An analytical expression for the determination of in situ stress state from borehole data accounting for breakout size

G. Della Vecchia [a,*], A. Pandolfi [a], G. Musso [b], G. Capasso [c]

[a] Politecnico di Milano, Department of Civil and Environmental Engineering, Piazza Leonardo da Vinci 32, 20133 Milano, Italy
[b] Politecnico di Torino, Department of Structural, Geotechnical and Building Engineering, Torino, Italy
[c] ENI E&P, Geomechanics & EOR Technologies Department, Italy

Received 6 November 2012 Received in revised form 29 August 2013
Accepted 30 December 2013

1. Introduction

Knowledge of the in situ stress state in rock and soil deposits is very important in many problems in civil, mining and petroleum engineering and energy development, as well as in geology and geophysics. The prediction of the response of rock masses interacting with underground structures is highly influenced by the stress field. For example, as pointed out in [1], in civil and mining engineering, in situ stresses control the distribution and magnitude of the stresses around underground openings such as tunnels, mines, shaft or caverns. Stress concentrations in the excavation walls may be large enough to overstress the rock, mobilize the strength of the rock mass and induce failure. On the other hand, tensile stresses in excavation walls may open existing fractures or create new ones which could result in block stability problems. An exact prediction of in situ stress acting over a rock mass, together with its spatial variation, is a very complex topic; the current stress state is a mixed consequence of tectonic conditions and of mechanical effects due to local thermo-chemo-hydraulic conditions. Due to the complex nature of rocks and rock masses, the stress field is rarely homogeneous and also its time evolution can be significant within a geological formation.

Stress state is a symmetric second-order tensor and so it is defined by six independent components, e.g. the three principal stresses and the three principal directions. Stresses in rocks cannot be measured directly and can only be inferred by disturbing the rock. Amadei and Stephansonn [2] presented a detailed summary of the available sources of information from which it is possible to obtain the in situ stress state, involving hydraulic methods (i.e. hydraulic and sleeve fracturing), relief methods, jacking methods, strain recovery methods, borehole failure methods as well as fault-slip data analysis and earthquake focal mechanisms.

Zoback [4] proposed an overview of a possible strategy to characterize the stress field: the vertical stress can be determined from the equilibrium in the vertical direction, i.e. by integration of the density logs, while observations of the geometrical arrangement of drilling-induced tensile fractures are an effective way to check whether the vertical stress is a principal stress. The orientation of the other principal stresses can be determined from wellbore observations, recent geologic observations and earthquake focal mechanisms, as well as from stress recovery methods. The magnitude of the minimum principal stress can be estimated from the analysis of hydraulic fracturing and leak-off tests, while the pore pressure can be either measured directly or estimated with some caution from geophysical logs or seismic data.

In this paper the vertical stress is assumed to be a principal stress, so that the remaining two directions are supposed to be horizontal. This assumption is reliable for non-active regions or regions already relaxed from the previous tectonical stress. As pointed out by Bell [3], the free surface of sedimentary basins is generally horizontal, so that the principal stress directions are, to a good approximation, horizontal and vertical. From a practical point of view, if the vertical direction is assumed to be a principal one, it is sufficient to know just a horizontal principal direction, since the remaining one is orthogonal to the plane on which the other two are lying. Throughout the paper, S_v will represent the principal vertical in situ stress, S_H represents the maximum horizontal in situ stress and S_h represents the minimum horizontal in situ stress, whose magnitude can be evaluated through hydraulic fracturing or leak off tests (except for reverse faulting regimes).

Assuming that S_v and S_h are known, the aim of the note is to present a methodology for the definition of some boundaries for

* Corresponding author.
E-mail addresses: gabriele.dellavecchia@polimi.it (G. Della Vecchia), anna.pandolfi@polimi.it (A. Pandolfi), guido.musso@polito.it (G. Musso), gaia.capasso@eni.com (G. Capasso).

the value of S_H, starting from compressive failure data recovered on circular borehole walls as a consequence of excavation and pressurization by means of drilling muds. Such boundaries can be determined from limitations imposed by the shear resistance of the material. In fact, when a well or a borehole is drilled, the stresses that were previously supported by the exhumed material are transferred to the region surrounding the well. The resultant stress concentration, well understood in terms of elastic theory, amplifies the difference between the far-field principal stresses. *Breakout* failures are related to a compressive failure process that occurs when the maximum hoop stress around the hole increases to such an extent that the shear resistance of the rock is exceeded. In the case of vertical wellbore and vertical principal stress, the azimuth of the breakout failures is coincident with the direction of the minimum horizontal principal stress. The possibility of multiple determination of stress in an individual well and the ability to check for regional consistency among numerous wells make breakout data valuable indicators of stress concentrations [2].

