Exergy based methods for economic and risk design optimization of energy systems: Application to a gas turbine

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1. Introduction

This paper presents an original implementation of the Thermoeconomic framework for analyzing and optimizing energy systems over different perspectives. The purpose of the study is to identify the possible optimal design configurations of a given energy system, including thermodynamic, economic and environmental perspectives.

For a major understanding of the procedure, a simple gas turbine operating in simple Joule–Brayton cycle is modelled and adopted as case study for the analysis.

First, simulation of the system is performed and energy and entropy balances and exergy accounting are applied to the system, in order to assess and to locate the main irreversibilities. Second, economic and environmental issues are assessed performing specific analyses based on exergy.

Thermoeconomic analysis is used to assess and to optimize economic cost of the system product, finding the design parameters that lead to the best trade-off between investment and operative economic costs [1,2]. The environmental impact is assessed in terms of risk on human health [3,4]: this issue is assessed using a model based on Risk Analysis proposed by the authors, developed to minimize the hazardous local impact of energy systems [5]. As will be shown, the optimal design configuration is strictly dependent on the considered objective and cost functions. The paper shows how a changing in the objective function of the analysis (exergy cost, monetary cost or risk cost of the system product) can influence the optimal design of the system. Finally, in order to avoid ambiguity among the terms used, Authors refer to the nomenclature used by Valero in Ref. [6]: the term exergy cost is used to represent the amount of exergy consumed by a system to produce an exergy product, and it is expressed in J/J. The exergoeconomic cost, on the other hand, is used in Thermoeconomics to indicate the economic cost of the exergy produced by the system, expressed in €/J. In a dual and novel way with respect to the exergoeconomic cost, the model developed by the authors in Ref. [2] defines a new sort of “cost” of the exergy product of the system, different from the previous two costs. It represents the potential damage associated to

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the system that is necessary to accept in order to produce the exergy product, expressed in injured/J. In the following, Authors will refer to this cost as *risk occurred per unit of exergy*.

2. Case study: simple cycle gas turbine

Gas turbines are well proven and reliable technology for electric power generation. For this paper, a simplified model of gas turbine is presented.

2.1. Description of the model

The power plant consist on a conventional open cycle for electric generation purpose ("Heavy Duty"), which has a constant net electrical power capacity of 115,9 MW and is natural gas fuelled. The system consists of an air compressor mechanically coupled with a gas expander, and a combustion chamber. Plant scheme is presented in Fig. 1 and represents the bases for the numerical model.

System was modelled through a code developed in MATLAB®, using average data and assumptions for gas turbine technology. Working fluids behavior was modelled with RefProp® [7] program, calling the thermophysical properties calculation functions from the Refprop.dll model libraries. Since gas *Pressure ratio* (β_C) and *Turbine Inlet Temperature* (T_3) are the main design parameters for gas turbine technology operating in simple cycle, the numerical model was conceived in order to perform parametric simulations.

The design optimization method adopted here consists in multiple runs of the MATLAB® code, calculating output performance results (Costs, Efficiencies, etc.) for different combinations of gas Pressure ratio and Turbine Inlet Temperature values, and visualising them as different surfaces. The optimal combinations of the two design parameters are thus found in a graphical way, respectively as the minimum or the maximum of the cost or efficiency surfaces.

Table 1
Gas Turbine variable parameters.

Variable parameters	Min value	Max value	Analysis step
Air pressure ratio [-]	8	34	0.1
Turbine inlet temperature [°C]	950	1300	5

Results of energy, exergy and Thermoeconomic analyses has been obtained for practical range of β_C and T_3, reported in Table 1, and are visualized as surfaces against these two parameters. A discussion is held for the optimum values and regions for each optimization case.

Reference environment was assumed to be the standard *Baehr environment* [8] at 25 °C and 1.01325 bar. Fuel is assumed to be pure methane. Both Air and Fuel are absorbed by the cycle at constant environmental temperature and pressure. The Gas Turbine operates in steady state at maximum load for 7500 h per year, which is a common assumption for designing gas turbine power plants. Off-design conditions were not investigated here.

3. Thermodynamic evaluation and exergy analysis

Energy, entropy and exergy analyses were performed according to literature [2]. A set of equations describe the thermodynamic behavior of the gas turbine; from the solution of these equations, the thermodynamic cycle can be completely determined.

As stated by Torres et Al. in Refs. [9], in order to perform the exergy analysis for a generic energy system, it is convenient to define its Productive Structure using the physical model of the system as reference. Considering the economic structure of the system, the incoming or outgoing currents for each piece of equipment and for the whole system can be grouped according to the *Fuel – Product – Losses* criterion: the F (fuel) represents every material or immaterial stream whose exploitation is one of the

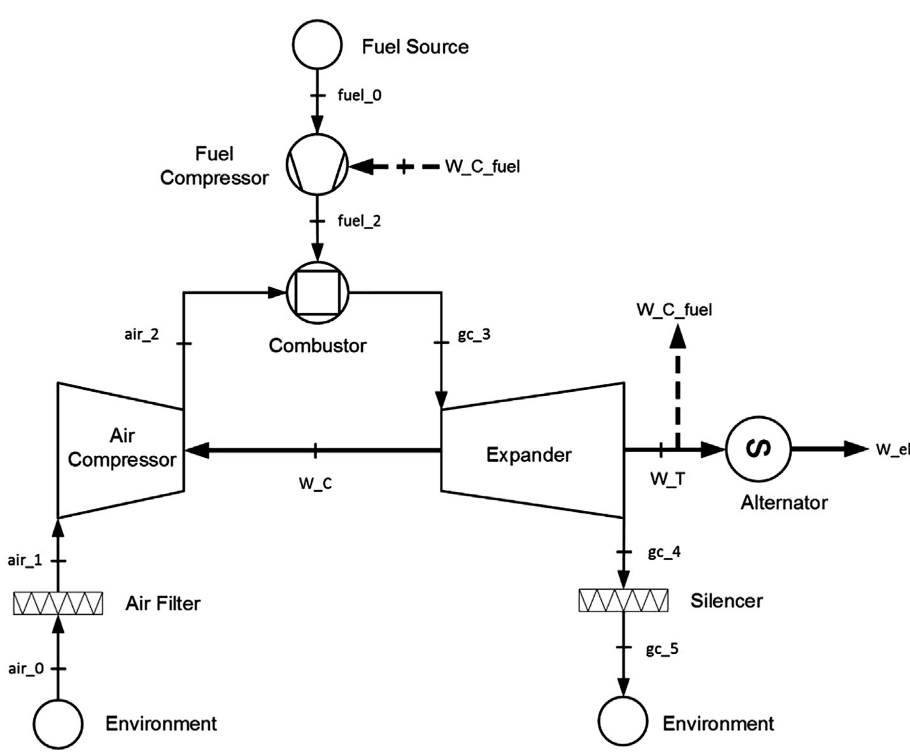

Fig. 1. Gas turbine plant layout.

Table 2
Productive structure. All terms represent power exergy (kW).

