

# Review of data and correlations for turbulent forced convective heat transfer of liquid metals in pipes

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## 1 Introduction

Liquid metals present in general large thermal conductivity, small kinematic viscosity, small vapor pressure and a wide temperature range over which they remain in the liquid phase. Therefore they are considered efficient heat transfer media in processes with limited heat exchange

surfaces and exceptionally high thermal loads. In the nuclear fusion, lithium or lithium alloys allow merging the fuel generation problem with the heat removal from the fusion reaction, see e.g. [32]. Fast fission reactors normally use sodium as coolant, while newer concepts, as well as accelerator-driven systems, rely on lead or lead alloys. A comprehensive study on the fuel and coolant aspects, as well as their advantages and disadvantages for fast neutron systems, may be taken from [7]. Liquid metals also find several applications in non-nuclear industries. As an early example, a pool of molten tin is used for the production of glass through the float-glass process, as described in [38]. Based on that experience, tin has also been proposed as a working medium for high-temperature chemical reactors, as described in [54] for coal gasification and in [43] for hydrogen production. Based on the experience from fast breeder reactors, sodium was selected as the heat transfer fluid in demonstration concentrated solar power plants (see [40]), where the sunlight is focused by numerous mirrors onto a receiver to obtain high discharge temperatures of the working fluid.

These applications require LM-specific correlations, which differ significantly from those of other conventional fluids, to calculate the heat transfer coefficient in a variety of configurations. Analytical and numerical solutions of laminar-flow heat transfer have been available for many years, whereas heat transfer coefficients for turbulent flow mostly rely on empirical equations based on experimental data.

As the underlying physical mechanisms of heat transfer to LMs significantly differ from those to gases or ordinary liquids, correlations developed for the latter are not applicable to the first. The reason lies in their low Prandtl number ( $Pr$ , always below 0.05), which can be thought as the ratio of the hydrodynamic-to-thermal viscous boundary

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layer thickness. Therefore, for these fluids, the contribution of the molecular thermal conduction to the total heat transfer is much higher than for order-one-and-higher Prandtl number fluids. According to Grötzbach [12], the turbulent thermal diffusivity overcomes the molecular thermal diffusivity only when  $Re > 60,000$  for  $Pr = 0.025$  (Lead Bismuth Eutectic, LBE) and  $Re > 214,000$  for  $Pr = 0.007$  (sodium, Na), corresponding in both cases to a Péclet number larger than 1,500. Indeed, for low-Pr fluids the thermal boundary layer is much thicker than the hydrodynamic one. Thus, the thermal resistance is distributed over the entire duct cross section. Consequently, rounded temperature profiles, similar to those for laminar flow, are obtained. Furthermore, these profiles are strongly influenced by the applied thermal boundary conditions, which is not the case for ordinary fluids.

The current study focuses on the data and correlations for round tubes, which are the most widely studied configuration for forced-convective heat transfer in general and for LM flows in particular. Moreover it is a configuration of high engineering importance and very often encountered in common practice. Recently, a similar analysis has been carried out for in-line tube bundles [35].

## 2 Review of data and correlations for $q_w = const$

### 2.1 Survey of experimental data

Most of the available data for this boundary condition corresponds to experiments performed before 1970. In this period, a large number of researchers in several countries studied the heat transfer of liquid metals for different geometries and operating conditions. Extensive lists of these early investigations can be found, for example, in [10, 28]

Nevertheless, it should be noticed that for several reasons related to the experimental conditions these reported results are not always directly comparable to each other. For example, while some authors report locally-measured heat transfer coefficients, others only presents average values and many researchers do not even indicate to which type their values correspond, thus failing to make their experiments reproducible. In this section, only locally-measured values for fully-developed conditions are considered. Thus, they must be properly filtered, as discussed below.

An important distinction must be made according to the thermal boundary condition in the test section. A uniform heat flux at the wall can be rather well established by means of electric heating, while an axially-constant wall temperature can be obtained by surrounding the test section with a temperature-controlled solid or liquid jacket. These

two thermal boundary conditions can be well reproduced, as summarized in [44]. In addition, several experiments, such as [29, 41] were performed with a heat-exchanger-type test section. These results are not included in the present survey because the test section was not long enough to obtain a thermal fully developed flow. Furthermore, for the purpose of this analysis, the available experiments shall be divided into three groups according to the particular fluid used:

- Sodium (Na) and sodium-potassium (NaK) alloys of several compositions;
- Pure lead (Pb) and lead-bismuth eutectic (LBE);
- Mercury (Hg), excluding cases with alkali additives.

The experimental data for uniform wall heat flux are summarized in Table 1. Over 1,100 data points from 21 different sources are considered, covering wide ranges of operating conditions (velocity, heat flux, diameter, among others) for both alkali and heavy LMs. In all cases, a vertical upward flow direction has been considered. The large extent of the considered dataset allows for a statistical analysis.

With  $Re$  ranging between 809 and a  $8.87 \cdot 10^5$ , the laminar, transition and turbulent flow regimes are covered by this dataset. Furthermore,  $Pr$  has a minimum of 0.007 (NaK) and a maximum of 0.046 (Hg), resulting in a Péclet number ( $Pe$ ) varying from 9.3 to 18,690.

According to a dimensional analysis applied to the forced-convection problem, it can be deduced that the Nusselt number ( $Nu$ ) has a functional dependence on  $Re$  and  $Pr$  as in Eq. (1). Within the LM-range ( $Pr < 0.05$ ), however, all the authors listed in Table 1 agree in representing their results in term of  $Pe$ , as in Eq. (2).

$$Nu = f_1(Re, Pr) \quad (1)$$

$$Nu = f_2(Re \times Pr) = f_2(Pe) \quad (2)$$

The complete data set is represented in Fig. 1a. Furthermore, in order to identify whether the Prandtl number has any individual effect or not, the results for each of the three categories of fluids defined above are represented individually in Fig. 1b, d. The following considerations can be derived through a detailed observation of Fig. 1.

In all Fig. 1b–d, some experimental results are observed below the theoretical limit for laminar flow, i.e.  $Nu = 48/11 = 4.36$ . These have been categorized by the experimenters themselves as well as by other authors as plainly wrong. In particular, this means that the experimental conditions do not fit to the assumptions of single-phase fully-developed flow due to different practical reasons, as discussed below.

