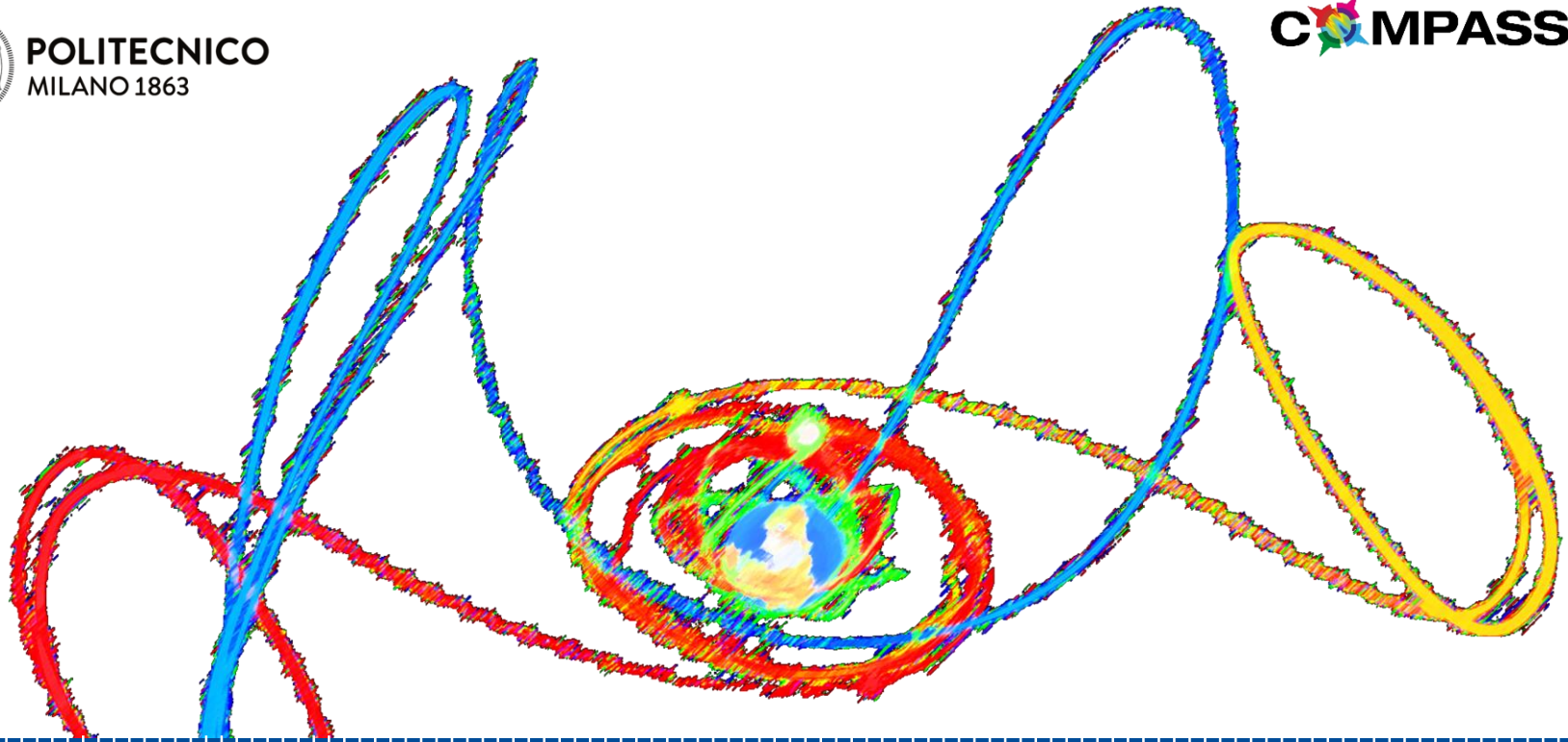




POLITECNICO  
MILANO 1863



# Luni-solar perturbations for missions design in highly elliptical orbits

Camilla Colombo

Barcelona Mathematical Days, 28 April 2017



# INTRODUCTION

## Highly Elliptical Orbits

### Why highly elliptical Orbits

- Astrophysics and astronomy missions (e.g., INTEGRAL and XMM-Newton)
- Earth missions (e.g., Molniya or Tundra orbits, Drim constellation, magnetotail mission)
- Geostationary transfer orbits

### Selection criteria

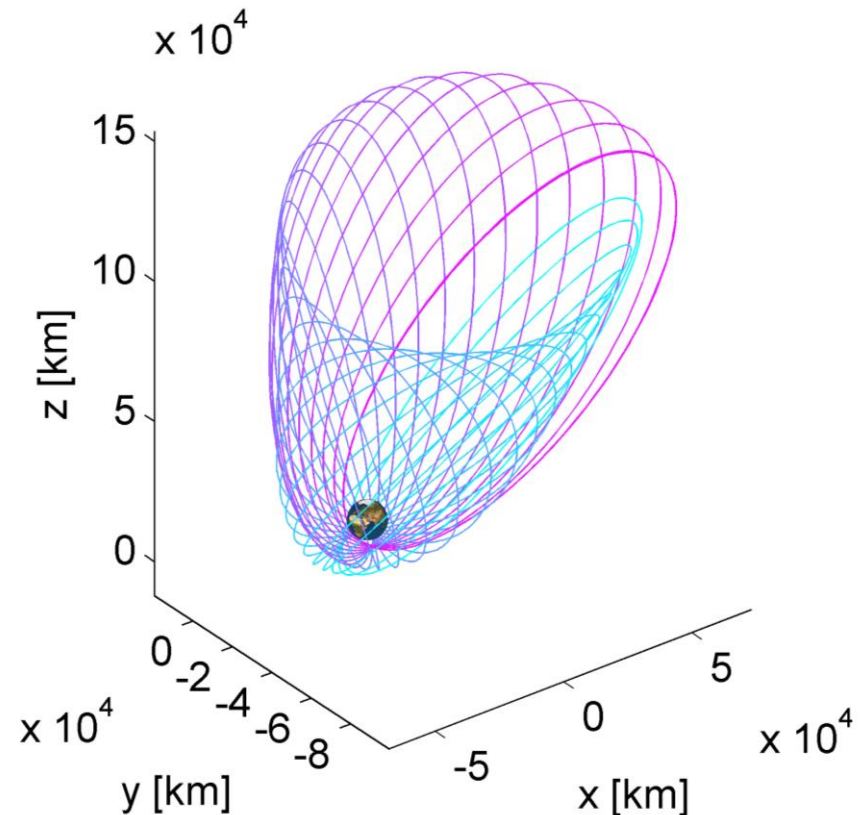
- Vantage points for the observation of the Earth and the Universe
- Avoid noise from radiation effects
- Geo-synchronicity to meet coverage requirements
- Inclination to minimise eclipse period

## Highly Elliptical Orbits

...Fascinating interaction between third body luni-solar perturbation and Earth's oblateness

...Perfect example on how we can leverage the natural dynamical effect through manoeuvres to obtain free long-term effect on the orbit:

- Frozen orbits
- End-of-life Earth re-entry
- End-of-life graveyard orbit injection



## Outline

- Dynamical model
- Analytical interpretation
- Dynamical maps
- Engineering Perturbation effects
- Applications



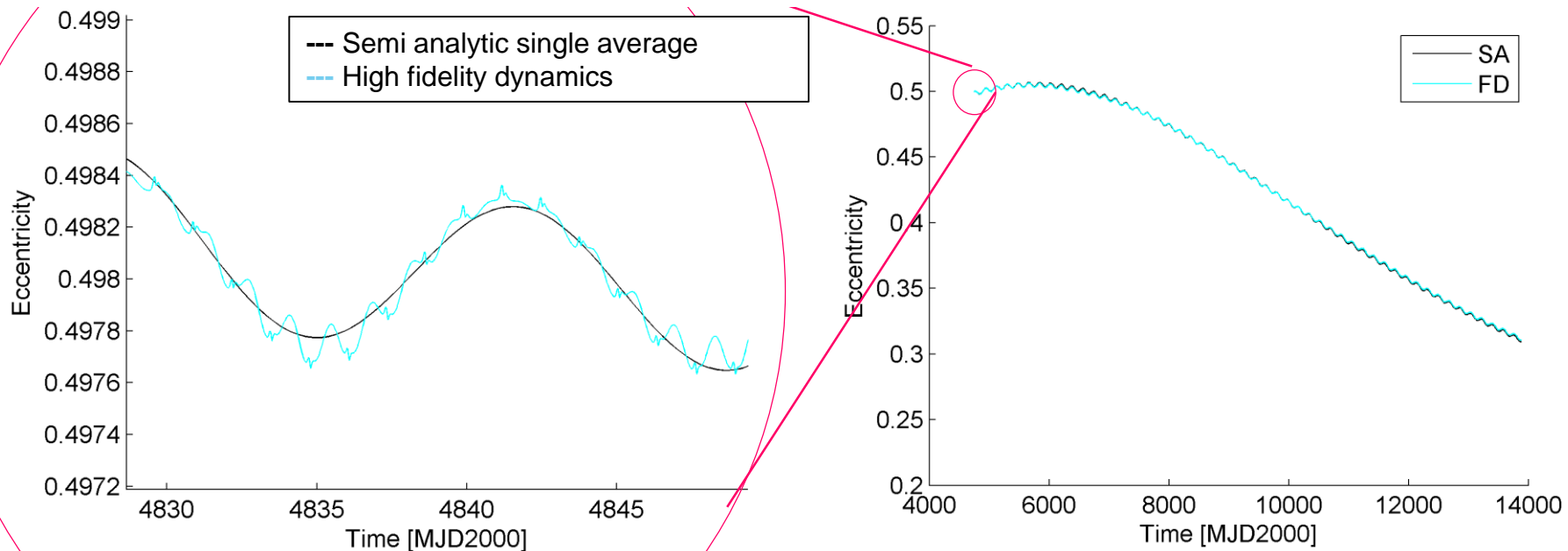
# DYNAMICAL MODEL

# Dynamical model

Orbit propagation based on averaged dynamics

Average variation of orbital elements over one orbit revolution

- Filter high frequency oscillations
- Reduce stiffness of the problem
- Decrease computational time for long term integration



# Dynamical model

PlanODyn suite



Space Debris Evolution, Collision risk, and Mitigation  
**FP7/EU Marie Curie grant 302270**



End-Of-Life Disposal Concepts for Lagrange-Point, Highly Elliptical Orbit missions, **ESA GSP**

End-Of-Life Disposal Concepts Medium Earth Orbit missions, **ESA GSP**



GEO disposal in “Revolutionary Design of Spacecraft through Holistic Integration of Future Technologies”  
**ReDSHIFT, H2020**



**COMPASS, ERC** “Control for orbit manoeuvring through perturbations for supplication to space systems”



Orbit propagation based on averaged dynamics

For conservative orbit perturbation effects

Disturbing potential function

$$R = R_{\text{SRP}} + R_{\text{zonal}} + R_{3\text{-Sun}} + R_{3\text{-Moon}}$$

Planetary equations in Lagrange form

$$\frac{d\mathbf{a}}{dt} = f\left(\mathbf{a}, \frac{\partial R}{\partial \mathbf{a}}\right) \quad \mathbf{a} = [a \quad e \quad i \quad \Omega \quad \omega \quad M]^T$$

$$\frac{da}{dt} = \frac{2}{na} \frac{\partial R}{\partial M}$$

$$\frac{de}{dt} = \frac{1}{na^2 e} \left( (1-e^2) \frac{\partial R}{\partial M} - \sqrt{1-e^2} \frac{\partial R}{\partial \omega} \right)$$

$$\frac{di}{dt} = \frac{1}{na^2 \sin i \sqrt{1-e^2}} \left( \cos i \frac{\partial R}{\partial \omega} - \frac{\partial R}{\partial \Omega} \right)$$

$$\frac{d\Omega}{dt} = \frac{1}{na^2 \sin i \sqrt{1-e^2}} \frac{\partial R}{\partial i}$$

$$\frac{d\omega}{dt} = -\frac{1}{na^2 \sin i \sqrt{1-e^2}} \cos i \frac{\partial R}{\partial i} + \frac{\sqrt{1-e^2}}{na^2 e} \frac{\partial R}{\partial e}$$

$$\frac{dM}{dt} = n - \frac{(1-e^2)}{na^2 e} \frac{\partial R}{\partial e} - \frac{2}{na} \frac{\partial R}{\partial a}$$

# Dynamical model

Orbit propagation based on averaged dynamics

For conservative orbit perturbation effects

Disturbing potential function

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Average over one orbit revolution of the spacecraft around the primary planet

$$\bar{R} = \bar{R}_{\text{SRP}} + \bar{R}_{\text{zonal}} + \bar{R}_{3\text{-Sun}} + \bar{R}_{3\text{-Moon}}$$

$$\frac{d\bar{\mathbf{a}}}{dt} = f\left(\bar{\mathbf{a}}, \frac{\partial \bar{R}}{\partial \bar{\mathbf{a}}}\right)$$

Single average



Average over the revolution of the perturbing body around the primary planet

$$\bar{\bar{R}} = \bar{\bar{R}}_{\text{SRP}} + \bar{\bar{R}}_{\text{zonal}} + \bar{\bar{R}}_{3\text{-Sun}} + \bar{\bar{R}}_{3\text{-Moon}}$$

$$\frac{d\bar{\bar{\mathbf{a}}}}{dt} = f\left(\bar{\bar{\mathbf{a}}}, \frac{\partial \bar{\bar{R}}}{\partial \bar{\bar{\mathbf{a}}}}\right)$$

Double average

# Dynamical model

## PlanODyn: Planetary Orbital Dynamics

Dynamics

Single average potential

+

Analytical  
Jacobian matrix

Double average potential

+

Analytical  
Jacobian matrix

For validation purposes

Full eqs. motion  
Cartesian

Full eqs. motion  
orbital elements

Integration method: explicit Runge-Kutta (4,5) method, Dormand-Prince pair

Implemented in Matlab

► *“Planetary Orbital Dynamics Suite for Long Term Propagation in Perturbed Environment,” ICATT, ESA/ESOC, 2016.*

# Dynamical model

## Perturbation model

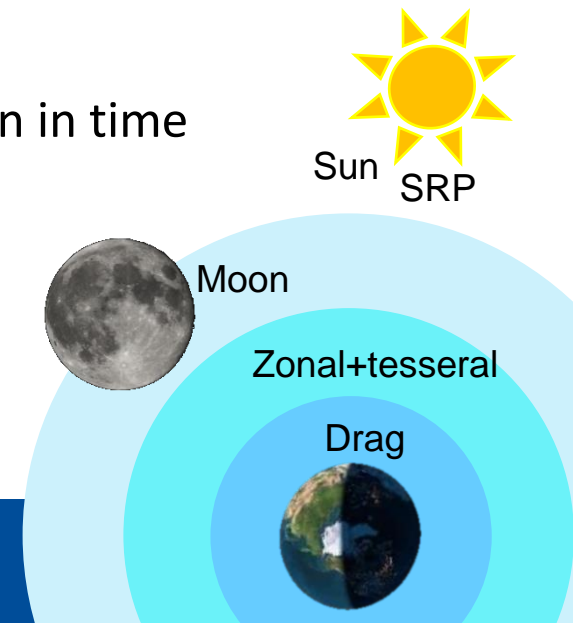
### Perturbations in planet centred dynamics

- Atmospheric drag (piece-wise exponential model)
- Zonal harmonics of the Earth's gravity potential,  $J_2^2$
- Solar radiation pressure (with eclipses)
- Third body perturbation of the Sun
- Third body perturbation of the Moon

### Ephemerides options

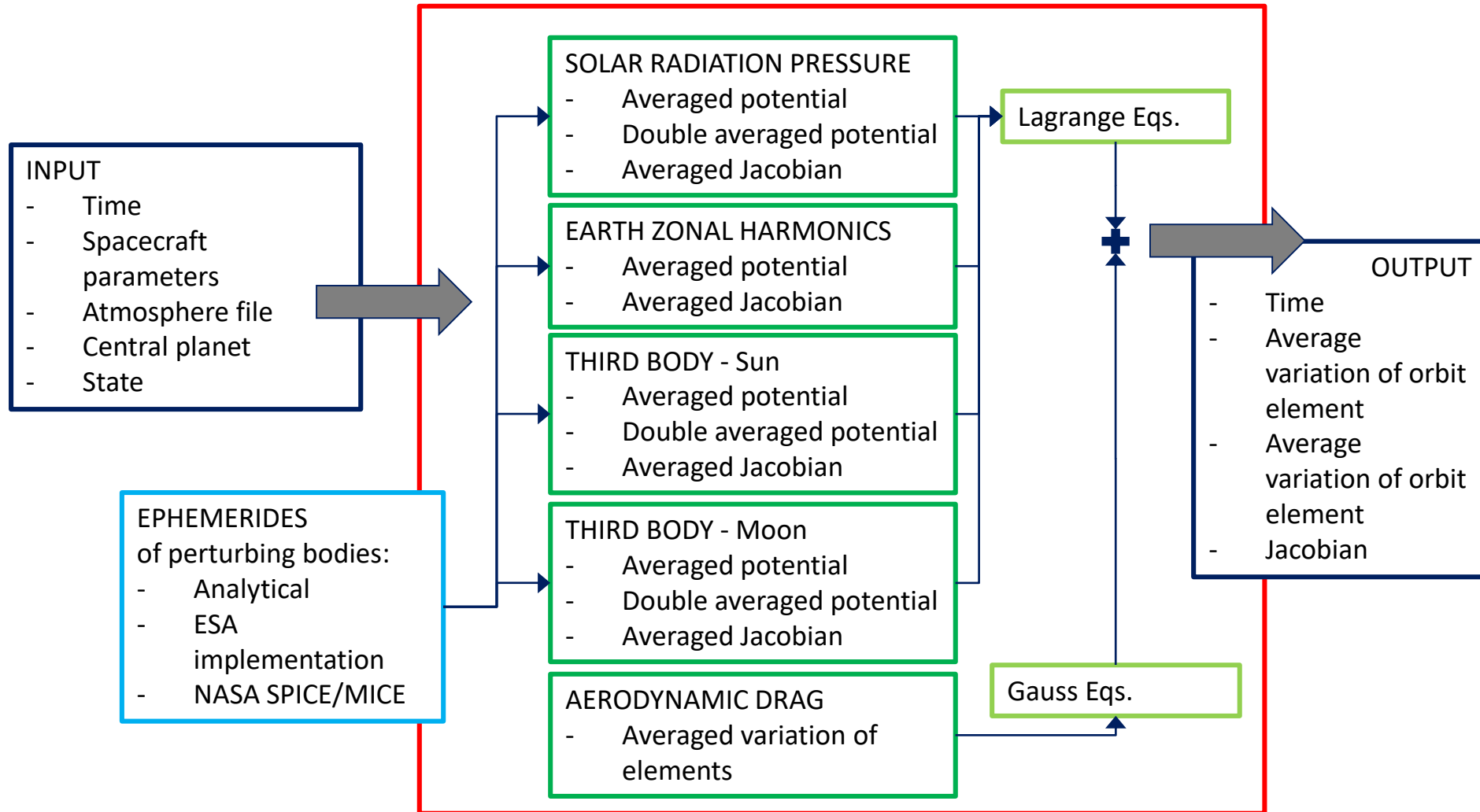
- Analytical approximation based on polynomial expansion in time
- Numerical ephemerides through the NASA SPICE toolkit
- Numerical ephemerides from an ESA implementation

### Orbital elements in Earth centred equatorial J2000 frame



# Dynamical model

## PlanODyn: Planetary Orbital Dynamics



## Third body potential

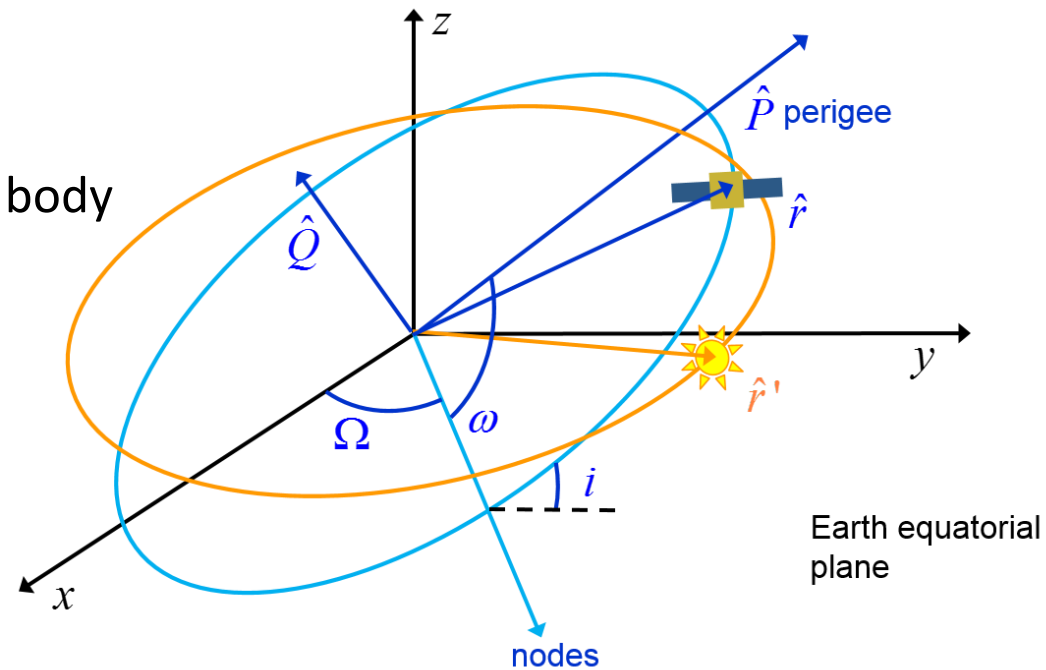
$$R_{3B}(r, r') = \frac{\mu'}{r'} \left( \left( 1 - 2 \frac{r}{r'} \cos \psi + \left( \frac{r}{r'} \right)^2 \right)^{-1/2} - \frac{r}{r'} \cos \psi \right)$$

