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A Singular Adaptive Attitude Control with active disturbance rejection

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Abstract

This paper develops a quaternion attitude tracking control with an adaptive gains parameter that can be tuned to compensate for disturbances with known bound. The adaptive gain is described by a simple, but singular, differential equation and the corresponding adaptive control is shown to asymptotically track a reference attitude. However, this control requires the bound on the disturbance torque to be known in order to appropriately tune the controller to compensate for it. Using a linear state observer to estimate the disturbance torque and compensating for the disturbance at each sampling period the adaptive control can achieve asymptotic tracking in the presence of an unknown disturbance torque. In this case the error in the estimation, rather than the entire disturbance, is compensated for by the adaptive gain at each sampling period. Simulations demonstrate that an improved tracking performance can be achieved when compared to standard quaternion tracking controls.

Keywords: Nano-spacecraft, Inverse Control, Quaternion Feedback Control, Linear Extended State Observer, Singular Quaternion Feedback Control

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1. Introduction

Attitude stabilization and tracking in the presence of disturbance torques is an active area of research due to the increased pointing and tracking requirements of many mission concepts. In the particular case of nano-spacecraft there is also a need to increase accuracy to enable missions that are currently only achievable with much larger spacecraft. There has been a range of different techniques to address the problem of attitude control in the presence of disturbance torques with a known bound [1, 2, 3]. These particular controls are useful when the order of magnitude of the environmental disturbances are well known such as with spacecraft in LEO or when the uncertainty in the spacecrafts moments of inertia can be computed experimentally within a known error tolerance. However, future missions to more complex environments, wear and tear in hardware (particularly evident in CubeSats) require robust controls in the presence of unknown disturbance torques. For example, future missions to asteroids or moons with uncertain gravitational fields could lead to uncertain gravity gradient torques or wear and tear of reaction wheels could lead to uncertain frictional torques. This paper addresses the problem of attitude tracking for spacecraft in the presence of such uncertain disturbance torques.

A number of adaptive control approaches have been developed to tackle the problem of unknown disturbance torques and modelling parameters, for example, using robust output regulation theory to develop a dynamic compensator for uncertain parameters in [4], adaptive sliding mode control[7] with an extended state observer [5], a non-regressor-based adaptive control in [6], an inverse optimality approach without the need to solve the associated Hamilton-Jacobi-Isaacs partial differential equation directly in [8] and adaptive backstepping in [9]. In this paper we use an adaptive gain parameter whose evolution can be described by a simple differential equation which effectively compensates for a known disturbance. Adaptive gains have been used effectively for small-time maneuvers, for example in [10], whereby simple adaptive laws adjust when a fault is detected. However, the differential equations describing the gain are unbounded and after a significant time will become excessively large. The adaptive parameters of this paper are bounded and can be designed specifically to counter a known disturbance. However, if the disturbance is unknown then the adaptive parameter has to be tuned based on a best guess which could consequently lead to poor tracking. However, by combining the adaptive control with an active disturbance rejection mechanism the disturbances can be estimated and compensated for in the controller. In this case the adaptive parameter does not need to compensate for a known disturbance but for the error in estimation whose order of magnitude is shown to be small. Therefore, combining an active disturbance rejection control (ADRC) with the singular adaptive control can asymptotically track a reference attitude in the presence of an unknown disturbance even with estimation errors.

ADRC is an effective control scheme that has been demonstrated both theoretically and in many applications [11, 12, 13, 14, 15]. The extended state observer (ESO), a key component of ADRC, estimates the uncertainty and external disturbance of the nonlinear system. It has previously been applied to spacecraft attitude control where a nonlinear ESO method combined with a sliding-mode controller [5] was used to develop an attitude control method that is robust to parametric uncertainties and disturbances. It has also been combined with a simple quaternion PID control that uses fuzzy logic to choose the gains to stabilize a spacecraft in the presence of uncertainty [16]. Nonlinear ESO uses a nonlinear function to achieve a high estimation performance but requires extensive tuning. In this paper we use a linear ESO which has the benefits of being simple to implement and tune and is computationally light while providing good estimations of the disturbances. In this paper we combine a singular adaptive control with a linear extended state observer (LESO) to develop an analytically verifiable and robust tracking control. The differential equation describing the adaptive parameter can be tuned to compensate for the estimation error as the order of magnitude of the estimation is small and bounded. The closed-loop system is proved to be asymptotically stable in the presence of the estimation error. In the papers [5**?**] the stability proofs of ADRC rely on the assumption that the ESO provides an exact estimation of the disturbance. The adaptive tracking control presented in this paper compensates for this error and therefore does not rely on such an assumption. In addition, the presented control does not suffer from the problem of chattering found in other robust controllers such as sliding mode control.

The attitude kinematics and dynamics of the spacecraft in the presence of disturbances are formulated in an appropriate form for the application of LESO in Section 2. Section 3 then addresses the problem of developing the singular adaptive control. It is shown that if the bound on the disturbance is known that a singular adaptive control can be designed to guarantee asymptotic stability. In Section 4, we propose an adaptive control based LESO to track a desired quaternion in the presence of external disturbance. For the linear ESO, we present an estimation model and prove its convergence and estimation error. It is shown that the adaptive control extended with LESO can asymptotically track a prescribed trajectory in the presence of an unknown disturbance torque even with an estimation error. In Section 5, simulations are undertaken using the model of a spacecraft which illustrates the robustness of the proposed control method to tracking desired quaternion and angular velocity with unknown disturbances.

