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Key Points:
- No consensus exists on the definition of particle motion/rest states in bed-load transport
- Use of a finite observation window biases the statistics of particle hop length and duration
- A sensitivity analysis of hop data to the above issues explains the scatter of literature points

Supporting Information:
- Supporting Information S1

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On Reasons of the Scatter of Literature Data for Bed-Load Particle Hops

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Abstract This Report examines two key issues emerging when one deals with high-resolution data for particle tracks in bed-load transport. The first relates to how a particle motion/rest state is defined; using different definitions of motion changes the mean values computed for particle hop length and duration, that are key properties for phenomenological analysis of bed-load transport and quantitative estimation of the associated rates. The second issue refers to the experimental bias due to considering a finite observation area within the channel bed. A conceptual presentation of the problem is here complemented by an analysis of experimental data, from both the literature and new experiments with weak bed-load transport. Four definitions of motion were employed, together with two different ways (correcting or not the bias due to the observation area) of computing mean values from measured hop data. The results demonstrate that using different definitions and operational methods highly changes the mean values of hop-related quantities. This may explain the scatter among literature data and challenges future Lagrangian investigation of bed-load transport.

1. Introduction

Several hydro-morphologic river processes depend on bed-load transport. Many theoretical and experimental studies were devoted to increasing our understanding of the dynamic processes governing bed load, from the pioneers (e.g., Du Boys, 1879; Einstein, 1950; Meyer-Peter & Müller, 1948; Shields, 1936) to many followers.

Bed-load transport is an intermittent process governed by alternations of particle motions and rests. This manuscript takes a Lagrangian approach to the investigation of bed-load transport and considers the individual motion of tracked particles (e.g., Campagnol et al., 2013; Fathel et al., 2015). Within the range of possible Lagrangian scales (from the scale of particle-bed collisions to that of an open-channel reach) we focus on particle hops, corresponding to the motions from entrainment to disentrainment. After Einstein (1950), particle hop has received extensive attention (e.g., Campagnol et al., 2015; Fathel et al., 2015; Furbish et al., 2016; Lajeunesse et al., 2010), due to both a fascinating phenomenology and its use to quantify sediment transport rates (which can be indeed obtained as a product of the hop length and an entrainment rate; see, e.g., Ballio et al., 2018; Garcia, 2008). Many studies have employed image-based methods, which are nonintrusive and can provide high-resolution data (e.g., the pioneering work of Francis, 1973, up to recent studies of Ballio et al., 2018; Bottacin-Busolin et al., 2008; Lajeunesse et al., 2010; Ramesh et al., 2011, among others). Furthermore, experiments have been performed in different conditions. For example, Niño et al. (1994), Drake et al. (1988), and Lajeunesse et al. (2010) ran experiments with a mobile bed; Campagnol et al. (2013) used both mobile and fixed beds; several others (e.g., Abbott & Francis, 1977; Hu & Hui, 1996; Lee & Hsu, 1994; Lee et al., 2000, 2006; Martin et al., 2012; Papanicolaou et al., 1999; Ramesh et al., 2011; Sechet & Le Guennec, 1999) employed fixed beds. Some experiments (e.g., Abbott & Francis, 1977) had particle motion recording performed from the side, whereas other scholars (e.g., Drake et al., 1988; Niño et al., 1994) used a top view. Transport modes included sliding, rolling, and saltation.

The mean values of the hop length and duration for the literature studies are scattered over more than one order of magnitude (see Figure 3, that will be extensively discussed later), for similar hydrodynamic conditions and even considering only uniform or nearly uniform sediment and stationary/uniform flows. This is a major shortcoming of current knowledge of bed-load transport, challenging future research. In particular, can we find possible reasons for the scatter of the available data? In turn, can guidance be proposed to reduce this scatter?

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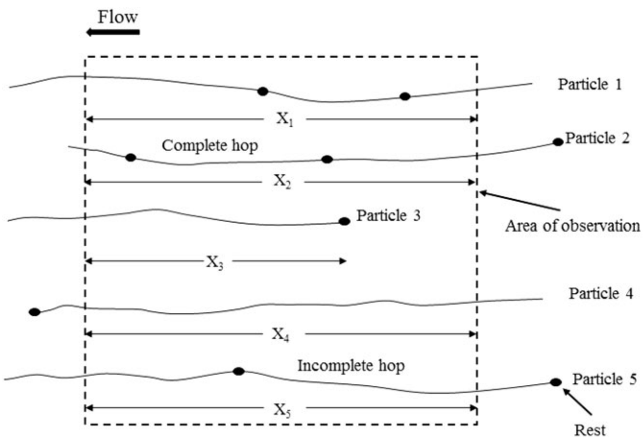

Figure 1. Sketch of complete and incomplete particle hops in a bed-load transport process. Black circles correspond to rest positions.