Some solutions put forward recently in the technical literature rely on the assumption that at failure the borehole assumes an elliptical shape and that all the failed material is spalled and removed from the borehole (e.g. Aadnoy et al. [5]). Under these assumptions it is then supposed that only the portion of material in correspondence of the major axis of the ellipse can lay on the failure envelope, and the maximum horizontal stress is then estimated on the basis of this assumption. In this note break out failures are considered as statically equivalent to the yielded zones arising on the contour of the circular borehole: in these volumes the material at yielding is assumed to persist. Assuming that the borehole is circular even after the occurrence of failure allows determining the amplitude of the yielded zones in a straightforward manner. The estimation of the maximum horizontal stress is then built upon this information.

Although not covered in this note, drilling-induced tensile fractures, typically associated to drilling in mud overbalance conditions, are another failure mechanism which could give significant information about the entity and the direction of the horizontal maximum principal stress, as shown, e.g. in [7,8].

2. Evaluation of the stress state for elastic-brittle materials

2.1. Borehole-induced perturbation

In the case of isotropic linear elastic behaviour, the perturbation to the stress field induced by a circular hole can be calculated with analytical solutions. In this case, the axi-symmetric problem of a circular hole (having an internal radius a and subjected to a uniform internal pressure p_i) in a linear elastic infinite rock mass is considered. The radial coordinate r, i.e. the distance from borehole center, ranges between a and ∞. The angle θ, positive counterclockwise, is defined as the angle between the radius considered and the direction of the maximum horizontal stress (see Fig. 1). The net pressure p_{net} is defined as the difference between p_i and the pressure of the pore fluid, p_w: $p_{\text{net}} = p_i - p_w$. Under the assumption of plane strain, the solution for the perturbation of the stress field due to the hole has been proposed by Kirsch (see, e.g. [6]), as a function of the maximum and the minimum horizontal far-field stresses, S_H and S_h. In terms of effective radial, hoop and shear stress, the Kirsch solution reads

$$\sigma'_r = \frac{1}{2}(S'_H + S'_h)\left[1 - \left(\frac{a}{r}\right)^2\right] + p_{\text{net}}\left(\frac{a}{r}\right)^2 + \frac{1}{2}(S'_H - S'_h)\left[1 - 4\left(\frac{a}{r}\right)^2 + 3\left(\frac{a}{r}\right)^4\right] \cos 2\theta,$$

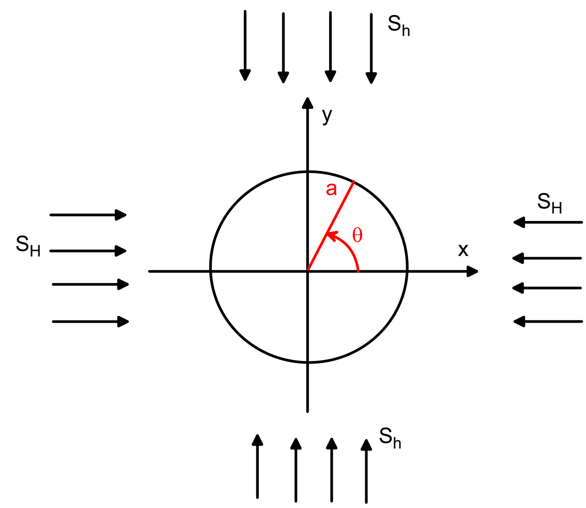

Fig. 1. Radial coordinates for the circular hole.

$$\sigma'_\theta = \frac{1}{2}(S'_H + S'_h)\left[1 + \left(\frac{a}{r}\right)^2\right] - p_{\text{net}}\left(\frac{a}{r}\right)^2 - \frac{1}{2}(S'_H - S'_h)\left[1 + 3\left(\frac{a}{r}\right)^4\right] \cos 2\theta,$$

$$\tau_{r\theta} = -\frac{1}{2}(S'_H - S'_h)\left[1 + 2\left(\frac{a}{r}\right)^2 - 3\left(\frac{a}{r}\right)^4\right] \sin 2\theta, \tag{1}$$

where $\sigma'_{ij} = \sigma_{ij} - p_w\delta_{ij}$ is the Terzaghi effective stress and δ_{ij} is the Kronecker's delta.

