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# Measuring reliability under epistemic uncertainty: Review on non-probabilistic reliability metrics



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# KEYWORDS

Belief reliability; Epistemic uncertainty; Evidence theory; Interval analysis; Possibility theory; Probability box; Reliability metrics; Uncertainty theory **Abstract** In this paper, a systematic review of non-probabilistic reliability metrics is conducted to assist the selection of appropriate reliability metrics to model the influence of epistemic uncertainty. Five frequently used non-probabilistic reliability metrics are critically reviewed, i.e., evidence-theory-based reliability metrics, interval-analysis-based reliability metrics, fuzzy-interval-analysis-based reliability metrics (posbist reliability) and uncertainty-theory-based reliability metrics (belief reliability). It is pointed out that a qualified reliability metric that is able to consider the effect of epistemic uncertainty needs to (1) compensate the conservatism in the estimations of the component-level reliability metrics caused by epistemic uncertainty, and (2) satisfy the duality axiom, otherwise it might lead to paradoxical and confusing results in engineering applications. The five commonly used non-probabilistic reliability metrics are compared in terms of these two properties, and the comparison can serve as a basis for the selection of the appropriate reliability metrics.

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# 1. Introduction

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Reliability refers to the capacity of a component or a system to perform its required functions under stated operating conditions for a specified period of time.<sup>1</sup> Reliability engineering has nowadays become an independent engineering discipline, which measures the reliability by quantitative metrics and controls it via reliability-related engineering activities implemented in the product lifecycle, i.e., failure mode, effect and criticality analysis (FMECA),<sup>2</sup> fault tree analysis (FTA),<sup>3</sup> environmental stress screening (ESS),<sup>4</sup> reliability growth testing (RGT),<sup>5</sup> etc. Among all the reliability-related engineering activities, measuring reliability is a fundamental one.<sup>6</sup> Measuring reliability

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refers to quantifying the reliability of a component or system by quantitative metrics. A key problem in measuring reliability is how to deal with the uncertainty affecting the product's reliability. Broadly speaking, uncertainty can be categorized as aleatory uncertainty which refers to the uncertainty inherent in the physical behavior of the system,<sup>7,8</sup> and epistemic uncertainty which refers to the uncertainty that is caused by incomplete knowledge.<sup>7,9</sup>

In the early years of reliability engineering, reliability has been measured by probability-based metrics, e.g., in terms of the probability that the component or system does not fail (referred to as probabilistic reliability in this paper<sup>10</sup>), and estimated by statistical methods based on failure data (e.g., see Ref.<sup>11</sup>). However, in engineering practice, the available failure data, if there are any, are often far from sufficient for accurate statistical estimates.<sup>12</sup> Also, the statistical methods do not explicitly model the actual process that leads to the failure. Rather, the failure process is regarded as a black box and assumed to be uncertain, which is described indirectly based on the observed distribution of the time-to-failure (TTF). From the perspective of uncertainties, the statistical methods do not separate the root causes of failures and uncertainties and therefore, they do not distinguish between aleatory and epistemic uncertainties.

As technology evolves, modern products often have high reliability, making it even harder to collect enough failure data, which severely challenges the use of statistical methods.<sup>13</sup> At the same time, as the knowledge of the failure mechanisms accumulates, deterministic models are available to describe the failure process based on the physical knowledge of the failure mechanisms (referred to as physics-of-failure (PoF) models<sup>14</sup>). An alternative method to estimate the probabilistic reliability is, then, developed based on the PoF models. In this paper, these methods are referred to as the model-based methods. Unlike statistical methods, model-based methods treat the actual failure process as a white box: the TTFs are predicted by deterministic PoF models, while the uncertainty affecting the TTF is assumed to be caused by random variations in the model parameters (aleatory uncertainty). The probabilistic reliability is, then, estimated by propagating aleatory uncertainties through the model analytically or numerically, e.g., by Monte Carlo simulation.<sup>15,16</sup> Compared to statistical methods, model-based methods explicitly describe the actual failure process (by the deterministic PoF models) and separate the root cause of failures (assumed to be deterministic) and the aleatory uncertainty (the random variation of model parameters). The separation of deterministic root causes and aleatory uncertainty allows the designer to implement parametric design for reliability, e.g., the reliability-based design optimization (RBDO),<sup>17,18</sup> tolerance optimization,<sup>19,20</sup> etc., which marks significant advancement in reliability engineering.