Component	Fuel	Product	Losses
Air filter	$Ex_{air,0}$	$Ex_{air,1}$	–
Air Compressor	W_C	$Ex_{air,2} - Ex_{air,1}$	–
Fuel Compressor	$W_{C,fuel}$	$Ex_{fuel,2} - Ex_{fuel,0}$	–
Combustor	$Ex_{air,2} + Ex_{comb,2}$	$Ex_{gc,3}$	–
Expander	$Ex_{gc,3}$	$W_T + Ex_{gc,4}$	–
Alternator	$W_T - (W_C + W_{C,comb})$	W_{el}	–
Silencer	$Ex_{gc,4}$	–	$Ex_{gc,5}$
Whole system	$Ex_{fuel,0}$	W_{el}	$Ex_{gc,5}$

design inputs, the P (product) includes any material or immaterial stream provided by the system as the useful effect and the L (losses), named also Wastes, are the material or energy flows released in the environment with no useful effect [10,11]. All these categories may include more than one incoming or outgoing currents.

According to this paradigm, assuming that the considered system operates at steady state, the traditional exergy budget can be rewritten as follows:

$$\dot{E}x_F = \dot{E}x_P + \dot{E}x_L + \dot{E}x_D \tag{1}$$

Since the Losses are associated to energy or material output streams, their exergy is in principle recoverable; by contrast, the exergy destruction ($\dot{E}x_D$) constitutes an irrecoverable loss.

The productive structure used for the system is represented in Table 2. It has to be noted that since the gas turbine operates in simple cycle, the flue gases (gc, 5) are release in the environment with no useful effect, thus they have been recognized as the losses of both the silencer and the whole system.

For each component of the system, an exergy efficiency is defined according to [12]. It provides a criterion for evaluating the thermodynamic performance of the component.

$$\eta_{ex,j} = \frac{\dot{E}x_{P,j}}{\dot{E}x_{F,j}} \tag{2}$$

The objective function of the energy and the exergy analysis is to find the combination of design parameters T_3 and β_C that provides the largest values of energy and exergy efficiency for the whole system, defined as follow:

$$\eta_{I,tot} = \frac{\dot{W}_{el}}{\dot{m}_{fuel} \cdot LHV_{fuel}} \tag{3}$$

$$\eta_{ex,tot} = \frac{\dot{E}x_P}{\dot{E}x_F} \rightarrow \eta_{ex,tot} = \frac{\dot{W}_{el}}{\dot{m}_{fuel} \cdot ex_{fuel}} \tag{4}$$

Fig. 2 shows energy and exergy efficiencies for every couple of T_3 and β_C. Results are in accordance to the literature, since higher values of efficiencies are obtained with higher values of T_3 and β_C. The maximum energy and exergy efficiency are reached respectively for $\beta_C = 34$ and $T_3 = 1300\,°C$. In particular, the best obtainable exergy efficiency is 36.1%.

4. Thermoeconomic approach: monetary cost analysis

4.1. Theoretical cost analysis of the system

Energy system requires capital investments for purchase, installation, operation and maintenance of plant equipment. At fixed electric power production rate, operative parameters of the system affect capital investment costs as well as operative costs.

Due to this fact, a thermoeconomic design analysis of the productive structure was performed in order to determine the couple of parameters T_3 and β_C at which the system produces the useful output (electricity) at the minimum monetary cost.

According to the theory presented by El-Sayed in Refs. [13], Thermoeconomic balance equations can be written for each component of the system:

$$\sum_{in} \dot{C}_{in} + \dot{Z}_{inv} = \sum_{out} \dot{C}_{out} \tag{5}$$

Due to the *Exergy Costing Principle* [14], a cost flow can be assigned to each exergy flow:

$$\dot{C}_i = c_i \cdot \dot{E}x_i \tag{6}$$

In expression (6), the term $\dot{C}_i$ is a cost per hour (€/h) and c_i represent the specific economic cost of exergy expressed in (€/MJ). According to expressions (4,5), the complete set of balance equations can be written (Table 3).

In order to solve the linear system equation presented in Table 3, auxiliary relations are needed. In absence of further estimates or

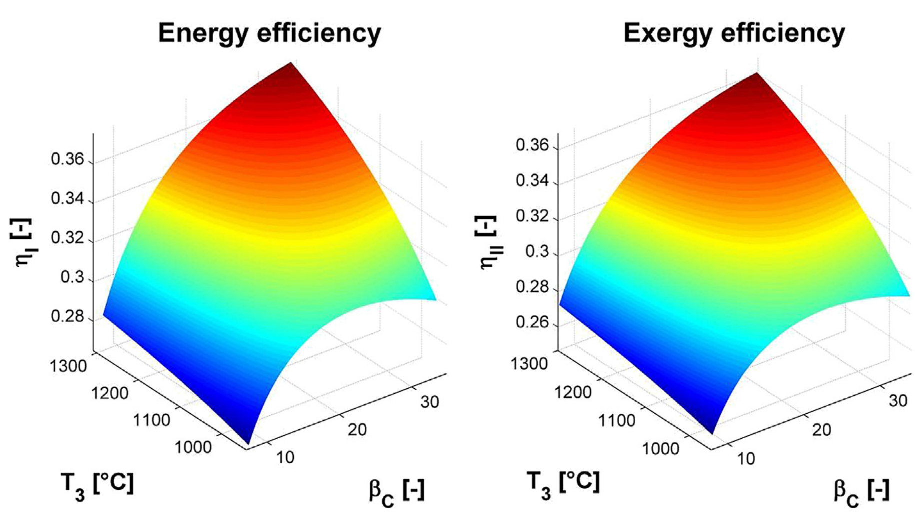

Fig. 2. Energy and Exergy efficiencies vs T_3 and β_C.

Table 3
Thermoeconomic balances for each component of the system.