According to (1), the perturbations to the in situ stress field due to the presence of the hole vanish proportionally to $(a/r)^2$ and thus they are localized to within a few radii of the hole. For example, for $r = 10a$, the value of σ'_θ is almost 1% of the corresponding value for $r = a$ (at the borehole wall), so that the perturbations induced by the hole can be neglected. Hence the principal effective stresses S'_H and S'_h actually denote the stresses that, in the absence of the hole, would exist in a region around the hole whose extent was about $10a$.

The hoop stress on the borehole wall reads

$$\sigma'_\theta(a, \theta) = (S'_H + S'_h) - p_{\text{net}} - 2(S'_H - S'_h) \cos 2\theta, \tag{2}$$

varying from a minimum value of $3S'_h - S'_H - p_{\text{net}}$, corresponding to $\theta = 0$ or $\theta = \pi$, and a maximum one of $3S'_H - S'_h - p_{\text{net}}$ for $\theta = \pi/2$, or $\theta = 3/2\pi$.

If the values of S_v, S_h, p_{net} and p_w were known a priori from other methods, together with the elastic parameters and the failure properties of the material, certain boundaries for S_H could be obtained basing on the occurrence of compression or tensile failure on the borehole wall.

2.2. Maximum horizontal stress S_H from breakout failure

If a breakout failure occurs, a first broad estimate of a lower boundary for the maximum horizontal stress $S_H^{\min}$ can be obtained assuming that failure just started, involving a single point on the borehole wall rather than a wider volume of rock. Breakout failure will start at $\theta = \pi/2$, where σ'_θ coincides with the local maximum principal stress. Expressing the principal stresses as a function of the far-field stresses, it follows that for $\theta = \pi/2$

$$\sigma'_\theta = 3S'_H - S'_h - p_{\text{net}},$$

$$\sigma'_z = S'_v + \Delta\sigma'_z,$$

$$\sigma'_r = p_{\text{net}}. \tag{3}$$

The increment $\Delta\sigma_z'$ due to borehole excavation has been evaluated assuming null vertical strain ($\Delta\varepsilon_z = 0$) during the drilling and mud pressurization processes, coherently with the elastic solution introduced. The increments of radial and hoop stress in this case are calculated as $\Delta\sigma_r' = p_{\text{net}} - S_h'$ and $\Delta\sigma_\theta' = \sigma_\theta' - S_H' = 2'S_H - \ S_h' - p_{\text{net}}$, respectively, so that $\Delta\sigma_z' = 2\nu(S_H' - S_h')$.

Once expressed the values of the principal effective stresses σ_θ', σ_r' and σ_z' as a function of the only unknown S_H', the problem to be solved reduces to

$$f_C(\sigma_z'(S_H'^{\text{min}}), \sigma_r', \sigma_\theta'(S_H'^{\text{min}})) = f_C(S_H'^{\text{min}}) = 0, \tag{4}$$

where f_C is a suitable failure criterion of the material.

For example, a Mohr–Coulomb failure criterion can be assumed, which in terms of maximum and minimum principal stress (σ_1' and σ_3', respectively) reads

$$\sigma_1' = C + N_\phi \sigma_3' \tag{5}$$

where C is the uniaxial compression strength and $N_\phi = (1 + \sin\phi')/(1 - \sin\phi')$, being ϕ' the internal friction angle. For $\theta = \pi/2$, where breakout failure firstly occurs, the maximum principal stress is σ_θ'. However, the minimum principal stress is not known a priori and two cases should be considered.

If the minimum principal stress was σ_r', a lower boundary of S_H' would be

$$S_H' \geq \tfrac{1}{3}[S_h' + (1 + N_\phi)p_{\text{net}} + C] \quad \text{if } \sigma_3' = \sigma_r', \tag{6}$$

while, if the minimum principal stress was σ_z', it would follow that

$$S_H' \geq \frac{C + N_\phi S_v' + S_h'(1 - 2\nu N_\phi) + p_{\text{net}}}{3 - 2\nu N_\phi} \quad \text{if } \sigma_3' = \sigma_z'. \tag{7}$$

In the case that no breakout failures occur, the values previously calculated for S_H' can be considered as upper boundaries.