Even neglecting those points below the laminar limit, a relatively large scatter in the data is observed. For

**Table 1** Summary of the available experimental data in literature for uniform wall heat flux conditions

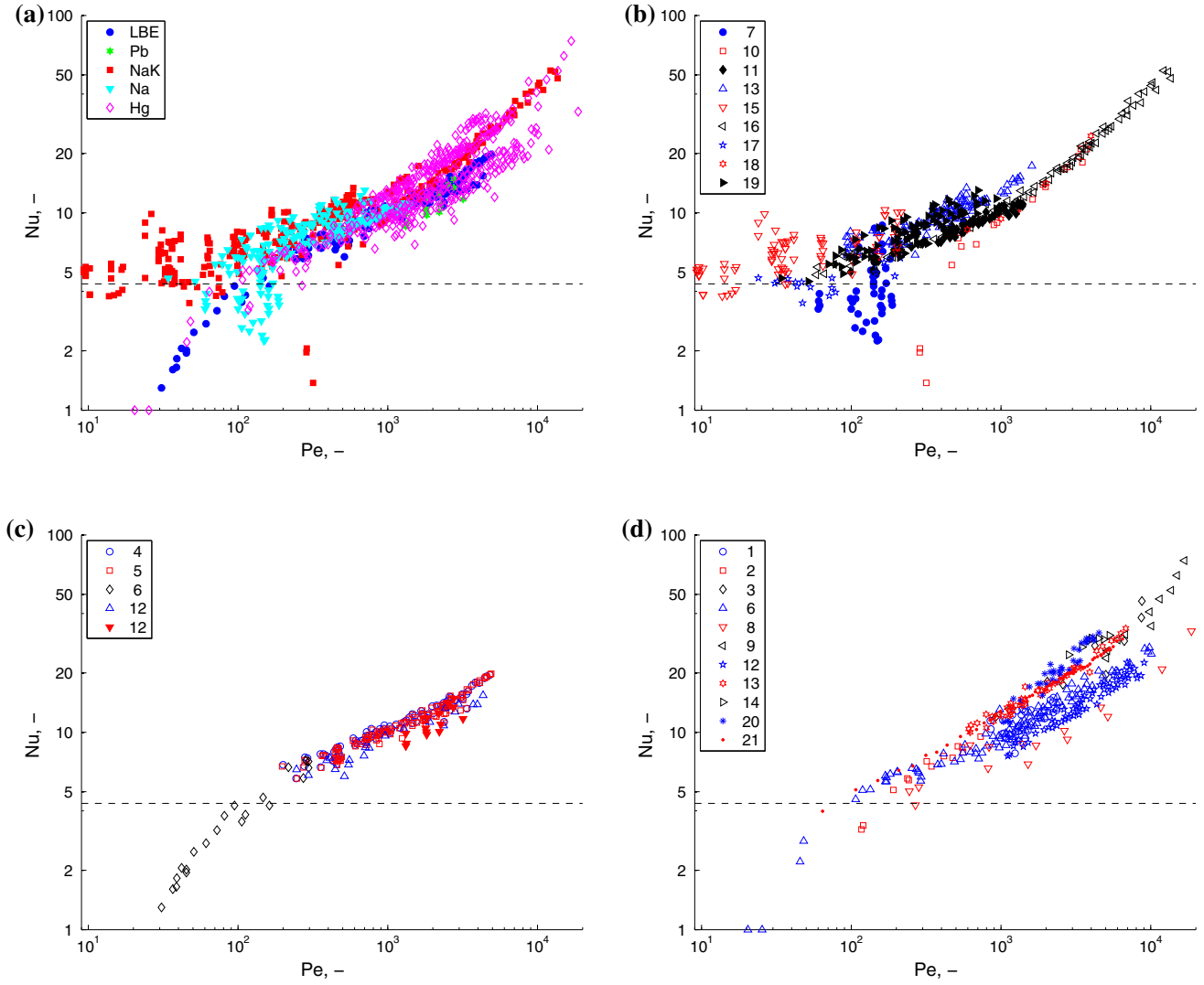
No.	Author(s)	Year	Refs.	Fluid	Points	min (Re)	max (Re)	min (Pe)	max (Pe)	Remarks
1	Styrikovich and Semenovker	1940	[50]	Hg	35	$8.49 \cdot 10^4$	$2.35 \cdot 10^5$	850	2,350	
2	English and Barrett	1951	[9]	Hg	13	$5.55 \cdot 10^3$	$3.47 \cdot 10^5$	117	731	Published in [28]
3	Isakoff and Drew	1951	[16]	Hg	9	$4.36 \cdot 10^4$	$4.14 \cdot 10^5$	920	8,720	Published in [28]
4	Johnson et al.	1952	[18]	LBE	92	$8.00 \cdot 10^3$	$1.92 \cdot 10^5$	200	4,800	Published in [20]
5	Johnson et al.	1953	[19]	LBE	127	$7.26 \cdot 10^3$	$1.67 \cdot 10^5$	197	4,905	Published in [20]
6	Johnson et al.	1953	[20]	Hg	98	$9.68 \cdot 10^2$	$4.79 \cdot 10^5$	204	10,100	
6	Johnson et al.	1953	[20]	LBE	21	$1.11 \cdot 10^3$	$1.06 \cdot 10^4$	31	296	
7	MacDonald and Quittenton	1953	[31]	Na	42	$8.60 \cdot 10^3$	$2.60 \cdot 10^4$	60	188	Published in [28]
8	Stromquist	1953	[49]	Hg	15	$1.16 \cdot 10^4$	$8.87 \cdot 10^5$	244	18,690	Published in [28]
9	Brown et al.	1957	[3]	Hg	10	$2.52 \cdot 10^5$	$8.40 \cdot 10^5$	5,050	16,800	
10	Kuczen and Bump	1957	[26]	NaK	23	$1.97 \cdot 10^4$	$1.60 \cdot 10^5$	286	3,828	
11	Novikov et al.	1957	[37]	Na	65	$1.41 \cdot 10^4$	$2.00 \cdot 10^5$	99	1,400	
12	Ibragimov et al.	1960	[15]	LBE	21	$1.05 \cdot 10^4$	$1.84 \cdot 10^5$	246	4,330	
12	Ibragimov et al.	1960	[15]	Hg	65	$4.53 \cdot 10^4$	$3.69 \cdot 10^5$	1,110	9,040	
12	Ibragimov et al.	1960	[15]	Pb	14	$6.60 \cdot 10^4$	$1.59 \cdot 10^5$	1,320	3,180	
13	Kirillov et al.	1960	[25]	Hg	45	$2.89 \cdot 10^4$	$3.23 \cdot 10^5$	609	6,814	
13	Kirillov et al.	1960	[25]	NaK	109	$6.56 \cdot 10^3$	$1.14 \cdot 10^5$	92	1,606	
14	Subbotin et al.	1963	[51]	Hg	7	$1.34 \cdot 10^5$	$2.51 \cdot 10^5$	2,830	5,300	Published in [10]
15	Holtz	1965	[14]	NaK	89	$8.09 \cdot 10^2$	$2.80 \cdot 10^4$	9.3	215	
16	Skupinski et al.	1965	[46]	NaK	89	$3.76 \cdot 10^3$	$8.79 \cdot 10^5$	58	13,630	
17	Emery and Bailey	1967	[8]	NaK	18	$1.70 \cdot 10^3$	$1.51 \cdot 10^4$	24	213	
18	Buhr et al.	1968	[4]	NaK	4	$2.39 \cdot 10^4$	$1.59 \cdot 10^5$	598	3,980	
19	Borishanskii et al.	1969	[2]	Na	79	$4.89 \cdot 10^3$	$1.33 \cdot 10^5$	34	932	Published in [10]
20	Borishanskii et al.	1969	[1]	Hg	35	$4.94 \cdot 10^4$	$2.14 \cdot 10^5$	1,040	4,510	Published in [10]
21	Kinoshita et al.	2013	[23]	Hg	63	$2.70 \cdot 10^3$	$2.71 \cdot 10^5$	64	6,484	