$\mu'$  gravitational coefficient of the third body

$r'$  position vector of third body

$r$  position vector of satellite

$\psi$  angle between satellite and third body



## Third body potential

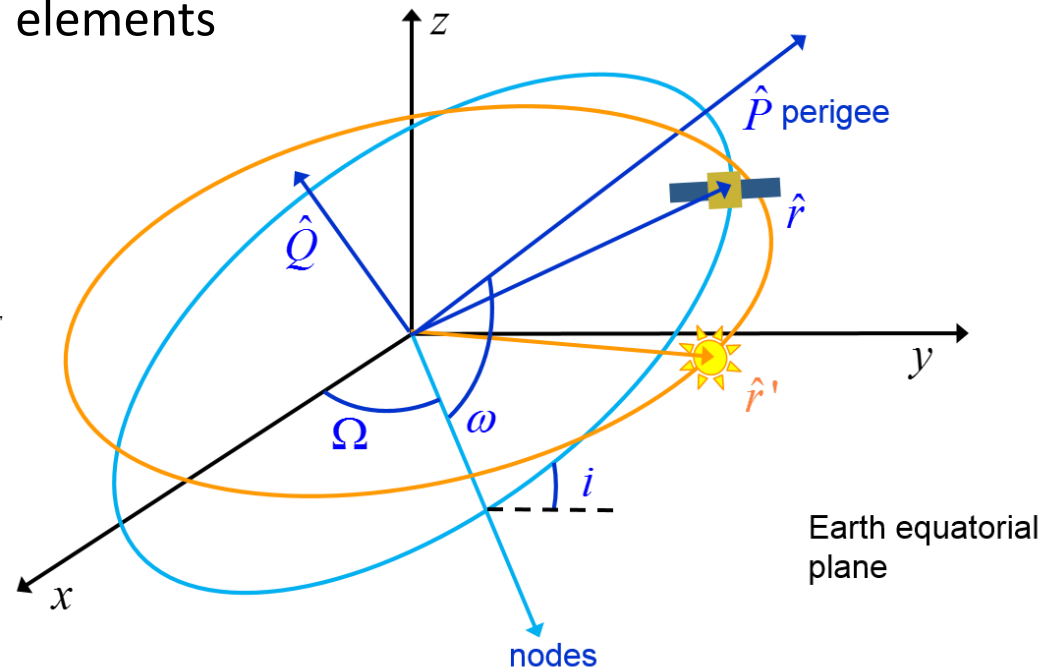
Third body potential in terms of:

- Ratio between orbit semi-major axis and distance of the third body  $\delta = \frac{a}{r'}$
- Orientation of orbit eccentricity vector with respect to third body  $A = \hat{P} \cdot \hat{r}'$
- Orientation of semi-latus rectum vector with respect to third body  $B = \hat{Q} \cdot \hat{r}'$
- Composition of rotation in orbital elements

$$\hat{P} = R_3(\Omega) R_1(i) R_3(\omega) \cdot [1 \ 0 \ 0]^T$$

$$\hat{Q} = R_3(\Omega) R_1(i) R_3(\omega + \pi/2) \cdot [1 \ 0 \ 0]^T$$

$$\hat{r}' = R_3(\Omega') R_1(i') R_3(\omega' + f') \cdot [1 \ 0 \ 0]^T$$



## Third body potential

Series expansion around  $\delta = 0$

$$R_{3B}(r, r') = \frac{\mu'}{r'} \sum_{k=2}^{\infty} \delta^k F_k(A, B, e, E)$$

$\mu'$  gravitational coefficient of the third body

$r'$  position vector of third body

$E$  eccentric anomaly

Average over one orbit revolution

$$\bar{R}_{3B}(r, r') = \frac{\mu'}{r'} \sum_{k=2}^{\infty} \delta^k \bar{F}_k(A, B, e)$$

$$\bar{F}_k(A, B, e) = \frac{1}{2\pi} \int_{-\pi}^{\pi} F_k(A, B, e, E) \overbrace{(1 - e \cos E)}^{dM} dE$$

Partial derivatives for Lagrange equations

$$A(\Omega, i, \omega, \Omega', i', u')$$

$$B(\Omega, i, \omega, \Omega', i', u')$$

$$\bar{F}_k(A, B, e)$$



$$\frac{\partial \bar{F}_k}{\partial \Omega} = \frac{\partial \bar{F}_k}{\partial A} \frac{\partial A}{\partial \Omega} + \frac{\partial \bar{F}_k}{\partial B} \frac{\partial B}{\partial \Omega}$$

$$\frac{\partial \bar{F}_k}{\partial i} = \frac{\partial \bar{F}_k}{\partial A} \frac{\partial A}{\partial i} + \frac{\partial \bar{F}_k}{\partial B} \frac{\partial B}{\partial i}$$

$$\frac{\partial \bar{F}_k}{\partial \omega} = \frac{\partial \bar{F}_k}{\partial A} \frac{\partial A}{\partial \omega} + \frac{\partial \bar{F}_k}{\partial B} \frac{\partial B}{\partial \omega}$$

$$\frac{\partial \bar{F}_k}{\partial a} = \frac{k}{a} F_k$$

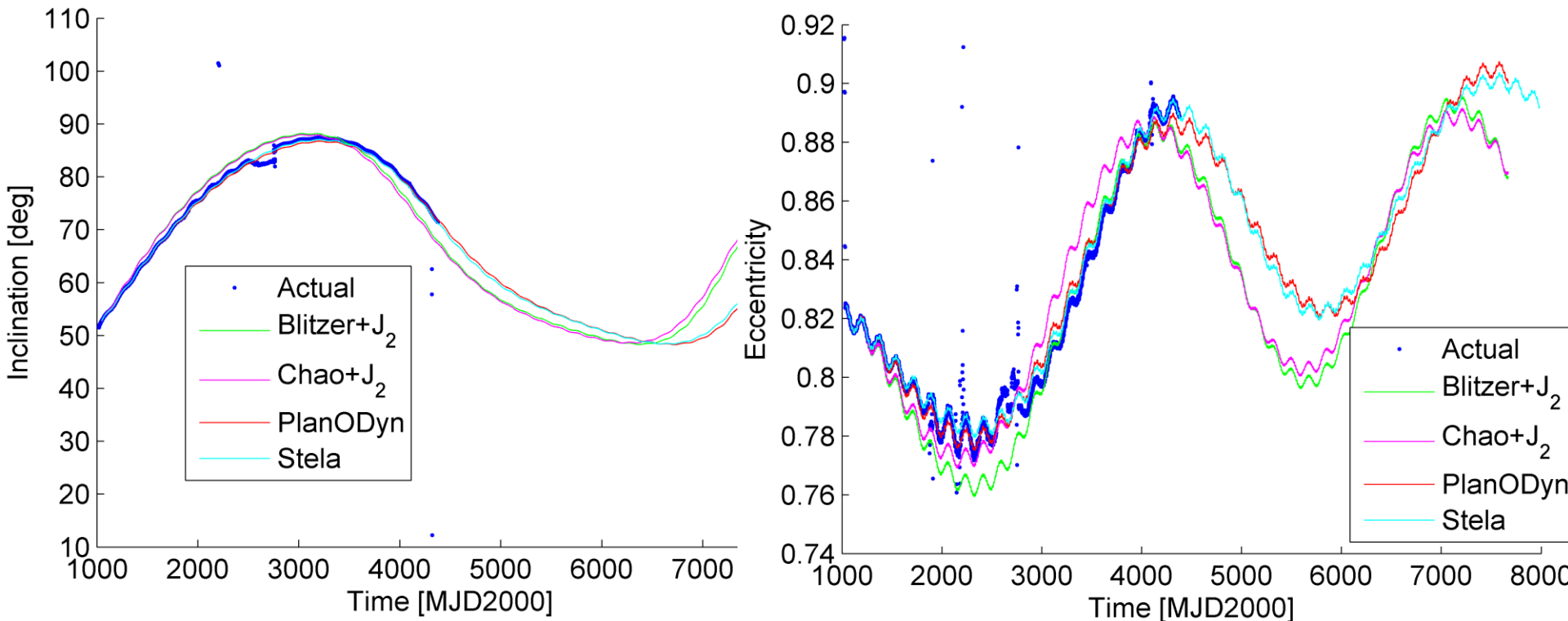
$$\frac{\partial \bar{F}_k}{\partial e}$$

► Kaufman and Dasenbrock, NASA report, 1979



## Order of the luni-solar potential expansion

Third-body perturbing potential of the Moon at least up to the fourth order of the power expansion



- ▶ *Blitzer L., Handbook of Orbital Perturbations, Astronautics, 1970*
- ▶ *Chao-Chun G. C., Applied Orbit Perturbation and Maintenance, 2005*

# Dynamical model

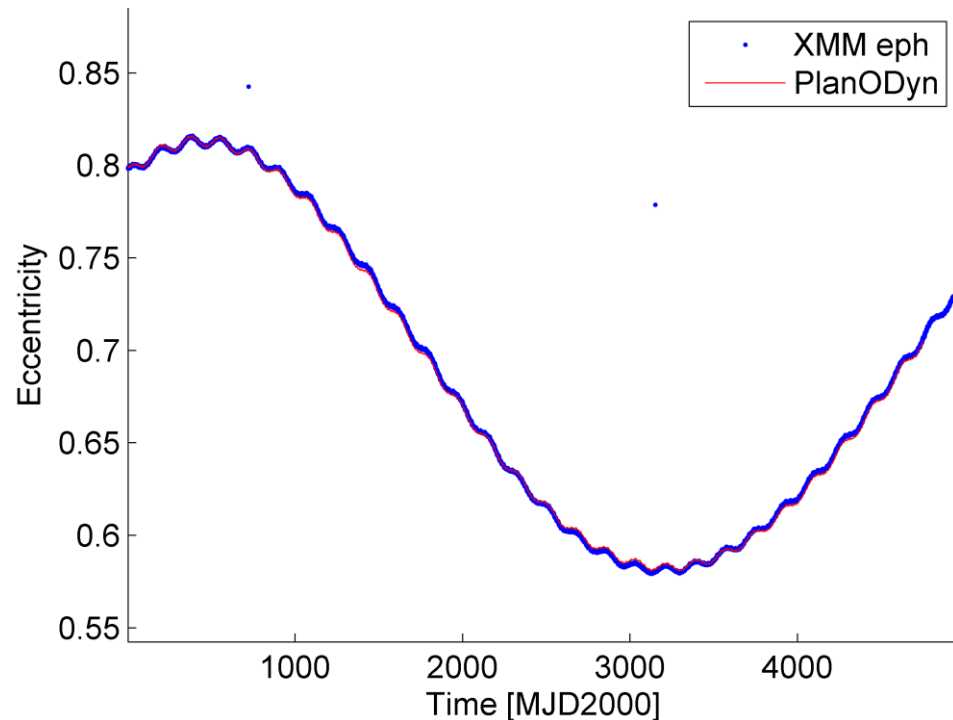
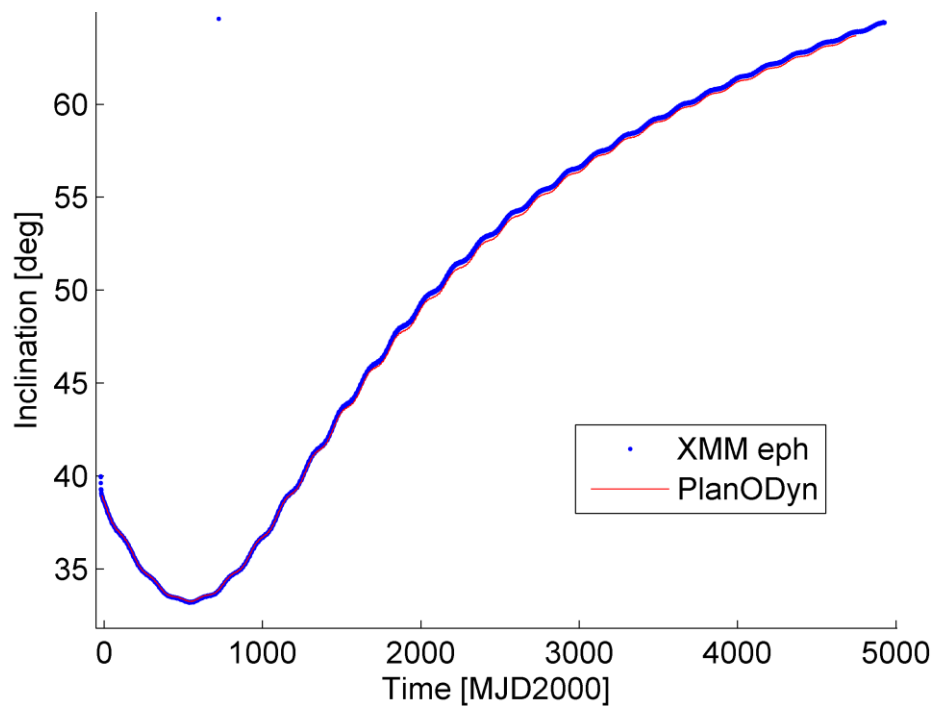
Validation: XMM Newton trajectory

Propagation time: 1999/12/15 to 2013/01/01

Initial Keplerian elements from ESA on 1999/12/15 at 15:00:

$a = 67045$  km,  $e = 0.7951$ ,  $i = 0.67988$  rad,  $\Omega = 4.1192$  rad,  $\omega = 0.99259$  rad

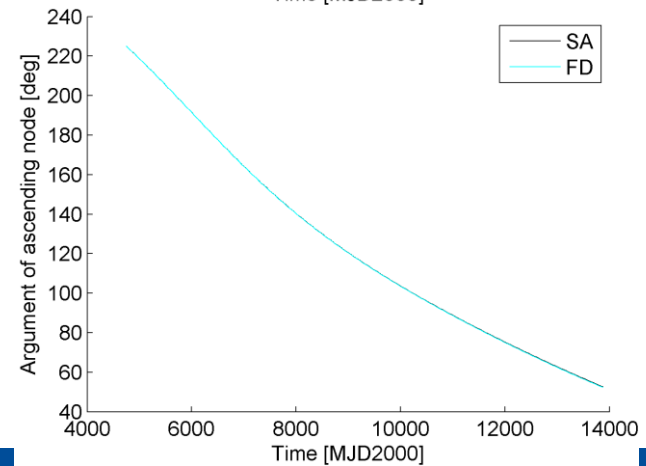
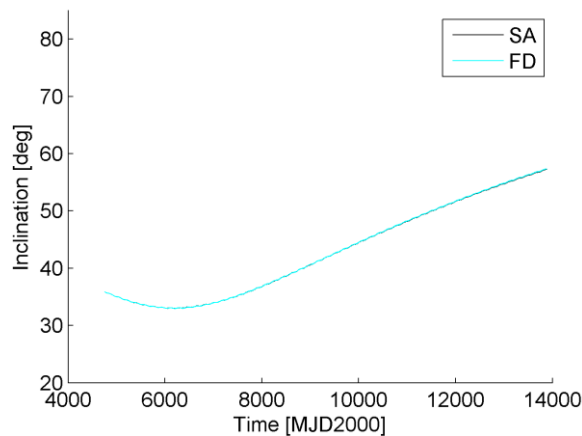
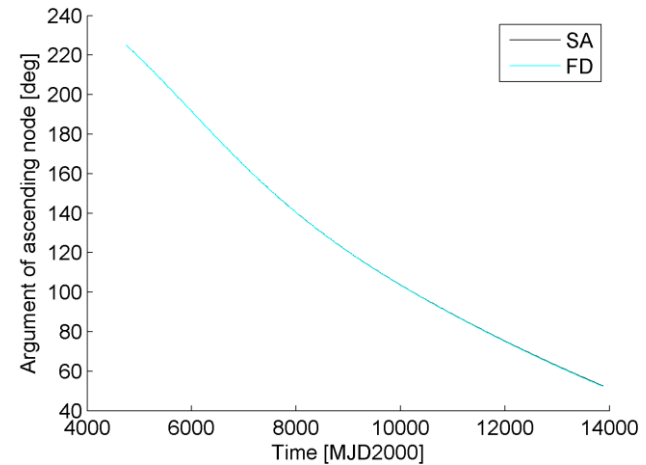
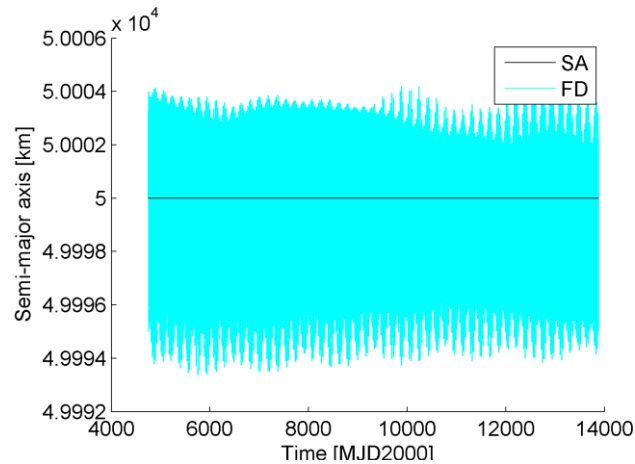
System: Earth centred, equatorial J2000



# Dynamical model

## Mean vs high fidelity dynamics

Initial Keplerian elements:  $a = 50000$  km,  $e = 0.5$ ,  $i = 35.88$  deg,  $\Omega = 225$  deg,  $\omega = 63.9$  deg



## Finite-time Lyapunov exponent calculation

$$\mathbf{y}' = \mathbf{f}(x, \mathbf{y}) \quad \mathbf{y}(x_0) = \mathbf{y}_0, \quad a \leq x \leq b$$

Linearise locally

$$\frac{d}{dt} \Delta \mathbf{y} = J \cdot \Delta \mathbf{y}$$

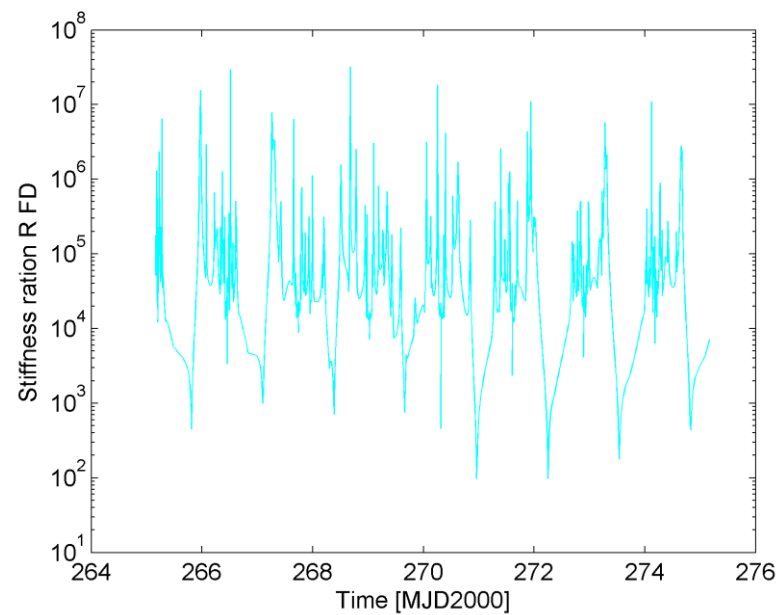
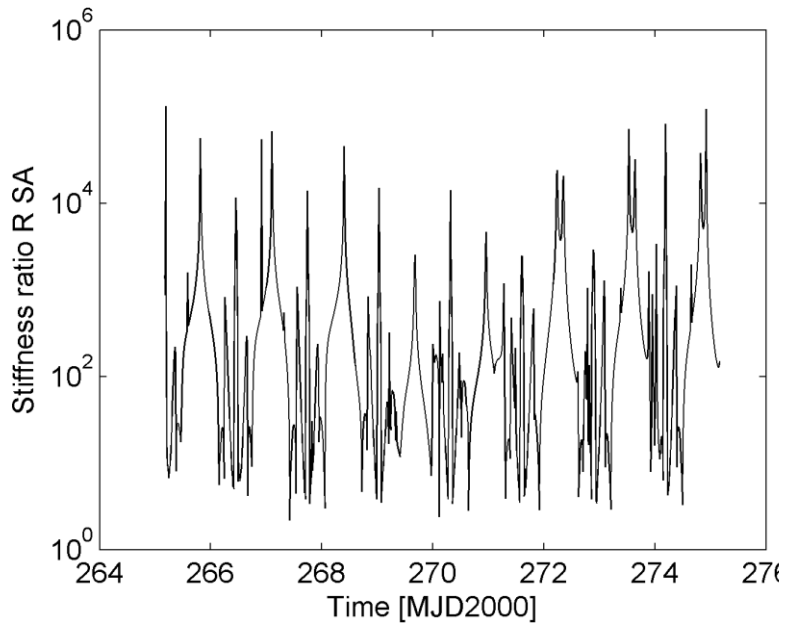
Jacobian matrix satisfy the following conditions

- For Stability:  $\operatorname{Re}[\lambda_s(x)] < 0 \quad s = 1, 2, 3, \dots, m$
- For Stiffness: stiffness ratio =  $R = \frac{\max \|\operatorname{Re}[\lambda_s(x)]\|}{\min \|\operatorname{Re}[\lambda_s(x)]\|} \gg 1 \quad s = 1, 2, 3, \dots, m$

$\lambda_s(x)$  are the eigenvalues of the Jacobian of the system

# Analytical interpretation

## Toward finite-time Lyapunov exponent calculation



*High fidelity dynamics orders of magnitude more stiff than semi-analytic ( $J_2$  and Luni-Solar)*



# ANALYTICAL INTERPRETATION

## Third-body double averaged potential

Double averaging over one orbit revolution of the s/c and one orbit evolution of the perturbing body (either Sun or Moon) around the Earth

$$\bar{\bar{R}}_{3B}(r, r') = \frac{\mu'}{r'} \sum_{k=2}^{\infty} \delta^k \bar{\bar{F}}_k(e, i, \Omega, \omega, i')$$

Earth's centred equatorial reference system.