This Report aims at providing possible answers to the above questions, by highlighting two operational issues that are related with measurement and analysis of bed-load particle tracks: (i) the recognition of a particle motion state and (ii) the experimental limitations induced by a finite observation area. These issues will be conceptually argued, and recent experimental data will be used to support the discussion. The manuscript is structured as follows. In section 2, different criteria for definition of particle motion/rest are introduced. The impact of a finite observation window on hop measurements is conceptually described in section 3, where the calculation of mean values for hop length and duration is also discussed. Section 4 describes the bed-load experiments and the particle-tracking algorithm adopted for this study. A sensitivity analysis of the values of mean hop properties on their estimation is provided in section 5, quantifying the individual and combined impacts of motion identification and focus area and explaining, accordingly, the scatter among literature data. The results of this study significantly challenge the research community willing to explore the bed-load transport process by considering particle hops.

2. Identification of a Particle Motion State

A hop length Δx is defined as the stream-wise distance traveled by one particle from entrainment to disentrainment, and the associated time of motion Δt^m is the time elapsed between the entrainment and disentrainment events. The state of a particle at each instant is identified by a Boolean variable M, with $M = 1$ and $M = 0$ representing motion and rest, respectively. Consequently, a change of M from 0 to 1 represents a particle entrainment, while the opposite indicates a disentrainment. Couples of these events bound hops with individual length and duration. While analyzing measured particle tracks (see below), it was thus necessary to determine if, at every instant, any particle was in motion or not. Several definitions have been proposed by different researchers to accomplish this requirement. For example, Roseberry et al. (2012) followed a principle that any particle motion be part of a hop and intended to measure all the hop lengths down to the shortest ones; coherently, they identified a state of motion with the particle velocity exceeding a cutoff value that was set as equal to 0. Differently, Heyman et al. (2016) used a combination of thresholds for velocity and elevation of particles to define a particle entrainment, that was associated with (i) the magnitude of instantaneous particle velocity exceeding 0.01 times the settling velocity and (ii) the distance from the particle center of mass to the estimated bed elevation being lower than the particle size (this definition is thus only applicable to experiments with a side view). Finally, other studies (e.g., Campagnol et al., 2013; Hosseini-Sadabadi et al., 2016b; Seizilles et al., 2014) defined the particle motion based on the particle positions. Campagnol et al. (2013) detected a state of motion at a certain instant if the stream-wise coordinate of the particle position at that instant was smaller than the stream-wise coordinates of all the positions taken later by the particle. Seizilles et al. (2014) identified a state of motion if the standard deviation of the last four positions taken by the particle was higher than 0.1 times the particle size. Hosseini-Sadabadi et al. (2016b) considered a particle at motion at a certain instant if the stream-wise coordinate of its position exceeded all the previous ones and was lower than all the following ones, amending in this way the earlier criterion of Campagnol et al. (2013). This brief review considered a few experimental studies, but the problem of defining an entrainment condition is also present in numerical investigations (e.g., Bialik, 2015; Wu & Chou, 2003; Wu & Jiang, 2007). The impact of using different criteria on the resulting statistics for hop properties will be explored in section 5.

3. Experimental Bias in Measuring Particle Hops

A predetermined working reach of a channel is (almost) unavoidably used in bed-load transport studies, for both experimental (e.g., Fathel et al., 2015; Lajeunesse et al., 2010; Radice et al., 2009) and numerical (e.g., Bialik & Karpiński, 2018; Lukerchenko et al., 2006; Oh & Tsai, 2010) investigations. The size of the observation area impacts the values measured for the relevant quantities, as illustrated by a sketch in Figure 1. Lines represent the particle tracks for a certain duration T, and the length along which a particle was observed is X. Circles correspond to rest positions, thus a line between two circles defines a hop. In this example, only two hops (one for particle 1 and one for particle 2) are completely observed, while other hops are only partially detected. Particle 4 makes a hop that is longer than the observation length, and several hops associated with particles 1, 2, 3, and 5 cross the area boundaries. These hops are identified as "incomplete."