comparable values of the Péclet number, deviations up to  $\pm 50\%$  in the measured Nusselt numbers can be observed. An interesting discussion of possible causes for these discrepancies has been given by Cheng and Tak [6]. Some of them are the following.

- *issues related to the experimental setup conditions.* In particular, the fluid-to-wall interaction can be largely affected by improper wetting and the presence of entrained gas. Achieving proper wetting conditions can be a particular challenge in liquid-metal flows, also according to the authors own experience. While these issues are evident in those results below the laminar limit, they might as well be present to any extent in the other measurements.
- *issues related to the evaluation of the measured data.* Both Nu and Pe are formed with measured values of heat flux, temperature, flow rate, fluid properties (i.e. density, thermal conductivity, heat capacity, viscosity), as well as geometrical parameters. In the absence of a common database, different authors (especially from different countries) presumably used different

values for determining the latter, which unfortunately were in most cases left unreported. Furthermore, the evaluation of the wall and bulk temperatures, as well as the heat flux is often performed by indirect methods and thus prone to larger uncertainties, particularly for experiments with large temperature gradients. As an example, Johnson et al. [20] reported a constant 6% error in the evaluation of the heat flux, due to thermal losses. Unfortunately, the results were not corrected accordingly and error bars were not reported.

- *issues related to mixed convection.* Buoyancy forces can become important in forced-convective LM heat transfer, especially for heavy LMs. Indeed, experimental results show that the velocity profile distorts rapidly by increasing the heat flux also at high Reynolds numbers, where pure forced convection is normally expected to occur. These effects influence the value of the measured Nusselt number. A detailed discussion about mixed convection in liquid metal flows can be found in [4, 17] and [33].



**Fig. 1** Experimental Nusselt versus Péclet data for uniform-wall-heat-flux conditions. The numbers (between 1 and 21) assigned to each symbol indicate the data source, as listed in Table 1. **a** Data for

all liquid metals. **b** Data for alkali liquid metals. **c** Data for lead and lead-bismuth eutectic. **d** Data for mercury

Within this context, it is remarkable that the difference in the Nusselt number at the same Pe between different fluids, see Fig. 1a, is not larger than that measured by different authors or even by a single author for the same fluid, see Fig. 1b–d and [31]. Furthermore, the relative magnitude of this scatter for alkali and heavy metals increases with the number of authors investigating those particular fluids, i.e. LBE and Pb < Na and NaK < Hg. Some previous authors have indicated that the Nusselt number for alkali-LM flows is usually larger than that for heavy-LMs at the same Pe, see e.g. Novikov et al. [37]. However, this statement was based on smaller databases and it is not supported by the data presented in Fig. 1. On the contrary, it can be observed in Fig. 1a that, within the uncertainties of the experiments (which were unfortunately not reported in

most cases, but can be estimated to be no lower than 5–10 %), all LMs present similar heat transfer behavior, i.e. comparable Nu at the same Pe.

## 2.2 Survey of available correlations

Early experiments, e.g. [50] already indicated that heat transfer coefficients for LM flows are not well represented by correlations developed for other fluids (e.g. water or air), such as the Dittus-Boelter equation. Therefore, specific models for these liquids are required and large research efforts were devoted to their development in several countries.

The pioneering works of Martinelli [34] and Lyon [29, 30] first gave a theoretical explanation of the experimental

observations. By considering the influence of the molecular heat conductivity in the core flow, they proposed Eq. (3), where  $Pr_t$  is the mean turbulent Prandtl number.

$$Nu = 7.0 + 0.025(Pe/Pr_t)^{0.8} \quad (3)$$

The exponent 0.8 in the above equation was derived from the assumed velocity and temperature profiles, and it was later found to represent well experimental values. The turbulent Prandtl number depends on Re, Pr and the distance from the wall, as represented by Eq. (4).

$$Pr_t = f_3(Re, Pr, y^+) \quad (4)$$

A constant value of  $Pr_t = 1$ , that is independent from Re and Pr, was suggested by Lyon [29, 30], while in general is  $Pr_t > 1$  for liquid metals (see [12]), thus leading to lower Nusselt numbers. Several empirical and semi-empirical models for  $Pr_t$  to be used in the Lyon-Martinelli correlation have been developed, see e.g. [22]. However, as shown by [6], they present large differences in  $Pr_t$  (and consequently in Nu) for a given Pe. Although they perform better than when considering  $Pr_t = 1$ , their reliability has not yet been confirmed. Therefore, Nusselt correlations including  $Pr_t$  are not considered in this survey. Instead, only equations expressed by the functional relationship of Eq. (5), are investigated.

$$Nu = a + bPe^c Pr^d \quad (5)$$

With few exceptions, the parameter  $c$  is close to the value of 0.8 recommended by Lyon [29, 30]. A few authors added a dependence on the Prandtl number, given by the coefficient  $d$ , in order to represent the difference between

alkali and heavy LMs. While this distinction might be justified by theoretical considerations, it is not observed in the experiments, as previously discussed in Sect. 2.1. Indeed, the values of  $d$  are rather small, for example Chen and Chiou [5] proposed  $d = 0.01$ , indicating that a tenfold change in the Prandtl number only results in a 2 % difference in the Nusselt number. Therefore, in principle, this dependence can be neglected (that is, assume  $d = 0$ ). Within the framework of this study, the available correlations reported in the open literature are summarized in Table 2.

Furthermore, Cheng and Tak [6] proposed a model, as a best fit to the LBE data in [20], where correlation No. 15 of Table 2 is recommended for  $Pe < 1,000$  and correlation No. 2 for  $Pe > 2,000$ , with a linear interpolation for the coefficient  $a$  in the intermediate region. However, such transition is not justified when the data from other authors are also considered (see Fig. 1c). Therefore, it is not included in the present survey.

