \frac{1}{\mu_{rev}} h_m} \tag{27}$$

being $A_\delta \simeq A_m = l \tau_p$. Finally Eq. (27) can be written considering $B_r = \mu_{rev} H_0$ and dividing both numerator and denominator for μ_{rev}/h_m:

$$B_\delta = k_{disp} \frac{B_r}{k_{disp} K_c \frac{\mu_{rev}}{\mu_0} \frac{\delta}{h_m} + 1} \tag{28}$$

where the flux density at the air gap depends on the residual flux density and by ratios μ_{rev}/μ_0 and δ/h_m.

4.3. Rotor moment of inertia

Assuming the rotor as a cylinder with equivalent density ρ_{eq}, diameter D and length l, its moment of inertia can be calculated as:

$$J_M = \frac{1}{2} M \frac{D^2}{4} = \frac{1}{2} \rho_{eq} \pi \frac{D^2}{4} l \frac{D^2}{4} = \frac{1}{32} \rho_{eq} \pi D^4 l \tag{29}$$

4.4. Linear current density

Consider a single stator's slot; τ_c is the slot pitch, b_c the slot width and h_c the sloth height. The total slot current:

$$I_t = \alpha_r b_c h_c S \tag{30}$$

where α_r is the slot filling factor and S is the current density. Since the linear current density does not change between the single slot and the whole stator circumference, Eq. (19) can be rewritten using Eq. (30):

$$A = \frac{I_t}{\tau_c} = \frac{\alpha_r b_c h_c S}{\tau_c} \tag{31}$$

Imposing that all the Joule effect heat in the conductors is exchanged through the air gap area of the slot, it results:

$$\rho_{cu} S^2 V_{cond} = \lambda A_{ex} \theta' \tag{32}$$

where ρ_{cu} is the resistivity of copper at nominal over temperature, $V_{cond} = \alpha_r b_c h_c l$ is the volume of conductors in a slot, λ is the thermal coefficient transfer, $A_{ex} = \tau_c l$ is the area of thermal transfer and θ' the conductor nominal over temperature (function of insulation and air temperature).

Thus, the following expression is obtained:

$$\rho_{cu} S^2 \alpha_r b_c h_c = \lambda \tau_c \theta' \tag{33}$$

So S can be calculated as a function of the constructive parameters and thanks to the (31) the linear current density is:

$$A = \alpha_r \frac{b_c}{\tau_c} h_c \sqrt{\frac{\lambda \theta' \tau_c}{\rho_{cu} \alpha_r b_c h_c}} = \sqrt{\frac{\lambda \theta' \alpha_r b_c h_c}{\rho_{cu} \tau_c}} \tag{34}$$

4.5. Final result

Substituting Eqs. (22) and (29) in (9), the accelerating factor can be expressed as a function of the motor constructive parameters:

$$\alpha = \frac{T_{M,n}^2}{J_M} = \pi^3 \frac{{k_B}^2 k_w^2}{\rho_{eq}} B_\delta^2 A^2 l \tag{35}$$

Considering a sinusoidal brushless motor ($k_B = 2/\pi$):

$$\alpha = 4\pi \frac{{k_w}^2}{\rho_{eq}} B_\delta^2 A^2 l \tag{36}$$

where B_δ is obtained from Eq. (28); substituting it in (36), it gives:

$$\alpha = 4\pi \frac{{k_w}^2}{\rho_{eq}} \left[k_{disp} \frac{B_r}{k_{disp} K_c \frac{\mu_{rev}}{\mu_0} \frac{\delta}{h_m} + 1} \right]^2 A^2 l \tag{37}$$

Finally, substituting the linear current density A from (34), the accelerating factor results:

$$\alpha = 4\pi \frac{{k_w}^2}{\rho_{eq}} \left[k_{disp} \frac{B_r}{k_{disp} K_c \frac{\mu_{rev}}{\mu_0} \frac{\delta}{h_m} + 1} \right]^2 \frac{\lambda \theta' \alpha_r b_c h_c}{\rho_{cu} \tau_c} l \tag{38}$$

4.6. Model verification

The verification is performed applying the model on a test motor (Fig. 5), that responds to target motor requirements. In Eq. (38) the relationship between α and hidden parameters is evident; fortunately, the test motor can be opened and deeply analyzed; the most of significant data have been found by measurement, additional datasheets and asking for the manufacturer. Table 3 reports test motor's data.