3. Extension to elastic-perfectly plastic materials

In order to improve the predictive capabilities of the methodology presented, a procedure to take into account the data about the size of breakout failure is proposed. Dipmeters or borehole televiewers can be used to obtain this information during breakout logging in vertical boreholes. Refined numerical and constitutive approaches have been proposed to simulate shear failure in compression (e.g. [9,10]), but their application is beyond the scope of the note, due to the significant number of parameters which require ad hoc laboratory testing. An approximate analytic approach is proposed here, based on the methodology suggested in [11], and compared with numerical results obtained by finite element analysis. The basic assumption is that the size of the breakout measured in situ at the well scale coincides with the size of the yielding zone that would originate in the same conditions in an elastic perfectly plastic material. Barton et al. [11] have introduced the angle α_b that subtends the breakout zone from the center of the hole and the angle θ_b, positive in counterclockwise direction, between the direction of the maximum horizontal principal stress and the radius passing from the extremity of the breakout zone. In this context, the breakout zone coincides with the zone where positive plastic strains develop. The geometry of the borehole and of the yielded zone is presented in Fig. 2.

The principal stresses on borehole wall for $\theta = \theta_b$ can be expressed as

$$\begin{aligned}
&\sigma_\theta' = S_H' + S_h' - p_{\text{net}} - 2(S_H' - S_h')\cos 2\theta_b,\\
&\sigma_z' = S_v' + \Delta\sigma_z',\\
&\sigma_r' = p_{\text{net}}.
\end{aligned} \tag{8}$$

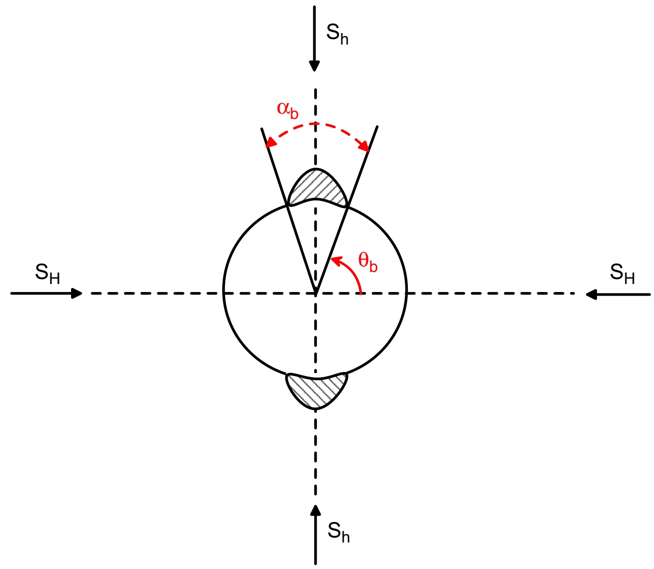

Fig. 2. Definition of α_b and θ_b.

The increment $\Delta\sigma_z'$ due to borehole excavation has been calculated, also in this case, assuming plane strain ($\Delta\varepsilon_z = 0$), so that $\Delta\sigma_z' = -2\nu(S_H' - S_h')\cos 2\theta_b$. It follows that

$$\begin{aligned}
&\sigma_\theta' = S_H' + S_h' - p_{\text{net}} - 2(S_H' - S_h')\cos 2\theta_b,\\
&\sigma_z' = S_v' - 2\nu(S_H' - S_h')\cos 2\theta_b,\\
&\sigma_r' = p_{\text{net}}.
\end{aligned} \tag{9}$$

Assuming that for $\theta = \theta_b$ the material is prone to yield, the vertical stress can be evaluated considering that the elastic solution (1) is still valid and that the stress satisfies also the yielding condition (i.e. Eq. (5) if a Mohr–Coulomb yield surface is assumed). The suitability of the Mohr–Coulomb criterion to predict breakout failure shape has been highlighted by Meier et al. [12]. The solution obtained is a function both of the size of the yield locus and of the particular yield function chosen. As in the elastic case, the maximum compression hoop stress acts for $\theta = \pi/2$. Being σ_θ' the maximum stress, the minimum is not know a priori. If σ_r' is the minimum effective stress, then a lower boundary for the maximum horizontal stress is

$$S_H' = \frac{C - S_h'(1 + 2\cos 2\theta_b) + (1 + N_\phi)p_{\text{net}}}{1 - 2\cos 2\theta_b}, \quad \sigma_3' = \sigma_r'. \tag{10}$$