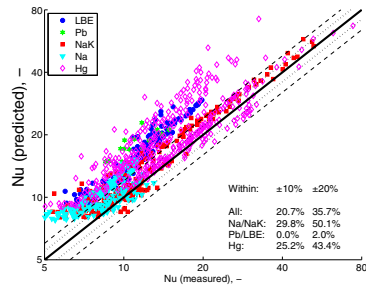
### 3 Analysis for $q_w = const$

It should be noted that all the correlations listed in Table 2 have been developed for Reynolds numbers in the broad range between  $10^4$  and  $10^6$ . Therefore, the complete dataset listed in Table 1 must be properly filtered in order to provide a fair comparison. In particular, only those experimental data points above the laminar limit ( $Nu > 48/11$ ) and in the fully turbulent regime ( $Re > 10^4$ ) are taken into account. With this consideration, the database is reduced roughly by 15 %.

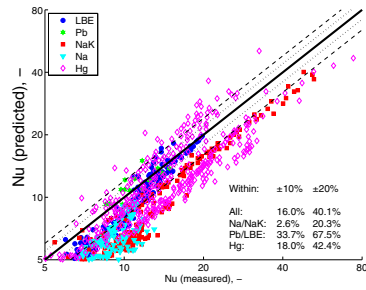
**Table 2** Survey of heat transfer correlations for liquid metal flows with a uniform heat flux

No.	Author	Year	Refs.	a	b	c	d	Remarks
1	Lyon	1949	[29]	7	0.025	0.8	0	Semi-analytical
2	Stromquist	1953	[49]	3.6	0.018	0.8	0	Fitting Hg data
3	Lubarsky and Kaufman	1955	[28]	0	0.625	0.4	0	Best fit of all data
4	Hartnett and Irvine	1957	[13]	5.33	0.015	0.8	0	Assuming a slug velocity profile
5	Sleicher and Tribus Jr.	1957	[47]	6.3	0.016	0.91	0.3	
6	Kutateladze et al.	1959	[27]	3.3	0.014	0.8	0	As a lower limit
7	Kutateladze et al.	1959	[27]	5	0.0021	1.0	0	As a lower limit
8	Kutateladze et al.	1959	[27]	5.9	0.015	0.8	0	Developed for Na
9	Ibragimov et al.	1960	[15]	4.5	0.014	0.8	0	
10	Subbotin et al.	1963	[52]	5	0.025	0.8	0	Developed for Na
11	Skupinski et al.	1965	[46]	4.82	0.0185	0.827	0	Developed for NaK
12	Notter and Sleicher	1972	[36]	6.3	0.0167	0.85	0.08	Based on (T, u) profiles for NaK
13	Chen and Chiou	1981	[5]	5.6	0.0165	0.85	0.01	Developed for Na and NaK eutectic
14	Siman-Tov et al.	1997	[45]	0	0.685	0.3726	0	Developed for Hg
15	Kirillov and Ushakov	2001	[24]	4.5	0.018	0.8	0	Developed for LBE

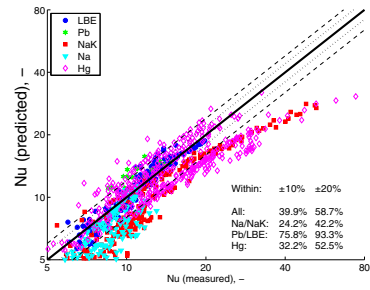
The coefficients  $a$  to  $d$  refer to Eq. (5)



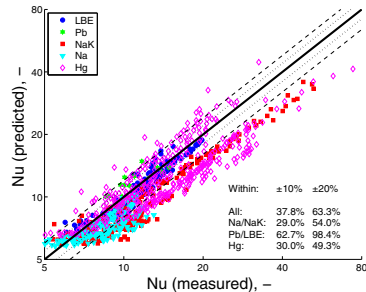
(a)



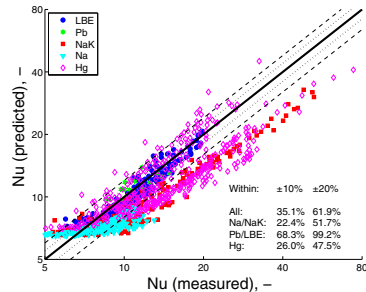
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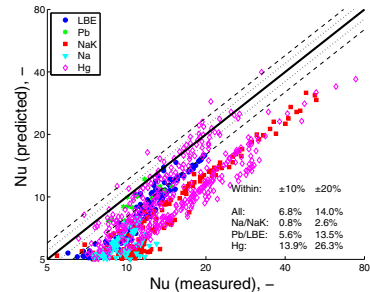
(c)



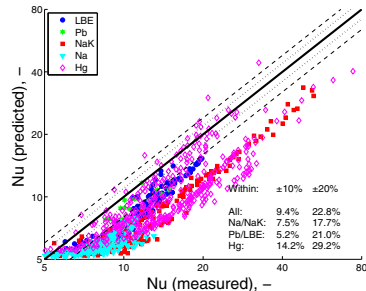
(d)



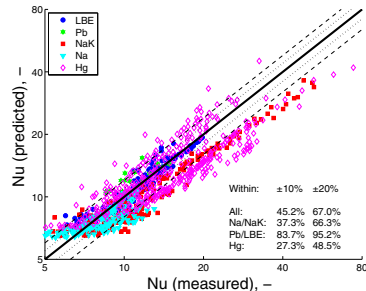
(e)



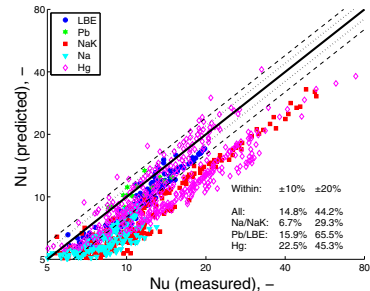
(f)



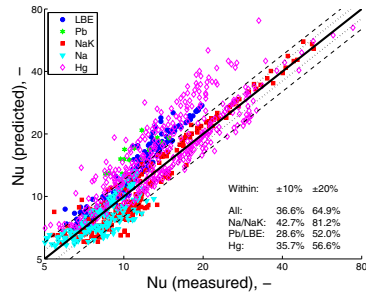
(g)



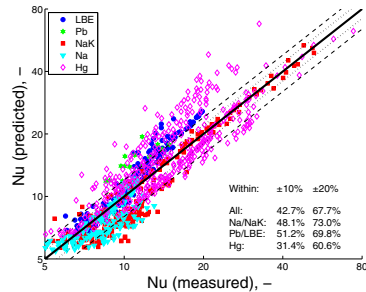
(h)