The result is the "proof" accelerating factor (α_p), that can be compared with the catalogue one (α_{cat}).

$$\alpha_p = 2.86 \cdot 10^4 \ [\mathrm{W/s}] \tag{39}$$

$$\alpha_{cat} = 2.96 \cdot 10^4 \ [\mathrm{W/s}] \tag{40}$$

$$\frac{\alpha_{cat} - \alpha_p}{\alpha_{cat}} = 0.036 \longrightarrow 4\% \tag{41}$$

The (41) suggests that Eq. (38) good represents the function (15) for the target motor. Of course such deep analysis is not possible nether reasonable for every motor in database. It's obvious that some hidden coefficients can be only approximative (λ as example), but this is not the right prospective in which the result must be evaluated. Another point of view, much more interesting, is instead the following:

"hidden values are deeply related to final motor performance, so it's strong interest of the motor manufacturer to make the best as possible to maximize their combined effect. It means that, once $\overline{\varepsilon}$ is given, whatever it's, one or more factors of $\overline{\gamma}$ good represents the global motor efficiency respect to α."

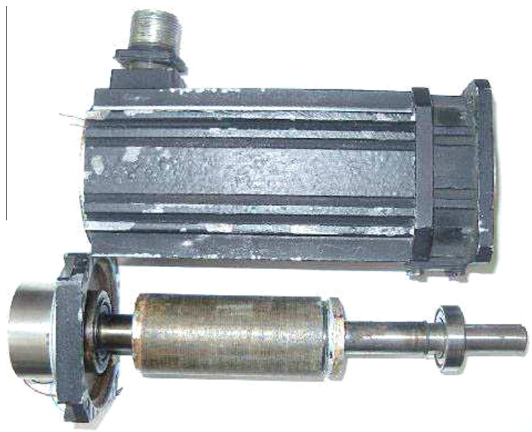

Fig. 5. Photo of the test motor with extracted rotor.

Table 3
Test motor's data.

Quantity	Value	Unit
$T_{M,N}$	2	(Nm)
J_M	1.35E−04	(kg m^2)
D	38E−03	(m)
z	180E−03	(m)
l	82E−03	(m)
p	6	
q	18	
δ	0.20E−03	(m)
h_m	2.60E−03	(m)
h_c	12E−03	(m)
$\tau_p = \frac{\pi D}{p}$	20E−03	(m)
$\tau_c = \frac{\pi D}{q}$	6.70E−03	(m)
b_c	3.35E−03	(m)
k_w	0.92	
k_{disp}	0.91	
K_c	1.10	
B_r	1.18	(T)
μ_{rev}	1.37E−06	(H/m)
λ	20	(W/(m^{2}C))
θ'	75	(C)
α_r	0.65	

As example, a motor with very efficient magnetic circuit needs less axial length to obtain certain α respect to one that is magnetically worse; or again, between two motor with the same α, the one that is smaller or lighter or thinner is usually better for the mechatronics, and means that the internal hidden structure is more efficient.Now next step is finding out the appropriate (γ) parameter that permits to rationalize the previous considerations during the sizing procedure.

5. The specific accelerating factor

It's interesting to observe how the accelerating factor linearly depends on the effective active length of the rotor l. This length can be expressed as product of the external axial depth z for a reductive coefficient k_l. Considering Eq. (36) results:

$$\alpha = 4\pi \frac{{k_w}^2}{\rho_{eq}} B_\delta^2 A^2 k_l z = \alpha_s z \quad (42)$$

where:

$$\alpha_s = 4\pi \frac{{k_w}^2}{\rho_{eq}} B_\delta^2 A^2 k_l = \frac{\alpha}{z} \quad (43)$$

is the *specific accelerating factor*.

It represents the motors' per-unit-length ability to develop the power; once motor's radial structure is given, it can be seen as the power density index for that structure, therefore the desired benchmark for the choice; moreover, α_s is easy to calculate with few fundamental data from catalogues; the higher is the resulting value, the better is the electro mechanical project of the motor.

The result of Eq. (43) becomes consistent observing Fig. 5: the test motor's active length is much shorter than the external depth; this causes an increasing of motor's dimensions without apparent performances advantage. α_s calculation should orient in choosing a suitable motor with maximal power density. As example, two machines are compared in Table 4.