From the perspective of uncertainties, only aleatory uncertainty is considered in the model-based methods. In practice, however, the trustfulness of the predicted reliability is severely influenced by epistemic uncertainty. As in today's highly competitive markets, it is more and more frequent to use the model-based method to measure reliability, due to the severe shortage on failure data. To better quantify the reliability with the model-based methods, the effect of epistemic uncertainty should also be considered. Epistemic uncertainty relates to the completeness and accuracy of the knowledge: if the failure process is poorly understood, there will be large epistemic uncertainty.<sup>21–23</sup> For instance, the deterministic PoF model might not be able to perfectly describe the failure process, e.g., due to incomplete understanding of the failure causes and mechanisms.<sup>21,24</sup> Besides, the precise values of the model parameters might not be accurately estimated due to lack of data in the actual operational and environmental conditions. Both of these two factors introduce epistemic uncertainty into the reliability estimation: the more severe the effect of these factors is, the less trustful the predicted reliability is.

In literature, there are various approaches to measure reliability under epistemic uncertainty, e.g., probability theory (subjective interpretation<sup>25,26</sup>), evidence theory,<sup>27</sup> interval analysis,<sup>28,29</sup> fuzzy interval analysis,<sup>30</sup> possibility theory,<sup>31,32</sup> uncertainty theory,<sup>33</sup> etc. In this paper, a critical review on these reliability metrics is conducted to assist the selection of appropriate metrics. Some researchers and practitioners use probability theory to describe epistemic uncertainty, taking a Bayesian interpretation of probability.<sup>25,26</sup> In recent years, problems in dealing with epistemic uncertainty by probabilistic methods have been pointed out.<sup>34,35</sup> Non-probabilistic metrics have, then, been proposed to model epistemic uncertainty. In this paper, we discuss these non-probabilistic reliability metrics.

More specifically, five reliability metrics are discussed in this paper, i.e., evidence-theory-based reliability metrics, interval-analysis-based reliability metrics, fuzzy-interval-analysis-based reliability metrics, possibility-theory-based reliability metrics (posbist reliability) and uncertainty-theory-based reliability metrics (belief reliability). They are classified, based on the mathematical essence of the metrics, as probability-interval-based and monotone-measure-based reliability metrics. The former refers to an interval that contains all the possible reliability metrics that are defined based on a monotone measure (or fuzzy measure<sup>36</sup>). A further classification is given in Fig. 1. The probability-interval-based and monotone-measure-based reliability interval-based reliability metrics are reviewed in Sections 2 and 3, respectively.

# 2. Probability-interval-based reliability metrics

Probability-interval-based reliability metrics (PIB metrics) describe the effect of epistemic uncertainty by an interval of values of failure probabilities/reliabilities. The width of the interval represents the extent of epistemic uncertainty: wide intervals represent large epistemic uncertainty. When there is no effect of epistemic uncertainty, the probability interval becomes a single distribution function of the TTFs. We consider three of the most popular non-probabilistic methods for epistemic uncertainty representation, i.e., evidence theory, interval analysis (probability box) and fuzzy interval analysis. We review each of these three methods separately in the remaining of this section.