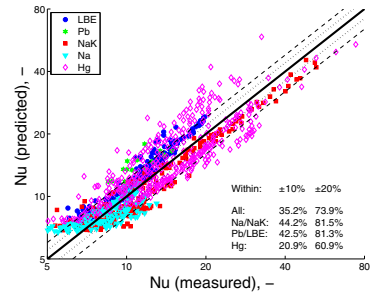
(i)



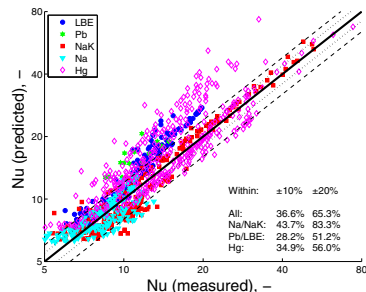
(j)



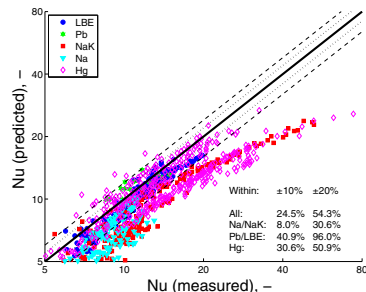
(k)



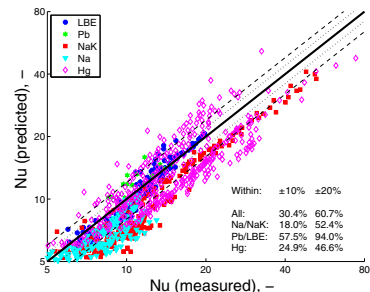
(l)



(m)



(n)



(o)



◀**Fig. 2** Performance of the correlations listed in Table 2. *Dashed line*  $\pm 20\%$  accuracy, *dot line*  $\pm 10\%$  accuracy. **a** Correlation 1, **b** Correlation 2, **c** Correlation 3, **d** Correlation 4, **e** Correlation 5, **f** Correlation 6, **g** Correlation 7, **h** Correlation 8, **i** Correlation 9, **j** Correlation 10, **k** Correlation 11, **l** Correlation 12, **m** Correlation 13, **n** Correlation 14, **o** Correlation 15

### 3.1 Individual evaluation

A comparison of the experimental and predicted Nusselt numbers for the correlations listed in Table 2 is presented in Fig. 2. The large number of data points plotted allow for some general conclusions to be derived.

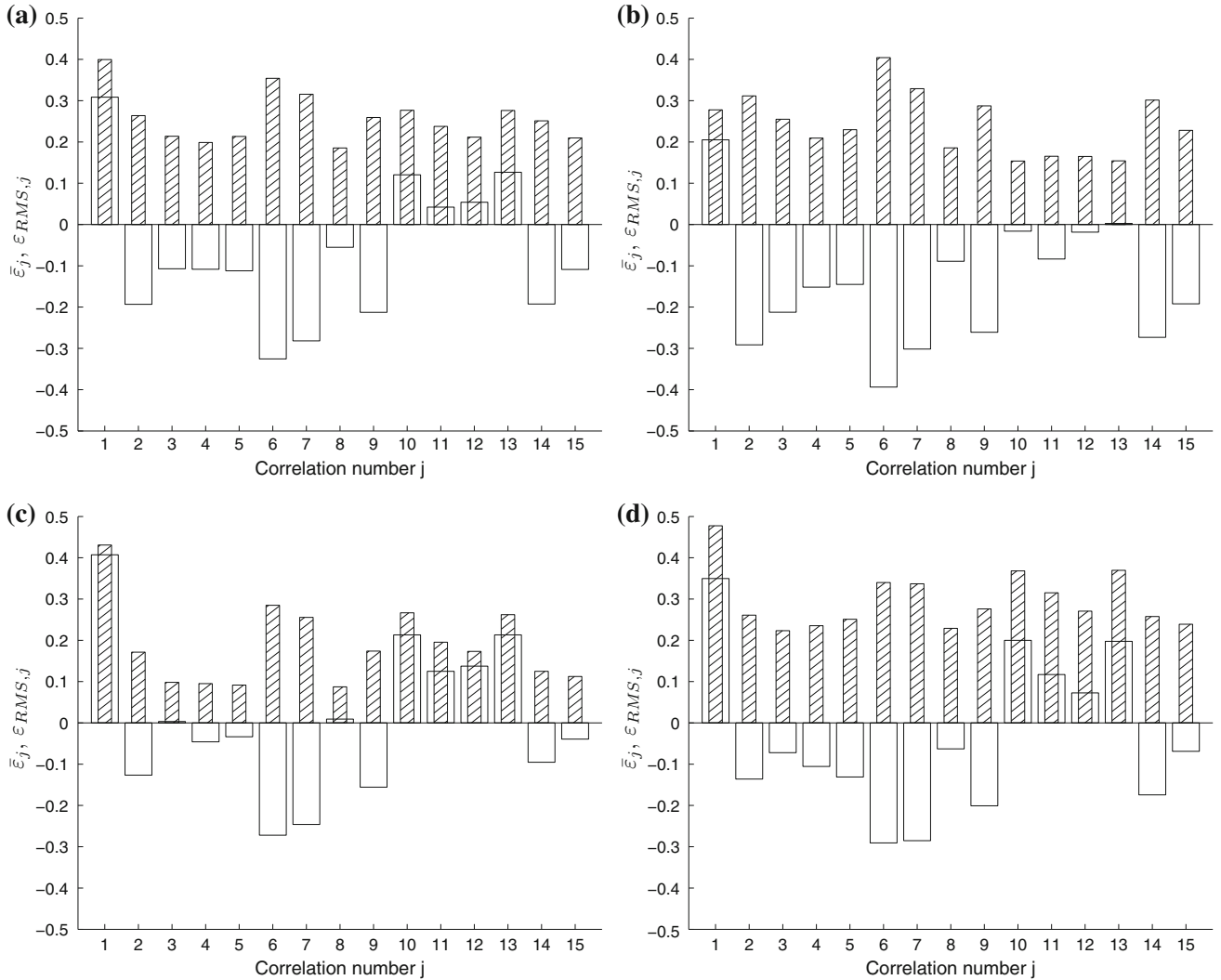
A clear over-prediction is observed for correlation No. 1, i.e. Eq. (3). Although this has been considered for long time the reference equation for liquid metals, it should instead be considered as an upper limit for the Nusselt

number, due to the higher  $Pr_r$  in LM flows, as shown for example by Kawamura et al. [21].