Component	Thermoeconomic balance	Exergy costing relations
Air filter + air compressor	$\dot{C}_{\mathrm{air},0} + \dot{C}_{W_c} + \dot{Z}_{\mathrm{inv},C} = \dot{C}_{\mathrm{air},2}$	$\dot{C}_{\mathrm{air},2} = c_{\mathrm{air},2} \cdot \dot{E}x_{\mathrm{air},2}$ $\dot{C}_{W_c} = c_{W_c} \cdot \dot{W}_C$
Fuel Compressor	$\dot{C}_{W_{C,fuel}} + \dot{C}_{fuel,0} + \dot{Z}_{inv,C,fuel} = \dot{C}_{fuel,2}$	$\dot{C}_{W_{C,\mathrm{fuel}}} = c_{W_{C,\mathrm{fuel}}} \cdot \dot{W}_{C,\mathrm{fuel}}$ $\dot{C}_{\mathrm{fuel},0} = c_{\mathrm{fuel},0} \cdot \dot{E}x_{\mathrm{fuel},0}$ $\dot{C}_{\mathrm{fuel},2} = c_{\mathrm{fuel},2} \cdot \dot{E}x_{\mathrm{fuel},2}$
Combustor	$\dot{C}_{air,2} + \dot{C}_{fuel,2} + \dot{Z}_{inv,comb} = \dot{C}_{gc,3}$	$\dot{C}_{\mathrm{air},2} = c_{\mathrm{air},2} \cdot \dot{E}x_{\mathrm{air},2}$ $\dot{C}_{\mathrm{fuel},2} = c_{\mathrm{fuel},2} \cdot \dot{E}x_{\mathrm{fuel},2}$ $\dot{C}_{gc,3} = c_{gc,3} \cdot \dot{E}x_{gc,3}$
Expander + Alternator + Silencer	$\dot{C}_{gc,3} + \dot{Z}_{inv,T} = \dot{C}_{gc,5} + \dot{C}_{W_c} + \dot{C}_{W_{C,\mathrm{fuel}}} + \dot{C}_{W_{el}}$	$\dot{C}_{gc,3} = c_{gc,3} \cdot \dot{E}x_{gc,3}$ $\dot{C}_{gc,5} = c_{gc,5} \cdot \dot{E}x_{gc,5}$ $\dot{C}_{W_c} = c_{W_c} \cdot \dot{W}_C$ $\dot{C}_{W_{C,\mathrm{fuel}}} = c_{W_{C,\mathrm{fuel}}} \cdot \dot{W}_{C,\mathrm{fuel}}$ $\dot{C}_{W_{el}} = c_{W_{el}} \cdot \dot{W}_{W_{el}}$

evaluations, these auxiliary relations have to be assumed according to specific criteria (propositions) from the literature [2]. For the present case study:

$$c_{\mathrm{air},0} = 0 \tag{7}$$

$$c_{\mathrm{fuel},0} = 10.902 \text{ €}/GJ \tag{8}$$

$$c_{gc,5} = 0 \tag{9}$$

$$c_{W_C} = c_{W_{C,\mathrm{fuel}}} = c_{W_{el}} = c_W \tag{10}$$

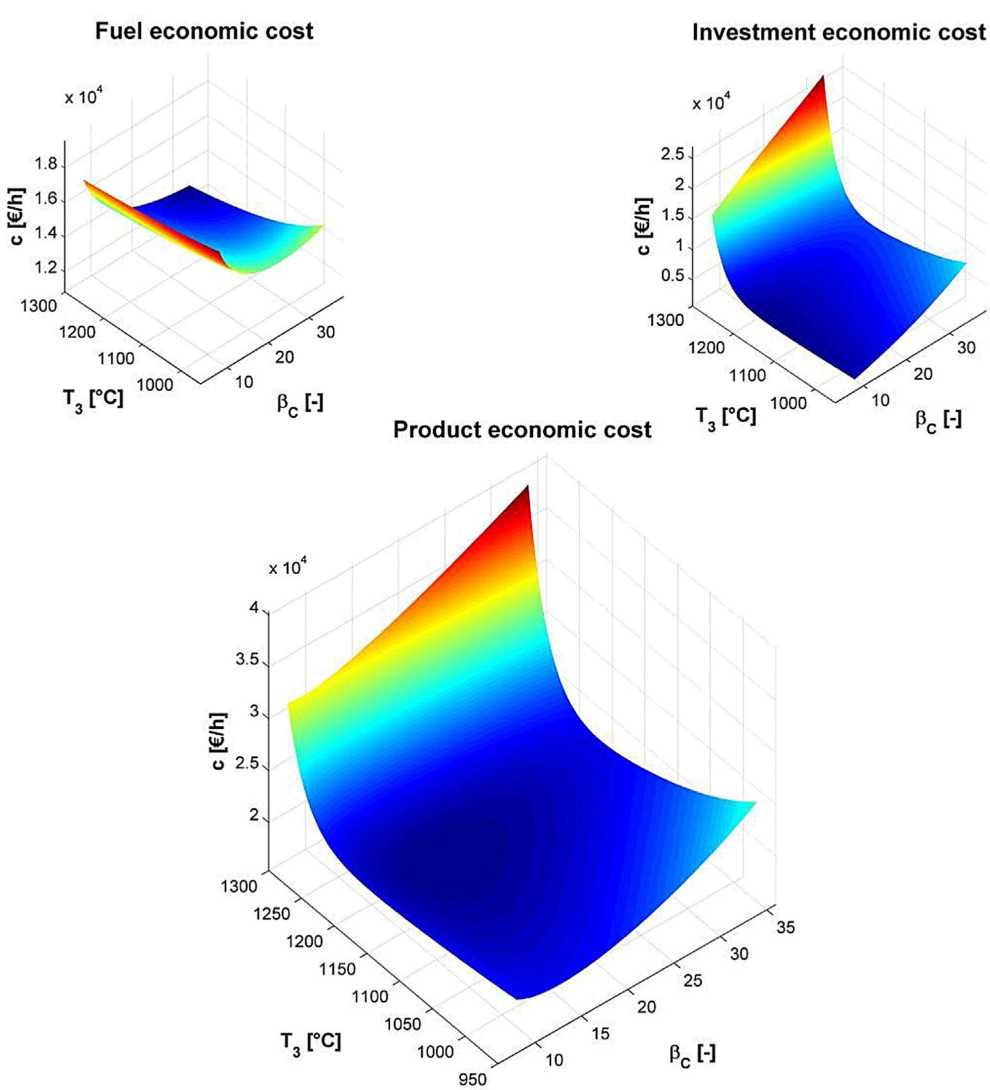

Fig. 3. Results of exergoeconomic analysis.

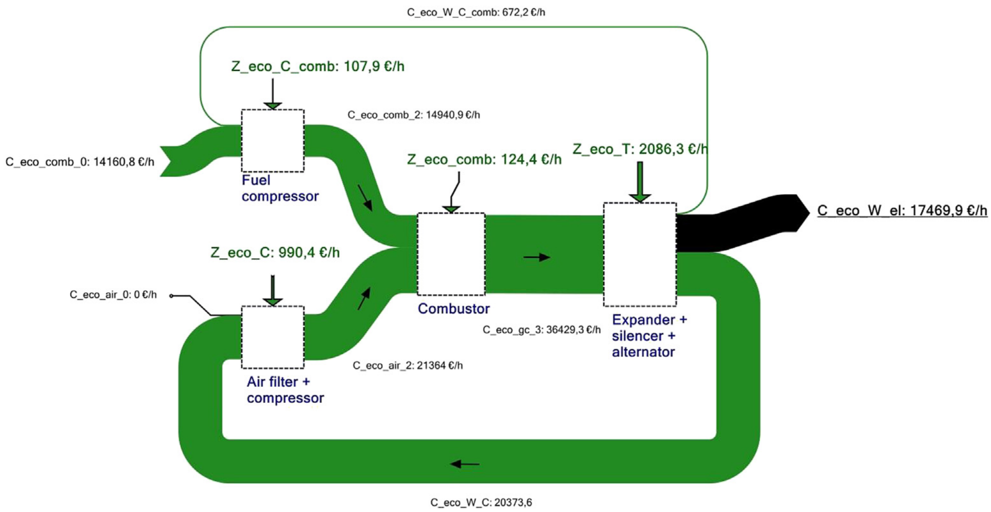

Fig. 4. Cost flow diagram for the gas turbine plant.