2. Attitude dynamics and kinematics of a rigid-spacecraft

In this section the attitude dynamics and kinematics in the presence of unknown disturbance torques are formulated in the appropriate form for the application of the LESO method. The spacecraft is assumed to be a rigid body described by the dynamics and kinematics [17]:

$$J\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}^{\times} J\boldsymbol{\omega} + \boldsymbol{u} + \boldsymbol{d} \tag{1}$$

$$\begin{aligned} \dot{\boldsymbol{q}} &= \tfrac{1}{2}\left(q_4\boldsymbol{\omega} - \boldsymbol{\omega}^{\times}\boldsymbol{q}\right) \\ \dot{q}_4 &= -\tfrac{1}{2}\boldsymbol{\omega}^T\boldsymbol{q} \end{aligned} \tag{2}$$

where, $\boldsymbol{\omega}=[\omega_1, \omega_2, \omega_3]^T$ indicates the angular velocity vector of nano-spacecraft with respect to the inertial frame and expressed in the body coordinates, $\boldsymbol{u} = [u_1, u_2, u_3]^T$ the control torque and $\boldsymbol{d}$ is the unknown external disturbance, $J \in \mathbb{R}^{3\times3}$ is the positive definite and symmetric inertia tensor. The unit quaternion is $\bar{\boldsymbol{q}} = [q_1, q_2, q_3, q_4]^T$, which can be expressed equivalently as $\bar{\boldsymbol{q}} = \left[\boldsymbol{q}^T, q_4\right]^T$ with $\boldsymbol{q} = [q_1, q_2, q_3]^T$ and such that $\boldsymbol{q}^T\boldsymbol{q} + q_4^2 = 1$, the $^{\times}$ denotes an operator, such that

$$\boldsymbol{\omega}^{\times} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}.$$

The unit quaternion $\bar{\boldsymbol{q}}$ and angular velocity $\boldsymbol{\omega}$ of spacecraft in Eq.(1) and Eq.(2) can be measured and available in control law design.

The attitude tracking control problem is to track a desired quaternion $\bar{\boldsymbol{q}}_d$, where $\bar{\boldsymbol{q}}_d = \left[\boldsymbol{q}_d^T, q_{d4}\right]^T$ with $\boldsymbol{q}_d = [q_{d1}, q_{d2}, q_{d3}]^T$ satisfying $\boldsymbol{q}_d^T\boldsymbol{q}_d + q_{d4}^2 = 1$ denotes the desired attitude quaternion and $\boldsymbol{\omega}_d$ is the target angular velocity.

Considering the error quaternion $\bar{\boldsymbol{q}}_e = \left[\boldsymbol{q}_e^T, q_{e4}\right]^T$ where $\boldsymbol{q}_e = [q_{e1}, q_{e2}, q_{e3}]^T$ and the error angular velocity $\boldsymbol{\omega}_e$ are defined as

$$\begin{array}{l} \boldsymbol{q}_e = q_{d4}\boldsymbol{q} - \boldsymbol{q}_d^\times \boldsymbol{q} - q_4 \boldsymbol{q}_d \\ q_{e4} = \boldsymbol{q}_d^T \boldsymbol{q} + q_4 q_{d4} \\ \boldsymbol{\omega}_e = \boldsymbol{\omega} - \boldsymbol{C}\boldsymbol{\omega}_d \end{array} \tag{3}$$

where $\boldsymbol{C} = \left(q_{e4}^2 - \boldsymbol{q}_e^T \boldsymbol{q}_e\right) I_3 + 2\boldsymbol{q}_e \boldsymbol{q}_e^T - 2q_{e4}\boldsymbol{q}_e^\times$ is the direction cosine matrix from inertial frame to body coordinate, where I_3 an identity matrix.

The kinematics can be expressed as error quaternion form [17]

$$\begin{array}{l} \dot{\boldsymbol{q}}_e = \frac{1}{2}\left(q_{e4}\boldsymbol{\omega}_e - \boldsymbol{\omega}_e^\times \boldsymbol{q}_e\right) \\ \dot{q}_{e4} = -\frac{1}{2}\boldsymbol{\omega}_e^T \boldsymbol{q}_e \end{array} \tag{4}$$

where $\bar{\boldsymbol{q}}_e$ satisfies $\boldsymbol{q}_e^T \boldsymbol{q}_e + q_{e4}^2 = 1$.

Substitution the $\boldsymbol{\omega}_e = \boldsymbol{\omega} - \boldsymbol{C}\boldsymbol{\omega}_d$ into the Eq.(1), considered $\dot{\boldsymbol{C}} = -\boldsymbol{\omega}_e^\times \boldsymbol{C}$ [18], the error dynamic equations is

$$\dot{\boldsymbol{\omega}}_e = -J^{-1}(\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d)^\times J\left(\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d\right) + \boldsymbol{\omega}_e^\times \boldsymbol{C}\boldsymbol{\omega}_d - \boldsymbol{C}\dot{\boldsymbol{\omega}}_d + J^{-1}\boldsymbol{u} + \boldsymbol{f} \tag{5}$$

where $\boldsymbol{f} = J^{-1}\boldsymbol{d}$ is an unknown disturbance will be estimated by LESO and the control inputs designed to guarantee asymptotic stability of the closed-loop system.

3. A Singular Adaptive Attitude Control Law

In this section, we propose an adaptive control law that can provide asymptotic tracking of a reference attitude in the presence of disturbances. In particular a singular differential equation defines the adaptive parameter (so called because the differential equation describing the adaptive parameter has a singularity $\sigma = 0$) and provides a simple means to compensate for disturbance torques. Adaptive parameters described by simple differential equations were used in [10] to compensate for disturbances. However, these adaptive parameters were unbounded i.e. they do not converge to a steady state. This causes problems when tracking over large time periods as the gain and corresponding required attitude torque input become too large. The adaptive control law is defined and proved to track a feasible attitude motion with asymptotic convergence.As the desired state of the closed-loop system $\boldsymbol{\omega}_{ed} = [0, 0, 0]^T$ and $\bar{\boldsymbol{q}}_{ed} = [\boldsymbol{q}_{ed}, q_{ed4}]^T = [0, 0, 0, \pm 1]^T$ provide a non-unique description of the desired tracking error, we define the desired error

state as any element of the set $E = \{(\boldsymbol{\omega}_e, q_{ed4}) \in D : \boldsymbol{\omega}_e = \mathbf{0}, q_{ed4} = \pm 1\}$. Then defining the sets $A_1 = \{(\boldsymbol{\omega}_e, q_{e4}) \in D : \boldsymbol{\omega}_e \in \mathbb{R}^3, q_{e4} \in [0,1]\}$ and $A_2 = \{(\boldsymbol{\omega}_e, q_{e4}) \in D :$
$\boldsymbol{\omega}_e \in \mathbb{R}^3, q_{e4} \in [-1,0)\}$ such that $A_1 \cap A_2 = \emptyset$ with the entire error domain $D = A_1 \cup A_2$ we state the following theorem:

Theorem 1. *For an initial tracking error $(\boldsymbol{\omega}_e(0), q_{e4}(0)) \in D$ of equations (5) for $\boldsymbol{d} = 0$, the adaptive feedback control law:*

$$\boldsymbol{u} = (\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d)^{\times} J (\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d) - J\boldsymbol{\omega}_e^{\times}\boldsymbol{C}\boldsymbol{\omega}_d + J\boldsymbol{C}\dot{\boldsymbol{\omega}}_d + \frac{1}{4}\sigma^2 J \frac{\partial H_i(q_{e4})}{\partial q_{e4}}\boldsymbol{q}_e - J\sigma\boldsymbol{\omega}_e \tag{6}$$

with adaptive parameter $\sigma > 0$ satisfying the singular differential equation:

$$\begin{cases} \dot{\sigma} = \frac{s_1 \boldsymbol{\omega}_e^T \boldsymbol{\omega}_e}{H_i(q_{e4})} - \frac{L\boldsymbol{\omega}_e^T \mathrm{sgn}(\boldsymbol{\omega}_e)}{H_i(q_{e4})\sigma} & if \quad H_i(q_{e4}) > 0 \\ \dot{\sigma} = 0 & if \quad H_i(q_{e4}) = 0 \end{cases} \tag{7}$$

where $\mathrm{sgn}(\boldsymbol{\omega}_\mathrm{e}) = [\mathrm{sgn}(\omega_{\mathrm{e}1}), \mathrm{sgn}(\omega_{\mathrm{e}2}), \mathrm{sgn}(\omega_{\mathrm{e}3})]^\mathrm{T}$, *$s_1 \leq 1$ is a scalar constant, L is a positive scalar and $i = 1$ if $q_{e4}(0) \in [0,1]$, $i = 2$ if $q_{e4}(0) \in [-1,0)$, then an element of the equilibrium set E is asymptotically stable on a subset $S \subseteq D$ if the following conditions hold: (a) $H_1(1) = 0$ and $H_1(q_{e4}) > 0$ for $q_{e4} \neq 1$ on A_1 and $H_2(-1) = 0$ and $H_2(q_{e4}) > 0$ for $q_{e4} \neq -1$ on A_2, (b) $H_1(q_{e4})$ is a C^1 function with respect to q_{e4} on A_1 and $H_2(q_{e4})$ is a C^1 function with respect to q_{e4} on A_2. (c) the only solution to $\frac{\partial H_i(q_{e4})}{\partial q_{e4}} = 0$ on S is either $q_{e4} = 1$ or $q_{e4} = -1$.*

Proof: Defining the Lyapunov function V_1 on A_1:

$$V_1 = \frac{1}{2}\boldsymbol{\omega}_e{}^T\boldsymbol{\omega}_e + \frac{\sigma^2}{2}H_1(q_{e4}) \tag{8}$$

we have that, at $\boldsymbol{\omega}_{ed} = [0,0,0]^T$ and $\bar{\boldsymbol{q}}_{ed} = [0,0,0,1]^T$, $V_1 = 0$ and $V_1 > 0$ in all other cases. Given condition (b) the time derivative of the Lyapunov function can be computed as

$$\dot{V}_1 = \boldsymbol{\omega}_e{}^T\dot{\boldsymbol{\omega}}_e + \frac{\sigma^2}{2}\frac{\partial H_1(q_{e4})}{\partial q_{e4}}\dot{q}_{e4} + \dot{\sigma}\sigma H_1(q_{e4}) \tag{9}$$

then initially assuming $H_i(q_{e4}) > 0$ and substituting Eq.(7) and Eq.(5) into

(9) gives:

$$\dot{V}_1 = \boldsymbol{\omega}_e{}^T \left[-J^{-1}(\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d)^\times J\left(\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d\right) + \boldsymbol{\omega}_e^\times \boldsymbol{C}\boldsymbol{\omega}_d - \boldsymbol{C}\dot{\boldsymbol{\omega}}_d + J^{-1}\boldsymbol{u} \right.$$
$$\left. -\frac{\sigma^2}{4}\frac{\partial H_1\left(q_{e4}\right)}{\partial q_{e4}}\boldsymbol{q}_e \right] + s_1\sigma\boldsymbol{\omega}_e{}^T\boldsymbol{\omega}_e - L\boldsymbol{\omega}_e^T \mathrm{sgn}\left(\boldsymbol{\omega}_e\right) \tag{10}$$

Substituting the control law Eq.(6) into Eq.(10) gives

$$\dot{V}_1 = \left(s_1 - 1\right)\sigma\boldsymbol{\omega}_e{}^T\boldsymbol{\omega}_e - L\boldsymbol{\omega}_e^T \mathrm{sgn}\left(\boldsymbol{\omega}_e\right) \tag{11}$$

then from condition (a), the function $H_1\left(q_{e4}\right) \geq 0$, $s_1 \leq 1$, therefore $\dot{V}_1 \leq 0$. In addition noting that the closed-loop dynamics when $\boldsymbol{\omega}_e = 0$ and ($\dot{V}_1 = 0$) is

$$\dot{\boldsymbol{\omega}}_{\boldsymbol{e}} = \frac{1}{4}\sigma^2\frac{\partial H_i(q_{e4})}{\partial q_{e4}}\boldsymbol{q}_e \tag{12}$$

then following from the Barbashin-Krasovskii theorem [25], if the only equilibria of the closed-loop system Eq.(12) is $\boldsymbol{q}_e = 0$ then the desired state is asymptotically stable. This is equivalent to the condition that $\frac{\partial H_i(q_{e4})}{\partial q_{e4}} = 0$ if and only if $q_{e4} = 1$ and that $\sigma > 0$. Defining an analogous Lyapunov function V_2 on A_2 similar computations show that $\boldsymbol{\omega}_{ed} = [0,0,0]^T$ and $\bar{\boldsymbol{q}}_{ed} = [0,0,0,-1]^T$ is asymptotically stable on A_2. □