Same approach as El'yasberg (and Kozai) with some improvements:

- Avoid simplification that Moon and Sun orbit on the same plane (very important for precise orbit evolution)
- Facilitate the introduction of the effect of the zonal harmonics

$$\bar{\bar{F}}_k(e, i, \Delta\Omega, \omega, i') = \frac{1}{2\pi} \int_0^{2\pi} \bar{F}_k(A(\Omega, i, \omega, \Omega', i', \omega' + f'), B(\Omega, i, \omega, \Omega', i', \omega' + f'), e) df'$$

► *Kozai, Secular Perturbations of Asteroids with High Inclination and Eccentricity, 1962*

► *El'yasberg, Introduction to the theory of flight of artificial Earth satellites - translated, 1967*

## Third body Kozai theory

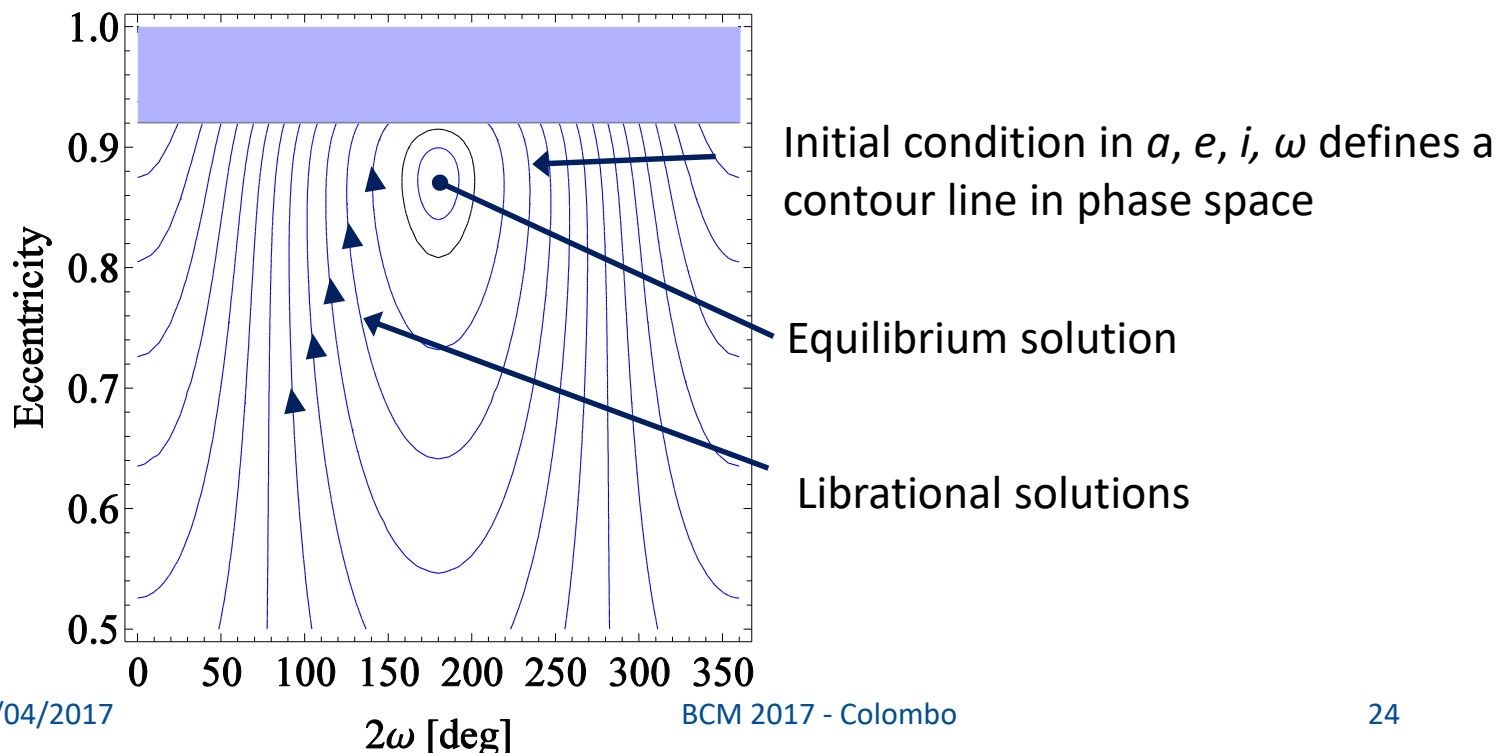
- Delaunay's transformation
- Time-independent Hamiltonian
- Double averaged potential
- Rotating reference system

$$W\left(\frac{a}{a'}, \Theta, e, 2\omega\right) = \text{const} \quad \Theta = (1 - e^2) \cos i^2$$

$$\bar{\bar{F}}_{3\text{Bsys},2}(e, \omega, i) = \frac{1}{32} \left( (2 + 3e^2)(1 + 3\cos(2i)) + 30e^2 \cos(2\omega) \sin^2 i \right)$$

► *Kozai, Secular Perturbations of Asteroids with High Inclination and Eccentricity, 1962*

► *El'yasberg, Introduction to the theory of flight of artificial Earth satellites - translated, 1967*





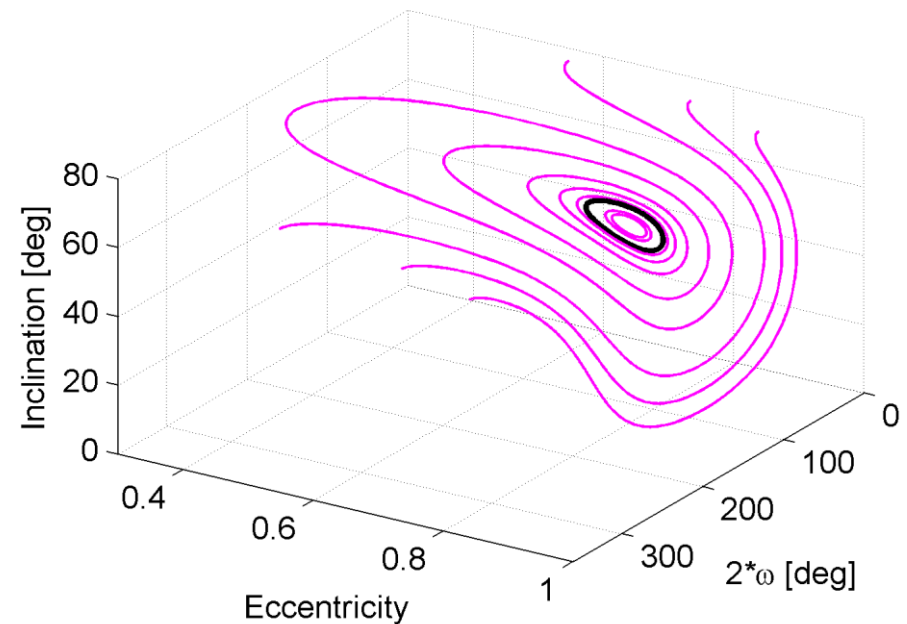
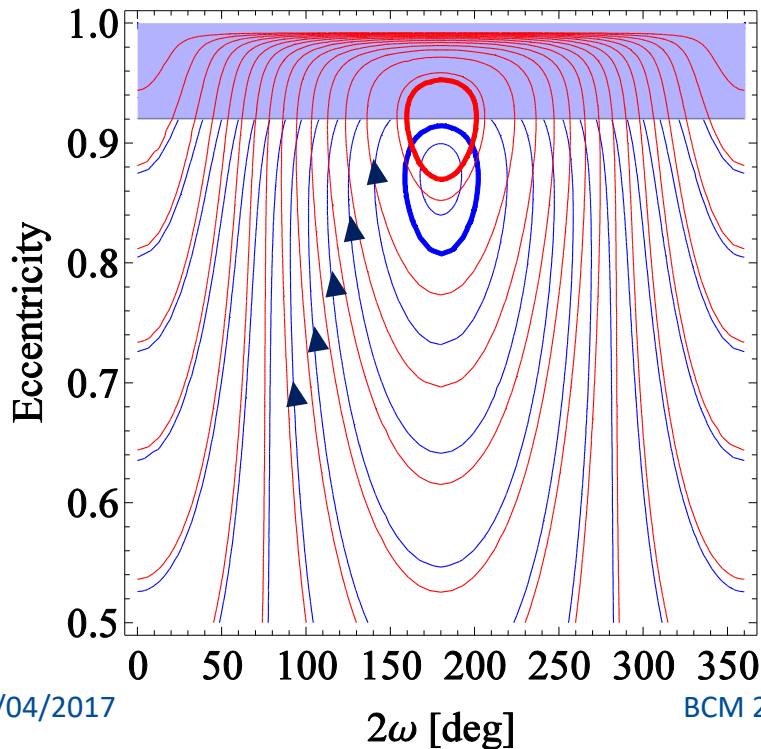
# Analytical interpretation

## Third body Kozai theory

$$W\left(\frac{a}{a'}, \Theta, e, 2\omega\right) = \text{const} \quad \Theta = (1 - e^2) \cos i^2$$

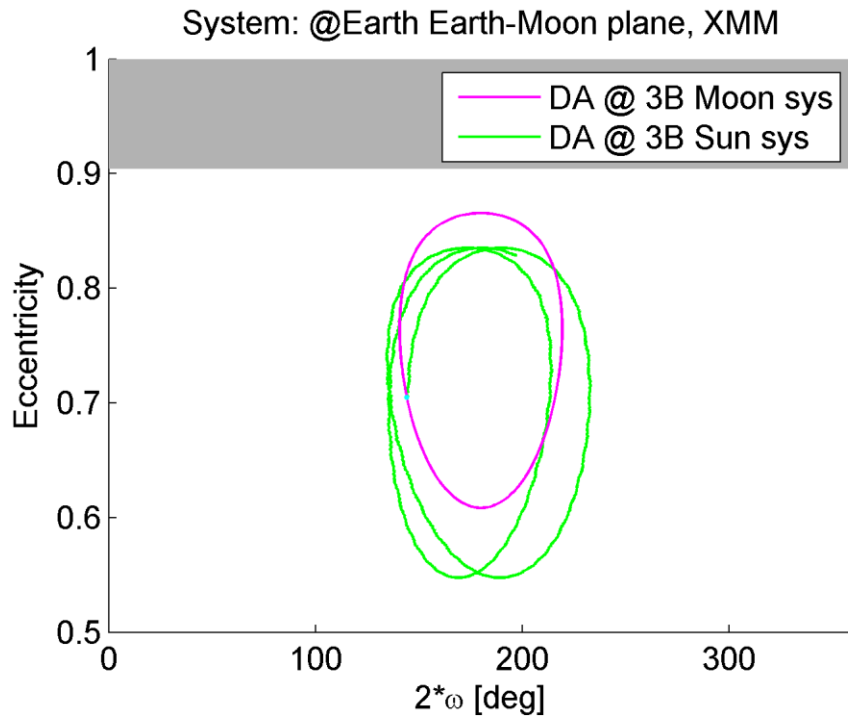
$a/a'$  increases

$\ominus$  decreases



# Analytical interpretation

## Third-body double averaged potential



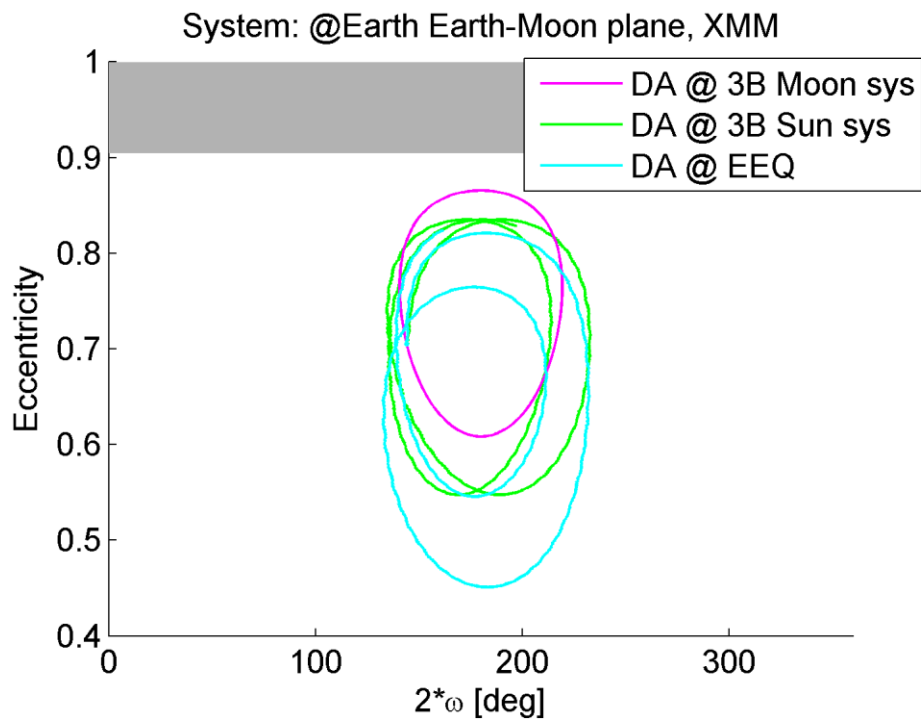
Reference system for figure:

- x-y plane lays on the Moon orbital plane
- z-axis in the direction of the Moon angular momentum

Kozai, El'yasberg:  $\bar{\bar{F}}_{3\text{Bsys},2}(e, \omega, i)$

# Analytical interpretation

## Third-body double averaged potential



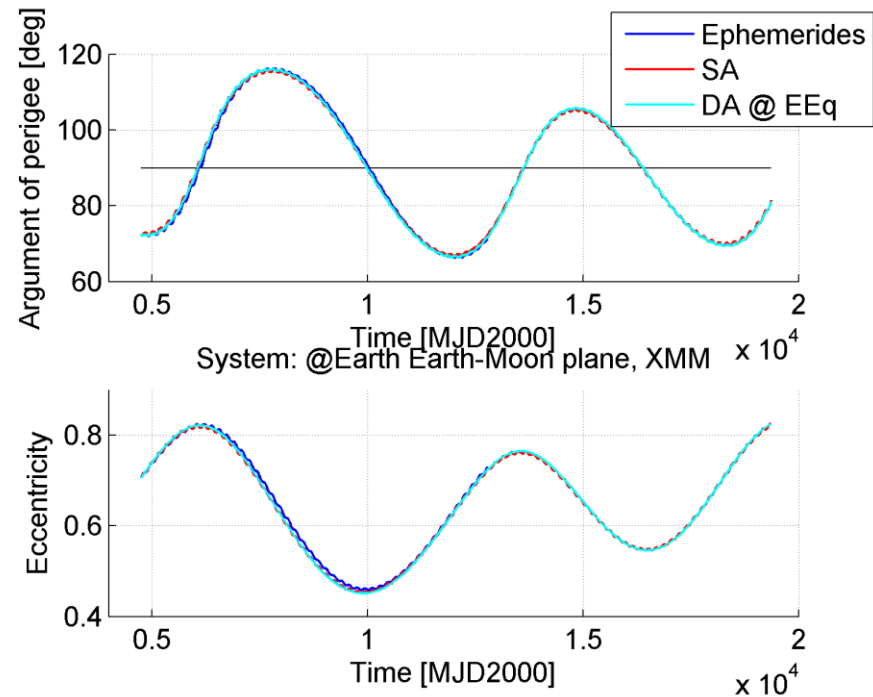
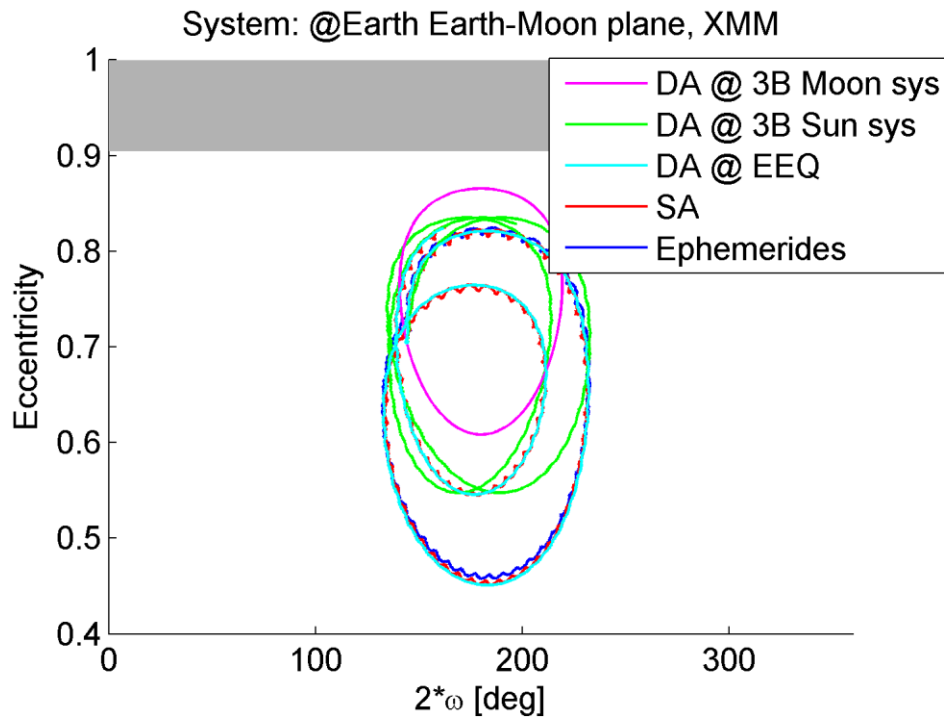
Kozai, El'yasberg:  $\bar{\bar{F}}_{3\text{Bsys},2}(e, \omega, i)$



$\bar{\bar{F}}_k(e, i, \Delta\Omega, \omega, i')$

# Analytical interpretation

## Third-body double averaged potential



Kozai, El'yasberg:  $\bar{\bar{F}}_{3\text{Bsys},2}(e, \omega, i)$



$\bar{\bar{F}}_k(e, i, \Delta\Omega, \omega, i')$

Non autonomous loops in the  $e$ - $\omega$  phase space!