With these auxiliary equations it comes out that all the costs of any residues are charged to the expander, and thus on the electrical power production final cost, as expressed by relation (9). The specific cost of the natural gas (8) was computed on the base of the Italian average market price, and it was considered constant for the entire lifetime of the plant. Moreover, the environmental air economic cost is zero (7) and the specific economic costs of all the mechanical power output produced by the gas expander are equal (10).

Fig. 3 shows the cost of fuel, investment and product. It is possible to notice how the cost of fuel is related to the gas flow rate $\dot{m}_{\text{fuel}}$ as T_3 and β_C increase, the energy and exergy efficiency increase and the required $\dot{m}_{fuel}$ reduces. On the contrary, the investment cost of components$\dot{Z}_i$ increases with the efficiency of the system, describing an opposite behavior compared to the cost of fuel. The values of T_3 and β_C that give the optimal cost of product are respectively 16.6 and 1175 °C. Finally, cost flow of the entire system is represented in Fig. 4.

4.2. Capital cost estimation

Estimation of the investment cost of components was performed starting from the PEC (purchased equipment cost). In this method, described by Bejan et Al. in Ref. [2] and by El-Sayed et Al. in Refs. [15], investment cost are obtained by mathematical cost correlations, which are influenced by operative parameters of the components. Table 4 resume the cost functions for PEC calculation proposed by Najjar and Suhayb in Ref. [16].

Starting from PEC values (€), it is possible to compute the TIC (total investment cost, in €) of the global system through the estimation of the direct, indirect and maintenance costs of the system (O&M, in €) with relation (11) [2].

$$\mathrm{TIC}_i \approx 6,32 \cdot \mathrm{PEC}_i \tag{11}$$

Assuming a plant lifetime of 10 years (N), for 7500 full load operative hours per year, it is possible to compute the cost rate associated to the capital investment for the *i-th* component $\dot{Z}_i$ (€/h) with relation (12).

$$\dot{Z}_i = \frac{\mathrm{TIC}_i \cdot \mathrm{CRF}}{N \cdot 7500} \tag{12}$$

Where CRF (*capital recovery factor*) takes into account the effect of the interest i (6% per year) on the invested capitals and it is computed with relation (13).

Table 4
Purchased equipment cost functions for gas turbine components. Mass flow rates and temperatures are expressed respectively in terms of kg/s and K.

Component	Purchased equipment cost (PEC)
Air filter + air compressor	$\mathrm{PEC}_C = \frac{30 \cdot \dot{m}_{air,0}}{1 - \eta_{iso,C}} \cdot \beta_C \cdot \ln(\beta_C)$
Fuel compressor	$\mathrm{PEC}_{C,\text{fuel}} = \frac{79 \cdot \dot{m}_{\text{fuel},0}}{1 - \eta_{iso,C,\text{fuel}}} \cdot \beta_{C,\text{fuel}} \cdot \ln\left(\beta_{C,\text{fuel}}\right)$
Combustor	$\mathrm{PEC}_{\text{comb}} = \frac{46,08 \cdot \dot{m}_{\text{air},0}}{0,995 - P_3/P_2} \cdot [1 + \exp(0,018 \cdot T_3 - 26,4)]$
Expander + alternator + silencer	$\mathrm{PEC}_T = \frac{479,34 \cdot \dot{m}_{gc,3}}{0,95 - \eta_{iso,T}} \cdot \ln\left(\beta_{C,\text{fuel}}\right) \cdot [1 + \exp(0,036 \cdot T_3 - 54,4)]$

$$\mathrm{CRF} = \frac{i \cdot (1+i)^N}{(1+i)^N - 1} \tag{13}$$

5. Coupling thermoeconomic framework and risk analysis to optimize the hazardous impact of the system

5.1. Model concept

To minimize the hazardous impact of the system, Authors propose a model that uses exergy as allocation criteria for risk analysis. This model has already been presented in a previous work [5].

The model has been developed by Authors and is based on a dual structure of the Thermoeconomics framework [17], in which the allocation criteria for cost analysis is exergy. The aim of the model is reducing the risk of the system by operating on the thermodynamics of the processes.

The hypothesis on which the model relies is that the risk of accident R of a system, expressed quantitatively as the product of probability p and fatality F of hazards [18], represents a "cost" for the society, expressed in terms of damage (number of individuals injured [19]), that needs to be accepted in order to benefit of the product of the system, in this case the electric power output of the gas turbine. This *social cost* is expressed as the total Risk of the system obtained by summing all the risks associated to all the hazards within the system itself. R is therefore expressed in number of individuals injured per year.

In industrial safety, the term *hazard* indicates an intrinsic property of a material, machinery, plant, situation, etc., that is able to damage things, environment or people [20]. Therefore, a hazard follows a binary logic: it is present or it is not present. It can be eliminated (by removing the material, the machinery, plant, situation, etc.), but it cannot be reduced. In every human activity, and more reasonably in industrial activities, there are intrinsically hazards to things, environment or people. The presence of a hazard can have very different consequences according to two variables: the fact that the hazard evolves in an undesired event or not, and the entity of the damage caused by the undesired event. The concept of risk therefore expresses the combination of the probability that the hazard generates a damage (the probability of occurrence p) and the entity of the damage F (fatality) [21].

This asset and the novelty of the method may be considered therefore the introduction of a new approach aimed at the extended analysis of the system: in a similar way of the exergoeconomic cost (€/J), the *risk occurred per unit of exergy* (injured/J) of a system is the potential damage, expressed in terms of injuries to human health, that the community has to accept to obtain the useful effect (the exergy) given by the product of the systems. In this vision, willingness to accept the risk is function of the obtainable utility of the process.

Such a use of the risk as objective function of a thermoeconomic approach represents an innovative application of exergy analysis. In literature there are examples of methods that, in a similar way to the proposed approach, use exergy beyond the thermodynamic frame by means of the Thermoeconomic framework: this is the case of Exergoenvironmental analysis [22–24]. In Refs. [22] and [23] the environmental impact of the systems analyzed is assessed by means of Life Cycle Analysis: here, the cost per unit of exergy of the product is computed by means of the Eco indicator [25]. The Eco indicator is internationally recognized as instrument for environmental assessment and declines the environmental impact in three damage categories: Human Health, Ecosystem Quality and Resources. The final indicator represents the synthesis of the three categories and the impact is expressed in terms of Pts (points). The environmental impact is then assigned to the exergy streams involved in the process as economic costs are assigned in Thermoeconomics [26].

As mentioned, in the current method used in this paper, the environmental impact is evaluated in terms of risk. Risk Analysis indeed is mandatory to estimate the potential impact that a system might have at local scale on human health and ecosystem of the surrounding environment during normal operations and in case of failures [18].