Remark 1. *If we consider a sliding surface $\boldsymbol{S} = \boldsymbol{\omega}_e + \kappa\boldsymbol{q}_e$ where $\kappa \geq 0$ is a scalar then Eq.(5) can be expressed as $J\dot{\boldsymbol{S}} = -(\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d)^\times J\left(\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d\right) + J\boldsymbol{\omega}_e^\times\boldsymbol{C}\boldsymbol{\omega}_d - J\boldsymbol{C}\dot{\boldsymbol{\omega}}_d + \boldsymbol{u} + \boldsymbol{d} + \frac{1}{2}J\kappa(q_{e4}\boldsymbol{\omega}_e - \boldsymbol{\omega}_e^\times\boldsymbol{q}_e)$. To guarantee that the sliding surface $\boldsymbol{S}$ converges to a zero vector, a control law of the form $\boldsymbol{u}_1 = -\boldsymbol{G} - \sigma\boldsymbol{S} - \gamma\mathrm{sgn}(\boldsymbol{S})$ (where σ is a constant) with $\boldsymbol{G} = \frac{1}{2}J\kappa(q_{e4}\boldsymbol{\omega}_e - \boldsymbol{\omega}_e^\times\boldsymbol{q}_e) - (\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d)^\times J\left(\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d\right) + J\boldsymbol{\omega}_e^\times\boldsymbol{C}\boldsymbol{\omega}_d - J\boldsymbol{C}\dot{\boldsymbol{\omega}}_d$, $\sigma \geq 0$ is a scalar, $\gamma \geq \rho_d$ is a gain, $\|\boldsymbol{d}\| \leq \rho_d$ and ρ_d is a constant can be used. To show stability of the closed loop system a Lyapunov function can be defined such that $V = \frac{1}{2}\boldsymbol{S}^T J\boldsymbol{S}$, with the time derivative of the Lyapunov function equal to $\dot{V} \leq -\sigma\boldsymbol{S}^T\boldsymbol{S} - \sum_{i=1}^{3}(\gamma - \rho_d)|S_i| \leq 0$. Therefore, if ρ_d is known, γ can be selected such that the system is stable in the presence of disturbances. The motivation for designing the adaptive parameter σ (Eq.(7)) is that the derivative of the Lyapunov function in the presence of disturbances using the adaptive control* *would be:*

$$\begin{aligned}\dot{V}_1 &= (s_1 - 1)\,\sigma \boldsymbol{\omega}_e{}^T \boldsymbol{\omega}_e + \boldsymbol{\omega}_e{}^T \boldsymbol{f} - L\boldsymbol{\omega}_e^T \mathrm{sgn}\,(\boldsymbol{\omega}_e) \\ &\le (s_1 - 1)\,\sigma \boldsymbol{\omega}_e{}^T \boldsymbol{\omega}_e + \sum_{i=1}^{3} (|\boldsymbol{f}_i| - L) \sum_{i=1}^{3} |\boldsymbol{\omega}_{ei}| \le 0\end{aligned} \tag{13}$$

which has a similar negative definite term to that found with the sliding mode controller, but where the control (6) does not contain the switching term $-\gamma \mathrm{sgn}(\cdot)$*. This implies that the adaptive control will have robustness to disturbances but without chattering. It can be seen that the scalar* L *can be chosen to be greater than* $|\boldsymbol{f}_i|$ *to yield a semi-negative definite derivative in the presence of disturbances. However, for appropriate tuning of* L *the bound of the disturbance* $|\boldsymbol{f}_i|$ *is required a priori.*

Remark 2. *There are a number of* $H_i(q_{4e})$ *functions which can be used:* $H(q_{e4}) = 1 - q_{e4}$ *or* $H(q_{e4}) = 1 - q_{e4}^2$ *in [20, 21, 22] and* $H(q_{e4}) = \ln(q_{e4})$ *in [23]. In addition the term* $H_i(q_{e4}) = \arccos^2[\mathrm{sgn}(q_{e4})q_{e4}]$ *is considered in [24] which under ideal conditions can provide faster convergence of the error to zero. In the case of a control designed such that* $H_i(q_{e4}) = 1 - \mathrm{sgn}(q_{e4})q_{e4}$ *this can lead to chattering around* $q_{e4} = 0$ *in the presence of noise and a hybrid control must be designed [26]. In this paper we consider the function* $H_i(q_{e4}) = 1 - \mathrm{sgn}[q_{e4}(0)]q_{e4}$*. Note that the choice of* $H_i(q_{e4})$ *is only dependent on the initial value* $q_{e4}(0)$ *and therefore will avoid any problems with switching in the presence of noise.*

In the following sections the adaptive controller is combined with an extended state observer to develop a control that tracks a reference trajectory in the presence of unknown disturbances such that the closed-loop system is stable. More, precisely if the estimator perfectly measures the disturbance then the desired state in asymptotically stable. In this case the control is adapted such that the tuning of the parameter L does not require knowledge of the bound of the disturbance but knowledge of the magnitude of the estimation error which is known to be small. Note that there is a singularity at $\sigma = 0$ which is theoretically possible. Moreover, the adaptive parameter when $H_i\,(q_{4e}) \neq 0$ can be expressed in the form:

$$\dot{\sigma} = A(t) - \frac{B(t)}{\sigma} \tag{14}$$

where $\sigma(0) > 0, A(t) \geq 0$, and $B(t) \geq 0$. Therefore, if $A(t) > \frac{B(t)}{\sigma}$ then σ is always increasing and thus positive definite. In the case that $\frac{B(t)}{\sigma} > A(t) \geq 0$ then σ will decrease and converge to a constant which is lower bounded by 0. However, it is possible to tune the system to avoid convergence to zero which can be demonstrated numerically in simulation. In addition, for practical implementation, a saturation limit can be placed on σ to avoid the singularity such that:

$$\text{sat}\left[\sigma\right] = \begin{cases} \varepsilon, & \sigma \leq \varepsilon \\ \sigma \;, & \sigma > \varepsilon \end{cases} \tag{15}$$

where ε is positive small constant.