#### 2.1. Evidence-theory-based methods

Evidence theory, also known as Dempster–Shafer theory or as the theory of belief functions, was established by Shafer<sup>37</sup> for representing and reasoning with uncertain, imprecise and incomplete information.<sup>38</sup> It is a generalization of the Bayesian theory of subjective probability in the sense that it does not require probabilities for each event of interest, but bases the



Fig. 1 Classification of existing non-probabilistic reliability metrics.

belief in the truth of an event on the probabilities of other propositions or events related to it.<sup>37</sup> Evidence theory provides an alternative to the traditional manner in which probability theory is used to represent uncertainty by means of the specification of two degrees of likelihood, belief and plausibility, for each event under consideration. The belief value of an event measures the degree of belief that the event will occur and the plausibility value measures the extent to which evidence does not support the negation of the event. Evidence theory is applied to describing uncertainty when the application of probability theory cannot be supported, e.g., when few samples of data are available to estimate the probability accurately.<sup>37</sup>

To obtain the evidence-theory-based reliability metrics, the first step is to define the frame of discernment:

$$\boldsymbol{\Theta} = \{\theta_1, \theta_2, ..., \theta_m\} \tag{1}$$

where the set  $\Theta$  includes all the possible and mutually exclusive elementary propositions or hypotheses with respect to the uncertain events. Let  $A_i$  ( $i = 1, 2, ..., 2^m$ ) denote the subsets of  $\Theta$ . All the subsets (also called focal sets) compose the power set of  $\Theta$ , which is denoted by  $2^{\Theta}$ . Next, basic probability assignment (BPA) is assigned to each focal set to represent our belief in the event associated to it. BPA is essentially a mapping function  $m: 2^{\Theta} \to [0, 1]$ , which satisfies

(1) 
$$m(\emptyset) = 0$$
  
(2)  $\sum_{A_i \subseteq \Theta} m(A_i) = 1$ 

In practice, the values of the BPAs are assigned by experts to represent the effect of epistemic uncertainty. Focal sets and their associated BPAs comprise the evidence, based on which the belief and plausibility of an event *B* can be calculated:

$$\begin{cases} \operatorname{Bel}(B) = \sum_{A_i \subseteq B} m(A_i) \\ \operatorname{Pl}(B) = \sum_{A_i \cap B \neq \emptyset} m(A_i) \end{cases}$$
(2)

where  $A_i$  denote the focal sets and  $m(A_i)$  is its BPA.

The belief in event B is quantified as the sum of the masses assigned to all sets enclosed by it; hence, it can be interpreted as a lower bound representing the amount of belief that supports the event. The plausibility of event B is, instead, the sum of the BPAs assigned to all sets whose intersection with event *B* is not empty; hence, it is an upper bound on the probability that the event occurs.<sup>39</sup> Thus,

$$\operatorname{Bel}(B) \leqslant P(B) \leqslant \operatorname{Pl}(B) \tag{3}$$

When the event B is the failure of a component or system, Eq. (3) leads to an interval that contains all possible failure probabilities/reliabilities, representing the effect of epistemic uncertainty on the reliability estimation: the larger the width of the interval, the greater the epistemic uncertainty is, and thus, the less we can trust the estimated reliability.

Rakowsky reviewed some early applications of evidencetheory-based reliability metrics constructed based on failure modes and effects analysis (FMEA), event tree analysis (ETA) and FTA.<sup>40</sup> Mourelatos and Zhou used evidence theory to construct failure probability intervals and applied them in engineering design optimization.<sup>41–43</sup> In reliability-based optimization (RBO), based on the interval of failure probability, Alyanak et al. developed a new method for projecting gradients in RBO when available data are not enough.<sup>44</sup> Yao et al. developed a sequential optimization and mixed uncertainty analysis method for RBO, where evidence theory is used to describe epistemic uncertainty.<sup>45</sup> Similar to Bayesian network, the evidential network was developed to construct the failure probability intervals.<sup>46</sup> Yang et al. applied the evidential network to FTA and calculated the failure probability intervals.<sup>47</sup> Bae et al. constructed failure probability intervals in large-scale structures based on evidence theory by identifying the failure region and expressing it as a function of the focus sets.<sup>27,48</sup> Considering the large computing cost, Bae et al. introduced an approximation method to calculate the failure probability intervals under the framework of evidence theory.<sup>49</sup> Jiang et al. developed an efficient evaluation method for structure reliability with epistemic uncertainty using evidence theory, which reduced the computation cost compared with traditional methods.<sup>50</sup> To solve the problem of constructing failure intervals with dependent parameters, Jiang et al. developed a multidimensional evidence-theory model, where the dependency is addressed by an ellipsoidal model.<sup>51</sup> Baraldi et al. studied the situation in which a number of experts provided different information about the imprecise parameters, and belief and plausibility functions are used to develop upper and lower bounds of cumulative probability functions.<sup>52,53</sup> Lo et al. assessed seismic probabilistic risk of nuclear power plants and built associated failure probability intervals based on evidence theory.<sup>54</sup> Khalaj et al. applied evidence theory to riskbased reliability analysis.<sup>55</sup> Yao et al. studied the uncertainty quantification in multidisciplinary optimization and developed a new method to calculate the failure probability intervals based on optimization in the framework of evidence theory.<sup>56,57</sup>