# DYNAMICAL MAPS

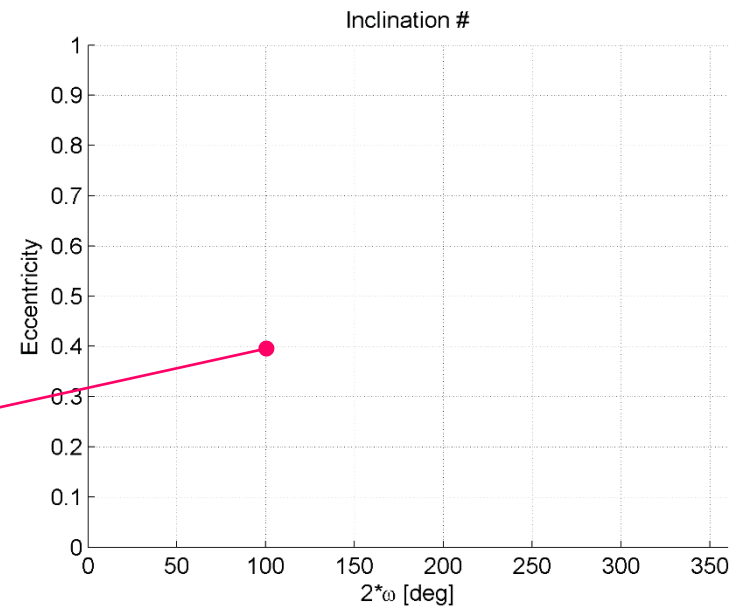
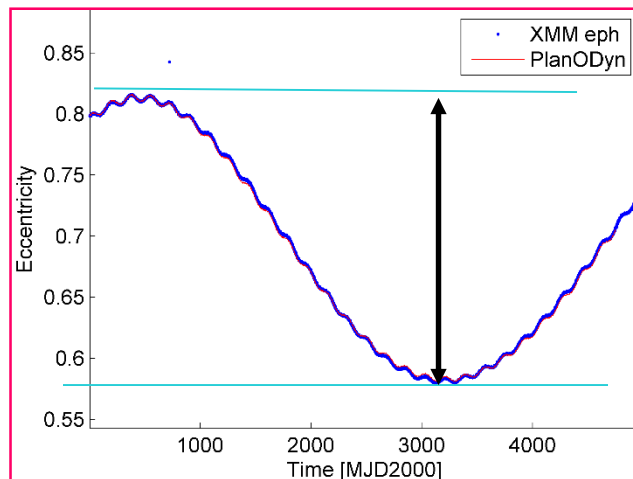
## Long-term orbit evolution

1. Grid in inclination, eccentricity and  $\omega$  (Moon plane reference system)
2. Propagation over  $\pm 30$  years with PlanODyn
3. Evaluate

$$\Delta e = e_{\max} - e_{\min}$$

$$e_{\max} = \max_t e(t) \quad t \in \left[ -\Delta t_{\text{graveyard}} \quad +\Delta t_{\text{graveyard}} \right]$$

$$e_{\min} = \min_t e(t) \quad t \in \left[ -\Delta t_{\text{graveyard}} \quad +\Delta t_{\text{graveyard}} \right]$$



## Long-term orbit evolution

### Luni-solar + zonal $\Delta e$ maps

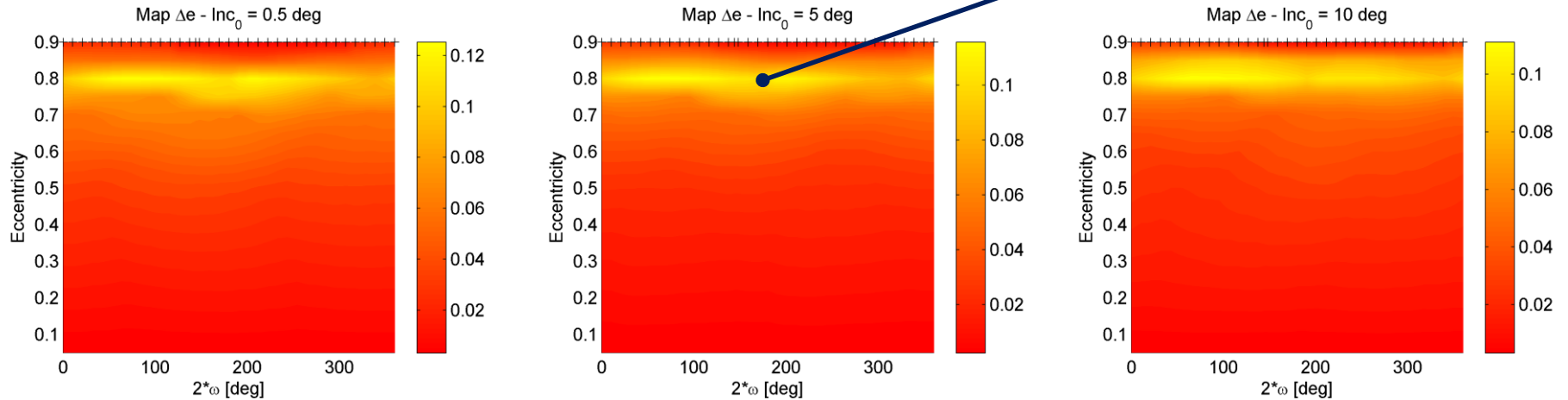
- Semi-major axis equal to 67045.39 km (XMM Newton's orbit)
- Different values of initial inclination with respect to the orbiting plane of the Moon
- Here: fixed  $t_0$  and fixed  $\Omega_0$  to analyse one loop in the phase space but different  $\Omega_0$  can be taken into account with  $2\omega + \Omega_0$

► Colombo "Long-Term Evolution of Highly-Elliptical Orbits: Luni-Solar Perturbation Effects for Stability and Re-Entry,"  
25<sup>th</sup> AAS/AIAA Space Flight Mechanics Meeting, 2015