However, if the adaptive parameter does saturate such that $\sigma = \varepsilon$ is constant, then during saturation the time derivative of the Lyapunov function Eq.(13) will be $\dot{V}_1 \leq -\varepsilon \boldsymbol{\omega}_e{}^T \boldsymbol{\omega}_e + \sum_{i=1}^{3} |\boldsymbol{f}_i||\boldsymbol{\omega}_{ei}|$. In this case, the stability proof depends on the choice of ε and the bound of the total disturbances $|\boldsymbol{f}_i|$. In the next section, we will utilize the ESO technique to estimate the uncertain total disturbance and guarantee the estimation error is bounded and that it converges to a small neighborhood even in the presence of the saturation.

4. Adaptive Attitude Controller with LESO

In this section, an attitude tracking control is designed which compensates for the estimation error at each sampling period via the adaptive controller. The external disturbances in the attitude dynamics of the spacecraft are estimated by LESO and compensated for in the control.

4.1. LESO Design for External Disturbance Estimation

The external disturbance term $\boldsymbol{f}$ of spacecraft error dynamics shown in Eq.(5) will be estimated as an extended state by the LESO. Let $\boldsymbol{x}_2$ be an extended state of the disturbance term $\boldsymbol{f}$. Note that although we focus on disturbance torques the problem can be reformulated to consider uncertainties in the inertia matrix. Noting that $\boldsymbol{f}$ is smooth and bounded then $\boldsymbol{\chi} = \dot{\boldsymbol{f}}$ is continuous and bounded then we can write Eq.(5) as

$$\begin{aligned} &\dot{\boldsymbol{x}}_1 = \boldsymbol{x}_2 - J^{-1}(\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d)^{\times} J\left(\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d\right) + \boldsymbol{\omega}_e^{\times}\boldsymbol{C}\boldsymbol{\omega}_d - \boldsymbol{C}\dot{\boldsymbol{\omega}}_d + J_0^{-1}\boldsymbol{u} \\ &\dot{\boldsymbol{x}}_2 = \boldsymbol{\chi} \end{aligned} \tag{16}$$

where $\boldsymbol{x} = [\boldsymbol{x}_1, \boldsymbol{x}_2]^T$, $\boldsymbol{x}_1 = \boldsymbol{\omega}_e = [\omega_{e1}, \omega_{e2}, \omega_{e3}]^T$ and $\boldsymbol{x}_2 = \boldsymbol{f} = [f_1, f_2, f_3]^T$.

To estimate the total disturbance $\boldsymbol{f}$ under the assumption that it is s-mooth and bounded we design a linear second-order extended state observer[19] and specialise it to the system (5) such that

$$\begin{aligned}\dot{\hat{\boldsymbol{x}}}_1 &= \hat{\boldsymbol{x}}_2 + \beta_1 I_3 (\boldsymbol{\omega}_e - \hat{\boldsymbol{x}}_1) - J^{-1}(\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d)^{\times} J (\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d) + \boldsymbol{\omega}_e^{\times}\boldsymbol{C}\boldsymbol{\omega}_d \\ &\quad - \boldsymbol{C}\dot{\boldsymbol{\omega}}_d + J_0^{-1}\boldsymbol{u} \\ \dot{\hat{\boldsymbol{x}}}_2 &= \beta_2 I_3 (\boldsymbol{\omega}_e - \hat{\boldsymbol{x}}_1)\end{aligned} \tag{17}$$

where $\hat{\boldsymbol{x}} = [\hat{\boldsymbol{x}}_1, \hat{\boldsymbol{x}}_2]^T$, $\hat{\boldsymbol{x}}_1 = [\hat{x}_{11}, \hat{x}_{12}, \hat{x}_{13}]^T$ is the estimation value of the $\boldsymbol{\omega}_e$ and $\hat{\boldsymbol{x}}_2 = [\hat{x}_{21}, \hat{x}_{22}, \hat{x}_{23}]^T$ is the estimation value of the unknown term $\boldsymbol{f}$. The I_3 denotes the identity matrix. β_1 and β_2 are the LESO gain parameters to be chosen. In particular, let us choose a special case of the LESO gains[19] as $\beta_1 = 2w_c$ and $\beta_2 = w_c^2$ where $w_c > 0$ is the bandwidth of LESO. Then defining the estimated error as $\tilde{\boldsymbol{x}} = [\tilde{\boldsymbol{x}}_1, \tilde{\boldsymbol{x}}_2]^T = \boldsymbol{x} - \hat{\boldsymbol{x}}$, the estimation error dynamics are

$$\begin{aligned}\dot{\tilde{\boldsymbol{x}}}_1 &= -2w_c I_3 \tilde{\boldsymbol{x}}_1 + \tilde{\boldsymbol{x}}_2 \\ \dot{\tilde{\boldsymbol{x}}}_2 &= -w_c^2 I_3 \tilde{\boldsymbol{x}}_1 + \boldsymbol{\chi}\end{aligned} \tag{18}$$

Lemma 1. *Assuming $|\boldsymbol{\chi}| \le M$, there exists some constant scalars $\delta_i > 0$ and a finite $T_1 > 0$ such that the estimation error is bounded $\|\tilde{x}_i\| \le \delta_i, i = 1, 2,$ where $\delta_i = O\left(\frac{1}{w_c^n}\right)$, n is positive integer, $\forall t > T_1 > 0$ and $w_c > 0$, where w_c is the band-width of Eq. (17).*

