- setting SU) reference signal strongly saturated, and
- setting SW) reference signal strongly saturated, error signal weakly saturated,

where weak and strong saturation refer respectively to a 90% and a 50% clipping of the signal with respect to its maximum value [6, 19].

Figure 11 reports the attenuation performance of the VFXLMS, BFXLMS, NFGLMS and DFLMS algorithms in the three settings. A full quadratic NARX model has been used in the last two cases. All models have the same L for a given setting ($L = 4$ for setting WU, $L = 8$ for settings SU and SW). A simple 10-tap FIR model (with random initial parameters) has been used as auxiliary model for the DFLMS. All algorithm gains have been optimized individually.

In all three settings, both the VFXLMS and BFXLMS achieve a noise reduction of only a factor 3, whereas the NMSE (after convergence) equals -38 dB for setting WU and -33 dB for settings SU and SW for both NFGLMS and DFLMS. Indeed, after a brief transient required for the estimation of $\tilde{S}$, the DFLMS provides almost equal performance to the NFGLMS. This is all the more remarkable, considering that the auxiliary system is initialized with random parameters.

7. CONCLUSIONS

A novel NANC scheme exploiting the flexibility of NARX models has been proposed. Its main feature is the commutation of the control filter block and the SP dynamics, as commonly assumed in the linear case, compensating the commutation error by means of an auxiliary filter also adapted online. Both the main and the auxiliary adaptation loops operate with weight update rules based on the use of suitable prefiltered signals. The control filter adaptation essentially requires the same computational effort as the standard FXLMS/FULMS algorithms, whereas the adaptation of the auxiliary filter, which is not time critical, can be slowed down and downsampled. In view of the dual adaptation loops, the algorithm is called DFLMS.

The proposed scheme configures the control filter adaptation task as a direct identification problem, which allows the use of an online model selection algorithm. The latter can be used to trim the model size and to adapt the model structure to account for the variability of the ANC setting. An extended version of the DFLMS has thus been implemented using the RFRP algorithm as model selection strategy.

The proposed methods have been tested on several scenarios of different complexity. The DFLMS achieves almost the same level of steady state disturbance noise cancellation as the NFGLMS, although with a slightly slower convergence rate. The RFRP-DFLMS, on the other hand, is very efficient in reducing the model size, thereby cutting to a minimum the computational load, and reacts much more rapidly than the NFGLMS to sudden variations of the system.

REFERENCES

1. Kuo SM, Morgan DR. *Active Noise Control Systems – Algorithms and DSP Implementations*. Wiley: New York, 1996.
2. Elliott SJ. Down with noise. *IEEE Spectrum* 1999; **36**:54–61.
3. Kuo SM, Morgan DR. Active noise control: a tutorial review. *Proceedings of the IEEE* 1999; **87**(6):943–973.
4. Elliott SJ. *Signal Processing for Active Control*. Academic Press: San Diego, CA, 2001.
5. Matsuura T, Hiei T, Itoh H, Torikoshi K. Active noise control by using prediction of time series data with a neural network. *Proceedings of the IEEE International Conference on Systems, Man and Cybernetics,* vol. 3, Vancouver, BC, Canada, 1995; 2070–2075.
6. Kuo SM, Wu H-T, Chen F-K, Gunnala MR. Saturation effects in active noise control systems. *IEEE Transactions on Circuits and Systems I: Regular Papers* 2004; **51**(6):1163–1171.
7. Snyder SD, Tanaka N. Active control of vibration using a neural network. *IEEE Transactions on Neural Networks* 1995; **6**(4):819–828.
8. George NV, Panda G. Advances in active noise control: a survey, with emphasis on recent nonlinear techniques. *Signal Processing* 2013; **93**:363–377.
9. Strauch P, Mulgrew B. Active control of nonlinear noise processes in a linear duct. *IEEE Transactions on Signal Processing* 1998; **46**(9):2404–2412.
10. Tan L, Jiang J. Adaptive Volterra filters for active control of nonlinear noise processes. *IEEE Transactions on Signal Processing* 2001; **49**(8):1667–1676.