The opposite trend, i.e. an under-prediction, is given by correlations 2, 4, 5, 6, 7, 9 and 14, especially at high  $Nu$  (corresponding to high  $Pe$ ). Similarly, these models shall only be considered as lower limits for the Nusselt numbers, although the under-predictions are relatively lower for LBE and Pb, as shown in Fig. 3c.

### 3.2 Statistical evaluation

The filtered dataset remains large enough (1,014 data points) to allow for a statistical description. For this analysis, the relative error of the  $j$ th correlation when predicting the  $i$ th datapoint ( $\varepsilon_{i,j}$ ), is defined in Eq. (6). Furthermore, the statistical mean and root-mean-square



**Fig. 3** Relative errors of the correlations listed in Table 2 for the different categories of LMs (a)–(d). The *dashed bars* indicate the mean relative error ( $\bar{\varepsilon}_j$ ) and the empty ones, the RMS error ( $\varepsilon_{RMS,j}$ ).

**a** Results for all liquid metals. **b** Results for alkali liquid metals. **c** Results for lead and lead-bismuth. **d** Results for mercury

(RMS) values of  $\varepsilon_{i,j}$ , are defined for a set of  $N$  data points in Eqs. (7) and (8), respectively.

$$\varepsilon_{i,j} = \frac{Nu_{\text{predicted},i,j}}{Nu_{\text{measured},i}} - 1 \quad (6)$$

$$\bar{\varepsilon}_j = \frac{1}{N} \sum_{i=1}^N \varepsilon_{i,j} \quad (7)$$

$$\varepsilon_{\text{RMS},j} = \sqrt{\frac{1}{N} \sum_{i=1}^N \varepsilon_{i,j}^2} \quad (8)$$

These statistical relative errors are shown in Fig. 3, as well as listed in Table 4 in the “Appendix”. It can be observed that the overprediction of correlation No. 1 and underprediction of correlations No. 2, 4, 5, 6, 7, 9 and 14 (as noticed in Sect. 3.1) are statistically large, except for Pb and LBE. Therefore, their use is not recommended.

In general, it can also be observed that some correlations developed for alkali metals (in particular, those numbered 10–13 in Table 2) tend to overpredict the heat transfer rates in heavy LMs. This observation, together with the above-mentioned overprediction of correlation No. 1, has been for long time the reason to believe alkali LMs yield higher Nusselt numbers than mercury and lead alloys at the same Péclet number. However, as discussed in Sect. 2.1, the cloud of points in Fig. 1a does not support this conclusion. On the contrary, this is a specific feature of correlations 10–13 alone and not of the experimental data themselves. For example, correlation No. 8, although developed for sodium, gives very good results for heavy LMs.

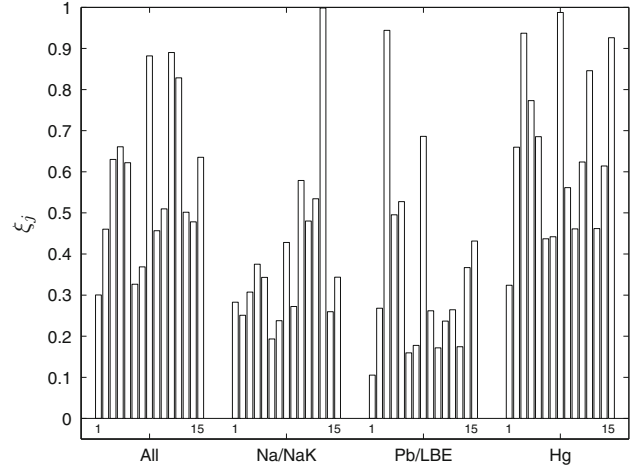
### 3.3 Selection of proper correlations

In order to univocally determine the best correlation for each category from the results presented in Fig. 3, a benchmark parameter  $\xi_j$  has been defined as in Eq. (9). This coefficient,  $0 \leq \xi_j \leq 1$ , accounts for both the absolute mean error and the RMS deviation of the correlation  $j$ .

$$\xi_j = 0.5 \left[ \frac{\min_j |\bar{\varepsilon}_j|}{|\bar{\varepsilon}_j|} + \frac{\min_j \varepsilon_{\text{RMS},j}}{\varepsilon_{\text{RMS},j}} \right] \quad (9)$$

With this consideration, the correlations giving the best results for each specific LM can be selected by comparing their benchmark parameter  $\xi_j$ , as in Fig. 4 and Table 4 in the “Appendix”. It should be noted that, as this is a relative coefficient, a correlation with  $\xi_j = 1$  is not a perfect one, but rather the one with simultaneously the smallest  $|\bar{\varepsilon}_j|$  and  $\varepsilon_{\text{RMS},j}$  within the selected group.

As the three subcategories (Na and NaK; LBE and Pb; and Hg) are evenly represented in this database, some conclusions on correlations for *all* LMs can be derived from



**Fig. 4** Comparison of the prediction performance of all correlations, in terms of the benchmark coefficient  $\xi_j$ . The detailed numerical values can be found in Table 4 in “Appendix”

Fig. 3a. The best results are given by correlation No. 11, by Skupinski et al. [46], while correlations 8 and 12 also perform well, with a similar coefficient  $\xi_j$ . It should be noted, however, that their RMS error values are roughly 20%. Therefore, it might be convenient to use individual equations for each specific LMs, instead of one single correlation encompassing all these fluids. The latter is recommended when other LMs not included in this survey are used.

Not surprisingly, the experimental data for alkali metals (Na and NaK) are best predicted by those models developed specifically for them. In particular, correlation 13 shows the best performance, followed by those numbered 10 and 12. Although this slight dependence on  $Pr$  ( $d = 0.01$ ) is, in principle, not justified by the experimental observations, this statistical analysis indicates that the best predictions are obtained with this correlation. Therefore, its use is recommended for this case. Nevertheless, it should be noted that good predictions are also obtained with correlation number 10, see Fig. 3b.

Lead and LBE measurements are best predicted by the correlation No. 3. In particular, this model, although developed as a best fit of relatively few datapoints for all LMs, has the smallest  $|\bar{\varepsilon}_j| = 0.3\%$ . With a significantly different functional form, correlation No. 8 gives also good results.

In general, the results for mercury follow the same trend as those for Pb and LBE, as it can be observed in Fig. 3c, d. Nevertheless, in all cases, a relatively large RMS error exceeding 20% is obtained. This circumstance is a consequence of the large scatter of the experimental data by different authors, as can be seen in Fig. 1d. In that context, the best results are obtained by correlation No. 8, followed closely by those numbered 3 and 15, respectively. It should be emphasized how these equations perform better than



others specifically developed for mercury, such as correlation No. 2. This is probably a consequence of the limited datasets considered by previous authors for developing their empirically-fitted relations.