#### 2.2. Interval-analysis-based methods

Another way to construct the interval of failure probabilities is to use interval analysis (or probability boxes). Given a model y = f(x), interval analysis assumes that the input variable x is subjected to epistemic uncertainty and is described by an interval (or convex sets if the input variables are multidimensional) comprised of an lower bound  $x_{\rm L}$  and an upper bound  $x_{\rm U}$ , so that  $x_{\rm L} \leq x \leq x_{\rm U}$ . Then, interval mathematics or numerical optimization methods are used to derive the upper and lower bounds of the output variable v.<sup>58</sup> When interval analysis is applied to probabilistic models, upper and lower bounds of the probability of interests can be calculated, which form a probability "box" (p-box) that contains all possible values of that probability. Since reliability is calculated by a probabilistic model, the p-box becomes a natural tool to describe the epistemic uncertainty influencing the calculated reliability.

Ferson et al. are among the first ones who apply the p-box to describing and propagating epistemic uncertainty in a reliability model, deriving intervals that contain all possible values of failure probabilities.<sup>59,60</sup> Karanki et al. applied p-box to evaluate the probability of system failure under the influence of epistemic uncertainty.<sup>61</sup> Using a similar method to describe epistemic uncertainty, Zhang et al. developed interval Monte Carlo simulation methods,<sup>62</sup> interval importance sampling methods<sup>63</sup> and quasi-Monte Carlo methods<sup>64</sup> to calculate the interval of failure probabilities when the structures are implicitly modeled based on a finite element model. Beer et al. developed a calculation method for failure probability intervals, which is specially designed for small sample size and is based on quasi-Monte Carlo simulations.<sup>65,66</sup> Xiao et al. put forward a saddle-point-based approximation method to enhance the computational efficiency in calculating the interval of structural failure probability.<sup>67</sup> Qiu et al. developed methods to construct the interval of failure probabilities with small sample size, using numerical optimization methods.<sup>68–70</sup> Crespo et al. applied p-box to the analysis of polynomial systems subject to parameter uncertainties.<sup>7</sup>

#### 2.3. Fuzzy-interval-analysis-based method

Fuzzy-interval-analysis-based method allows the consideration of both aleatory and epistemic uncertainty simultaneously.<sup>34</sup> The method can be regarded as the combination of probability theory and fuzzy set theory, where the effect of aleatory uncertainty is described by probability distributions, while the effect of epistemic uncertainty is described by possibility distributions. For instance, in a model z = f(x, y), the input variable x might be subject to aleatory uncertainty and described by a probability density function  $f_X(\cdot)$ ; while the other variable y might be subject to epistemic uncertainty and described by a possibility distribution  $\Pi_y(\cdot)$  (often through expert opinion elicitations).