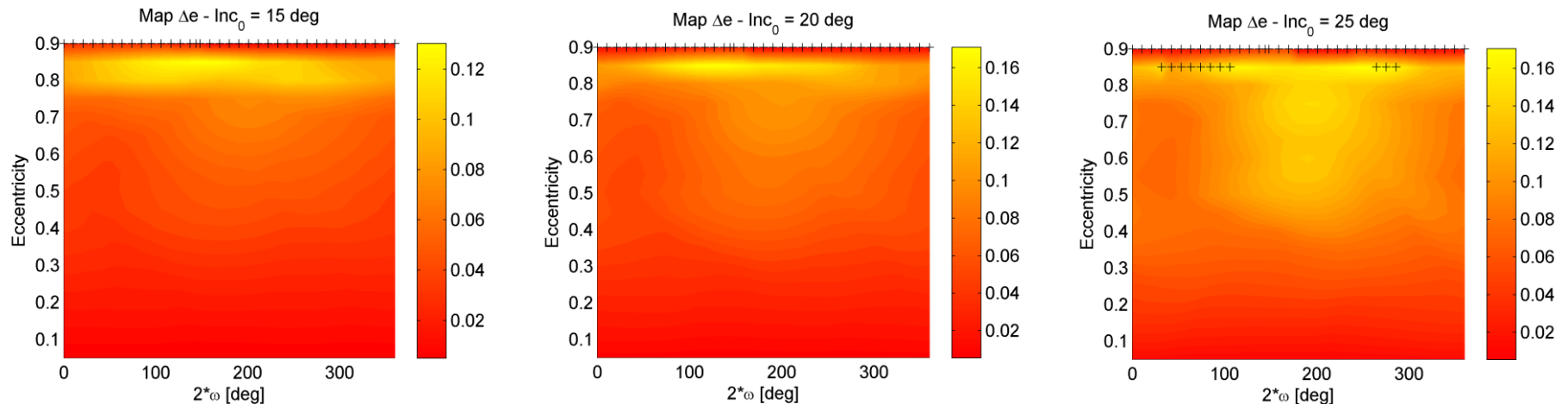
# Dynamical maps

## Long-term orbit evolution

Higher  $\Delta e$  variation for high initial  $e$



## Rotational solutions in $\omega$

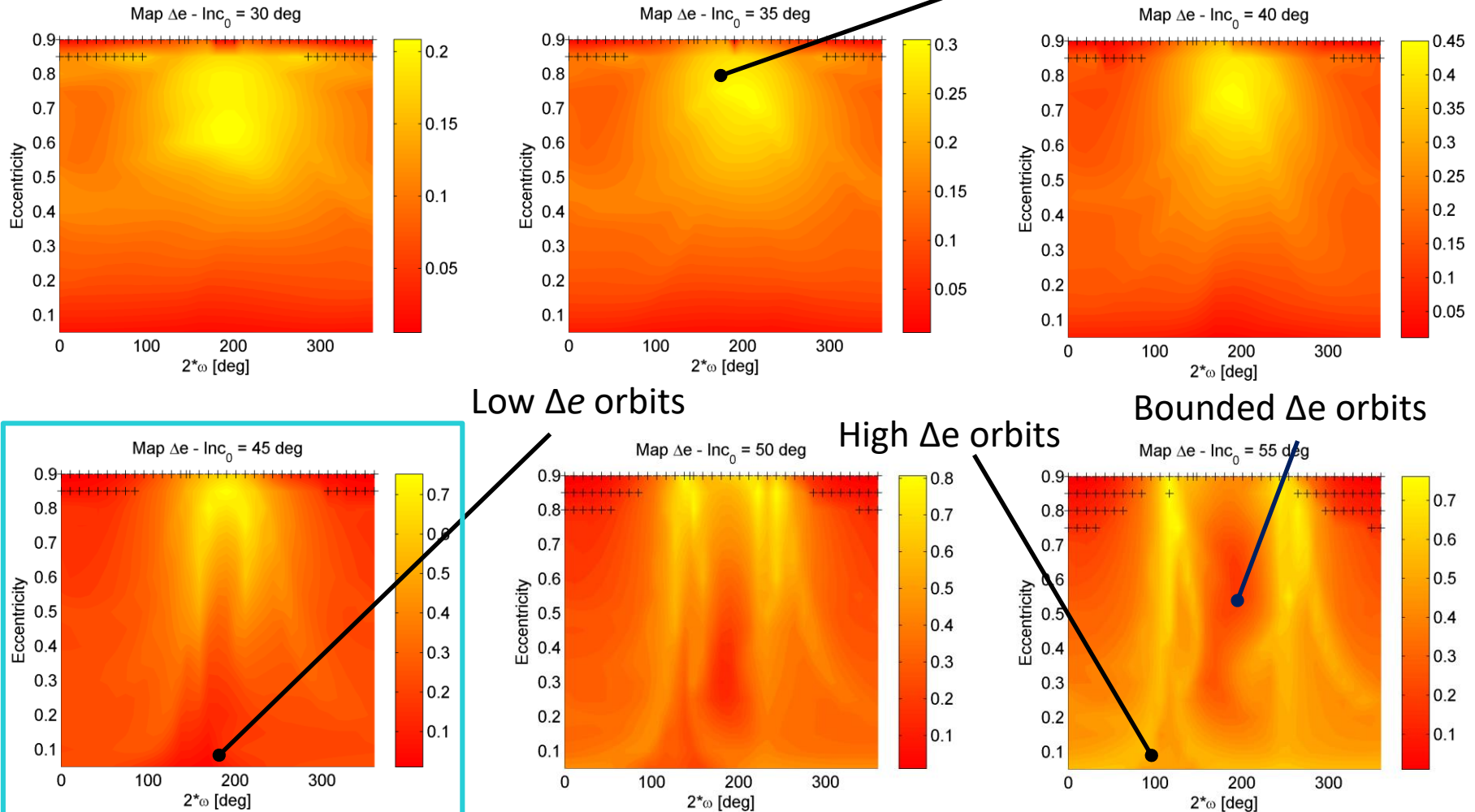




# Dynamical maps

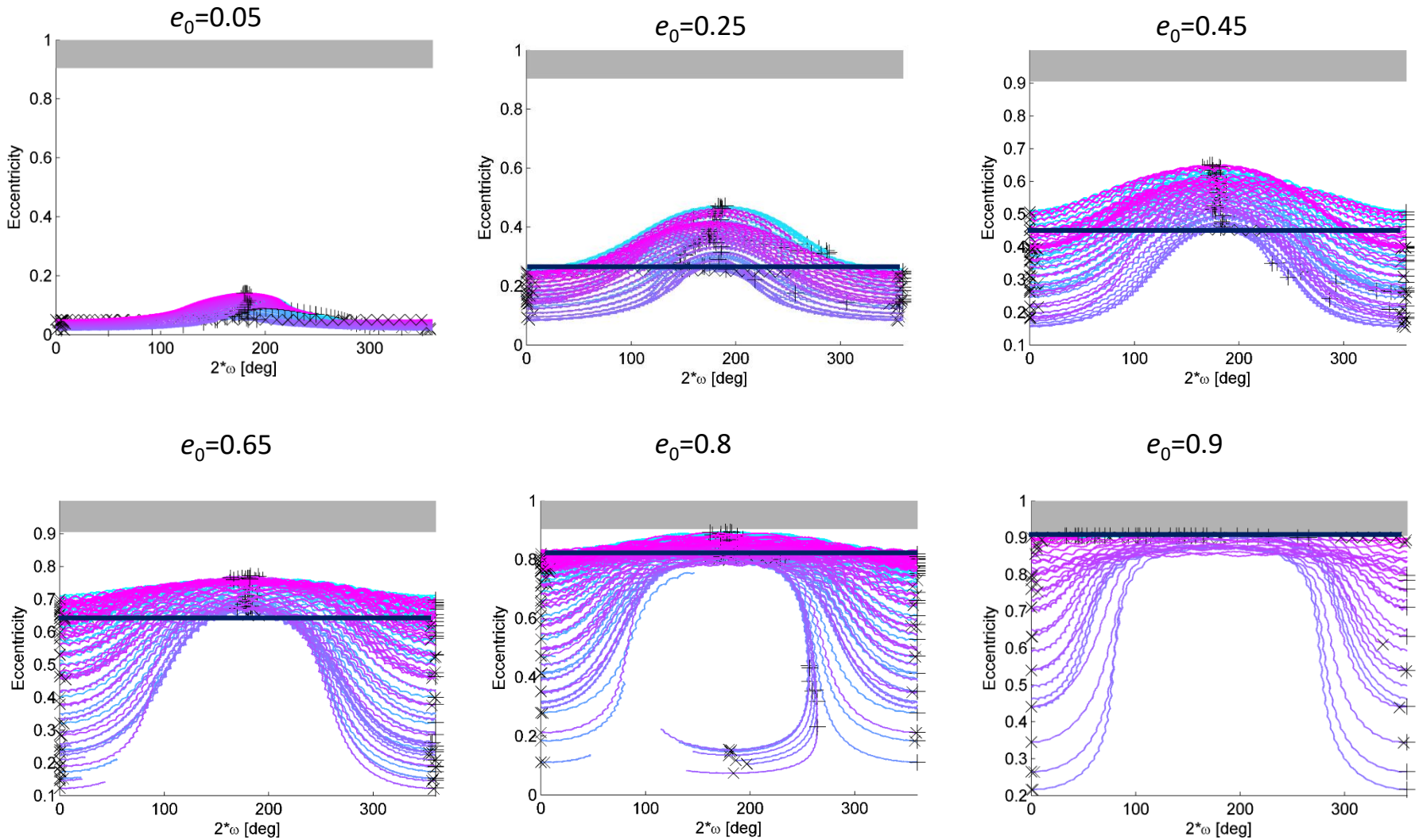
## Long-term orbit evolution

Higher  $\Delta e$  variation for high initial  $e$



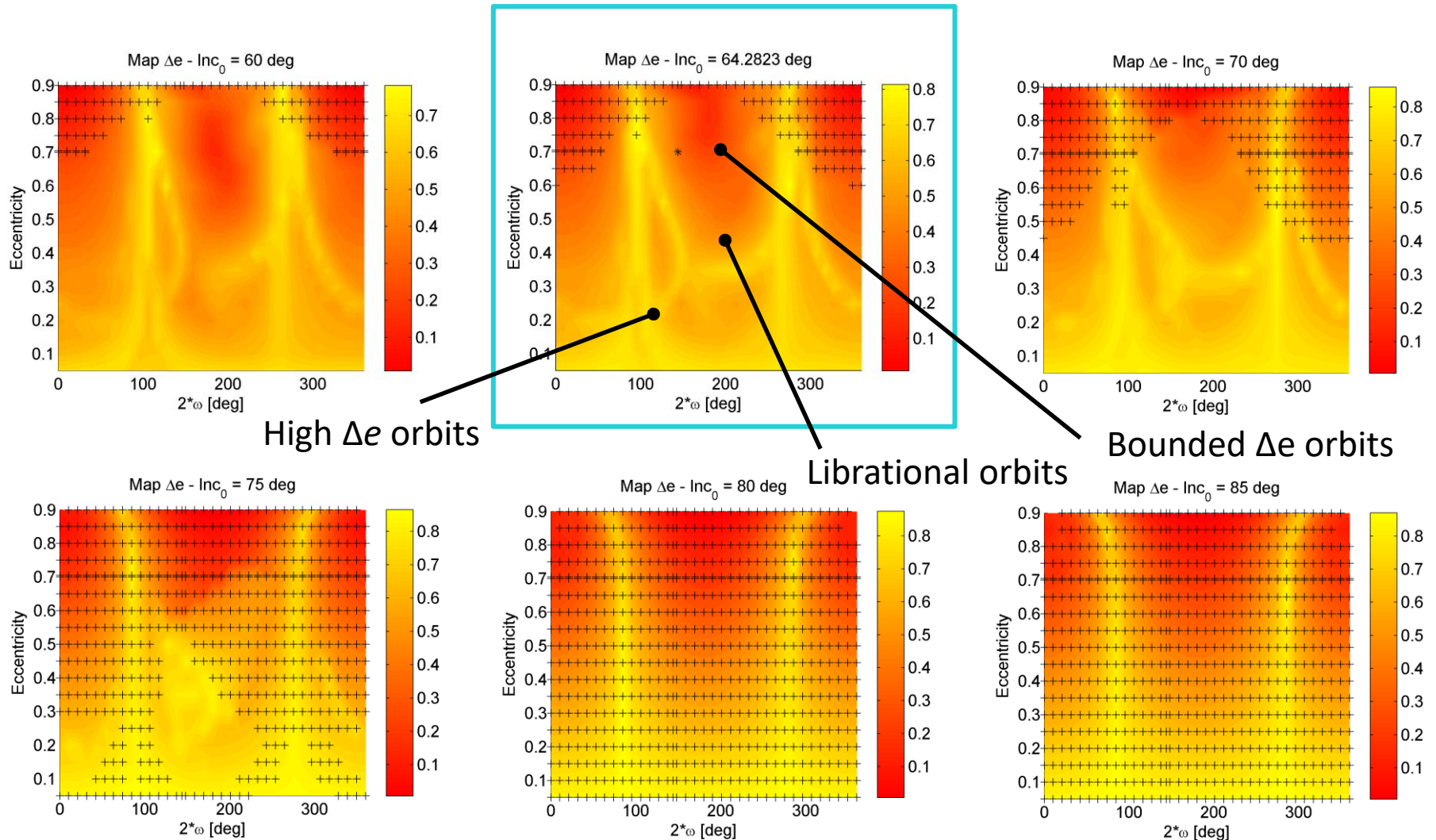
# Dynamical maps

Long-term orbit evolution - Initial inclination 45 degrees



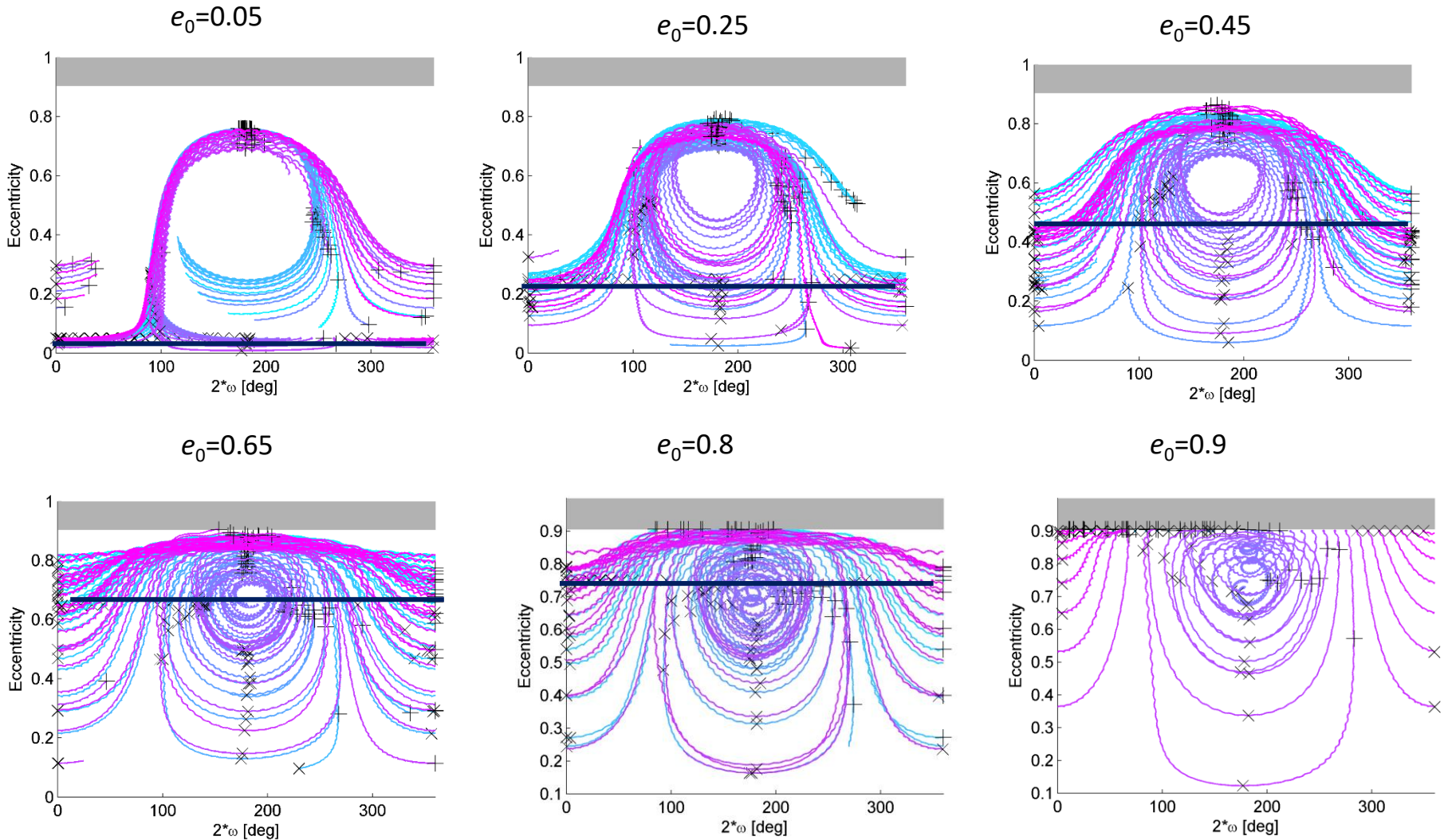
# Dynamical maps

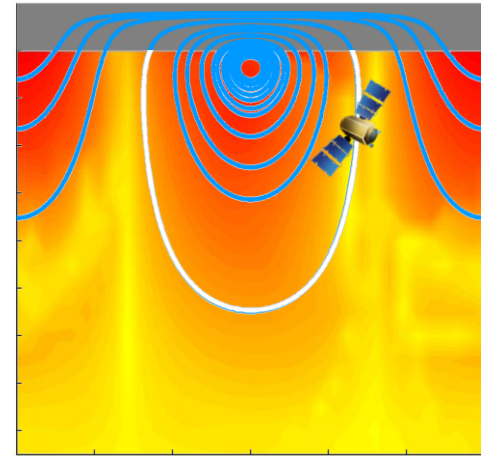
## Long-term orbit evolution



# Dynamical maps

Long-term orbit evolution - Initial inclination 64.28 degrees





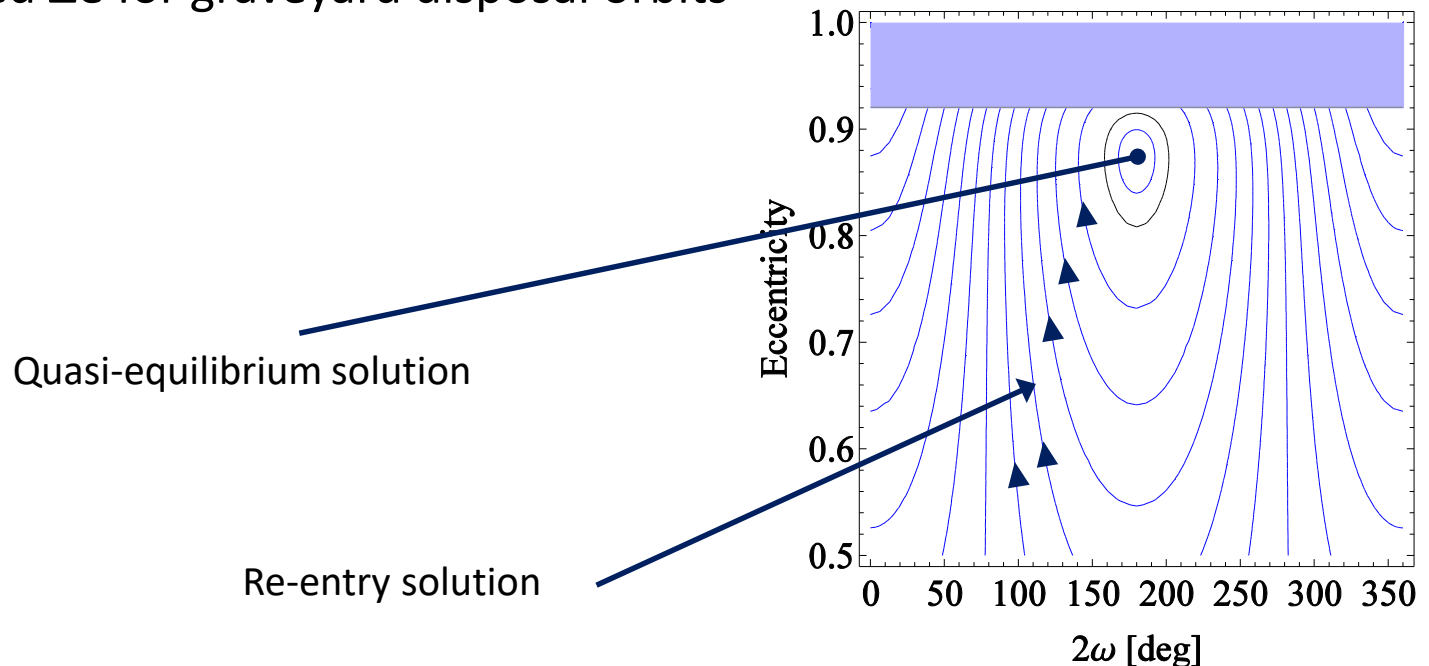
Design of disposal manoeuvres

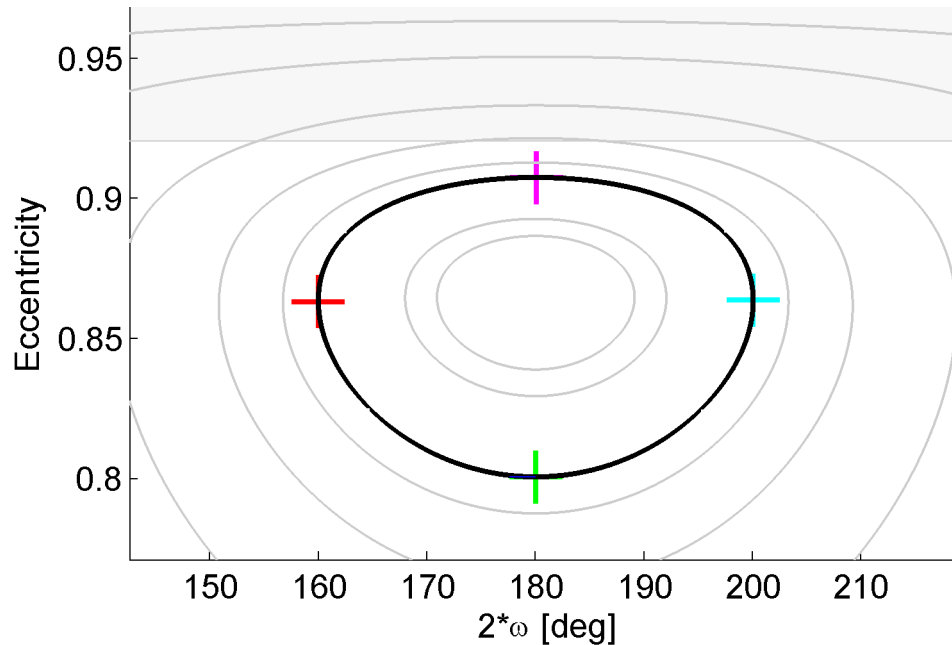
# ENGINEERING PERTURBATION EFFECTS

## Design disposal manoeuvre in the phase space

### Design manoeuvre in the phase space

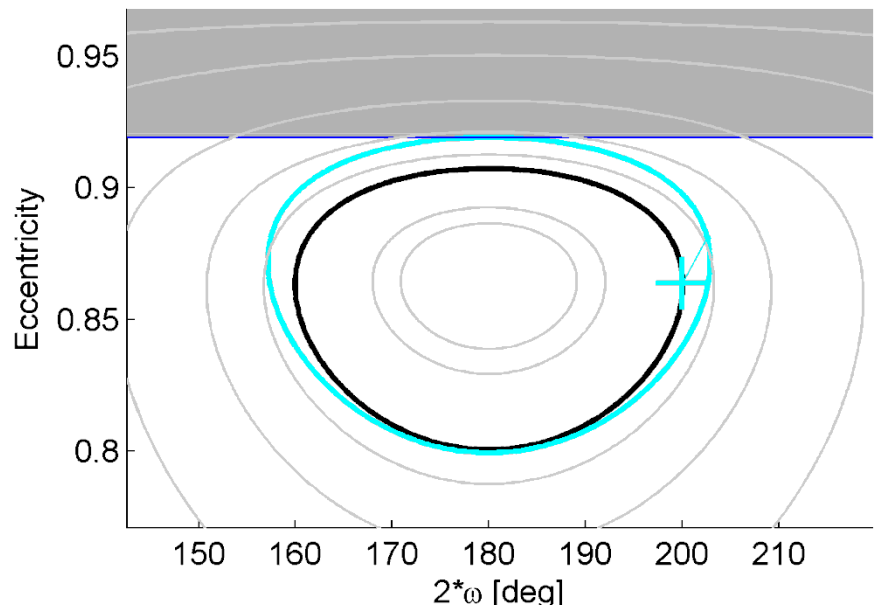
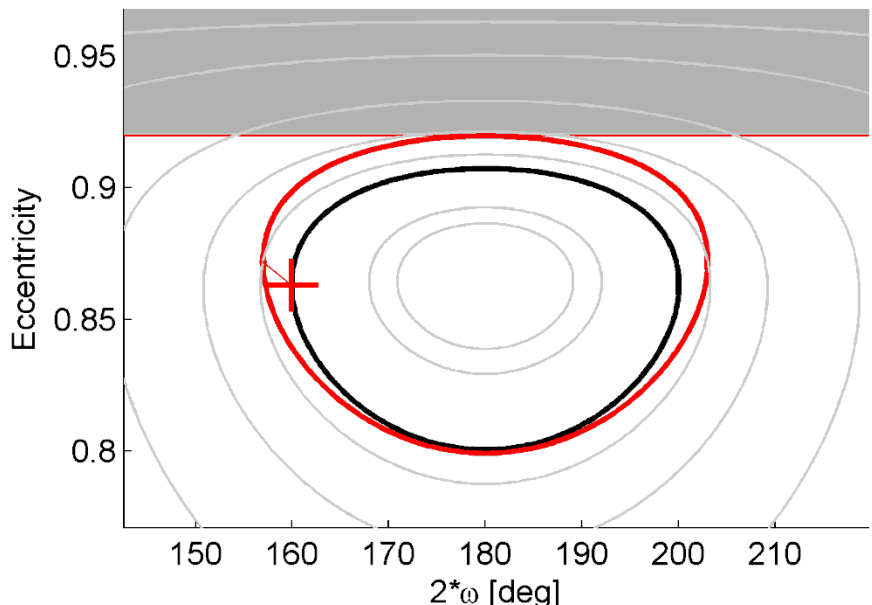
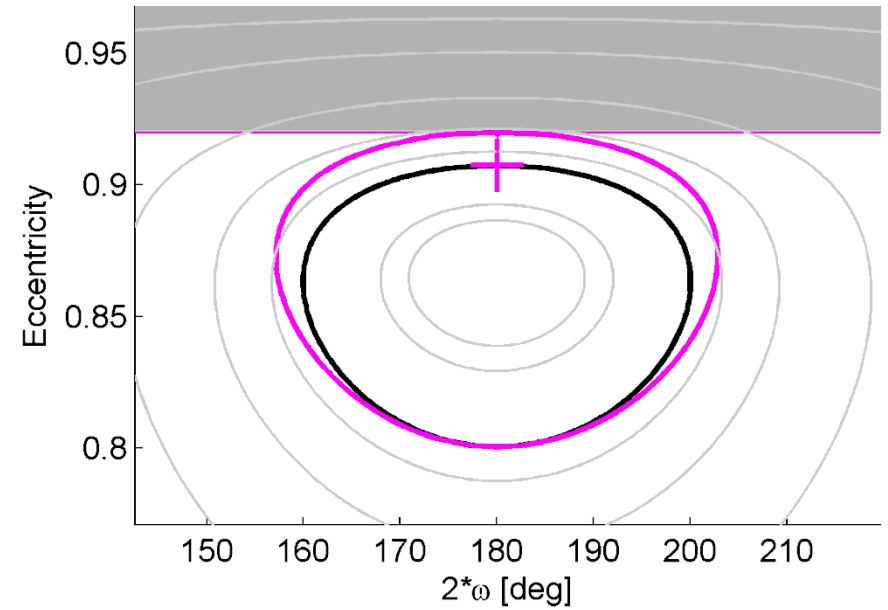
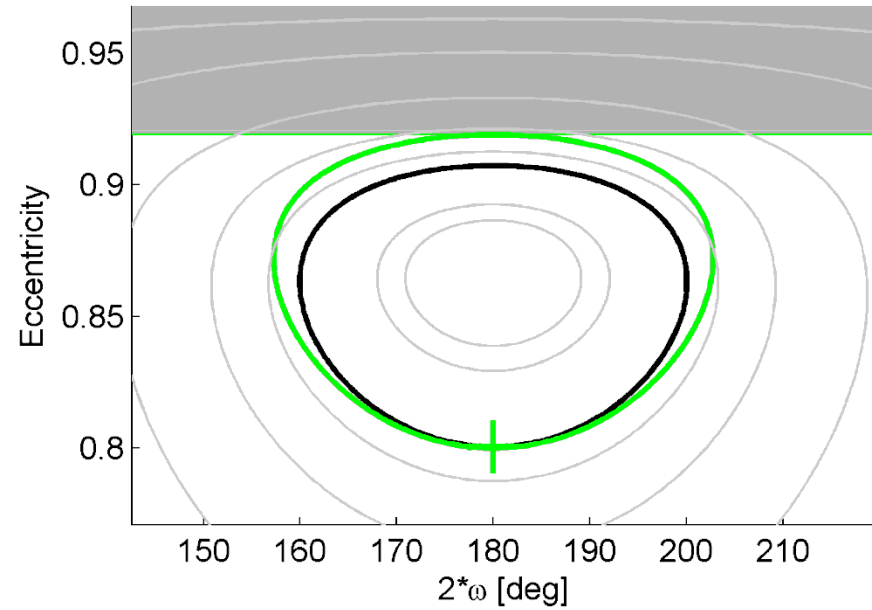
- Re-entry transfer on trajectories in the phase space to reach  $e_{\text{crit}} = 1 - (R_{\text{Earth}} + h_{p, \text{drag}}) / a$   
Maximum  $\Delta e$  exploitable for re-entry or free orbit change
- Graveyard: transfer to quasi-stable point in the phase space  
Bounded  $\Delta e$  for graveyard disposal orbits





- Optimisation 
$$\min_{\{\Delta v, \delta, \beta, f\}} \Delta v \quad C : \max [e(t)] = e_{\text{crit}}$$
- Multi-start method plus local constrained optimisation
- Gauss planetary eqs. for finite differences to compute change in orbital elements
- Orbit evolution computed with double average eqs.

# Preliminary analysis Earth re-entry





# Engineering the perturbation effects

Design disposal manoeuvre in the phase space

Single manoeuvre

$$\Delta \mathbf{v} = \Delta v \begin{bmatrix} \cos \alpha \cos \beta \\ \sin \alpha \cos \beta \\ \sin \beta \end{bmatrix}$$



$$\Delta kep = G(kep(t_m), f_m, \Delta \mathbf{v})$$

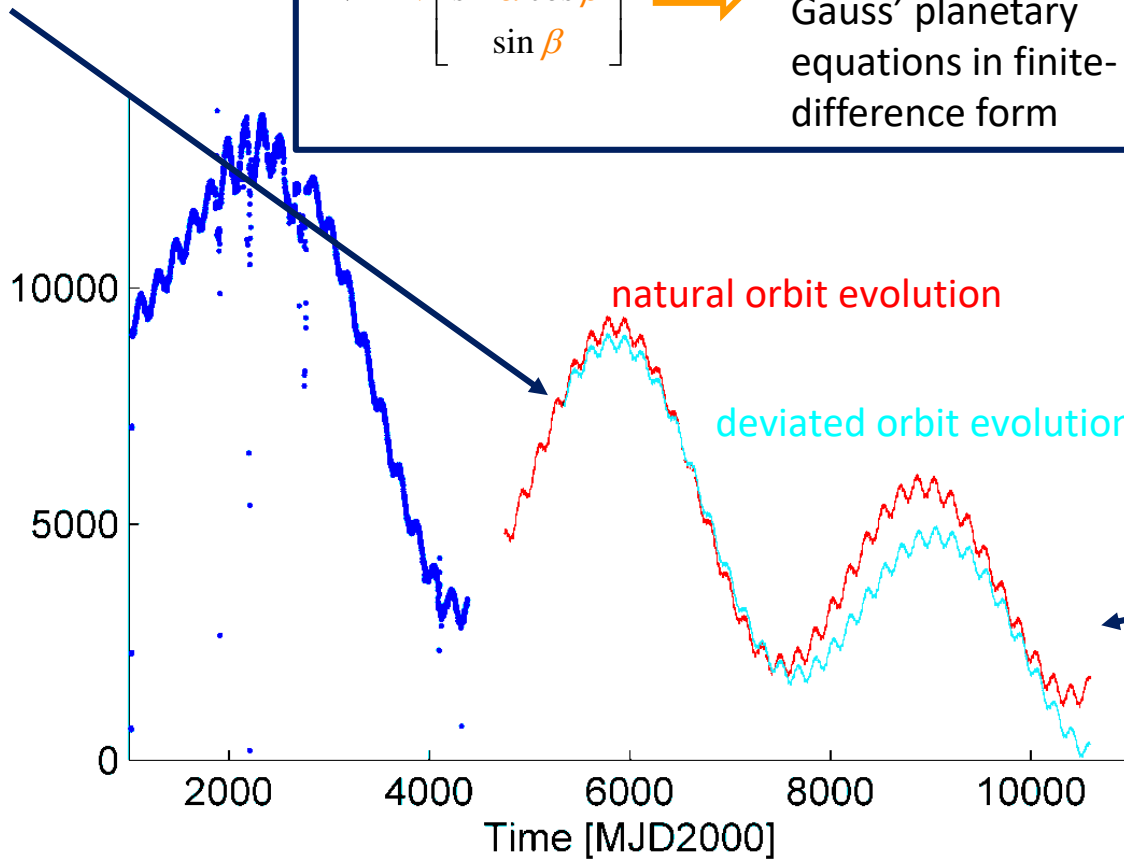
Gauss' planetary equations in finite-difference form



$$kep_d = kep(t_m) + \Delta kep$$

Deviated condition

Perigee altitude [km]



Minimum perigee within selected time interval for disposal

$$h_{p,\min} = \min_{t \in \Delta t_{\text{disposal}}} h_p(t)$$

► Colombo, Letizia, Alessi, Landgraf, 24th AAS/AIAA 2014



# APPLICATIONS



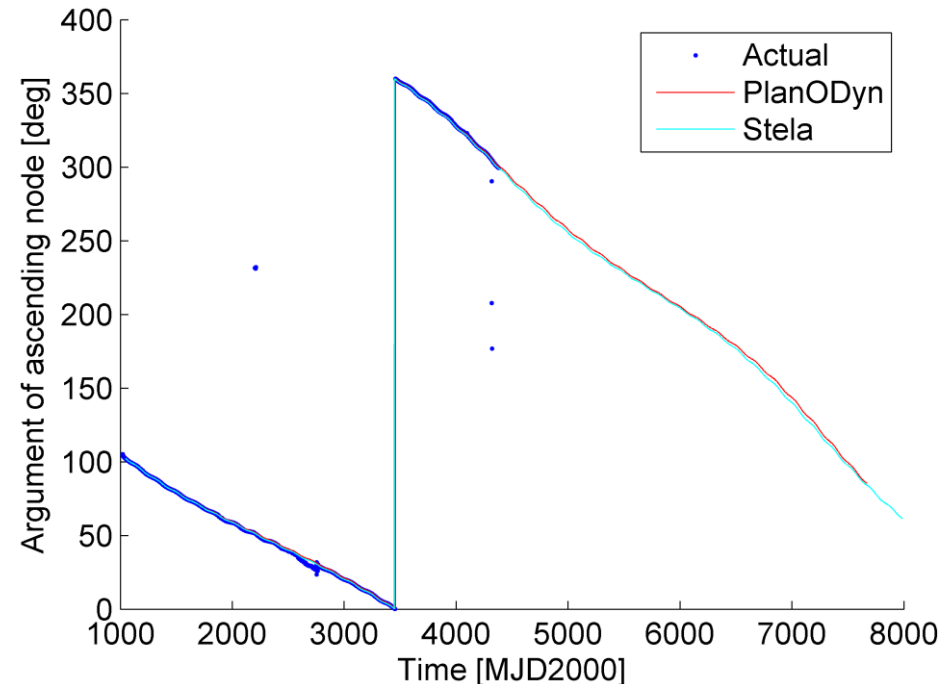
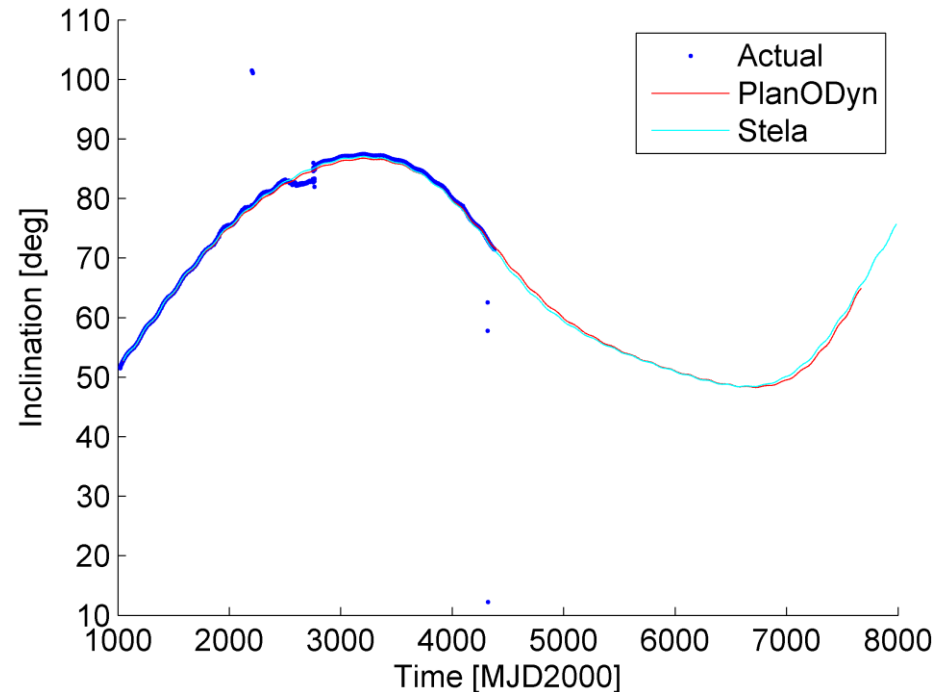
Integral: gamma-ray observatory

ESA's Integral observatory is able to detect gamma-ray bursts, the most energetic phenomena in the Universe