Risk analysis hence supplies a complementary information with respect to Life Cycle Analysis: this evaluate the impact of the system at the global level [27], while risk analysis is focused on local impact and on the safety of the plant [19]. Such information is not given by the former indicator. Hence, using risk analysis as objective function allows assessing hazardous local impacts, complementary to resource consumption and global impact, in terms of:

- Effects on human health [3]
- Contamination of atmosphere, hydrosphere and lithosphere [28]
- Ecosystem quality and biodiversity loss.

The numerical value of R is calculated by mathematical models that estimate the consequences and the probability of system failures that may lead to hazard for human health such as leaks, emissions, dispersions, fires and explosions [4,29,30].

According to the literature [31], to applying a conservative approach it is generally possible to assume that all the risks in the system are mutually independent; the risk R of the system is thus the sum of all the risks related to all the hazards:

$$R = \sum_{i=1}^{n} R_i \tag{14}$$

where i represents the i-*th* risk related to i-th hazard.

In the model, each i-*th* risk is defined according to the general expression [21]:

$$R_i = p_i \cdot F_i \tag{15}$$

Where F_i is the fatality from the hazard i and p the probability of occurrence [32,33]. Risk is expressed as fatalities (or individuals injured) on time interval (y^{-1} generally): therefore, considering the mathematical symbolism and in coherence with symbolism used in Thermoeconomics, the expression $\dot{R}$ is used. Considering $\dot{R}$ as a risk for producing only the product of the system ($\dot{E}x_P$). The risk occurred per unit of exergy produced by the system is therefore obtained as:

$$r_P = \frac{\dot{R}}{\dot{E}x_P} \tag{16}$$

It is important to mention that $\dot{R}$ does not represent the risk related to the exergy content of the product, but it is the risk associated to the system (i.e. involving the processes and components) required to produce the product.

5.2. Risk functions identification

Usually, in Thermoeconomic design optimization procedures the investment costs involved in the systems are defined by cost functions [2,13]. In the model presented, the objective functions are obtained directly by risk analysis.

In (15) indeed, F_i may be evaluated by means of Probit functions, which are an alternative way of expressing the probability of injury from accidents [31]. Probit functions have usually the following form [19]:

$$Y_i = k_1 + k_2 \ln x_i \tag{17}$$

where Y_i is the Probit value relative to hazard i, x is causative variable (e.g. thermal radiation kW/m^2, peak overpressure N/m^2, impulse Ns/m^2, concentration ppm) of hazard i, k_1 and k_2 are defined constants for the specific hazard. Examples of Probit functions are in Ref. [31]. Y_i is then converted in the fatality F_i by means of the function [19]:

$$F_i = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{Y-5} \exp\left(-\frac{u^2}{2}\right) du \tag{18}$$

where Y_i has a mean 5 and variance 1. Mean 5 is adopted to avoid the necessity of computing with negative numbers of the distribution [34].

Equation (18) may be also expressed through the Finney chart [35,36] to convert the probit in F_i. By interpolating the Finney chart through a polynomial function, a simpler formulation of F_i is obtained:

$$F_i = a \cdot Y_i^2 + b \cdot Y_i - c \tag{19}$$

The entity of the causative variable x_i is related to the entity of the physical effects generated because of the failure, e.g. a thermal radiation from a fire or a wave shock due to an overpressure. Such physical effects will be demonstrated to be function of process parameters of the system, as temperatures, pressures and flow rates.

The dependency of the risk $\dot{R}$ from the process parameters can be seen considering the causative variable $x_{i,k}$ of the i-th hazard in the generic component *k-th* within a plant [35]. Referring to the component k in thermodynamic equilibrium at steady state, the causative variable $x_{i,k}$ may be expressed with Equation (20) [5].

$$x_{i,k} = x_{i,k}\left(op_{\alpha,k}\right) \tag{20}$$

where $op_{\alpha,k}$ are the independent operating parameters such as pressure P_k, temperature T_k, or mass flow rate $\dot{m}_{\theta,k}$, θ being the type of fluid used with its specific toxicological profile. The causative variables of each hazard are related to the respective parameters they depend on (i.e., T for thermal radiation, P for overpressure, $\dot{m}$ for release). These parameters are evaluated in the point where the accident occurs and are therefore related to those of the stream of flow rates crossing the component k [20].

From relations (17) and (18) it is therefore possible to suppose a direct relation between the fatality F_i and the operating parameters of the system. From (20), it is consequently possible to associate the risk of the k-th component $\dot{R}$ to the independent operating parameters of the system:

$$\dot{R} = f_1\left(op_{\alpha,k}\right) \tag{21}$$

where f_1 represents a general function expressing the relation between $\dot{R}$ and the independent operating parameters of the *k-th* component.

For the exergy balance, two considerations need to be added. Exergy associated to heat and work may depend on (some of) the parameters of the component k, while the exergy associated to a stream flow rate depends directly on the parameters of the stream itself entering or exiting component k [27]. These are necessarily related to the operating parameters of k such as P_k, T_k, $\dot{m}_{\theta,k}$. The exergy balance, and thus the exergy destructions$\dot{E}x_D$, depends on the same parameters. From the Fuel – Product allocation, also the exergy of the system product $\dot{E}x_P$ comes to be linked to system operating parameters and therefore to $\dot{R}$. On the consequence it is possible to state:

$$\dot{E}x_{P,k} = f_2\left(op_{\alpha,k}\right) \tag{22}$$

Therefore a link between $\dot{R}$ and $\dot{E}x_P$ is recognized, and this allows to evaluate how the entity of risk is modified by varying these parameters. $\dot{R}$ and $\dot{E}x_P$ are therefore both connected to the operating parameters of the systems, hence it is possible to find the value for each parameter that minimizes the risk of the system starting from the thermodynamics of its processes.

5.3. *Application to gas turbine*

In the case studied presented the hazard analyzed are the following [31]:

1. thermal radiation from accidental ignition of fuel from a pipe rupture (*jet fire*);
2. thermal radiation from combusted gas from pipe rupture (*jet of combusted gases*);
3. pressure and thermal effect generated by overpressure in the combustion chamber (*overpressure*).

These hazards were identified through a simplified Hazard Operability Analysis [31,37]. A synthesis of the analysis is in Table 5. In this case study, the fatality F of the different hazards has been evaluated with the polynomial function (19), where the values of the constants a,b and c obtained by extrapolation from Finney chart are respectively 0.002, 29.3 and 96.4.

5.3.1. *Jet fire*

In the case of *jet fire* the thermal radiation I [kW/m^2] is expressed as:

$$I_{jf}(d) = \frac{\dot{Q}_{jf}}{4\pi d^2} \cdot \tau_A \tag{23}$$

where d is the distance from the heat source and τ_A is the fraction of heat not absorbed by atmosphere.

The heat of the fuel combustion is computed as:

$$\dot{Q}_{jf} = \dot{m}_{\mathrm{fuel,leak}} \cdot \mathrm{LHV}_{\mathrm{fuel}} \tag{24}$$

Where LHV_{fuel} represents the lower heating value of fuel, in kJ/kg. The parameters of the system on which the risk of jet fire depends are therefore the flow rate of fuel $\dot{m}_{\mathrm{fuel}}$ and the lower heating value of fuel $\mathrm{LHV}_{\mathrm{fuel}}$.In (24), $\dot{m}_{\mathrm{fuel,leak}}$ is the fuel flow rate that is released from a pipe rupture and its value depends from the particular system and failure. In our case, it is assumed as $\dot{m}_{\mathrm{fuel,leak}} = \alpha \cdot \dot{m}_{\mathrm{fuel}}$[38], being α between 0,2 and 1.