Kaufmann and Gupta introduced the basic idea of expressing randomness (probability) in combination with imprecision (possibility) via hybrid numbers.<sup>72</sup> Ferson et al.<sup>73,74</sup> extended Kaufmann's work by developing computational rules of hybrid numbers (i.e., the probability distributions are fuzzily known), which can be applied in risk assessment. Through the computational method, the random fuzzy sets can be obtained and converted to the upper and lower bounds of failure probability. Guyonnet et al. introduced a hybrid method to propagate both aleatory and epistemic uncertainties using fuzzy interval analysis.<sup>75</sup> In this method, the possibility distribution function of the output variable z can be first calculated based on the Monte-Carlo sampling method and the possibility extension principle, and then used to derive the upper and lower bounds of failure probabilities based on fuzzy interval analysis.<sup>76</sup> Baudrit et al. developed a postprocessing method based on belief functions (evidence theory) to extract useful information and to construct the failure probability bounds based on the results of the hybrid method,<sup>34</sup> and they proved that the method improved the work of Ferson et al.<sup>73,74</sup> and Guvonnet et al.<sup>75</sup> Baraldi and Zio summarized the hybrid method that jointly propagates probabilistic and possibilistic uncertainties, and compared the method with pure probabilistic and pure fuzzy methods.<sup>77</sup> Based on the work of Baudrit et al.<sup>34</sup> Li and Zio applied the fuzzy interval analysis method to assess the reliability of a distributed generation system, which is affected by serious epistemic uncertainty.<sup>30</sup> The hybrid fuzzy interval analysis method has also been applied successfully in other areas, e.g., reliability assessment of a flood protection dike<sup>78</sup> and a turbo-pump lubricating system.<sup>7</sup> Flage et al. used probabilistic-possibilistic computational framework to propagate uncertainties in FTA, giving rise to the failure probability bounds of top event.<sup>80</sup> Li et al. developed a hybrid-universal-generating-function-based (HUGF) method for the fuzzy interval analysis of multi-state systems.<sup>81</sup>

# 2.4. Problem with PIB metrics

Although differences exist in the way that the interval of failure probabilities is constructed, all the three methods reviewed in Sections 2.1–2.3 use this interval as the reliability metrics. The width of the interval reflects the extent of epistemic uncertainty. One important problem in reliability theory is how to calculate the system-level reliability metrics based on the reliability metrics of the components. Since PIB metrics are intervals of probabilities, the system-level PIB metrics are calculated based on the laws of probability theory. This fact causes a common problem for the PIB metrics when applied to calculate system reliability metrics. Consider the following example.

**Example 1.** Consider a series system composed of 30 components. Suppose that the real reliability of each component is 0.95. Since the system is subject to epistemic uncertainty, the PIB metrics are used to quantify the reliability of the components. We suppose that the reliability interval for each component is [0.9, 1]. Then, following the laws of probability theory, the system's PIB reliability metric will be  $[0.9^{30}, 1^{30}] = [0.04, 1]$ . This interval is not representative of the actual uncertainty on the system reliability and obviously too wide to provide any valuable information in practical applications.

The reason for the unsatisfactory result in Example 1 is that the imprecision in the component reliability metrics (the width of the interval) is amplified by the product law of probability theory that calculates the intersection of events. The systemlevel reliability metric should be able to compensate for the conservatism in the component-level reliability metrics caused by the consideration of epistemic uncertainty. Monotonemeasure-based reliability metrics are developed for this aim.

#### 3. Monotone-measure-based reliability metrics

Monotone measure was defined by Choquet as a generalization of the classical measure theory.<sup>82</sup> Let X be a finite universal set, and let l be a non-empty family of subsets of X. Then g:  $l \rightarrow [0, \infty]$  is a monotone measure on (X, l) if it satisfies the following requirements:

(1)  $g(\emptyset) = 0$ ; (2)  $\forall A, B \in l$ , if  $A \subseteq B$ , then  $g(A) \leq g(B)$ .

Probability measure is a special case of the monotone measure, which is also additive. As pointed out by Klir and Smith,<sup>83</sup> non-additive monotone measures might be able to represent broader types of uncertainty than the addictive probability theory. Therefore, they are applied to developing reliability metrics that model epistemic uncertainty. Typical monotone-measure-based reliability metrics include posbist reliability which is based on possibility theory, and belief reliability which is based on uncertainty theory.