## Operational orbit

### Mission scenario

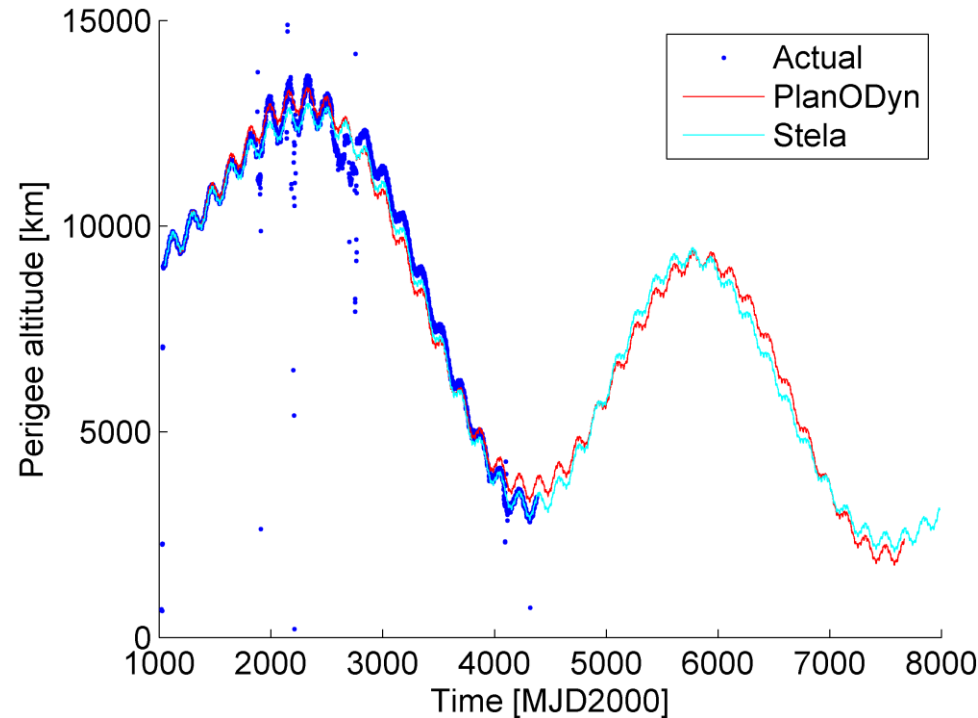
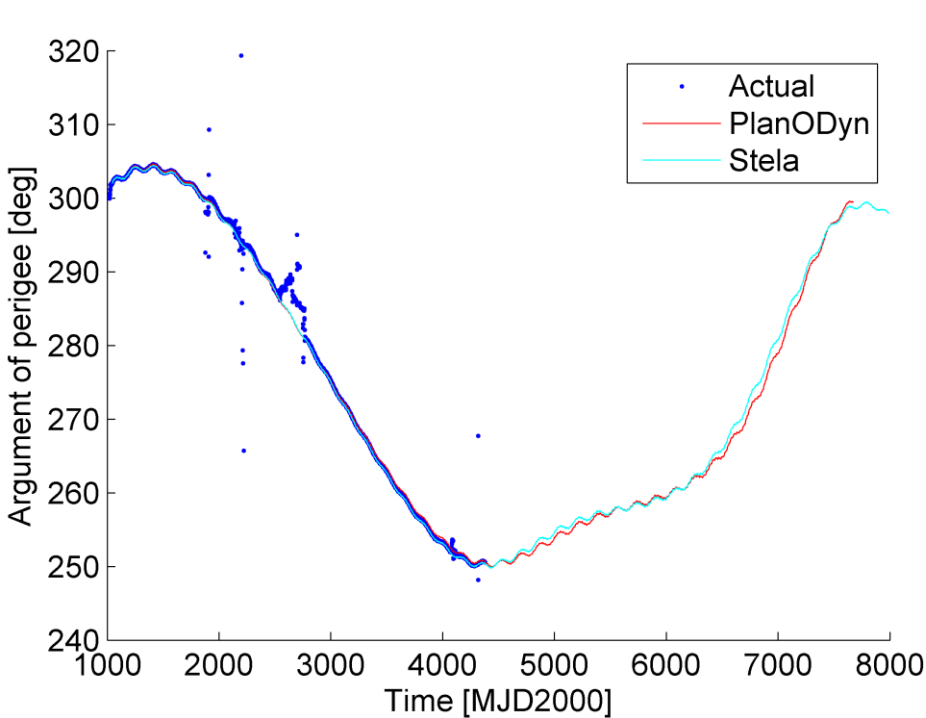
- Propagation time: 2002/11/13 to 2021/01/01
- Initial Keplerian elements from Horizon NASA on 2002/11/13 at 00:00:  
 $a = 87736$  km,  $e = 0.82403$ ,  $i = 0.91939$  rad,  $\Omega = 1.7843$  rad,  $\omega = 5.271$  rad



## Operational orbit

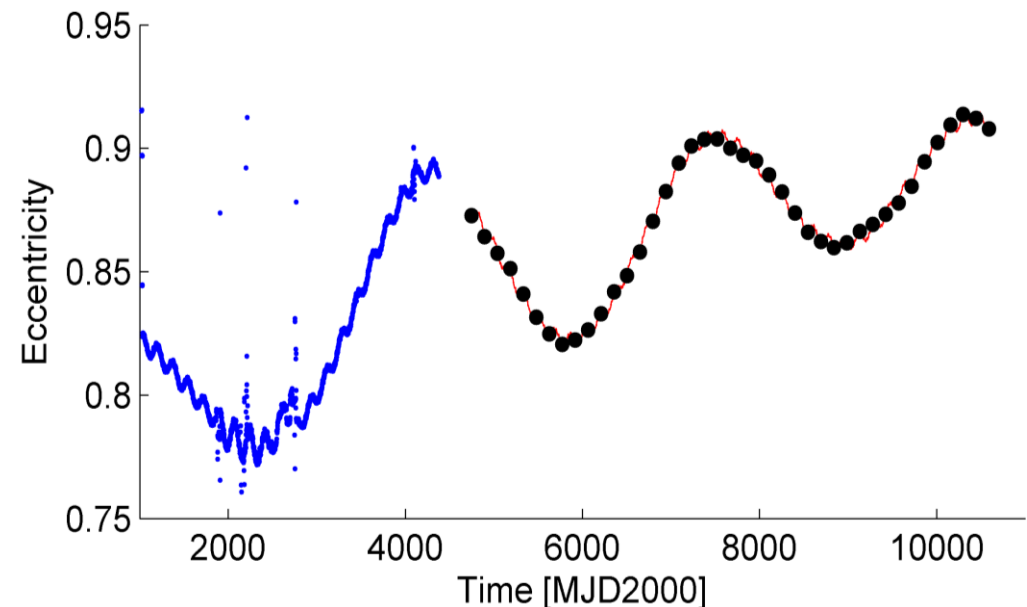
### Mission scenario

- Propagation time: 2002/11/13 to 2021/01/01
- Initial Keplerian elements from Horizon NASA on 2002/11/13 at 00:00:  
 $a = 87736$  km,  $e = 0.82403$ ,  $i = 0.91939$  rad,  $\Omega = 1.7843$  rad,  $\omega = 5.271$  rad



## Design disposal manoeuvre in the phase space

- Only 5 Keplerian elements are propagated:  $a$ ,  $e$ ,  $i$ ,  $\Omega$ ,  $\omega$
- Optimal true anomaly  $f_M$  where the manoeuvre is applied is selected through optimisation
- Dynamics of the mean/true anomaly is much faster than the evolution
- Single manoeuvre considered at different dates within a wide disposal window [2013/01/01 to 2029/01/01]



## Re-entry disposal design

For each initial condition global optimization  $x = [\Delta v \quad \alpha \quad \beta \quad f]$

$$J = \max(h_{p,\min} - h_{p,\text{target}}, 0)^2 + w \cdot \Delta v \quad \text{minimise } \Delta v$$

target minimum perigee

$$h_{p,\min} = \min_{t \in \Delta t_{\text{disposal}}} h_p(t) \qquad h_{p,\text{target}} = 50 \text{ km}$$

## Genetic algorithm

- Population of 100 individuals and a maximum of 200 generations
- The tournament selection is applied to identify the best individuals and the mutation is used 10% of the times to maintain genetic diversity

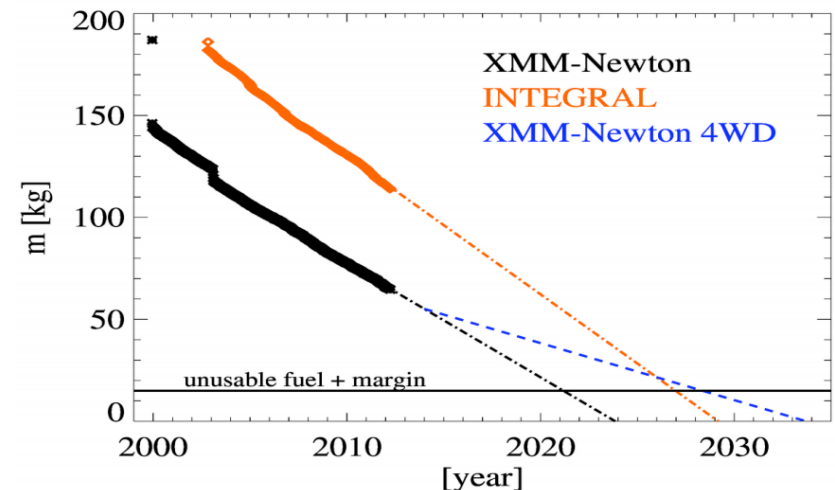
## Constraints

### Mission constraints

Parameter	Value
Dry mass	3414 kg
No. thrusters thrust	8 x 20 N
Specific impulse $I_{sp}$	235 s
Propellant	Hydrazine
Available fuel (01/01/2013)	61.5 kg
Equivalent $\Delta v$	61.9 m/s
Fuel consumption	8 kg/year
Pointing constraints	Telescope never points closer than 15 deg from the Sun
$c_R$ at BOL	1.3
Max $A/m$ EOL	0.013 m <sup>2</sup> /kg

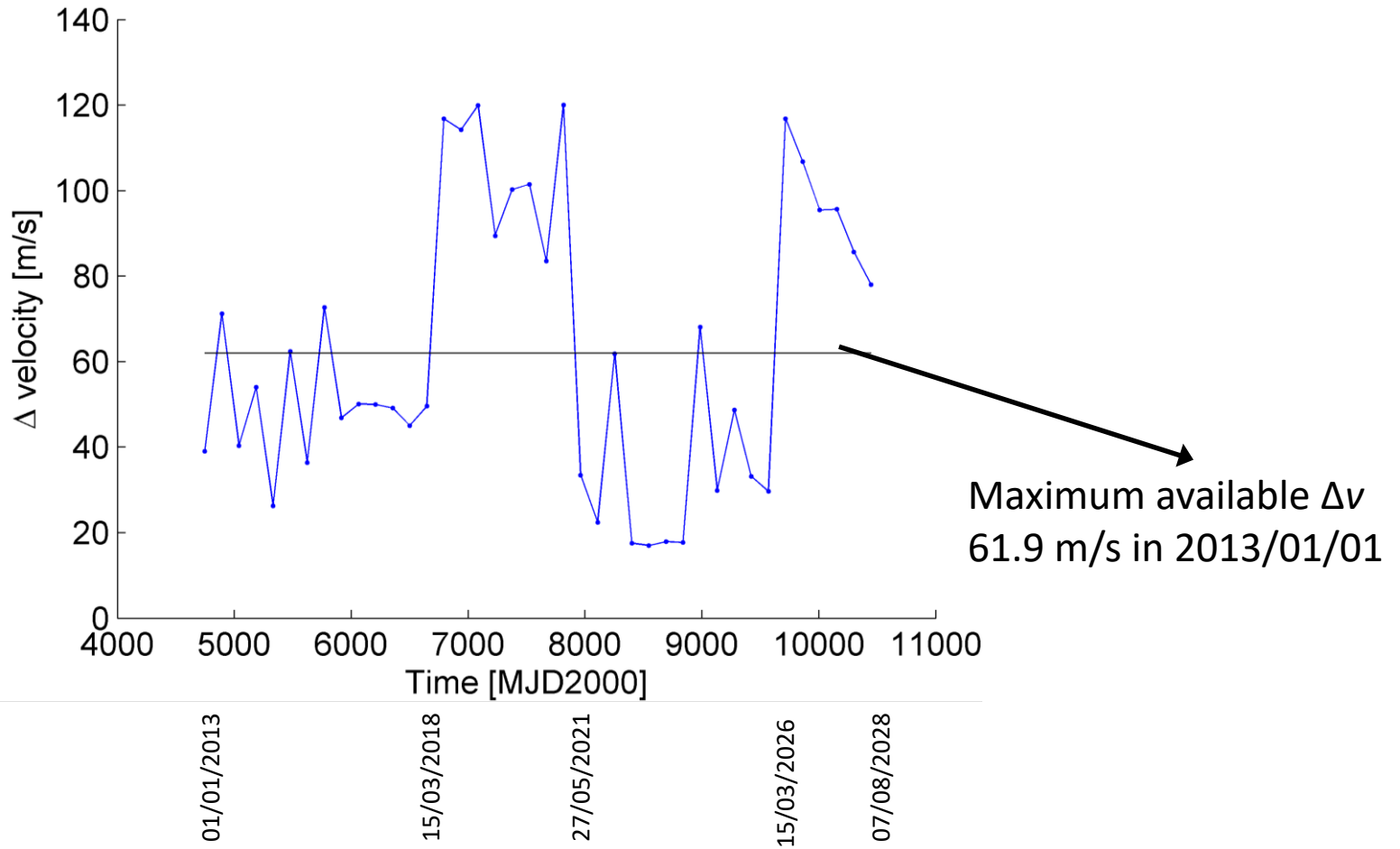
### Mission extended

- Change in attitude and orbital control system (four reaction wheels instead of three nominal + one redundancy)
- Limiting the degradation of the wheels
- Reduction of the fuel consumption.
- Spacecraft lifetime would increase of 6-8 years (going from 2020 to 2026-2028)





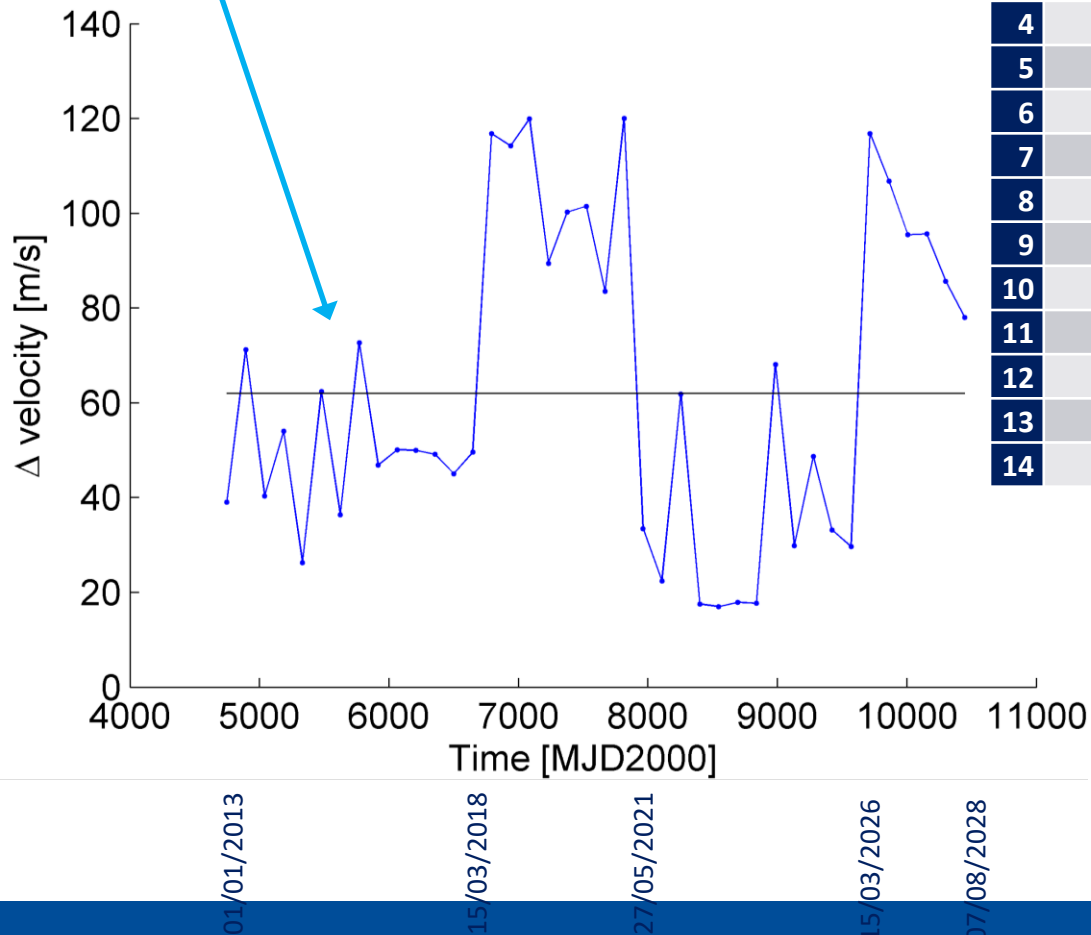
## Results



## Results

### Family 1

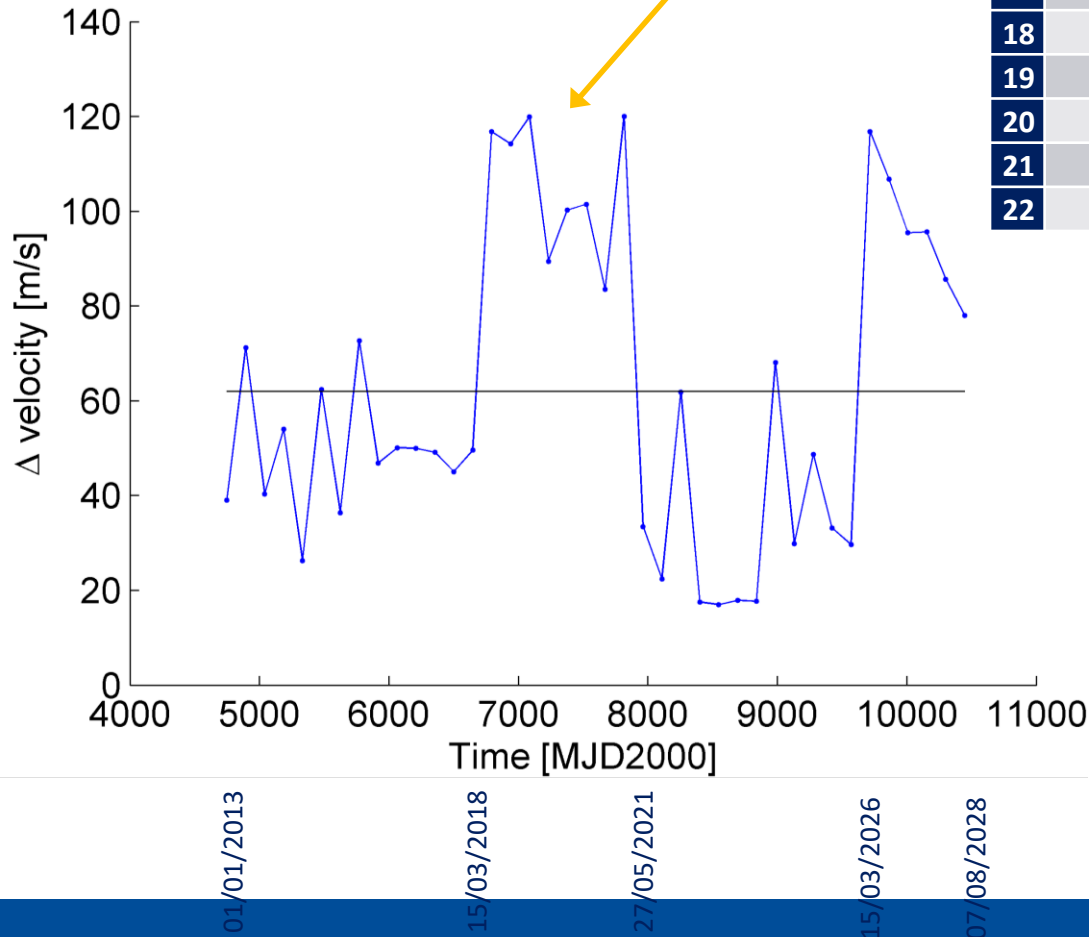
re-entry in 2028,  $\Delta v$  between 27 and 73 m/s



N	Re-entry manoeuvre [date]	$\Delta v$ [m/s]	Re-entry epoch [date]	Minimum perigee [km]
1	01/01/2013	39.03	19/09/2028	49.86
2	27/05/2013	71.23	19/09/2028	49.72
3	20/10/2013	40.28	20/09/2028	49.21
4	15/03/2014	54.03	19/09/2028	49.72
5	08/08/2014	26.26	16/10/2028	49.99
6	01/01/2015	62.39	18/09/2028	49.62
7	27/05/2015	36.33	19/09/2028	49.10
8	20/10/2015	72.66	19/09/2028	50.33
9	14/03/2016	46.80	08/04/2028	49.56
10	07/08/2016	50.06	19/09/2028	49.83
11	01/01/2017	49.99	19/09/2028	49.03
12	27/05/2017	49.11	08/04/2028	49.04
13	20/10/2017	45.03	08/04/2028	46.25
14	15/03/2018	49.55	18/09/2028	49.82