One could compute a mean value for the hop length considering only the hops that are completely observed. In this way, arithmetic averages of hop lengths $\langle \Delta x \rangle$ and hop durations $\langle \Delta t^m \rangle$ are obviously

$$\langle \Delta x \rangle = \frac{1}{N_c} \sum_{j=1}^{N_c} \Delta x_j, \tag{1}$$

$$\langle \Delta t^m \rangle = \frac{1}{N_c} \sum_{j=1}^{N_c} \Delta t_j^m, \tag{2}$$

with j as a hop counter and N_c as the number of completely observed hops. However, equations (1) and (2) are *biased* estimates of the mean values because several incomplete hops are excluded from the computation. Roseberry et al. (2012), Fathel et al. (2015), and Furbish et al. (2016) were probably the first ones recognizing the impact of a finite window on the investigation of hop properties. While proposing a conceptual framework for definition of several bed-load quantities, Ballio et al. (2018) mentioned that an *unbiased* estimate of the mean hop length should be computed as

$$\xi = \frac{1}{N_e} \sum_{i=1}^{N_p} X_i, \tag{3}$$

where i is a particle counter, N_p is the number of tracked particles (equal to 5 in the sketch of Figure 1), and N_e is the number of entrainment events observed (equal to 6 in Figure 1). This method pools together all the measured particle tracks (including both complete and incomplete hops), as if they were portions of a single super-track. Furthermore, Heyman et al. (2016) and Ballio et al. (2018) also computed unbiased mean values of the hop duration τ^m as

$$\tau^m = \frac{1}{N_e} \sum_{i=1}^{N_p} T_i^m, \tag{4}$$

where T^m is the total time of motion for a tracked particle. Equations (3) and (4) are equations (52) and (50) of Ballio et al. (2018), to which the reader is directed for the theoretical developments. In section 5, a sensitivity analysis is performed to investigate how the mean value of particle hop properties changes depending on the method used for its computation (on the one hand, using equations (1) and (2); on the other hand, using equations (3) and (4)).

4. Experiments and Image Processing

The bed-load experiments for this study were conducted in a pressurized duct at the Hydraulics Laboratory of the Politecnico di Milano. The channel length is 5.8 m, while the rectangular cross section is 0.4 m wide and 0.11 m high. Uniform, quasi-spherical Polybutylene Terephthalate grains with a dimension $d = 3.0$ mm and a density $\rho_g = 1.27 \times 10^3$ kg/m^3 were used as bed-load sediment. A fixed, rough bed covering the entire length of the flume was employed. The channel is pressurized for its entire length by a transparent lid, avoiding image distortion that could have been caused by a wavy free surface.

The present study included four experiments with steady water discharges ranging between 12 and 18 L/s, the threshold flow rate being equal to 10 L/s as determined by Campagnol et al. (2012). Sediment was released into the duct by an automatic feeder located at the inlet. During each experiment, the particle motion was recorded for 50 s at 32 fps using a camera installed above the channel, at approximately 1 m upstream of the outlet (thus, ensuring a developed flow at the working section). Presented results are related to a focus area of 270 and 200 mm in the stream-wise and transverse directions, respectively (corresponding to 90 and 67 times the particle size). One image pixel corresponded to 1.47 mm in reality (half the particle

Table 1
Experimental Conditions

Run	Q (l/s)	U (mm/s)	Re (10^4)	$u*$ (mm/s)	Q/Q_c	Re*	q_s (mm^2/s)	N_t
R1	12.1	272.7	2.8	7.5 ± 2.9	1.2	22.4	0.047	60
R2	14.1	318.2	3.3	15.6 ± 2.7	1.4	46.8	0.181	192
R3	16.1	363.6	3.8	17.9 ± 1.5	1.6	53.6	0.439	416
R4	18.1	409.1	4.3	20.7 ± 2.3	1.8	61.9	1.487	1,421

size). For a better tracking process, the entire bed was painted in black, while the released bed-load grains were white.

The vertical profile of the stream-wise flow velocity component was measured at the center of the recording section using a package of two Ultrasonic Velocity Profilers. The shear velocity $u*$ was estimated based on the time-averaged velocity profile. The main characteristics of the runs are summarized in Table 1, where Q is the flow rate, U is the bulk flow velocity, Re is the Reynolds number, and Re^* is the particle Reynolds number.