Proof. Let $\boldsymbol{\varepsilon}_i = \frac{\tilde{\boldsymbol{x}}_i}{w_c^{i-1}}, i = 1, 2$, then the Eq.(18) becomes

$$\begin{cases}\dot{\boldsymbol{\varepsilon}}_1 = w_c I_3 \boldsymbol{\varepsilon}_2 - 2w_c I_3 \boldsymbol{\varepsilon}_1 \\ \dot{\boldsymbol{\varepsilon}}_2 = -w_c I_3 \boldsymbol{\varepsilon}_1 + \boldsymbol{\chi}/w_c\end{cases} \Rightarrow \dot{\boldsymbol{\varepsilon}} = w_c A_0 \boldsymbol{\varepsilon} + B\frac{\boldsymbol{\chi}}{w_c} \tag{19}$$

where $A = \begin{bmatrix} -2I_3 & I_3 \\ -I_3 & \mathbf{0}\end{bmatrix}$ and $B = \begin{bmatrix}\mathbf{0} \\ I_3\end{bmatrix}$.

Solving the Eq.(19), the result is

$$\boldsymbol{\varepsilon}(t) = \boldsymbol{\varepsilon}(0)\exp(w_c A t) + \int_0^t \exp[w_c A (t-\tau)]\frac{B\boldsymbol{\chi}}{w_c}d\tau \tag{20}$$

For equations of the form Eq.(20) there exists a constant $\delta_i = O\left(\frac{1}{w_c^n}\right)$, n is positive integer, such that the estimation error $\|\tilde{x}_i\| \le \delta_i, i = 1, 2$ [19] . □

Defining the set $B(\delta_2, \hat{\boldsymbol{x}}_2) = \{\hat{\boldsymbol{x}}_2 : \|\hat{\boldsymbol{x}}_2 - \boldsymbol{f}\| \le \delta_2\}$ $\forall\delta_2 > 0$, it follows that, as $t \to \infty$, $\hat{\boldsymbol{x}}_2 \in B$.

4.2. Adaptive Feedback control law with LESO

In Subsection 4.1, it has been proved the unknown term $\boldsymbol{f}$ can be estimated by LESO. However, the estimation error bounded by δ_2 can cause inefficiencies in the control where at each sampling time it will be correcting also for the error. In this subsection, the design of an adaptive feedback controller with LESO is shown that can compensate for the estimation error rather than the entire disturbance and can track a prescribed reference trajectory such that the closed-loop system is stable. Let $\boldsymbol{E}_\delta = \boldsymbol{f} - \hat{\boldsymbol{x}}_2$ denote the error estimation of LESO. The $\boldsymbol{E}_\delta$ is bounded and $\|\boldsymbol{E}_\delta\| \leq \delta_2$. Recalling here the error dynamics described by Eq.(5) with desired equilibrium $\boldsymbol{\omega}_{ed} = 0$ and $\bar{\boldsymbol{q}}_{ed} = [0,0,0,1]^T$ with $\boldsymbol{q}_{ed} = [0,0,0]^T$ and $q_{e4} = 1$ then we state the following theorem.

Theorem 2. *The adaptive feedback control law Eq.(6) can be modified such that:*

$$\boldsymbol{u} = \boldsymbol{u}_p - J\hat{\boldsymbol{x}}_2 \tag{21}$$

with

$$\boldsymbol{u}_p = (\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d)^\times J(\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d) - J\boldsymbol{\omega}_e^\times \boldsymbol{C}\boldsymbol{\omega}_d + J\boldsymbol{C}\dot{\boldsymbol{\omega}}_d + \frac{1}{4}\sigma^2 J\frac{\partial H_i(q_{e4})}{\partial q_{e4}}\boldsymbol{q}_e - J\sigma\boldsymbol{\omega}_e \tag{22}$$

where σ is the adaptive parameter defined by Eq.(7), with $L \geq \delta_2$ and where $\hat{\boldsymbol{x}}_2$ is the estimated value of $\boldsymbol{f}$, then the adaptive feedback controller (21), under the conditions (a),(b) and (c) of Theorem 1, asymptotically stabilizes the desired closed-loop equilibrium $\boldsymbol{\omega}_{ed}$ and $\boldsymbol{q}_{ed}$ such that $\lim_{t\to\infty}\boldsymbol{q}_e = \boldsymbol{q}_{ed}$, $\lim_{t\to\infty}\boldsymbol{\omega}_e = \boldsymbol{\omega}_{ed}$ as the gain of the estimator $w_c \to +\infty$.

Proof: Using the same Lyapunov function (8) and differentiating with respect to time and substituting in the control law Eq.(21) gives

$$\begin{aligned}\dot{V}_1 &= (s_1 - 1)\sigma\boldsymbol{\omega}_e^T\boldsymbol{\omega}_e + \boldsymbol{\omega}_e^T(\boldsymbol{f} - \hat{\boldsymbol{x}}_2) - L\boldsymbol{\omega}_e^T\operatorname{sgn}(\boldsymbol{\omega}_e)\\ &\leq (s_1 - 1)\sigma\boldsymbol{\omega}_e^T\boldsymbol{\omega}_e + \sum_{i=1}^{3}(|\boldsymbol{E}_{\delta i}| - L)\sum_{i=1}^{3}|\boldsymbol{\omega}_{ei}|\end{aligned} \tag{23}$$

Since $L \geq \delta_2$ and $\|\boldsymbol{E}_\delta\| \leq \delta_2$, $|\boldsymbol{E}_{\delta i}| - L \leq 0$. Then from condition (a), the function $H_i(q_{e4}) \geq 0$, $s_1 \leq 1$ and $\sigma > 0$, therefore $\dot{V} \leq 0$. It follows that the closed-loop dynamics when $\boldsymbol{\omega}_e = 0$ and $\dot{V}_1 = 0$ are

$$\dot{\boldsymbol{\omega}}_{\boldsymbol{e}} = \frac{1}{4}\sigma^2\frac{\partial H_i(q_{e4})}{\partial q_{e4}}\boldsymbol{q}_e + \boldsymbol{E}_\delta \tag{24}$$

then as $w_c \to +\infty$ we have $\boldsymbol{E}_\delta \to 0$ and following the Barbashin-Krasovskii theorem and an analogous argument to Theorem 1 then the state $(\boldsymbol{\omega}_{ed}, \boldsymbol{q}_{ed})$ is asymptotically stable on D. □

In practise w_c is finite and constrained by the saturation of the actuators, so the best that can be achieved in practise is asymptotic stability of a small bounded region around the desired state which is demonstrated in the following simulation example.