## Results

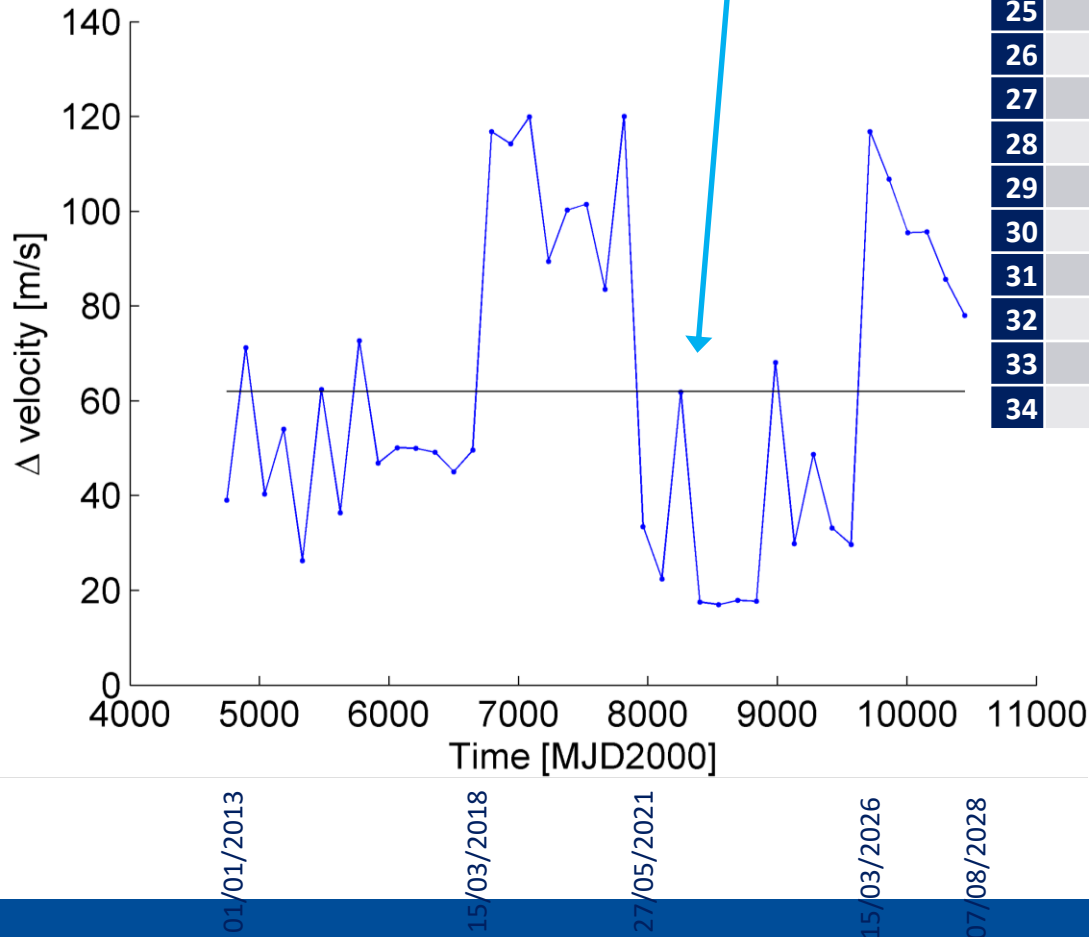
**Family 2**  
reach minimum perigee  
between 2019 and 2020  
(quicker re-entry)



N	Re-entry manoeuvre [date]	$\Delta v$ [m/s]	Re-entry epoch [date]	Minimum perigee [km]
15	08/08/2018	116.7488	18/10/2028	50.08
16	01/01/2019	114.23	14/10/2019	49.37
17	27/05/2019	119.89	09/11/2019	48.42
18	20/10/2019	89.40	17/10/2028	49.99
19	14/03/2020	100.26	22/04/2020	49.99
20	07/08/2020	101.48	02/10/2020	47.89
21	01/01/2021	83.48	19/09/2028	49.65
22	27/05/2021	120.02	13/09/2026	48.68

## Results

**Family 3**  
re-entry in 2028,  
 $\Delta v$  between 17 and 70 m/s

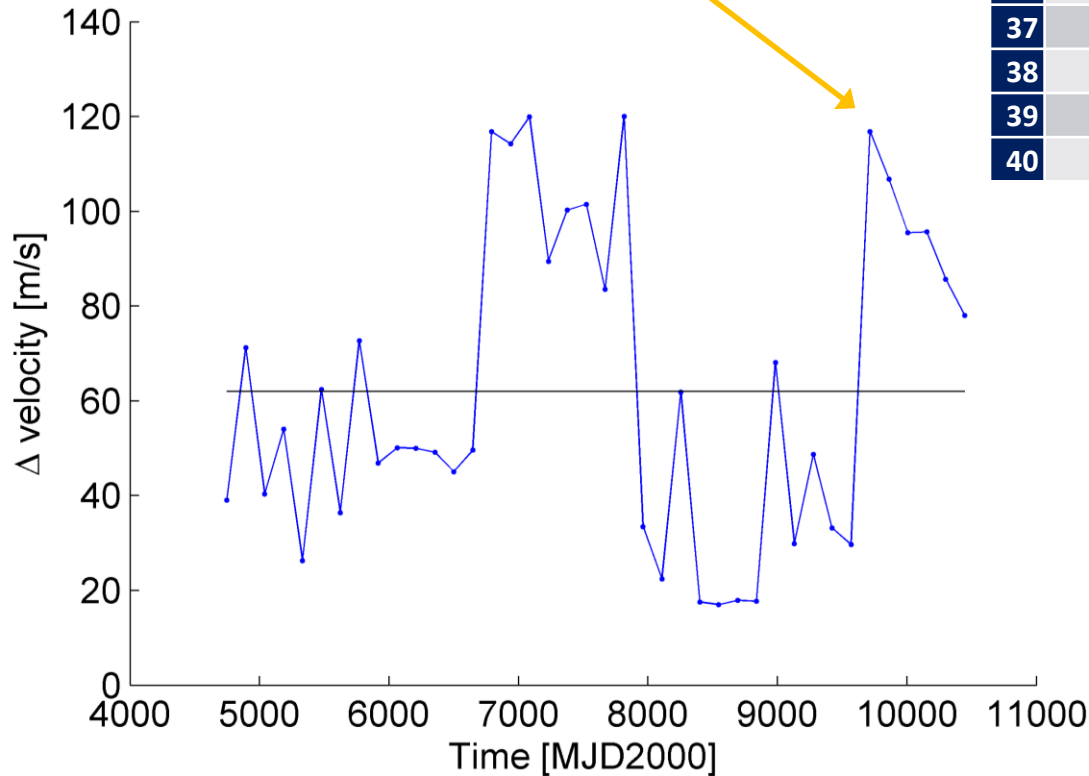


N	Re-entry manoeuvre [date]	$\Delta v$ [m/s]	Re-entry epoch [date]	Minimum perigee [km]
23	20/10/2021	33.38	18/09/2028	48.24
24	15/03/2022	22.38	18/09/2028	45.53
25	08/08/2022	61.83	08/04/2028	49.84
26	01/01/2023	17.51	19/09/2028	46.46
27	27/05/2023	16.97	18/09/2028	49.71
28	20/10/2023	17.92	18/09/2028	46.13
29	14/03/2024	17.68	19/09/2028	45.28
30	07/08/2024	68.04	18/09/2028	49.94
31	01/01/2025	29.79	08/04/2028	48.81
32	27/05/2025	48.71	18/09/2028	39.33
33	20/10/2025	33.13	18/09/2028	28.31
34	15/03/2026	29.65	19/09/2028	49.09

## Results

### Family 4

have a quick re-entry in 2028 but requires higher  $\Delta v$  than family 3



N	Re-entry manoeuvre [date]	$\Delta v$ [m/s]	Re-entry epoch [date]	Minimum perigee [km]
35	08/08/2026	116.78	20/04/2027	48.88
36	01/01/2027	106.77	22/04/2027	48.40
37	27/05/2027	95.43	02/10/2027	48.14
38	20/10/2027	95.60	12/03/2028	49.83
39	14/03/2028	85.58	08/04/2028	48.47
40	07/08/2028	78.04	18/09/2028	48.78

01/01/2013

15/03/2018

27/05/2021

15/03/2026

07/08/2028

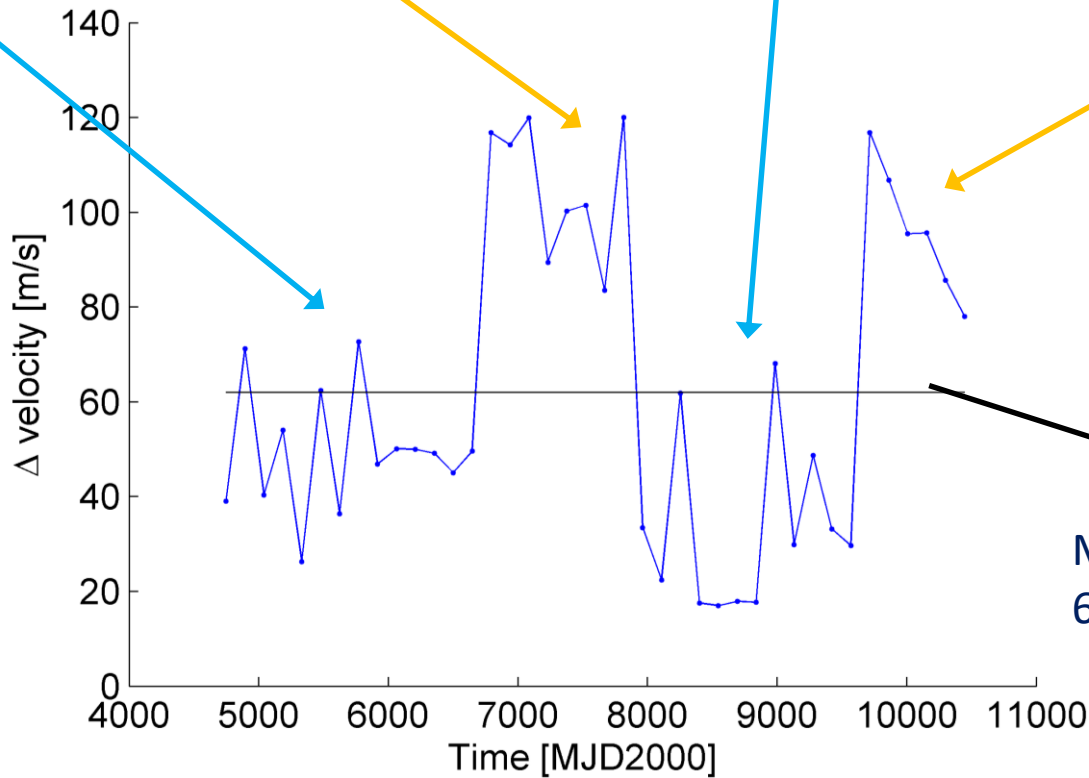
# INTEGRAL mission

**Family 1**  
re-entry in 2028,  $\Delta v$  between 27 and 73 m/s

**Family 2**  
reach minimum perigee between 2019 and 2020 (quicker re-entry)

**Family 3**  
re-entry in 2028,  $\Delta v$  between 17 and 70 m/s

**Family 4**  
have a quick re-entry in 2028 but requires higher  $\Delta v$  than family 3



Maximum available  $\Delta v$   
61.9 m/s in 2013/01/01

01/01/2013

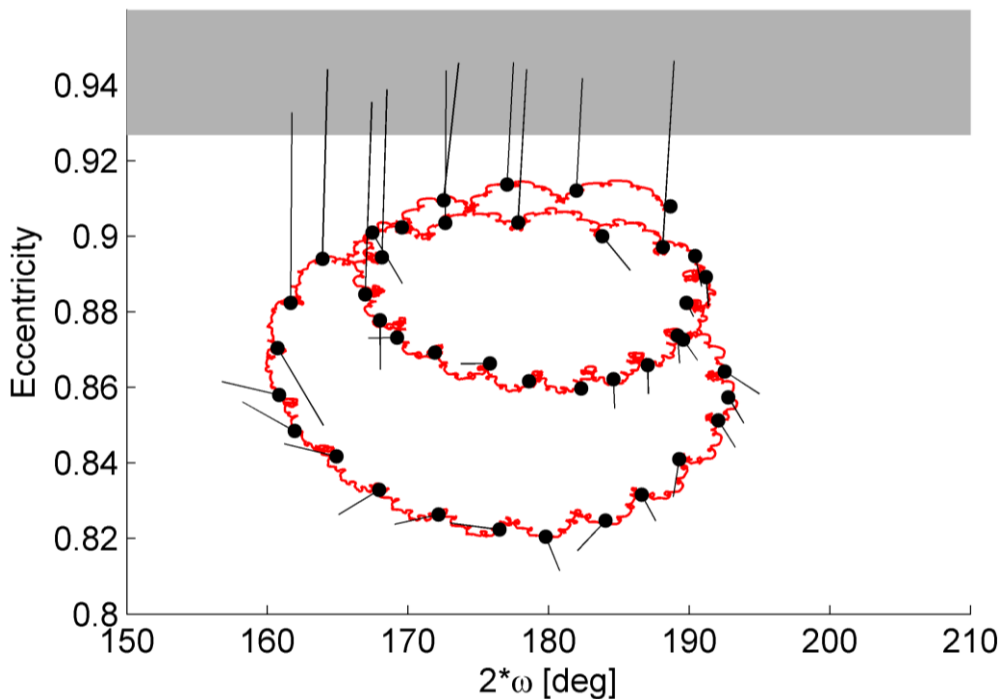
15/03/2018

27/05/2021

15/03/2026

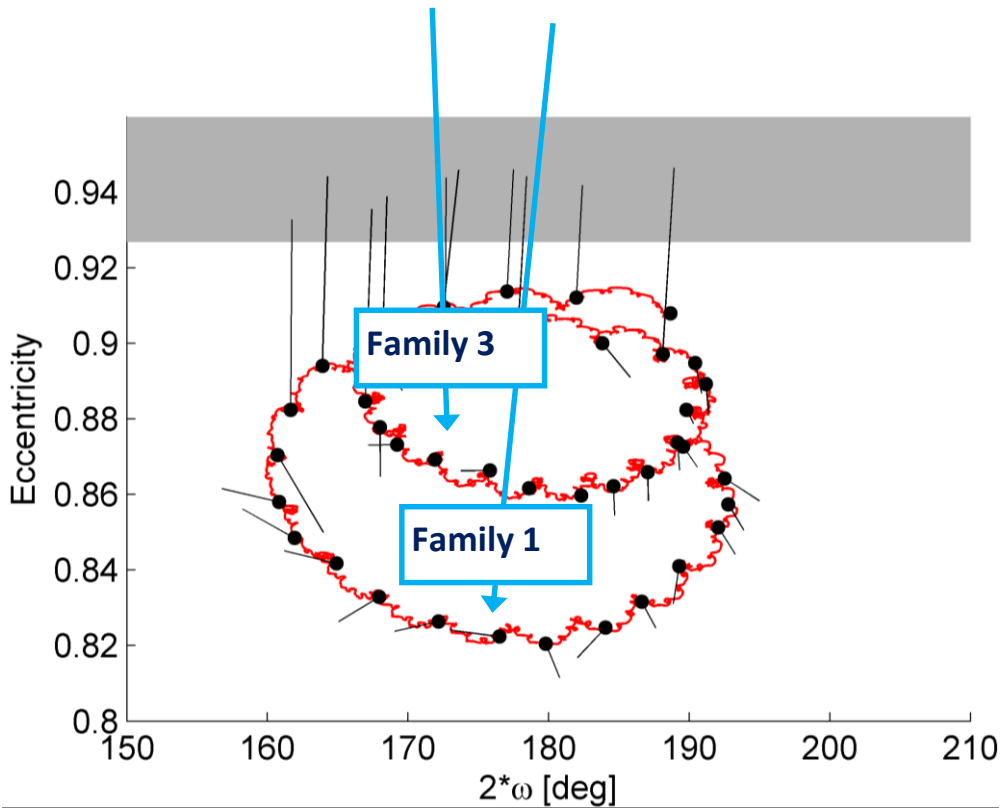
07/08/2028

## Results



## Results

Low eccentricity conditions





## Results

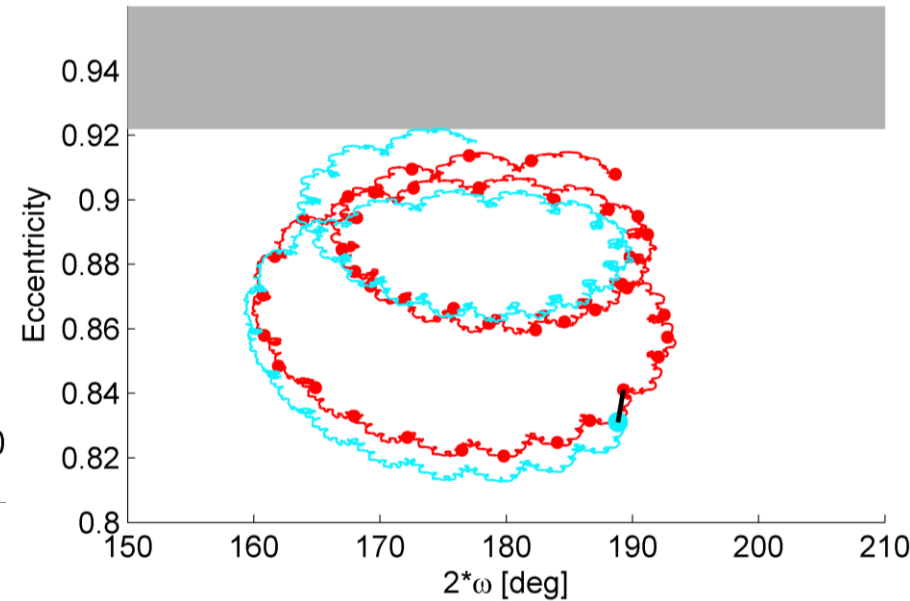
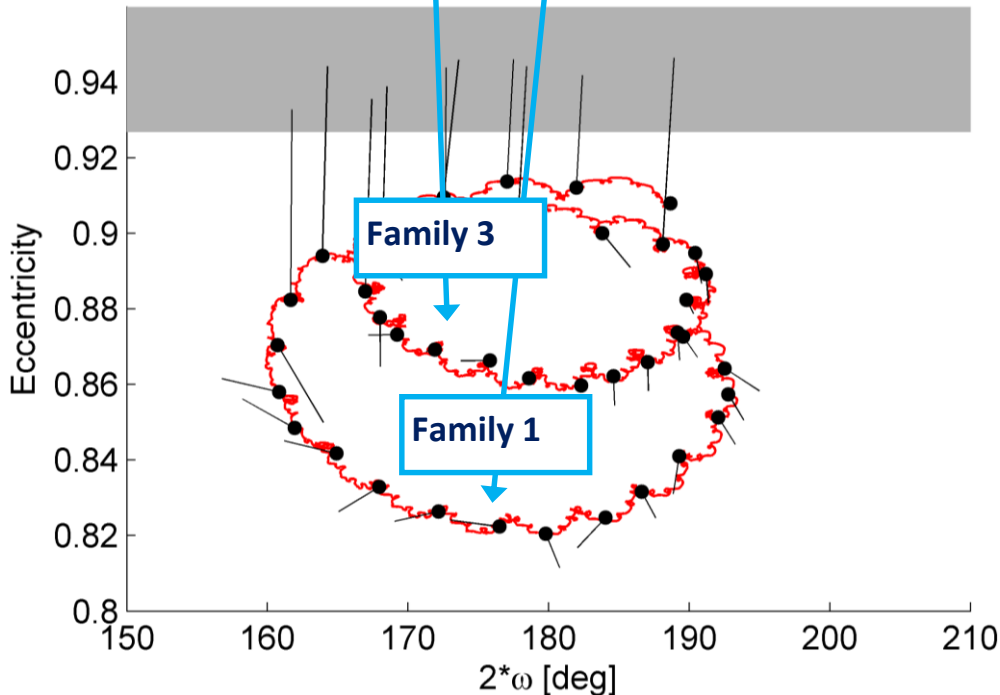
Low eccentricity conditions  
Manoeuvre performed on 08/08/2014  
Re-entry in 2028



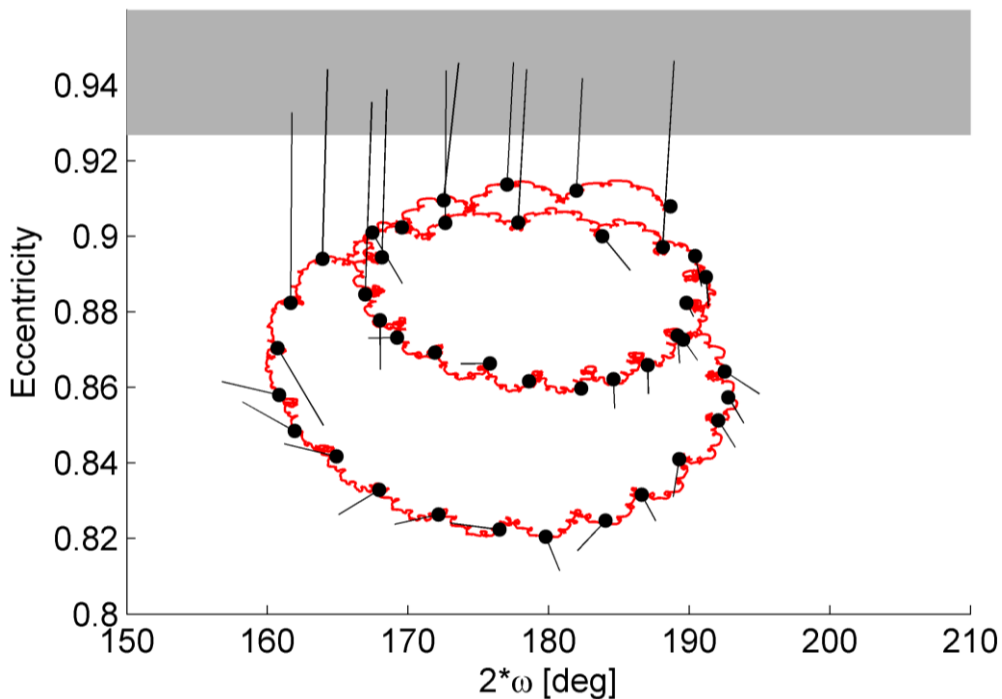
The manoeuvre tends to further decrease  $e$

→ the following long term propagation will reach a higher eccentricity (re-entry).