If the minimum stress is σ_z', then

$$S_H' = \frac{C + N_\phi S_v' + S_h'[-1 - 2\cos 2\theta_b(1 - \nu N_\phi)] + p_{\text{net}}}{1 + 2\cos 2\theta_b(\nu N_\phi - 1)}, \quad \sigma_3' = \sigma_z'. \tag{11}$$

The approach proposed does not take into account explicitly stress redistribution around the well due to plastic strains which occur in the yielded zone. To check the reliability of the solution in linking θ_b to the applied far field stresses, some FEM simulations have been performed.

3.1. Numerical validation

The approximated analytical solution has been compared to finite element numerical simulations to evaluate the relationship between θ_b and S_H'. Several combinations of far-field stresses and net pressures have been taken into account, in order to reproduce all the possibilities in terms of faulting regimes (normal, strike-slip and reverse). The *normal faulting* regime is defined by the inequalities $S_v > S_H > S_h$ that have reproduced considering the case:

$$S_v'/p_{\text{net}} = 4, \quad S_h'/p_{\text{net}} = 2. \tag{12}$$

The *strike-slip faulting* regime is defined by $S_H > S_v > S_h$ and it has been simulated assuming

$$S'_v/p_{\text{net}} = 2, \quad S'_h/p_{\text{net}} = 1.6. \tag{13}$$

The *reverse faulting* regime is defined by $S_H > S_h > S_v$. This situation has been reproduced assuming

$$S'_v/p_{\text{net}} = 1.6, \quad S'_h/p_{\text{net}} = 2.4. \tag{14}$$

In all the three cases, θ_b was assumed to range from $\pi/4$ to $\pi/2$.

In the simulations, some reference values for the material parameters have been adopted: $C=0$, $N_\phi = 4.6$, corresponding to a friction angle of 40°, and $\nu = 0.3$. The analytical method has been used to calculate, through Eqs. (10) and (11), the value of S'_H as a function of θ_b, given the value of the other principal stress S'_h and S'_v and the net pressure p_{net}. The numerical simulation has been performed through a sequence of static analyses which reproduce three subsequent situations: (1) imposition of the initial stress field, due to the vertical, S'_v, and horizontal, $S'_H = S'_h$, far field stresses; (2) modelling of the borehole (neglecting the simulation of the drilling process) through the application of the net pressure p_{net} inside the borehole in plane strain conditions; (3) increment of the maximum horizontal stress S'_H in plane strain conditions, keeping fixed both the minimum stress S'_h and the net pressure p_{net}. During this stage yielding in some regions around the borehole is anticipated.

During the third stage, the size of the yielding zone as a function of S'_H has been evaluated, by identifying the element on the borehole wall which experienced plastic strains. Due to the finite size of the elements, the result of this procedure is mesh dependent.

The reference analysis considered a hole with radius a in a parallelepiped domain: the plane size of the domain was $20a \times 20a$ and the height was $2a$. To evaluate the influence of the mesh, different analysis have been performed with decreasing element size on borehole wall. A refined mesh has been obtained by using elements of size $0.05 \cdot a$ on borehole wall. The results of the FEM analysis with the refined mesh are presented in Figs. 3–5, confirming the reliability of the analytical approach proposed.

The evolution of the predicted maximum horizontal stress (normalized with reference to p_{net}) as a function of the internal friction angle of the rock is presented in Fig. 6, for different amplitudes of the breakouts, with reference to the normal faulting