Particle Tracking Velocimetry was conducted using the Streams software (Nokes, 2012) which was successfully applied in earlier studies of Campagnol et al. (2013, 2015), Ballio and Radice (2015), Hosseini-Sadabadi et al. (2016a, 2016b), and Radice et al. (2017). The stream-wise and transverse coordinates (x and y, respectively) of the fed particles were determined for each movie frame corresponding to an instant in time. The tracks of individual grains were consequently derived using a criterion of minimum distance for particle matching. Most of the tracks (on average, 81%) crossed the upstream and downstream boundaries of the focus area, while few crossed the sides. After Particle Tracking Velocimetry, eye-based inspection revealed that some of the particle tracks were incorrect; therefore, these tracks were excluded from the samples. The time evolutions of the stream-wise position and instantaneous velocity (u_s) are depicted for a sample particle in Figure 2. Table 1 includes q_s as the time-averaged sediment transport rate per unit width (measured by equation (7), see later) and N_t as the number of valid tracks.

5. Sensitivity Analysis Results and Discussion

5.1. How Do Operational Choices Affect the Mean Values of Hop Length and Duration?

Two operational issues affecting the measured values have been introduced in this manuscript: First, the definition used to identify the particle motion state and, second, using biased and unbiased estimates for

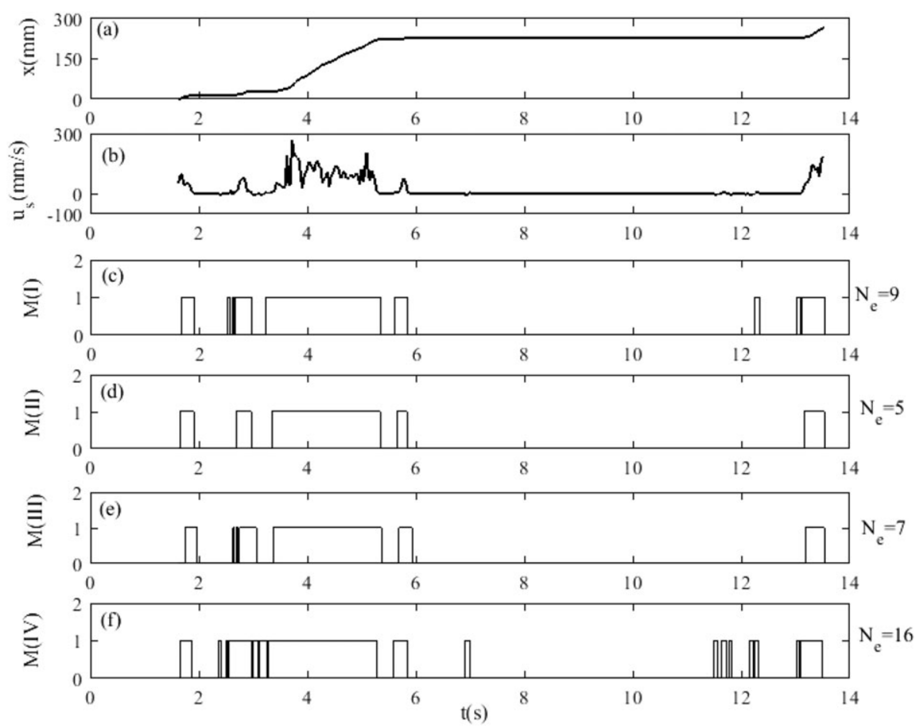

Figure 2. (a) The x coordinate of a sample tracked particle and (b) its stream-wise velocity u_s (experiment R2). The other plots (c to f, respectively) present the Boolean variable M as returned by the criteria of (*I*) Campagnol et al. (2013), (*II*) Hosseini-Sadabadi et al. (2016b), (*III*) Seizilles et al. (2014), and (*IV*) a criterion based on a cutoff velocity. N_e is the total number of entrainment events detected using a definition of motion.

the mean values of hop length and duration. The two issues are first considered separately, and then together to quantify their impact on the mean values of particle hop properties.