According to these observations, the individual correlations recommended for each specific group of LMs are summarized in Table 3. Furthermore, the detailed results of this statistical analysis are listed in Table 4 in “Appendix”.

It can be concluded that both alkali- and, particularly, lead-alloys data are very well predicted by the best-performing correlations, with  $|\bar{\varepsilon}_j| < 1\%$  and  $\varepsilon_{\text{RMS},j} < 15\%$  and more than 75% of the datapoints within a  $\pm 20\%$  interval (within  $\pm 10\%$  for Pb/LBE). Such is not the case, however, for mercury, as less than half of the data points can be predicted within a  $\pm 20\%$  confidence interval. As previously discussed in Sect. 2.1, this fact may be attributed to the large scatter between data from different authors for this material.

#### 4 Review and analysis for $T_w = \text{const}$

Few reliable experimental data are available for this thermal boundary condition. As previously discussed in Sect. 2.1, those experimental results obtained with heat-exchanger-type test sections are not included in this survey, because they do not always fulfill the thermally fully developed condition. With this consideration, only two datasets (one for Hg [11] and one for NaK [48], altogether 22 data points) are evaluated and shown in Fig. 5. For all these experimental points the Péclet number is in the range 450–8,000. Therefore, the existence of measured Nusselt numbers below the laminar limit, for  $\text{Pe} < 300$  (as observed for the uniform wall heat flux condition, see Fig. 1), cannot be confirmed nor denied for this boundary condition.

The relatively small number of data points does not allow for a thorough statistical analysis. Assuming a dependence of the type  $\text{Nu} = a + b\text{Pe}^{0.8}$ , Reed [39] proposed Eq. (10) as the best fit for the data of Sleicher et al. [48]. However, this correlation slightly overestimates the data by Gilliland et al. [11]. Including both datasets in the analysis, Eq. (11) is developed in the present work as a best fit of the complete dataset.

$$\text{Nu} = 3.30 + 0.02\text{Pe}^{0.8} \quad (10)$$

$$\text{Nu} = 2.75 + 0.02\text{Pe}^{0.8} \quad (11)$$

While the accuracy of Eq. (11) is good ( $\bar{\varepsilon} = -0.95\%$ ,  $\varepsilon_{\text{RMS}} = 5.28\%$ ), it should be reminded that it is derived from a best-fit of the few available data. Therefore, it should not be used beyond the range of these data, as any extrapolation would not be justified.

Several correlations with the functional dependence as in Eq. (5) have been proposed by different authors following analytical and numerical studies. A critical statistical evaluation of the performance of these correlations as for the uniform-heat-flux case is not suitable here due to the limited number of experimental data points. As a reference, the lower and upper limits are given by the correlations by Sleicher and Tribus [47] and Seban and Shimazaki [42], as in Eqs. (12) and (13), respectively. It should be noted that all the available data lay between these two limits.

$$\text{Nu} = 4.8 + 0.015\text{Pe}^{0.91}\text{Pr}^{0.3} \quad (12)$$

$$\text{Nu} = 5.0 + 0.025\text{Pe}^{0.8} \quad (13)$$

Therefore, for estimating heat transfer coefficient beyond this restricted range theoretical considerations, as the ones described below, are crucial.

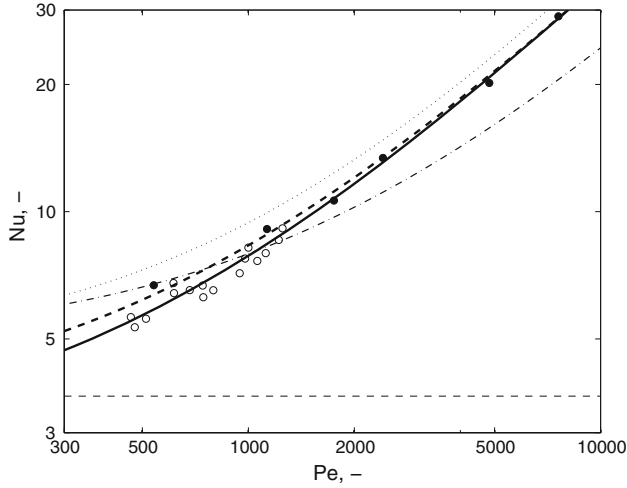
In order to evaluate the heat transfer coefficient beyond the restricted range of validity of Eq. (11), the alternative method proposed by Tricoli [53] can be used. He obtained that, for incompressible flows at high Pe and low Pr, the ratio of local temperature gradients at the wall for both boundary conditions remains constant and Eq. (14) was proposed. Consequently, the Nusselt numbers for a uniform wall temperature boundary condition can be estimated from the correlations recommended in Table 3, corrected by Eq. (14).

$$\text{Nu}|_{T_w=\text{const}} = \frac{\pi^2}{12} \text{Nu}|_{q_w=\text{const}} \quad (14)$$

The predictions of this alternative method, in combination with the best-performing correlations for  $q_w = \text{const}$ , summarized in Table 3 for Hg and NaK accordingly, are also included in Fig. 5. In both cases, the Nusselt numbers are overpredicted by Eq. (14), although the agreement is

**Table 3** Performance of the best correlations for each fluid category for  $q_w = \text{const}$ : mean and RMS error values, benchmark coefficient  $\xi_j$  and percentage of the data predicted with  $\pm 10\%$  and  $\pm 20\%$  accuracy

Fluid	No.	Author(s)	Year	Refs.	$\bar{\varepsilon}_j$ (%)	$\varepsilon_{\text{RMS},j}$ (%)	$\xi_j$	In $\pm 10\%$ (%)	In $\pm 20\%$ (%)
All	11	Skupinski et al.	1965	[46]	+4.20	23.77	0.890	42.7%	67.7
Na,NaK	13	Chen and Chiou	1981	[5]	+0.25	15.42	0.998	43.7%	83.3
Pb,LBE	3	Lubarsky and Kaufman	1955	[28]	+0.34	9.83	0.944	75.8%	93.3
Hg	8	Kutateladze et al	1959	[27]	-6.30	22.91	0.988	27.3%	48.5



**Fig. 5** Experimental  $Nu$  versus  $Pe$  data for uniform wall temperature. *Open circle* Hg data of [11]; *filled circle* NaK data of [48]; *solid line* Eq. (11), *dark dashed line* Eq. (10), *dashed-dot line* Eq. (14) with corr. No. 8 (Hg), *dotted line* Eq. (14) with corr. No. 13 (NaK), *light dashed line*  $Nu_{t,um} = 3.66$

better at higher  $Pe$ . Therefore, this model is recommended for use outside of the range of the available experimental data. Within the latter Eq. (11) is more suitable.