5. Numerical Example

In this section, the proposed singular adaptive control with active disturbance rejection is demonstrated in simulation and compared to a standard quaternion tracking controller described by

$$\boldsymbol{u}_{qfc} = (\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d)^{\times} J (\boldsymbol{\omega}_e + \boldsymbol{C}\boldsymbol{\omega}_d) - J\boldsymbol{\omega}_e^{\times}\boldsymbol{C}\boldsymbol{\omega}_d + J\boldsymbol{C}\dot{\boldsymbol{\omega}}_d - k_{q_e} J\boldsymbol{q}_e - k_{\omega_e} J\boldsymbol{\omega}_e \tag{25}$$

we implement the control law (21) in simulation on a representative rigid spacecraft which is with the inertial matrix as

$$J = \begin{bmatrix} 20 & 1.2 & 0.9 \\ 1.2 & 17 & 1.4 \\ 0.9 & 1.4 & 15 \end{bmatrix} \quad kg \cdot m^2 \tag{26}$$

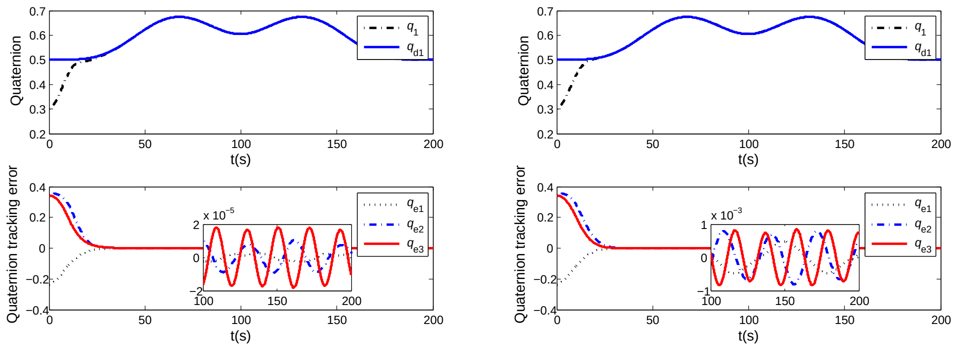
The external disturbance torque $\boldsymbol{d}$ is assumed:

$$\boldsymbol{d} = \begin{bmatrix} 1 \sin 0.1t \\ 2 \sin 0.2t \\ 3 \sin 0.3t \end{bmatrix} \times 10^{-3} N \cdot m \tag{27}$$

In order to test the effectiveness of the proposed control law (21), the desired angular velocity $\boldsymbol{\omega}_d$ will be chosen as

$$\boldsymbol{\omega}_d = [\sin(\pi t/100); \sin(2\pi t/100); \sin(3\pi t/100)] \times 10^{-2} rad/s \tag{28}$$

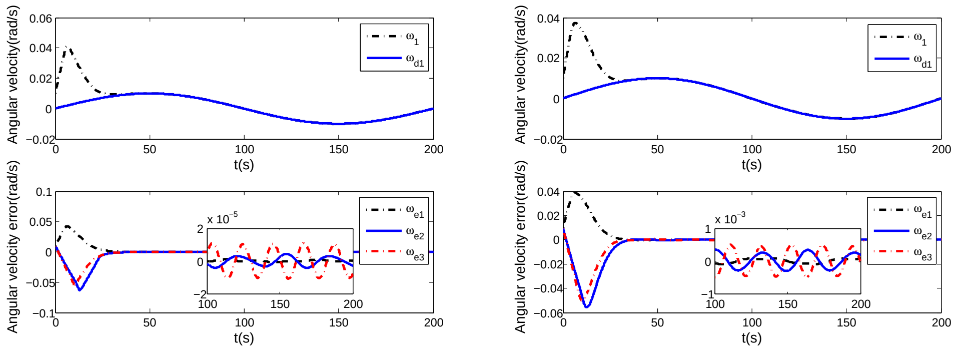
and let the desired quaternion is computed by integrating the kinematic equation for $\bar{\boldsymbol{q}}_d(0) = [0.5, -0.5, -0.5, 0.5]^T$, the initial angular velocity $\boldsymbol{\omega}(0) = [0.01, 0.01, 0.01]^T \, rad/s$, initial quaternion $\bar{\boldsymbol{q}}(0) = [0.3, -0.3, -0.2, 0.8832]^T$. The comparison control gains are $k_{q_e} = 0.1$ and $k_{\omega_e} = 0.4$. The gain parameters of adaptive control are $s_1 = 1$, $L = 0.02$, $\varepsilon = 0.1$, $\sigma(0) = 1$, the



(a) Quaternion tracking and error of proposed control

(b) Quaternion tracking and error of compared control

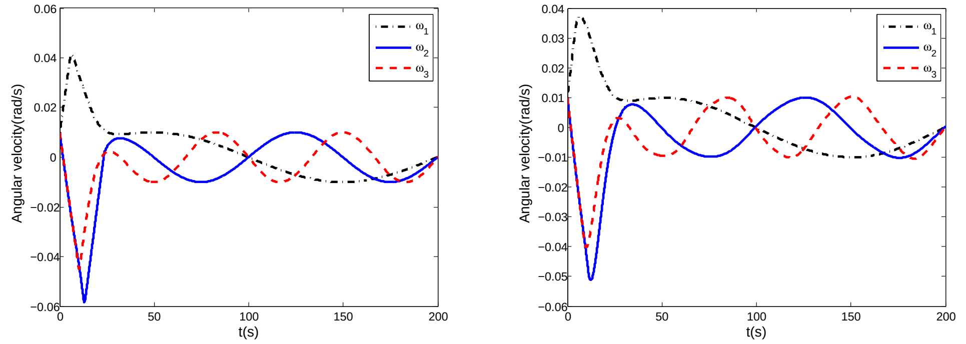
Figure 1: Quaternion tracking comparison