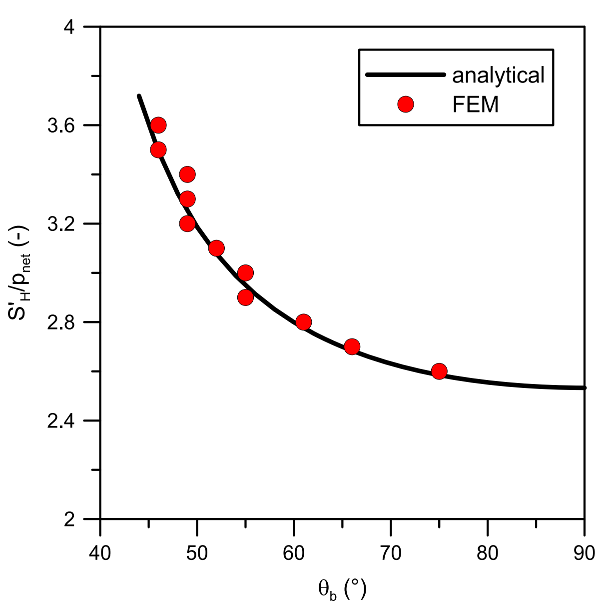

Fig. 3. Analytical estimate of S'_H as a function of θ_b, compared to numerical results: normal faulting.

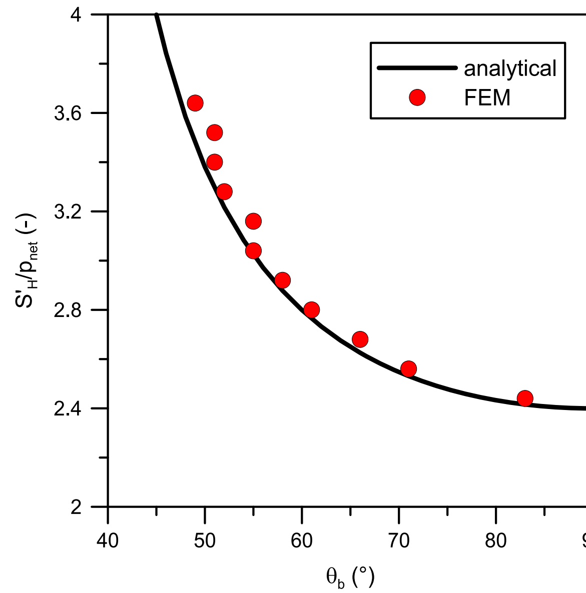

Fig. 4. Analytical estimate of S'_H as a function of θ_b, compared to numerical results: strike-slip regime.

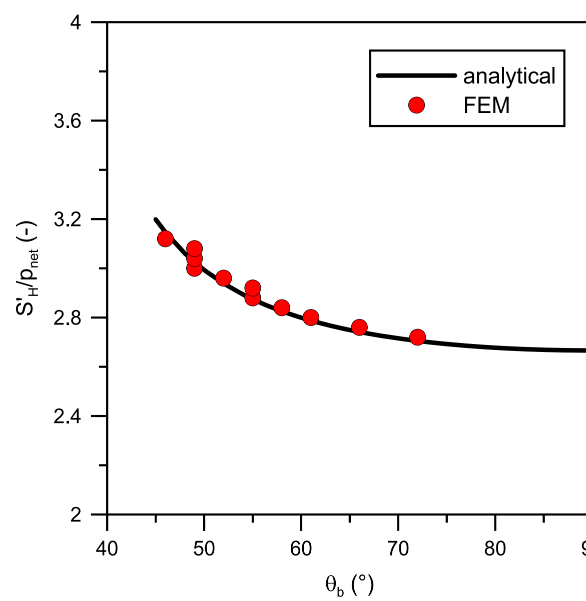

Fig. 5. Analytical estimate of S'_H as a function of θ_b, compared to numerical results: reverse regime.

example described in Eq. (12). Correctly the model predicts that the greater the friction angle, the greater the horizontal stress needed to obtain a given breakout size. The solution for $\theta_b = \pi/2$ coincides with the solution of Eq. (6), which does not account for breakout size.

4. Conclusion

An approximate analytical solution has been proposed to account for the amplitude of breakout failures to estimate the maximum in situ horizontal stress. The solution is based on the assumptions that the borehole is circular and that stress redistribution occurs accordingly to the elastic solution of Kirsch. These hypotheses coincide with those of Zoback et al. [13], although some modifications are introduced, namely, the influence of the pressure of the mud inside the borehole on stress redistribution on the breakout failure has been