Four definitions were applied to explore the sensitivity of measured hop properties to the identification of particle motion states. Definitions (*I*), (*II*), and (*III*) were those proposed by Campagnol et al. (2013), Hosseini-Sadabadi et al. (2016b), and Seizilles et al. (2014), respectively. Definition (*IV*) was based on a cutoff velocity, chosen as equal to $(1/30) \times d/\Delta t$ (where Δt is the time interval between frames). The four definitions are applied to a sample particle track in Figure 2, where Boolean variables $M(I)$ to $M(IV)$ identify the states of motion and rest obtained for the particle. When the particle vibrated around a relatively stable position (see, e.g., velocity fluctuations at around 12 s), it was considered still by definitions (*II*) and (*III*), whereas some short hops were detected by definitions (*I*) and (*IV*). Figure 2 also shows that the definitions returned different numbers N_e of the entrainment events detected for the particle.

Figure 3a presents the dimensionless mean hop lengths (using equation (1)) for the experiments of this study, as a function of the ratio u^*/u^*_c between the shear velocity and its threshold value. To check the statistical stability of the mean values, the Kolmogorov-Smirnov test (Massey, 1951) was applied to the experimental Probability Density Functions of hop length for different sample sizes (details are in the supporting information and in Hosseini-Sadabadi, 2017). The plot of Figure 3a evidences the significant impact of the definition used to identify particle motion on the mean values that are obtained. For example, for experiment R1 ($u^*/u^*_c = 1.2$), the mean hop length returned by the definition of Seizilles et al. (2014) is almost 20 times the mean value obtained using the one based on a cutoff velocity. This is the largest difference detected; globally, the points scatter over one order of magnitude. In general, the highest mean values are obtained employing the definitions of Seizilles et al. (2014) and Hosseini-Sadabadi et al. (2016b). For comparison, Figure 3a also includes the results of several literature studies (Abbott & Francis, 1977; Campagnol et al., 2013; Drake et al., 1988; Hu & Hui, 1996; Lajeunesse et al., 2010; Lee et al., 2000, 2006; Lee & Hsu, 1994; Martin et al., 2012; Niño et al., 1994; Niño & Garcia, 1998; Papanicolaou et al., 1999; Ramesh et al., 2011; Sechet & Le Guennec, 1999). Definitions for identification of motion are rarely reported; on the other hand, the sensitivity analysis described above demonstrates that the use of different definitions may be one reason for the large scatter of points from various scholars.

In the second part of the analysis, the relationship between the biased (equation (1)) and unbiased (equation (3)) estimates for the mean hop length was explored. Figure 3b provides a comparison between the biased and unbiased estimates for our experiments using the definitions of Hosseini-Sadabadi et al. (2016b) for the identification of particle motion. The ratio between the unbiased and biased estimates of the mean hop length is 2.9 and 17.7 for the lowest and the highest flow discharges, respectively. This demonstrates that, in this work, the mean length of particle hops was significantly affected by the window size. Figure 3b also shows that the ratio between the two values grows for increasing shear velocity (and consequently increasing mean hop lengths), indicating that mean hop lengths for experiments with more intense transport conditions are more prone to bias by a finite area of observation. Again, the points for the present study were compared to those for earlier literature analyses. The scatter of the literature points might also be explained by the impact of a finite window size on the computation of mean values, even though no definite argument can be proposed because these studies often lacked a detailed description of how the mean values were computed.

Finally, Figure 3c presents a comprehensive comparison of results, considering all the definitions of motion for both biased and unbiased estimates of the mean values. The same set of analyses is performed for the hop duration in Figure 3d, including data of Campagnol et al. (2013), Lajeunesse et al. (2010), Martin et al. (2012), and Papanicolaou et al. (1999). The summary of results in Figures 3c and 3d confirms that mean values are highly dependent on operational criteria, answering in this way the question raised in this study. The scatter of the literature results is fully explainable considering differences in (i) definitions used to define particle motion and rest states and (ii) how the mean hop properties are computed. In turn, the present analysis suggests that, in the absence of a clear description of all the methods used in bed-load transport studies, it would not be possible to reduce the scatter of points from different authors.