11. Reddy EP, Das DP, Prabhu KMM. Fast adaptive algorithms for active control of nonlinear noise processes. *IEEE Transactions on Signal Processing* 2008; **56**(9):4530–4535.
12. Zhao H, Zeng X, Zhang X, He Z, Li T, Jin W. Adaptive extended pipelined second-order Volterra filter for nonlinear active noise controller. *IEEE Transactions on Audio, Speech, and Language Processing* 2012; **20**(4):1394–1399.
13. Bouchard M, Paillard B, Dinh CTL. Improved training of neural networks for the nonlinear active control of sound and vibration. *IEEE Transactions on Neural Networks* 1999; **10**(2):391–401.
14. Das DP, Panda G. Active mitigation of nonlinear noise processes using a novel filtered-s LMS algorithm. *IEEE Transactions on Speech and Audio Processing* 2004; **12**(3):313–322.
15. Das DP, Mohapatra SR, Routray A, Basu TK. Filtered-s LMS algorithm for multichannel active control of nonlinear noise processes. *IEEE Transactions on Audio, Speech and Language Processing* 2006; **14**(5):1875–1880.
16. Sicuranza GL, Carini A. A generalized FLANN filter for nonlinear active noise control. *IEEE Transactions on Audio, Speech and Language Processing* 2011; **19**(8):2412–2417.
17. Zhao H, Zeng X, Zhang J. Adaptive reduced feedback FLNN nonlinear filter for active control of nonlinear noise processes. *Signal Processing* 2010; **90**(3):834–847.
18. Sicuranza GL, Carini A. Piecewise–linear expansions for nonlinear active noise control. *Proceeding of the IEEE International Conference on Acoustics, Speech and Signal Processing*, Toulouse, France, 2006; 209–212.
19. Kuo SM, Wu H-T. Nonlinear adaptive bilinear filters for active noise control systems. *IEEE Transactions on Circuits and Systems I: Regular Papers* 2005; **52**(3):1–8.
20. Zhou D, DeBrunner V. Efficient adaptive nonlinear filters for nonlinear active noise control. *IEEE Transactions on Circuits and Systems I: Regular Papers* 2007; **54**(3):669–681.
21. Zhao H, Zeng X, He Z, Li T. Adaptive RSOV filter using the FELMS algorithm for nonlinear active noise control systems. *Mechanical Systems and Signal Processing* 2013; **34**(1–2):378–392.
22. Sicuranza GL, Carini A. On the BIBO stability condition of adaptive recursive FLANN filters With application to nonlinear active noise control. *IEEE Transactions on Audio, Speech and Language Processing* 2012; **20**(1): 234–245.
23. Napoli R, Piroddi L. Nonlinear active noise control with NARX models. *IEEE Transactions on Audio, Speech and Language Processing* 2010; **18**(2):286–295.
24. Das DP, Panda G, Nayak DK. Development of frequency domain block filtered–s LMS (FBFSLMS) algorithm for active noise control system. *Proceedings of the IEEE International Conference on Acoustics, Speech and Signal Processing*, Toulouse, France, 2006; 289–292.
25. Feintuch PL. An adaptive recursive LMS filter. *Proceedings of the IEEE* 1976; **64**(11):1622–1624.
26. Söderström T, Stoica P. *System Identification*. Prentice-Hall: London, 1989.
27. Aguirre LA, Billings SA. Dynamical effects of overparametrization in nonlinear models. *Physica D: Nonlinear Phenomena* 1995; **80**:26–40.
28. Piroddi L, Spinelli W. An identification algorithm for polynomial NARX models based on simulation error minimization. *International Journal of Control* 2003; **76**(17):1767–1781.
29. Bjarnason E. Active noise cancellation using a modified form of the filtered-x. *Proceedings of the EURASIP European Signal Processing Conference (EUSIPCO),* vol. 2, Amsterdam: Elsevier, 1992; 1053–1056.
30. Liao C, Lin J. New FIR filter-based adaptive algorithms incorporating with commutation error to improve active noise control performance. *Automatica* 2007; **42**(2):325–331.
31. Lin J, Liao C. New IIR filter-based adaptive algorithm in active noise control applications: commutation error-introduced LMS algorithm and associated convergence assessment by a deterministic approach. *Automatica* 2008; **44**(11):2916–2922.
32. Cantelmo C, Piroddi L. Adaptive model selection for polynomial NARX models. *IET Control Theory and Applications* 2010; **4**(12):2693–2706.
33. Leontaritis IJ, Billings SA. Input–output parametric models for non-linear systems – Part II: stochastic non-linear systems. *International Journal of Control* 1985; **41**:329–344.
34. Mumolo E, Carini A. On the stability of discrete time recursive Volterra filters. *IEEE Signal Processing Letters* 1999; **6**:230–232.
35. Mathews VJ, Sicuranza GL. *Polynomial Signal Processing*. Wiley: New York, 2000.
36. Mumolo E, Carini A. A stability condition for adaptive recursive second-order polynomial filters. *Signal Processing* 1996; **54**(1):85–90.
37. Roy E, Stewart RW, Durrani TS. Stability condition for certain recursive second-order polynomial filters. *Electronics Letters* 1996; **32**(16):1522–1523.
38. Billings SA, Chen S, Korenberg MJ. Identification of MIMO non-linear systems using a forward-regression orthogonal estimator. *International Journal of Control* 1989; **49**:2157–2189.
39. Li K, Peng J-X, Irwin GW. A fast nonlinear model identification method. *IEEE Transactions on Automatic Control* 2005; **50**(8):1211–1216.
40. Tibshirani R. Regression shrinkage and selection via the lasso. *Journal of the Royal Statistical Society B* 1996; **58**(1):267–288.
41. Tobias OJ, Seara R. Performance comparison of the FXLMS, nonlinear FXLMS and leaky FXLMS algorithms in nonlinear active noise applications. *Proceedings of 11th European Signal Processing Conference,* vol. 1, Toulouse, France, 2002; 155–158.