#### 3.1. Possibility-theory-based reliability metrics

The most widely applied possibility-theory-based reliability metric is the posbist reliability. The two basic assumptions of posbist reliability are<sup>32,84</sup>

- (1) Possibility assumption: the system failure behavior is fully characterized in the context of possibility measures.
- (2) Binary-state assumption: the system demonstrates only two crisp states, i.e. fully functioning or fully failed. At any time, the system is in one of the two states.

In posbist reliability theory, lifetime of a system (or a component) is a non-negative real-valued fuzzy variable, and the posbist reliability of a system (or a component) is defined as the possibility measure that the system (or the component) performs its assigned functions properly during a predefined exposure period in a given environment.<sup>84</sup> The epistemic uncertainty is, then, described and propagated based on possibility theory.

Following the definition of posbist reliability, Cai et al. developed posbist reliability analysis methods for series, parallel, series–parallel, parallel–series and coherent systems.<sup>84,85</sup> Huang et al. proposed detailed posbist reliability analysis methods for *k*-out-of-*n*: G systems.<sup>86</sup> Cai et al. studied posbist reliability behavior of cold stand-by and warm stand-by systems, considering both full reliable and non-full reliable conversion switches.<sup>87</sup> Utkin et al. extended Cai's work to repairable systems and developed a posbist reliability analysis method based on state transition diagram.<sup>88,89</sup> Huang et al. introduced a posbist reliability fault tree analysis (posbist

FTA) method for coherent systems to evaluate reliability and safety.<sup>90</sup> He et al. developed calculation methods of posbist reliability for typical systems when the components are symmetric Gaussian fuzzy variables.<sup>91</sup> Bhattacharjee et al. investigated the posbist reliability of *k*-out-of-*n* systems and pointed out that the posbist reliability does not depend on the number of components.<sup>92</sup>

In essence, posbist reliability is a possibility measure. In possibility theory, the possibility measure  $\Pi(\cdot)$  satisfies the following three axioms:<sup>93</sup>

Axiom 1. For the empty set  $\emptyset$ , there is  $\Pi(\emptyset) = 0$ .

**Axiom 2.** For the universal set  $\Gamma$ , there is  $\Pi(\Gamma) = 1$ .

**Axiom 3.** For any events  $\Lambda_1$  and  $\Lambda_2$  in the universal set  $\Gamma$ , there is  $\Pi(\Lambda_1 \cup \Lambda_2) = \max(\Pi(\Lambda_1), \Pi(\Lambda_2))$ .

Axiom 3 shows that the operation laws of possibility theory differ from those of probability theory. Therefore, the system reliability analysis method is also different from that based on probability theory. For instance, Cai et al. proved that the system posbist reliability is the minimum one among all the posbist reliabilities of its components.<sup>32</sup> This difference makes it possible for possibility theory to compensate the conservatism caused by epistemic uncertainty in component-level reliability estimations.

**Example 2.** Consider a series system composed of 300 components. An extreme case is considered where all the components are designed with sufficient margins, so that they are completely reliable and the real reliability should be 1. It is easy to verify that the system's reliability is also 1, which means that the system is highly reliable. However, since the system is subject to epistemic uncertainty, the estimates of component-level reliabilities are likely to be conservative. We suppose, for example, the reliability of each component is estimated to be  $R_1 = R_2 = \cdots = R_{300} = 0.99$ . If we use probability theory to model the reliability metric, the system reliability is

# $R_{\rm S} = R_1 R_2 \cdots R_{300} = 0.04$

It can be seen from the result that the conservatism in component-level reliability estimates is amplified by the operation laws of probability theory, which contradicts with our intuitions since a highly reliable system is judged as highly unreliable.

If we use the posbist reliability, however, the system reliability is

$$R_{\rm S} = \min(R_1, R_2, \cdots, R_{300}) = 0.99$$

which avoids the previous counter-intuitive result and demonstrates that possibility theory can compensate the conservatism in the component-level reliability estimates caused by epistemic uncertainty.