Assuming the heat source as punctual, it is adopted to consider the radiation as equally distributed on a sphere of ray d. Fraction of heat not absorbed by atmosphere (τ_A) generally assumes values between 0,6 and 0,8 [38,39].

Table 5
Hazard Operability analysis of the gas turbine.

Deviation	Possible causes	Consequences
LESS fuel flow LESS pressure of fuel	Fuel line leakage	Possible line rupture and release of fuel, possible fire, possible jet fire
LESS pressure of combusted gas	Line leakage	Possible release of combusted gas
MORE pressure of combusted gas	Combustor blockage	Possible overpressure

Another assumption is that the combustion is incomplete, so in (24) a thermal radiation efficiency η_{rad} is added, hence:

$$\dot{Q}_{jf} = \dot{m}_{fuel,leak} \cdot LHV_{fuel} \cdot \eta_{rad} \tag{25}$$

For natural gas η_{rad} is 0,19 ÷ 0,34 [38]. The Probit function for evaluating the fatality of jet fire is then (26) [31].

$$Y_{jf} = -14,9 + 2,56 \cdot ln\left(t_e \cdot I_{jf}(d)^{4/3} \Big/ 10^4\right) \tag{26}$$

Where t_e is the exposition time to the jet fire. The product $t_e \cdot I_{jf}(d)^{4/3}$ is called tdu (thermal dose unit). By means of (19), Y_{jf} can be converted in Ref. F_{jf} and therefore it is possible to evaluate the risk of the jet fire $\dot{R}_{jf}$ with relation (27).

$$\dot{R}_{jf}(d) = p_{jf} \cdot F_{jf}(d) \tag{27}$$

Where p_{jf} is the probability of occurrence of the jet fire, that in our case it is the product of the probability of fuel leakage from the pipe p_{leak} and the probability of ignition p_{ign}, as relation (28) shows.

$$p_{jf} = p_{leak} \cdot p_{ign} \tag{28}$$

Values of p_{leak} and p_{ign} are taken from literature [40,41].

5.3.2. *Jet of combusted gas*

In the case of jet of combusted gas the expressions of thermal radiation and the Probit function are the same of jet fire, but in this case:

$$\dot{Q}_{jcg} = \dot{m}_{cg,leak} \cdot (h_{gc} - h_0) \tag{29}$$

where h_0 is the enthalpy at reference temperature and

$$\dot{m}_{cg,leak} = \alpha \cdot \dot{m}_{cg} \tag{30}$$

The parameters of the system involved are therefore $\dot{m}_{cg}$, h_{cg} and h_0.

The steps to estimate the risk of jet of combusted gas $\dot{R}_{jcg}$ are therefore:

$$I_{jgc}(d) = \frac{\dot{Q}_{jcg}}{4\pi d^2} \cdot \tau_A \tag{31}$$

$$Y_{jcg} = -14,9 + 2,56 ln\left(t_e I_{jcg}(d)^{4/3} \Big/ 10^4\right) \tag{32}$$

$$\dot{R}_{jcg}(d) = p_{jcg} \cdot F_{jcg}(d) \tag{33}$$

where $p_{jcg} = p_{leak}$ for the pipe.

5.3.3. *Overpressure*

The Probit for evaluating the fatality from overpressure is:

$$Y_{over} = -39,1 + 4,45 \cdot ln\, J \tag{34}$$

being J the impulse in Ns/m^2 over the peak overpressure P_0. The peak overpressure depends on the pressure P in the combustion chamber and is evaluated with the procedure in Ref. [31]. In the overpressure also the thermal effect of an instantaneous expansion of combustion gases has been considered and evaluated with the Probit function (26), where:

Table 6
Table of synthesis for risk calculation.

Risk	Variables	Set of equations (in order of solving)
Jet fire from pipe	$\dot{m}_{fuel}$, LHV_{fuel}	(18),(22), (24–27)
Jet of combusted gas from pipe	$\dot{m}_{gc}$, h_{gc}, h_0	(18), (28–32)
Overpressure in combustion chamber	ρ, V, v_{air}, u_{gas}	(18), (33–36)

$$Q_{heat,over} = \rho \cdot V \cdot (u_{gas} - u_0) \tag{35}$$

$$I_{heat\_over}(d) = \frac{Q_{heat\_over} \cdot v_{air}}{4/3\pi d^3} \cdot \tau_A \tag{36}$$

where ρ is the density of gas at combustion temperature, V the volume of combustor and v_{air} is the velocity of sound through air (360 m/s). The risk of overpressure $\dot{R}_{over}$ is hence:

$$\dot{R}_{over}(d) = p_{over}\left(F_{p_0}(d) + F_{heat_{over}}(d)\right) \tag{37}$$

also in this case p_{over} is taken from literature [42].

Parameters involved in the evaluation of the risk of overpressure are hence the pressure P in combustion chamber and the internal energy of gas u_{gas}.

In Table 6, the list of the different variable and equations solved to obtain the final results is presented.

5.3.4. *Total risk*

Once calculated the risks related to the hazards identified, the total risk $\dot{R}$(d) of the system is.

$$\dot{R}(d) = \dot{R}_{jf}(d) + \dot{R}_{jcg}(d) + \dot{R}_{over}(d) \tag{38}$$

and it is expressed as the probability of fatality due to hazards in the system.

In the simulation of the case studied the values of$\dot{R}_{jf}(d)$, $\dot{R}_{jcg}(d)$ and $\dot{R}_{over}(d)$ have been calculated for different values of β_C and T_3 and therefore integrated over the d distance from the source of risk where the effects of accidents reduce to zero, according to literature [4]. The results have been obtained using the values of parameters in Table 7 and supposing the constant presence of individuals in the vicinity (one person per meter) of the system.

Hence $\dot{R}$ is defined as:

$$\dot{R} = \int_{d=0}^{\infty} \dot{R}(d) dd \tag{39}$$

$\dot{R}$ represents the total risk associated to the system, whose utility is finally the electric power produced by the turbine therefore it is

Table 7
Values of parameters.