## 5 Conclusions

Liquid metals (LMs) are efficient heat transfer fluids considered for a broad range of application in nuclear and non-nuclear systems. Engineering heat transfer correlations play a major role for the proper design and operation of such systems. In that context, round pipes have been widely studied for direct uses and as a representative geometry for many configurations.

Since the 1940s, extensive information has been published in literature regarding experimental data and correlations for LM heat transfer under uniform-wall-heat-flux conditions. However, as they remained disperse and individual authors often dealt with limited datasets, no definitive statement on their validity has so far been derived. Filling this gap is the main contribution of the present work.

Over 1,100 data points from 21 different sources (see Table 1; Fig. 1) and 15 correlations (see Table 2) were considered in this work for  $q_w = \text{const}$ .

Within the significant scatter of the data, it can be observed that the long-believed fact that alkali metals yield larger Nusselt numbers than heavy LMs is not supported by the experimental evidence. On the contrary, it can be observed in Fig. 1 that, within the presumable uncertainties of the experiments, all LMs present similar heat transfer behavior, i.e. comparable  $Nu$  at the same  $Pe$ .

Based on the pioneering theoretical considerations by Martinelli [34] and Lyon [29, 30], most authors proposed

Nusselt number correlations based on the same functional dependence as in Eq. (5). In that context, it should be noted that the original Lyon-Martinelli model, i.e. Eq. (3) with  $Pr_t = 1$ , is the only one that tends to overpredict the experimental data for all fluids, see Figs. 2a and 3. In principle, this can be explained by  $Pr_t$  exceeding unity in LMs. In turn, several constitutive models for  $Pr_t$  have been proposed in the literature. However, their use in combination with Eq. (3) is not considered in this work, because their reliability has not yet been confirmed and because of the different predicted  $Nu$  at the same  $Pe$ . Instead, only those correlations listed in Table 2 are considered in the present work.

From the results of a statistical analysis and the definition of a proper benchmark coefficient as in Eq. (9), a set of most suitable correlations is recommended in Table 3, to be used in the range of the filtered dataset, that is  $Re > 10^4$  and  $0.007 < Pr < 0.046$ . While a generic correlation with an acceptable accuracy for all fluids is available, better results are obtained using individual equations for each case. Their predicting errors are well within acceptable engineering ranges for both sodium and lead alloys. For mercury, the large scatter of the experimental data leads in all cases to  $\epsilon_{RMS} > 20\%$  and to an unsatisfactory prediction within an accuracy of  $\pm 20\%$ .

Such a detailed statistical analysis is not suitable for the uniform-wall-temperature condition, due to the limited experimental and theoretical information available. Instead, a best-fit-model is presented in Eq. (11), along with lower and upper limits as in Eqs. (12) and (13), respectively. Furthermore, due to the influence of the boundary condition on LM heat transfer, the Nusselt numbers for  $T_w = \text{const}$  result slightly lower than those for  $q_w = \text{const}$ . An alternative description, based on this observation was presented by Tricoli [53], where  $Nu$  for a uniform wall temperature boundary condition can be estimated from the correlations recommended in Table 3 for  $q_w = \text{const}$  and corrected by Eq. (14).

In conclusion, the performance of the selected available correlations for uniform-heat-flux can be defined as satisfactory. For uniform-wall-temperature, a further validation through new experimental data is necessary before reaching the same conclusions. In general, a broad range of operating conditions is covered by the experiments for  $q_w = \text{const}$ . Nevertheless, the observed Nusselt numbers below the laminar limit at  $Pe < 300$  remain unclear, and further experimental investigations in this laminar and transition regimes would be most beneficial. Furthermore, the largest data scatter is found at low  $Pe$  for Na/NaK and at high  $Pe$  for Hg, which could be ascribed to the presence of mixed convection possibly due to too high heat fluxes. The same behavior is almost absent in Pb and LBE, or restricted to a very narrow interval, as described in [33].

## Appendix

### Tabulated results

The numerical indicators for all cases studied in Sects. 3.2 and 3.3 are listed in Table 4.

**Table 4** Results from the statistical analysis presented in Sections 3.2 and 3.3, in terms of the mean ( $\bar{\epsilon}_j$ ) and RMS ( $\epsilon_{\text{RMS},j}$ ) errors, as well as the benchmark coefficient ( $\xi_j$ ) given by Eq. (9)

$j$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
<i>All fluids</i>															
$\bar{\epsilon}_j$	0.308	-0.193	-0.107	-0.108	-0.112	-0.326	-0.282	-0.055	-0.213	0.120	0.042	0.054	0.127	-0.193	-0.109
$\epsilon_{\text{RMS},j}$	0.399	0.264	0.214	0.199	0.213	0.354	0.316	0.186	0.259	0.277	0.238	0.212	0.276	0.251	0.210
$\xi_j$	0.300	0.460	0.630	0.661	0.622	0.327	0.369	0.882	0.457	0.510	0.890	0.829	0.502	0.478	0.635
<i>Na, NaK</i>															
$\bar{\epsilon}_j$	0.205	-0.292	-0.213	-0.152	-0.145	-0.394	-0.302	-0.089	-0.261	-0.016	-0.083	-0.018	0.003	-0.273	-0.192
$\epsilon_{\text{RMS},j}$	0.278	0.312	0.255	0.209	0.230	0.404	0.329	0.186	0.287	0.154	0.165	0.165	0.154	0.301	0.228
$\xi_j$	0.283	0.251	0.307	0.375	0.343	0.193	0.238	0.428	0.272	0.579	0.480	0.534	0.998	0.260	0.344
<i>Pb, LBE</i>															
$\bar{\epsilon}_j$	0.407	-0.127	0.003	-0.046	-0.034	-0.272	-0.246	0.009	-0.156	0.213	0.125	0.137	0.213	-0.095	-0.039
$\epsilon_{\text{RMS},j}$	0.431	0.171	0.098	0.095	0.091	0.285	0.256	0.087	0.174	0.267	0.195	0.173	0.262	0.125	0.112
$\xi_j$	0.105	0.268	0.944	0.495	0.527	0.159	0.178	0.686	0.262	0.172	0.237	0.264	0.174	0.367	0.431
<i>Hg</i>															
$\bar{\epsilon}_j$	0.350	-0.136	-0.072	-0.105	-0.131	-0.291	-0.285	-0.063	-0.201	0.200	0.117	0.073	0.198	-0.174	-0.069
$\epsilon_{\text{RMS},j}$	0.477	0.261	0.223	0.236	0.251	0.340	0.337	0.229	0.276	0.368	0.315	0.271	0.369	0.258	0.239
$\xi_j$	0.324	0.660	0.937	0.773	0.685	0.437	0.442	0.988	0.562	0.461	0.624	0.846	0.462	0.614	0.926

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