These motors are similar concerning the accelerating factor, but one has a specific accelerating factor higher than the other. Number 142, whose mass is 1 kg, could be considered a better choice than 165, whose mass is 6 kg and its axial length is almost three times higher. In Fig. 6 their pictures have been reported; they have been also confined in two boxes in scale for better comparison. The geometric differences are evident, and although the same α, for 142 α_s is three times the other.

The specific accelerating factor adds an important information on the motor quality; crossing α_s with other catalogue's data (α, mass, volume, etc.) gives a significant contribution in answering question a).

5.1. Maximal valued α_s

Sometimes it's difficult estimate if a motor able to realize the given task is existing or not. Theoretically, the needed value of accelerating factor can be always reached, but since the (11) does not consider motor dimensions, the resulting machine could be huge, or extremely deep, for example. The specific accelerating factor, instead, as expression of the power density of the machine, cannot be increased till infinite. This consideration is interesting, because finding a practical maximum of α_s is a way for answering questions (b and c). By the practical point of view, claim to obtain a definitive maximum value of α_s with described model is naturally excessive; other machines, other electromechanical structures, other models must be analyzed to upgrade the knowledge about this parameter. Result below is a first step in that direction; further development will hopefully overtake this value.

The possibility to evaluate $\alpha_{s,max}$ is related to the maximal estimation of the terms in Eq. (43); top performance values of coefficients/quantities have been collected from experience and literature; they are reported in Table 5.

With that values, the maximal valued specific accelerating factor is:

$$\alpha_{s,max} = 2.05 \cdot 10^6 \ [\text{W}/(\text{ms})]$$

All the database motors are below, but one machine reaches about 70% of this maximum value ($1.41 \cdot 10^6$ [W/(ms)]); this is a good indication because it means the maximal result is reasonable and it can be used as reference for top quality accelerating performances.

Table 4
Comparison between motors 142 and 165.

N		142	165
$T_{M,N}$	(Nm)	0.64	3.20
J_M	(kg m^2)	2.19E−05	5.53E−04
α	(W/s)	18.53E+03	18.52E+03
α_s	(W/(ms))	26.47E+04	95.45E+03
z	(m)	70E−03	194E−03
M	(kg)	0.84	5.90

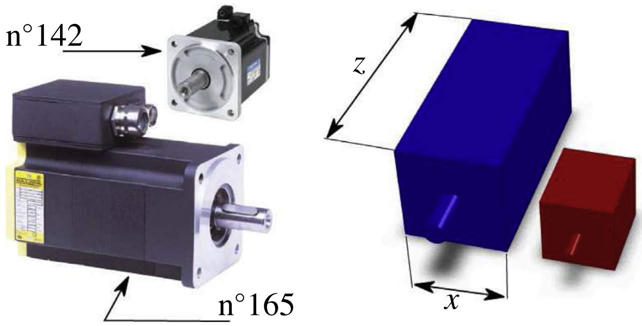

Fig. 6. Dimensional comparison between the motors nos. 142 and 164.

Table 5
Maximal constructive parameter of an electrical machine.

Name	Unit	Value
A	(A/m)	40E+03
k_w		0.95
k_{disp}		0.91
ρ_{eq}	(kg/m^3)	0.9·78.74E+02
K_c		1.1
μ_{rev}	(H/m)	1.1μ_0
B_r	(T)	1.2
δ/h_m		1/5
k_l		0.90

6. Conclusions

This work analyzed brushless motors starting from few data usually available on catalogs. The collected database revealed how accelerating performances of such machines are extremely heterogeneous; a mathematical model was realized and verified to find a theoretical expression of the accelerating factor in function of a wide number of constructive parameters. For the target machine considered, the model highlighted a significant dependance between the accelerating factor and motor's depth: the specific accelerating factor, a per-unit-length power density index, was found. The model was then maximized using the linear current density to release as much as possible the result from stator hidden parameters (like slot geometry and cooling).

Considering mechatronics needs, the specific accelerating factor is easy to use during the sizing process, easy to calculate from catalogs and can be crossed with other data (α, mass, volume, etc.) to improve the optimal motor choice; it provides a relative benchmark when experience, instinct and a wide motor database are not enough anymore; his superior practical limit is intended as a tool to evaluate the motor in absolute, comparing it with a top quality performance machine.

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