Description	Symbol	Value
Percentage of release from pipe [ad.]	α_{jf}, α_{jcg}	0,2
Radiation efficiency [ad.]	η_{rad}	0,34
Heat not absorbed by environment [ad.]	τ_A	0,8
Frequency of pipe rupture [year^{-1}]	p_{leak}	1,6* 10^{-5}
Frequency of ignition [year^{-1}]	p_{ign}	0,0339
Frequency of overpressure [year^{-1}]	p_{over}	3* 10^{-6}
Exposure time [s]	$t_{e,jf}$, $t_{e,jcg}$	1
Exposure time [s]	$t_{e,over}$	0.0028
Volume [m^3]	V_c	1

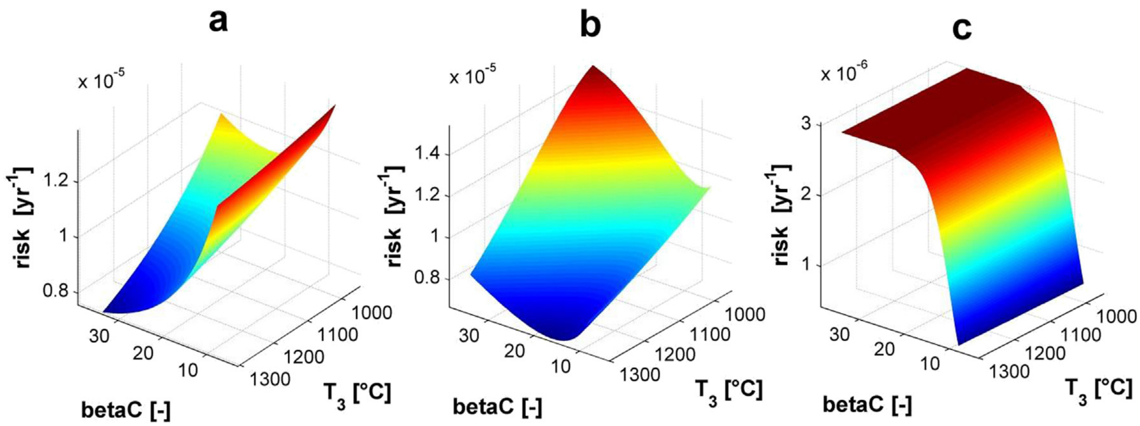

Fig. 5. Risk sources.

possible to estimate the risk of the unit of exergy of the product r_P as in (16).

In Fig. 5 the risks associated to the three separated components are reported. It is possible to notice how the risk associated to jet fire decreases according to T_3 and β_C: the reason is related to the improvement of η_{II} observed with higher T_3 and β_C and the reduction of $\dot{m}_{fuel}$ required, that bring to a smaller $\dot{m}_{\mathrm{fuel,leak}}$. For the same reason the risk associated to the jet of combusted gas decreases with T_3 and increases with β_C, values according to which $\dot{m}_{gc}$ is higher. The risk of overpressure instead is independent from T_3 and increases according only to $\beta_{C.}$ For this type of risk a constant maximum value is reached when the fatality F reaches the value for which the probability of injury for a person located in the vicinity of the system is 100%. The combination of T_3 and β_C values corresponding to the minimum of the surface of Fig. 6 is then the couple that minimizes the total risk of the gas turbine product, and it is the ultimate value to investigate. From the result obtained indeed the total risk appears to be driven more significantly by T_3 rather than β_C.

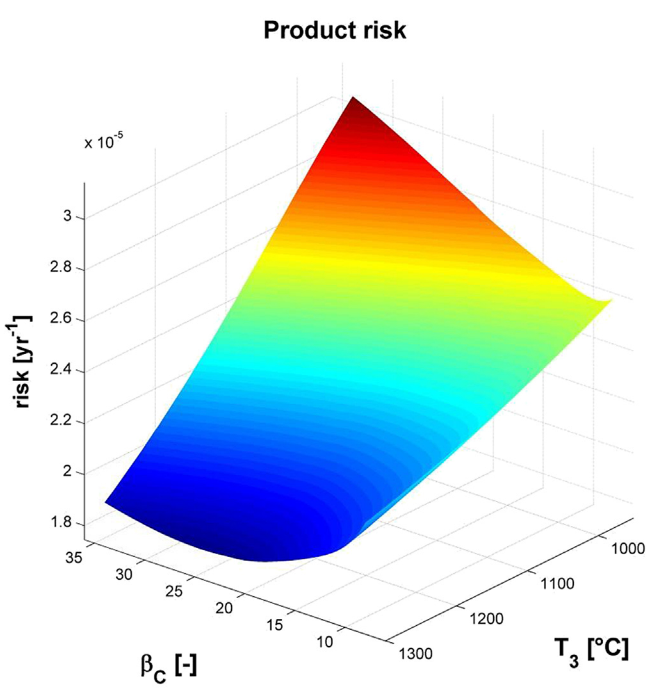

Fig. 6. Risk Of product.

6. Results and discussion

In Table 8 the results of the different analyses are presented. It appears how the three optimal design configurations have been found: thermodynamic, economic and risk optimal designs. The values of T_3 and β_C that minimize each objective function of analysis (exergy efficiency, economic cost and risk of the product) differ from each other.

In Fig. 7 the comparison histogram of the three configurations according to each objective function is reported.

Thermodynamic optimum: as predictable, the configuration of the system that is characterized by the highest efficiency is the configuration where the values of T_3 and β_C are the highest. On the consequence, according to the cost correlations used, also the monetary cost is the highest among the three configurations. The cost of fuel $\dot{C}_{\mathrm{eco},F}$ is the half of the total investment cost rate $\dot{Z}_{\mathrm{inv,tot}}$. This means that the total cost of product $\dot{C}_{\mathrm{eco},P}$ is strongly driven by the former. This relation is well known in industry and it is at the base of Thermoeconomic optimization [10]. From Table 8 it is noticeable how $\dot{R}_{\mathrm{therm,opt}}$ is mainly due to the risk of jet fire and the risk of combusted gas, that together represent almost the totality of the risk associated to this configuration. Since the causative variable of these two risks have demonstrated to be correlated to $\dot{m}_{\mathrm{fuel}}$ and $\dot{m}_{air}$ respectively, we can foresee that the risk can be affected by efficiency.

Economic optimal design: the economic optimal design indeed is characterized by a lower T_3 and very low β_C with respect to the other two optimal configurations: on the consequence, η_{II} is smaller. The total cost is driven by fuel cost rate $\dot{C}_{\mathrm{eco},F}$ and that investment cost rate of components $\dot{Z}_{\mathrm{inv,tot}}$ becomes relevant only for higher temperatures and pressure ratio. The product cost of this configuration is thus almost 50% lower than the other two configurations. According to the previous consideration on the dependence of $\dot{R}_{\mathrm{therm,opt}}$ from $\dot{m}_{\mathrm{fuel}}$ and $\dot{m}_{\mathrm{air}}$, the total risk associated to the economic optimum $\dot{R}_{\mathrm{eco,opt}}$ achieves the highest value since $\dot{m}_{\mathrm{fuel}}$ and $\dot{m}_{\mathrm{air}}$ are the highest.

Minimum risk design: the couple of T_3 and β_C values that define the risk optimal design is much closer to the thermodynamic optimal configuration rather than the economic optimal configuration.