(a) Angular velocity tracking and error of proposed control

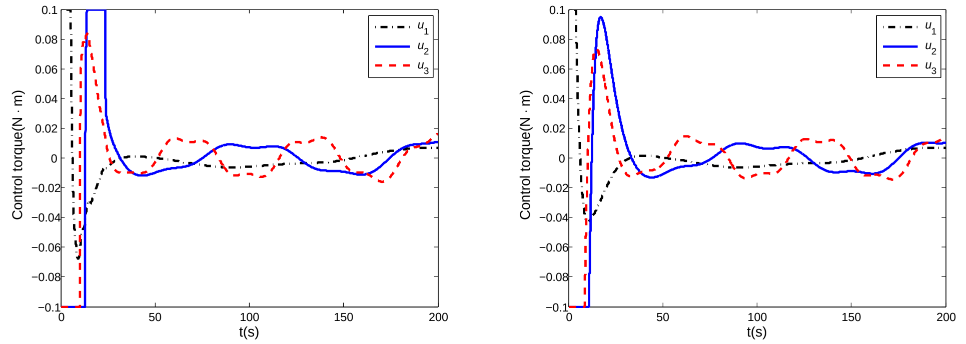
(b) Angular velocity tracking and error of compared control

Figure 2: Angular velocity tracking comparison



(a) Angular velocity of proposed control (b) Angular velocity of compared control

Figure 3: Angular velocity comparison



(a) Control torque of proposed control (b) Control torque of compared control

Figure 4: Control torque comparison

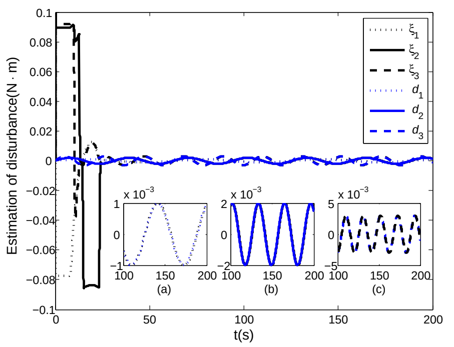


Figure 5: Estimation of disturbance via linear ESO

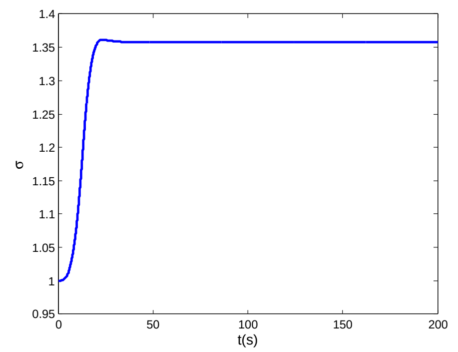


Figure 6: Adaptive gain σ of proposed control

linear ESO parameter $\beta_1 = 10, \beta_2 = 40$, $\hat{\boldsymbol{x}}_1 = [0,0,0]^T$, $\hat{\boldsymbol{x}}_2 = [0,0,0]^T$ and maximum of control torque $u_{max} \leq 0.1N \cdot m$.

The tracking of the quaternion component q_1 and quaternion error $\boldsymbol{q}_e$ are shown in Fig.1. It can be seen that the quaternion tracking error of the proposed control is significantly less than 2×10^{-5} in magnitude in contrast to 1×10^{-3} for a standard quaternion controller. Fig.2 illustrates the angular velocity tracking and error of the proposed control. The angular velocity of spacecraft can great track the desired angular velocity with extremely small error. The angular velocity of the spacecraft is presented in Fig. 3 and the corresponding control torque in Fig.4. Fig.5 indicates the estimation performance of LESO where $[\xi_1, \xi_2, \xi_3]^T = J\hat{\boldsymbol{x}}_2$ and external disturbance $\boldsymbol{d} = [d_1, d_2, d_3]^T$. Fig.5(a) is the estimation value ξ_1 of d_1, Fig.5(b) denotes the estimation value ξ_2 of d_2 and Fig.5(c) indicates the estimation value ξ_3 of d_3. In Fig.5, the estimate of the disturbance has a significant error during the first two oscillations, but converges quickly after the second oscillation. The cause of this large oscillation is that the estimator has large initial errors and that the control torque of the actuator saturates at the beginning of the motion (maximum torque is $0.1N \cdot m$). Fig.6 illustrates the adaptive gain σ which is shown to converge to a steady state. In this example, it can be seen from the figures that the proposed controller is effective at dealing with unknown disturbances and shows a significant improvement in tracking capability in the presence of uncertainty when compared to a standard quaternion tracking controller.

6. Conclusion

In this paper a singular adaptive control has been presented which can be tuned to track a reference attitude in the presence of disturbance torques with a known bound such that the closed loop system is stable. By combining the control with an active disturbance rejection controller the disturbances can be estimated at each sampling period and compensated for in the controller. In this case the adaptive parameter only needs to compensate for the much smaller estimation error rather than the entire disturbance torque. Therefore, the new control can provide robust tracking in the presence of unknown disturbances. The proof is demonstrated by using a Lyapunov function and does not need to make the assumption that the disturbance estimator has no error. The control is simple to implement in that it extends standard quaternion tracking control to include a simple linear estimator and a singular adaptive gain parameter described by a 1-D differential equation. The only complication is that it is possible for the adaptive parameter to converge to zero where the stability proof breaks down and the singularity can cause numerical issues. Although careful tuning can avoid this situation, adding a simple lower bound saturation on the parameter avoids this problem. Simulations demonstrate that the control is robust to unknown disturbances and provides more accurate tracking when compared to a conventional quaternion feedback tracking controller.

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