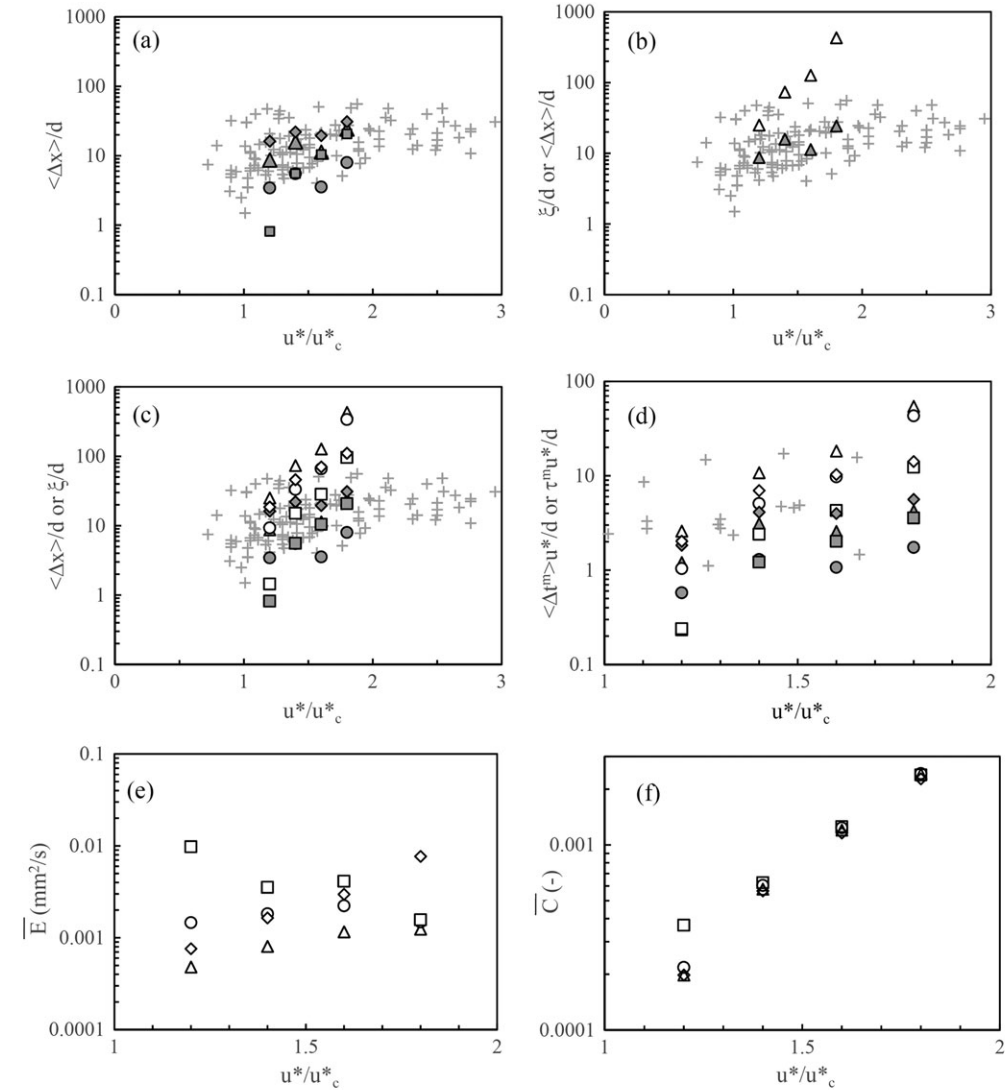

Figure 3. (a) Dimensionless mean hop length (using the biased estimate) as a function of u^*/u^*_c. The motion was identified using the definitions of: ● Campagnol et al. (2013), ▲ Hosseini-Sadabadi et al. (2016b), ◆ Seizilles et al. (2014), and ■ using a cutoff velocity. (b) Dimensionless mean hop length as a function of u^*/u^*_c. Gray and hollow triangles show biased and unbiased estimates, respectively. Motion was identified using the definition of Hosseini-Sadabadi et al. (2016b). (c) Dimensionless mean hop length as a function of u^*/u^*_c. Gray and white symbols are for the biased and unbiased estimates, respectively. Symbol shapes are as in Figure 3a. (d) Dimensionless mean time of motion as a function of u^*/u^*_c. Gray and white symbols are for the biased and unbiased estimates, respectively. Symbol shapes are as in Figure 3a. In panels (a) to (d), previous literature findings are pooled together and symbolized with +. (e) The time-mean entrainment rate $\overline{E}$ as a function of u^*/u^*_c for each definition of motion used in this study. (f) The time-mean concentration of moving particles as a function of u^*/u^*_c for each definition of motion used in this study. In panels (e) and (f), symbol shapes are as in Figure 3a. Note that the horizontal axis has different range in the panels to maximize readability.