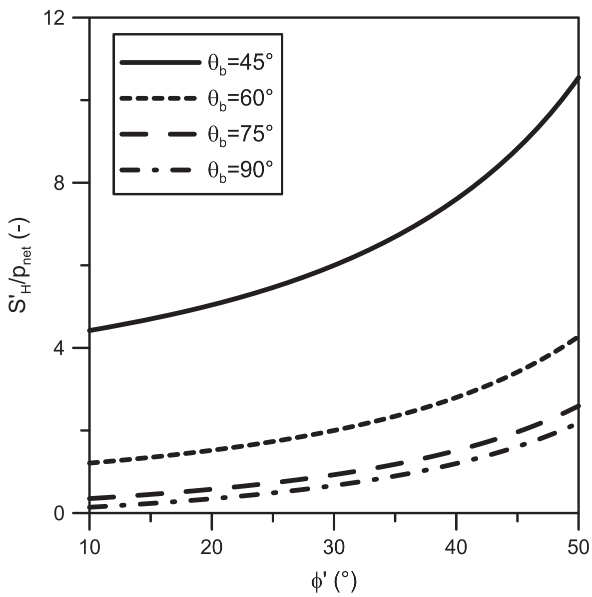

Fig. 6. Prediction of S'_H/p_{net} as a function of ϕ' for different values of θ_b.

taken into account explicitly; the solution is valid for every faulting regime, including the case in which the radial stress is the intermediate principal stress (and so it does not contribute to the shear resistance if a Mohr–Coulomb criterion is assumed); the information about the deepest radius reached by breakout failure is not needed.

The approximate analytical solution does not consider explicitly the stress redistribution around the wellbore due to the inelastic deformation. For this reason some FEM analyses were performed, which validate the approach proposed. The results suggest that the solution proposed leads to a direct estimate of the exact value of S'_H, rather than to a lower boundary. However, this deterministic estimate of the magnitude of the in situ stress by using breakout geometry should be used with caution. As pointed out in [2], breakouts can be enlarged due to various phenomena, like the reduction of rock strength induced by thermal or chemical actions, the weathering of borehole wall, the intensity of drilling and the drilling method itself, so that the actual geometrical and physical conditions could be different from the ones considered by the method proposed. These factors can influence the reliability of the method, which is intended to be used for a fast preliminary characterization of in situ stress based on a limited number of parameters.

References

[1] Aadnoy BS. In situ stress directions from borehole fracture traces. J Pet Sci Eng 1990;4:143–53.

[2] Amadei B, Stephansson O. Rock stress and its measurements. London: Chapman and Hall; 1997.

[3] Bell J. Practical methods for estimating in situ stress for borehole stability applications in sedimentary basins. J Pet Sci Eng 2003;38:111–9.

[4] Zoback MD. Reservoir geomechanics. Cambridge: Cambridge University Press; 2007.

[5] Aadnoy BS, Kaarstad E, de Castro Goncalves CJ. Obtaining both horizontal stresses from wellbore collapse. In: SPE/IADC drilling conference and Exhibition, IADC/SPE 163563; 2013. p. 1–12.

[6] Jaeger J, Cook N, Zimmermann R. Fundamentals of rock mechanics. Oxford: Blackwell Publishing; 2007.

[7] Lucie AM, Zoback MD, Heesakkers V, Reches Z, Murphy SK. Constraining the far-field in situ stress near a deep South African gold mine. Int J Rock Mech Min Sci 2009;46:555–67.

[8] Sanaee R, Shadizadeh SR, Riahi MA. Determination of the stress profile in a deep borehole in a naturally fractured reservoir. Int J Rock Mech Min Sci 2010;47:599–605.

[9] Zheng Z, Kemeny J, Cook NGW. Analysis of borehole breakouts. J Geophys Res 1989;94:7171–82.

[10] Li G, Mizuta Y, Ishida T, Li H, Nakama S, Sato T. Stress field determination from local stress measurements by numerical modelling. Int J Rock Mech Min Sci 2009;46:138–47.

[11] Barton CA, Castillo DA, Moos D, Peska P, Zoback M. Characterising the full stress tensor based on observations of drilling-induced wellbore failures in vertical and inclined boreholes leading to improved wellbore stability and permeability prediction. APPEA J 1998;38:466–87.

[12] Meier T, Rybacki E, Reinicke A, Dresen G. Influence of borehole diameter on the formation of borehole breakouts in black shale. Int J Rock Mech Min Sci 2013;62:74–85.

[13] Zoback MD, Moos D, Mastin L, Anderson RN. Well-bore breakouts and in situ stress. J Geophys Res 1985;90:5523–30.