# 3.1.1. Problems with posbist reliability

A major drawback of the possibility-theory-based reliability metrics is that the possibility measure does not satisfy the duality axiom, which might lead to counter-intuitive results when applied in practical reliability-related applications. **Example 3.** Let event  $\Lambda_1 = \{\text{The system is working}\}$  and  $\Lambda_2 = \{\text{The system fails}\}$ . It is obvious that the universal set  $\Gamma = \Lambda_1 \cup \Lambda_2$ . Also, we have the posbist reliability and posbist unreliability to be  $R_{\text{pos}} = \Pi(\Lambda_1)$  and  $\overline{R_{\text{pos}}} = \Pi(\Lambda_2)$ , respectively. According to Axioms 2 and 3, we have

$$\Pi(\Gamma) = \Pi(\Lambda_1 \cup \Lambda_2) = \max(R_{\text{pos}}, \overline{R_{\text{pos}}}) = 1$$
(4)

Therefore, if  $R_{\text{pos}}$  does not equal to 1, e.g.,  $R_{\text{pos}} = 0.8$ ,  $\overline{R_{\text{pos}}}$ must equal to 1. Vice versa, if  $\overline{R_{\text{pos}}}$  does not equal to 1, e.g.,  $\overline{R_{\text{pos}}} = 0.8$ ,  $R_{\text{pos}}$  must equal to 1. This is a counterintuitive result and easily confuses the decision maker in real applications. Hence, even though designed to consider epistemic uncertainty, a reliability metric should still satisfy the duality axiom.

#### 3.2. Uncertainty-theory-based reliability metrics

As just explained in Section 3.1.1, one major drawback of the possibility-theory-based reliability metrics is that possibility theory does not satisfy the duality axiom. To overcome this drawback, belief reliability has been developed based on uncertainty theory. Founded by Liu,<sup>33,94</sup> uncertainty theory relies on the uncertain measure to describe the belief degree of events affected by epistemic uncertainty, which is a monotone measure based on the following four axioms:

- 1) Normality axiom:  $\mathcal{M}{\Gamma} = 1$  for the universal set  $\Gamma$ .
- 2) Duality axiom:  $\mathcal{M}{\Lambda} + \mathcal{M}{\Lambda^c} = 1$  for any event  $\Lambda$ .
- 3) Subadditivity axiom: for every countable sequence of events Λ<sub>1</sub>, Λ<sub>2</sub>,..., we have M{<sup>☉</sup><sub>i=1</sub> Λ<sub>i</sub>} ≤ ∑<sup>∞</sup><sub>i=1</sub> M{Λ<sub>i</sub>};
  4) Product axiom: Let (Γ<sub>k</sub>, L<sub>k</sub>, M<sub>k</sub>) be uncertainty spaces
- 4) Product axiom: Let (Γ<sub>k</sub>, L<sub>k</sub>, M<sub>k</sub>) be uncertainty spaces for k = 1, 2, .... The product uncertainty measure M is an uncertain measure satisfying M{∏<sub>k=1</sub><sup>∞</sup> A<sub>k</sub>} = <sup>∞</sup><sub>A=1</sub> M{Λ<sub>k</sub>}, where L<sub>k</sub> are σ -algebras over Γ<sub>k</sub>, and Λ<sub>k</sub> are arbitrarily chosen events from L<sub>k</sub> for k = 1, 2, ..., respectively.