5.2. Can One Give Preference to any Definition for Identification of Motion?

Since anyone would prefer an unbiased estimate to a biased one, it is evident that equations (3) and (4) are to be preferred to equations (1) and (2). Beyond providing conceptual arguments, Ballio et al. (2018) also took advantage of a proof-of-concept result comparing the values of the time-averaged sediment transport rate derived from their primitive quantities following two methods. A first evaluation of the sediment transport rate is

$$q_s = \overline{E}\xi, \tag{5}$$

which is an Einstein-type formula with

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$$\overline{E} = N_e w/(AT) \tag{6}$$

as the time-averaged entrainment rate (w is the volume of one particle and A is the area of observation). The second expression determines the sediment transport rate from the concentration $C = N_m w/(Ad)$ (with N_m as the total number of moving particles at each instant within A) and the velocity, u_A, of the same particles:

$$q_s = \overline{Cu_A}d, \tag{7}$$

where again an overbar denotes time averaging. It was shown by Ballio et al. (2018) that equations (5) and (7) returned the same value for the sediment transport rate when equation (3) was used to determine the mean hop length while, when hop lengths are evaluated with equation (1), resulting sediment discharge values differ as a consequence of the bias. This further motivates the choice of formulas (3) and (4) with respect to the more straightforward (1) and (2) for the evaluation of mean values of hop properties. On the other hand, one cannot prefer any definition for identification of motion: All those used here are conceptually reasonable, in spite of the highly different results obtained from each of them. Moreover, equations (5) and (7) are in this case unable to give any proof of concept. In fact, the two equations were employed to compute the sediment transport rate for the experiments of the present study; the exercise returned values from (5) always equal to those from (7), independently of the different definitions. Reasons for this result are different for the two approaches used for the evaluation of the transport rate (equations (5) and (7)), as briefly discussed in the following.

Let one first consider the two terms, $\overline{E}$ and ξ, appearing in equation (5): Figures 2, 3c, and 3e show that the definition of motion significantly affects the values of both terms. This is because a definition selecting very small movements as "motion" also indicates a high number of events, N_e, and therefore a high value for $\overline{E}$ (equation (6)); at the same time, the resulting average hop length, ξ, would be small (and proportional to $1/N_e$, equation (3)). When combined in equation (5), the variations of the two terms ($\overline{E}$ and ξ) are exactly compensated.

We now analyze the effect of the motion definition on the concentration and velocity values appearing in equation (7). As mentioned above, $\overline{Cu_A}$ is also insensitive to the definition of motion. It is worth exploring if the primitive quantities are both independent of how the particle states are identified. Figure 3f demonstrates that the time-mean value of C was insensitive to the definition of motion that was used. The same held also for the Eulerian velocity of the moving particles (not shown). This result was likely obtained because the area of observation was large enough to obtain relatively definition-independent instantaneous values of C and u_A (thanks to a sufficient number of particles laying within A and determining these Eulerian quantities). The only experiment for which the concentration was somehow variable with the definition of motion was R1 with a u^*/u^*_c ratio of 1.2, for which indeed the number of moving particles was the lowest.

Based on the above comparison, one must conclude that a simple double-check on sediment transport rate values is not useful to identify an "optimal" definition for identification of motion. For the time being, the issue is unsolved and affects all the literature values of hop properties. Therefore, a necessity emerges of focusing on conceptual arguments and methods that might provide more definitive and reasonably grounded results.

6. Conclusions

Several studies have aimed at investigating bed-load particle hops tracking individual grains, and results from different scholars are largely scattered. In order to measure hop-related quantities, scholars employed different definitions for identification of motion. In addition, the experiments were conducted using finite observation windows, within which only a part of the particle hops could be completely observed.

The mean values of hop length and duration change depending on how a particle motion state is defined at any given instant, and the variation may be of more than one order of magnitude. A finite observation window also significantly biases the mean values that are computed. These considerations can explain the scatter among literature data for bed-load particle hops.

The experimental bias related with a finite observation window can be fixed with an appropriate computation of the mean hop properties; the unbiased estimates are remarkably higher than the biased ones, the ratio between the two increasing with more intense transport conditions.

By contrast, no preference can be given to any definition of particle motion and stillness states. However, in the absence of commonly accepted definitions, the research community can hardly target to reach a scatter of particle hop data below one order of magnitude.

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