In particular, the optimal risk is obtained at highest T_3 and however high β_C. This enforces the conclusion that the risk is mainly driven by hazards that are further reduced by efficiency increase rather than variation in component design parameters, i.e., the major risks are related to flow rates ($\dot{m}_{\mathrm{fuel}}$ and $\dot{m}_{gc}$) rather than pressure p. In this configuration indeed $\dot{m}_{\mathrm{fuel}}$ and $\dot{m}_{gc}$ present the lowest values among the three optimal design options. Energy and Exergy

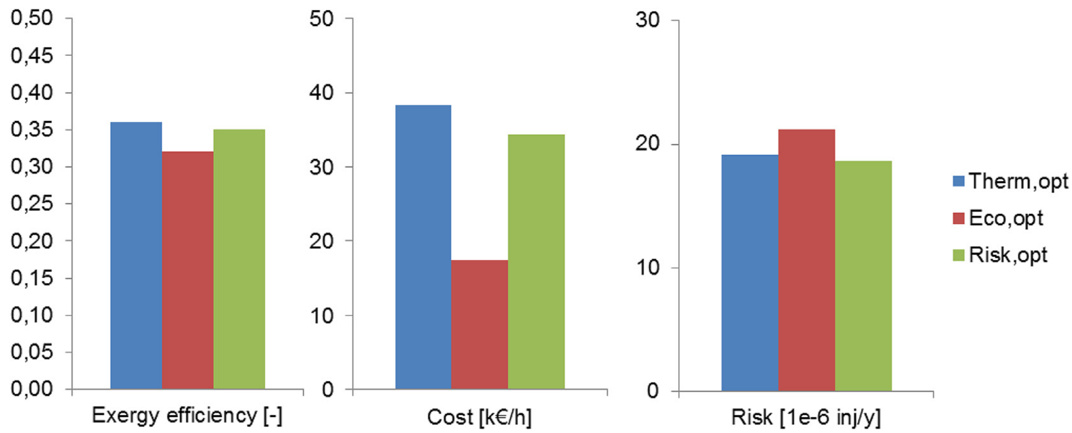

Fig. 7. Comparison histogram of configurations.

efficiencies of this configuration are very close to thermodynamic optimum but they are not optimum because β_C is not maximum. The risks of jet fire and combusted gas are however the lowest.

According to these results, the objective of minimizing the risk seems to be aligned with the objective of improving the efficiency of the system up to the point in which the risk related to overpressure becomes relevant compared to the other two. This is a significant indication for improving the safety of the system. In this configuration indeed, the system is intrinsically safer compared to the economic optimum design. Considering that every system requires additional safety systems to respect the limits of risk imposed by regulations, in the configuration characterized by lower risk the added safety system would be reduced. This could have an important impact on the final economic assessment of the system. However, it must be remarked that the results of risk analysis are strictly related to the considered hazards and set hypotheses. A more accurate risk analysis on a real plant could lead to the identification of other hazards that might change the results achieved in this paper.

On the contrary, the objective of minimizing the monetary cost of the system appears therefore in contrast with the objective of maximizing the efficiency and minimizing the risk of the system. This could represent a critical issue for deciding the final configuration of the system, since the decision maker has obviously both economic and safety constraints to respect.

The selection of the final design that take into account all the three objective functions here considered might be achieved by a multi-objective approach as proposed in Refs. [24], depending on the purpose of the decision makers.

Table 8
Results of the analyses.

Parameters	Units	$R_{therm,opt}$	$R_{eco,opt}$	$R_{risk,opt}$
β_C	–	34	17	25
T_3	°C	1.300	1.175	1.300
m_{air}	kg/s	311	339	293
m_{fuel}	kg/s	6,32	7,12	6,52
m_{gc}	kg/s	317	346	299
η_I	–	0,38	0,33	0,36
η_{ex}	–	0,36	0,32	0,35
$\dot{Z}_{\text{eco,plant}}$	€/h	25.665	3.294	19.297
$\dot{C}_{\text{eco,fuel}}$	€/h	12.596	14.176	12.983
$\dot{C}_{\text{eco,product}}$	€/h	38.261	17.470	34.371
$\dot{R}_{over}$	individuals injured *1e-6/yr	3,00	2,79	3,00
$\dot{R}_{jf+jcgs}$	individuals injured *1e-6/yr	16,18	18,38	15,61
$\dot{R}$	individuals injured *1e-6/yr	19,18	21,17	18,61

7. Conclusion

Objective of the article is to propose exergy based methods for designing different possible configurations to supporting decision making.

These methods are then applied to evaluate the performance of a simplified gas turbine. Thermodynamic, economic and risk optimal configurations are identified according to three different objective functions. For evaluating the thermodynamic optimum design an exergy analysis and optimization is performed, whilst for identifying the economic optimum design a Thermoeconomic analysis is implemented. The hazardous impact of the system is evaluated through a new analysis proposed by the Authors. The analysis uses exergy to allocate and minimize the risk associated to the system, and it is structured as a dual model of Thermoeconomics. Risk analysis is preferred to Life Cycle Assessment as indicator of impact since it focuses on local rather than global impact of the system and includes impact on human health. Such model allows moreover to improve the safety of the system.

In conclusion, according to the hypotheses set in the modeling (par. 2) and in the analyses (par. 3,4 and 5), the study proposes three methodologies able to identifying appropriate design configurations of energy systems. This opens interesting scenarios for the inclusion of externalities (economic, safety) in analysis and optimization of energy systems.

Nomenclature

T	temperature, °C
LHV	lower heating value of fuel, kJ/kg
$\dot{E}x$	exergy, kW
$\dot{W}$	work, kW
h	enthalpy, kJ/kg
s	entropy, kJ/(kg K)
ex	specific exergy, kJ/kg
$\dot{m}$	mass flow rate, kg/s
$\dot{C}$	economic cost per hour, €/h
c	specific economic cost, €/MJ
$\dot{Z}$	investment economic cost per hour, €/h
p	probability of occurrence of the failure, yr^{-1}
P	pressure, bar
F	probability of injury from an hazard, ad.
$\dot{R}$	risk, injured/yr
r	specific risk, injured/(yr*kW)
Y	probit
x	causative variable

$\dot{Q}$	heat, kW
I	radiation, kW/m^2
d	distance, m
v	velocity, m/s
V	volume, m^3
τ_A	fraction of heat not absorbed by atmosphere
t	time, s
u	general thermodynamic parameter of the system
a,b,c	fatality constants
k_1,k_2	Probit constants

Greek symbols

α	(as symbol) flow fraction
β	pressure ratio
η	efficiency

Subscripts

j	j-th stream
k	k-th component
i	i-th hazard
el	electric
air	air
fuel	fuel
gc	combusted gases
env	environment
C	compressor
T	expander
F	fuel
P	product
I	first law
II	second law
tot	total
in	inlet
out	outlet
z	component of the plant
leak	leakage
jf	jet fire
jcg	jet combusted gases
over	overpressure
rad	radiation
e	exposition
0	reference
cum	cumulative
ign	ignition
eco	economic
α	(as subscript) identifier of general thermodynamic property

Abbreviations

TIT	turbine inlet temperature
af	air filter
C	compressor
C,comb	fuel compressor
c	combustor
T	expander
al	alternator
sil	silencer
PEC	purchased equipment cost
TIC	total investment cost
CRF	capital recovery factor

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