Belief reliability was defined by Zeng et al. as the uncertainty measure of the system to perform specified functions within given time under given operating conditions.<sup>95</sup> Zeng et al. developed an evaluation method for component belief reliability, which incorporates the influences from design margin, aleatory uncertainty and epistemic uncertainty.96 The issue of quantifying the effect of epistemic uncertainty is addressed by developing a method based on the performance of engineering activities related to reducing epistemic uncertainty.<sup>97,98</sup> The reason why uncertainty theory should be chosen as the theoretical foundation of belief reliability was explained by Zeng et al.<sup>99</sup> by comparing it with other commonly encountered theories to deal with epistemic uncertainty, i.e., evidence theory, possibility theory, Bayesian theory, etc. system reliability analysis methods are also developed for coherent systems.95,99

Compared to the PIB metrics, belief reliability uses the minimum operation to calculate the belief degree of the intersection events, and therefore can compensate for the conservatism in the component-level reliability metrics caused by the consideration of epistemic uncertainty. Compared to the possibility-theory-based reliability metrics, belief reliability satisfies the duality axiom, which avoids the possible paradoxical results often encountered in engineering applications of the possibility-theory-based reliability metrics. Therefore, belief reliability is a promising reliability metric to measure the reliability affected by epistemic uncertainty. However, the researches in the theory of belief reliability are far from mature. In fact, as shown in the classical probability-based reliability theory, there are four major topics in the research of reliability theory:

- (1) How to measure reliability (measurement).
- (2) How to evaluate the reliability of a system based on the reliability of its components (analysis).
- (3) How to design the system so that the desired reliability level can be fulfilled (design).

Table 1 Comparison of five reliability metrics.

Non-probabilistic metrics		Theory basis	Representative literature	Method to obtain metric	Existing problems
PIB reliability metrics	Evidence-theory-based reliability metric	Evidence theory	42	Use belief and plausibility functions to express the lower and upper bounds of failure probability.	The metrics are not able to compensate the conservatism in
	Interval-analysis-based reliability metric	Interval analysis	59, 61	Calculate the maximum and minimum of failure probability through interval analysis, given the range of input parameters.	the estimated component-level reliability metrics, arising from the
	Fuzzy-interval-analysis- based reliability metric	Fuzzy interval analysis	30, 34	First establish the possibility distribution of failure probability through Monte Carlo simulation and fuzzy interval analysis, and then obtain the bounds of failure probability via evidence theory.	consideration of epistemic uncertainty.
Monotone- measure- based	Posbist reliability	Possibility theory	85	Use possibility measure to calculate products' reliability.	The metric does not satisfy duality axiom.
reliability metrics	Belief reliability	Uncertainty theory	95	Obtain the belief reliability through calculating products' design margin, aleatory uncertainty factor and epistemic uncertainty factor.	The research is far from mature.

(4) How to demonstrate that the system satisfies its reliability requirements (demonstration).

Among the four topics, measurement is the most fundamental one. Since belief reliability is an entirely different reliability metric from the classical probability-based reliability metrics, new analysis, design and demonstration methods are also needed for the theory of belief reliability. As reviewed before, however, current researches on belief reliability only concentrate on the first two problems. The problems of design and demonstration are still relatively unexplored and deserve further investigations.

To summarize, we make a comparison of the five reviewed reliability metrics (see Table 1) in terms of theory basis, methods to obtain metric, and existing problems. This will help people to choose appropriate reliability metric according to different demands and situations.

# 4. Conclusions

In this paper, a systematic review is conducted on the nonprobabilistic reliability metrics that are used to describe the effect of epistemic uncertainty. Five reliability metrics are discussed, i.e., the evidence-theory-based, interval-analysis-based, fuzzy-interval-analysis-based, possibility-theory-based (posbist reliability) and uncertainty-theory-based reliability metrics (belief reliability). Among them, the former three provide, in essence, an interval that contains all the possible values of the reliabilities/failure probabilities whereas the latter two give monotone measures.

An investigation of the five metrics reveals two important features that a qualified reliability metric under epistemic uncertainty should possess: (1) it should be able to compensate the conservatism in the component-level reliability metrics caused by the consideration of epistemic uncertainty, and (2) it should satisfy the duality axiom, otherwise it might lead to paradoxical and confusing results in engineering applications.

Finally, the five reliability metrics are compared with respect to the above two features, as well as other important characteristics which can be used to assist the selection of appropriate reliability metrics considering the effect of epistemic uncertainty.

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