**The manoeuvre is more efficient (i.e., lower  $\Delta v$  is required).**



## Results

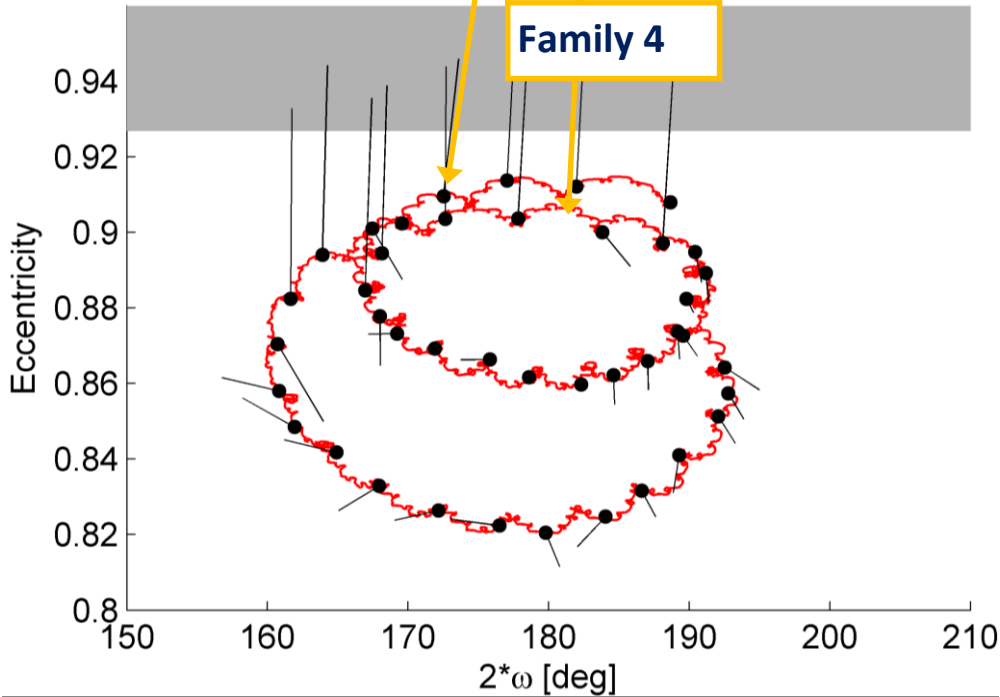


## Results

High eccentricity conditions

Family 2

Family 4

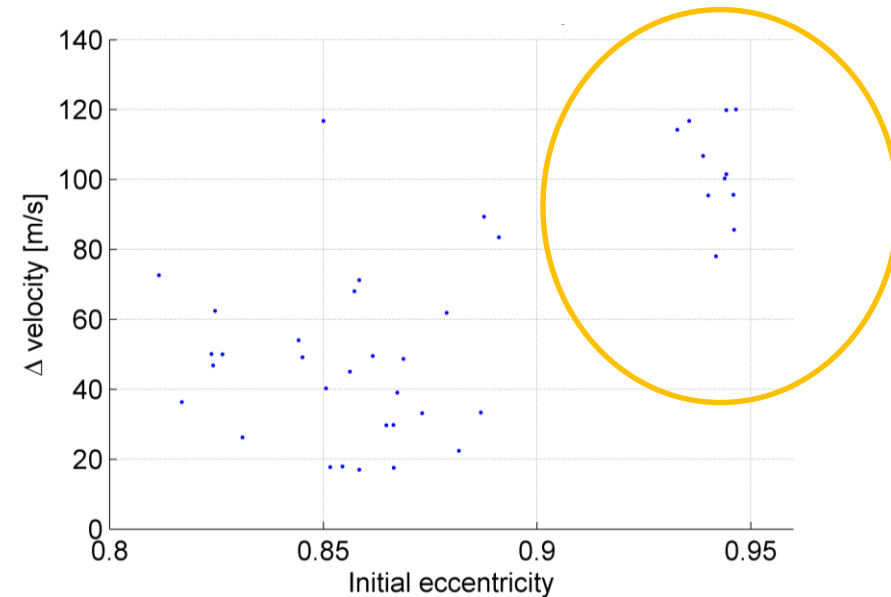
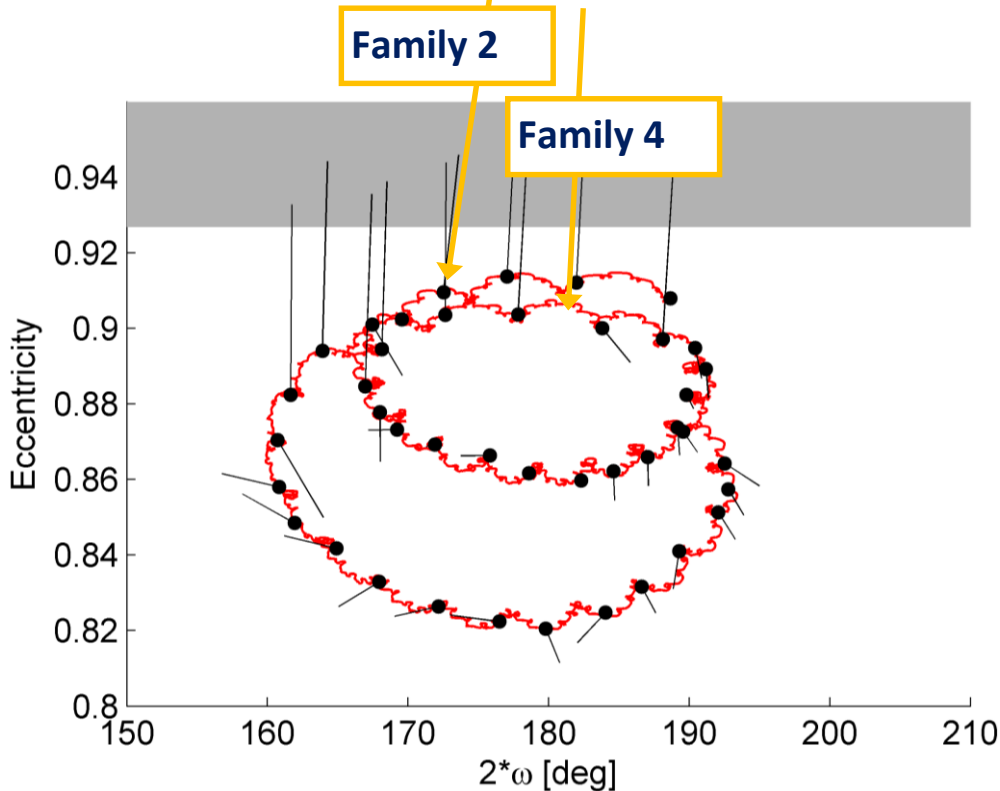


## Results

High eccentricity conditions



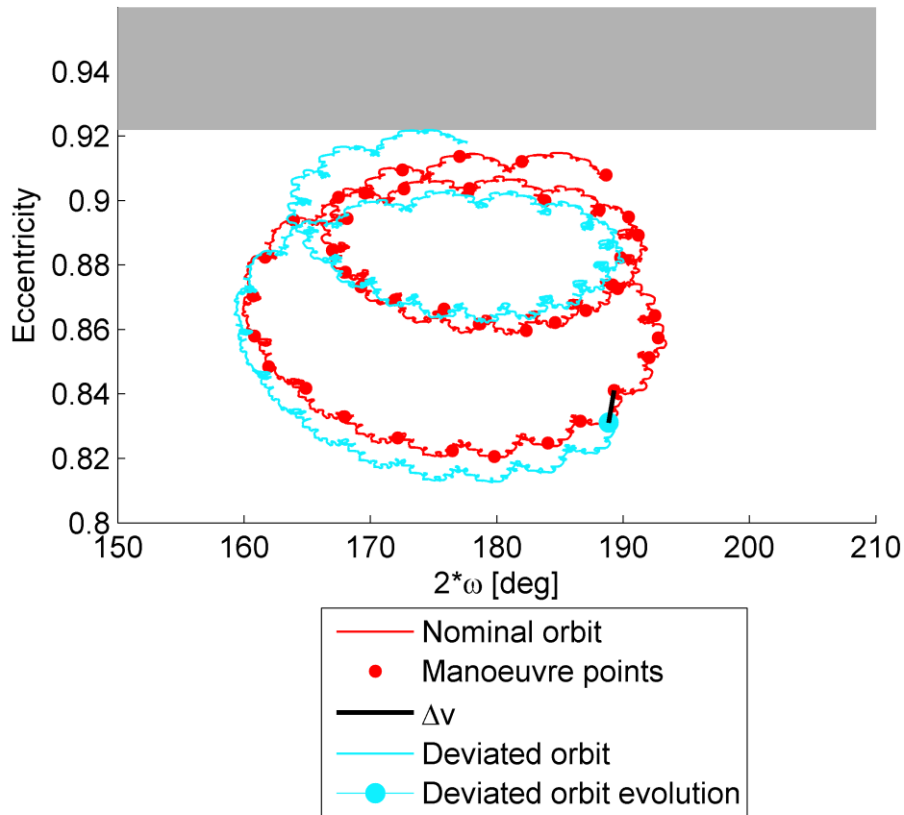
The re-entry manoeuvre aims at further increasing the eccentricity, so that the target minimum perigee is reached after a relatively shorter time



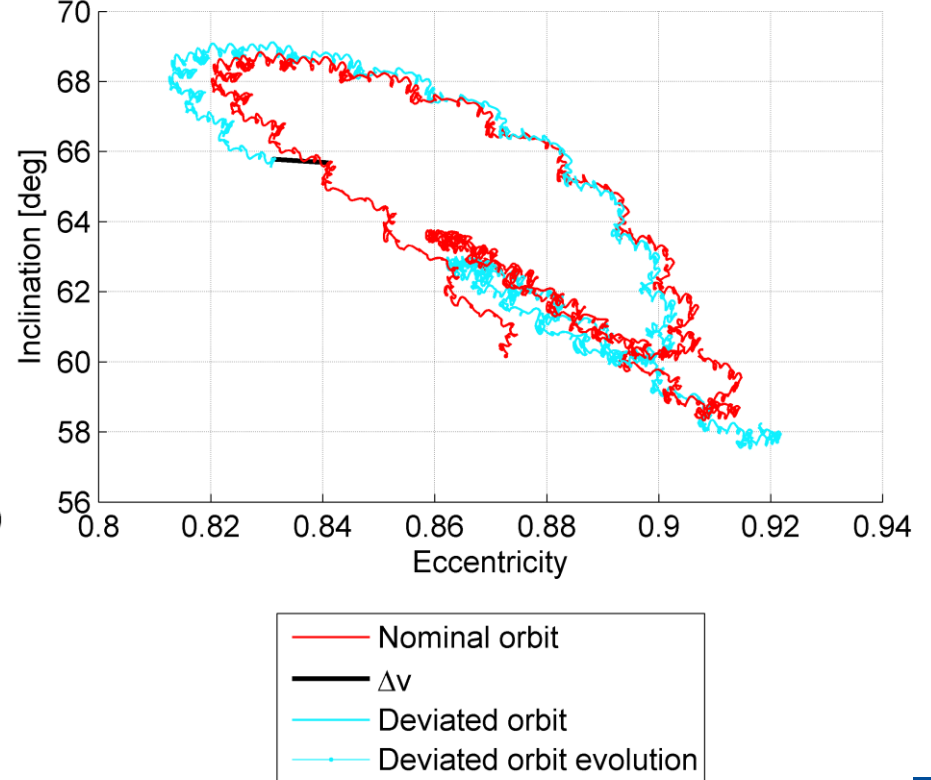
## Re-entry manoeuvre

Example: manoeuvre performed on 08/08/2014

INTEGRAL, System: @Earth Earth-Moon plane



INTEGRAL, System: @Earth Earth-Moon plane

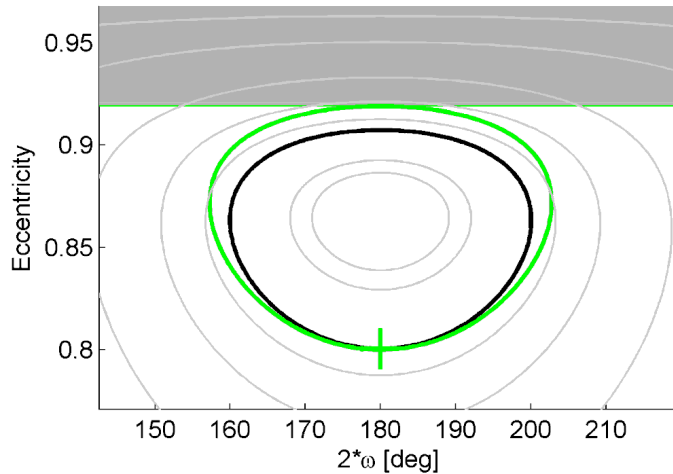


## Re-entry manoeuvre

### Preliminary mission design

Moon effect only

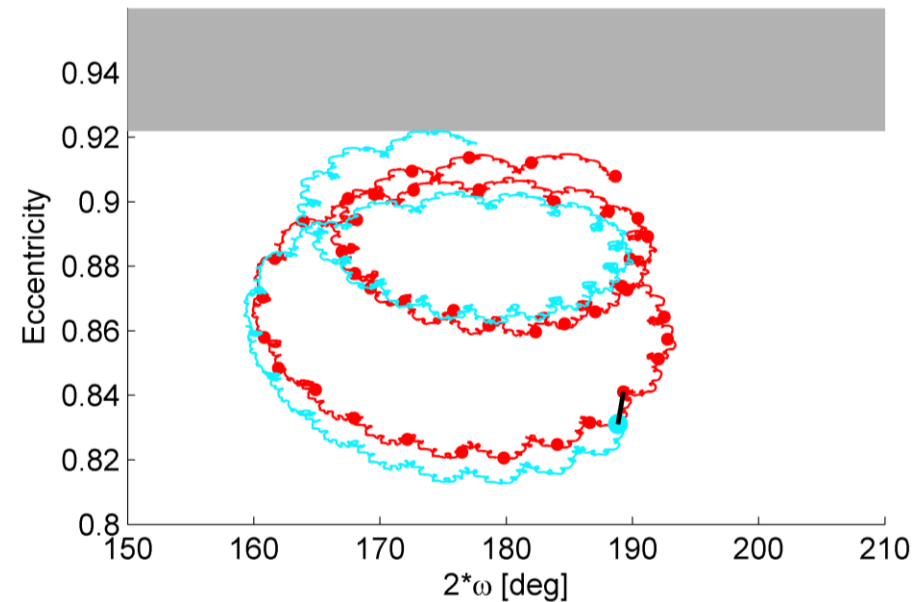
Double averaged potential



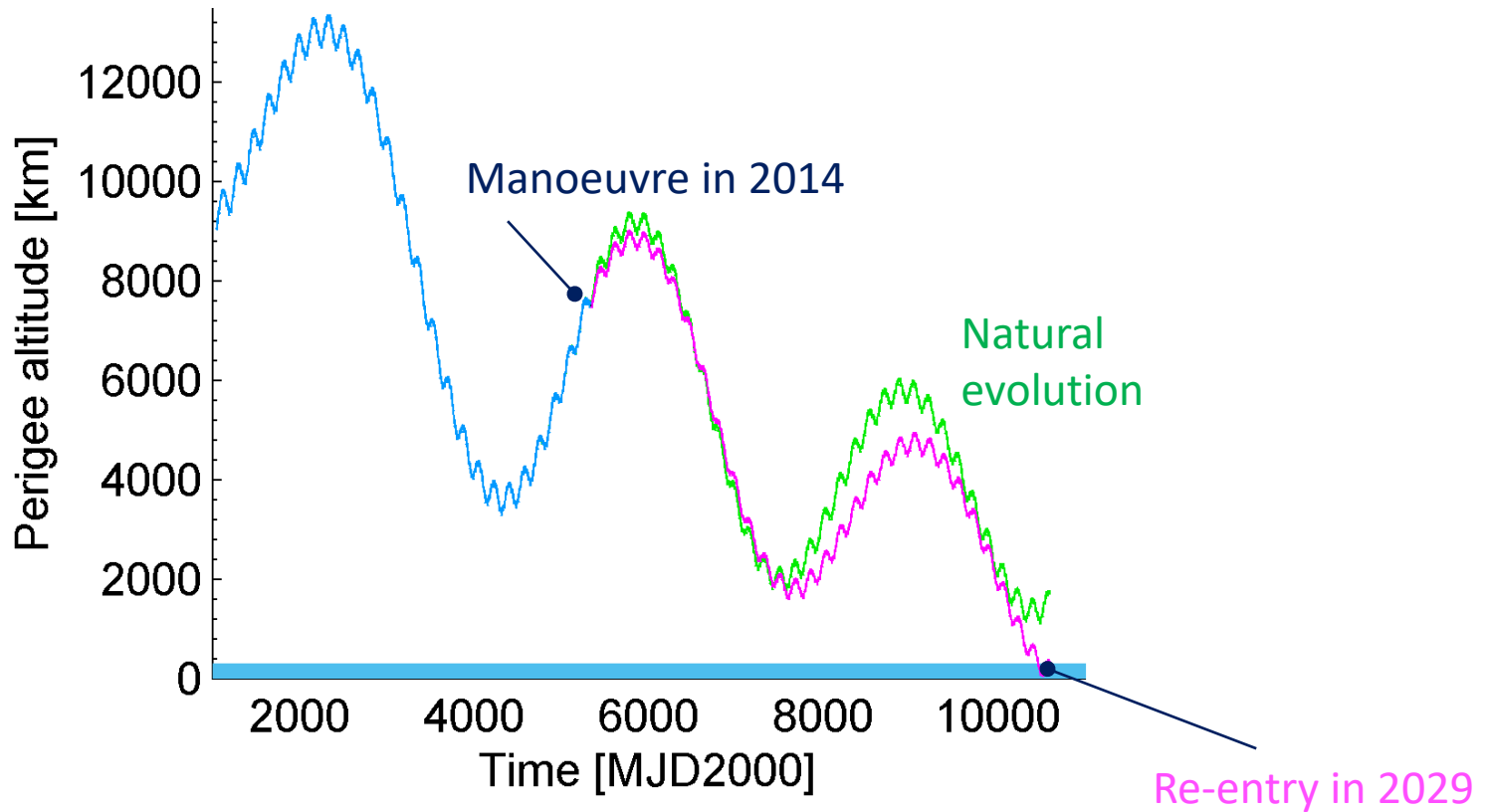
### Optimised solution

Moon + Sun + J2:

Single averaged dynamics + global optimisation



## Re-entry manoeuvre

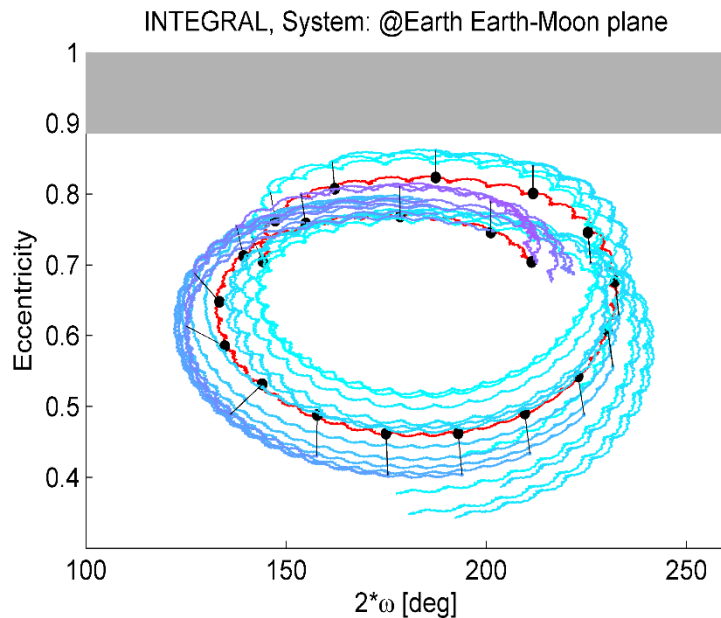


## Graveyard disposal

- Time window for starting the disposal manoeuvre [2013/01/01 to 2035/01/01].
- Max  $\Delta v$  available for the manoeuvre 81 m/s

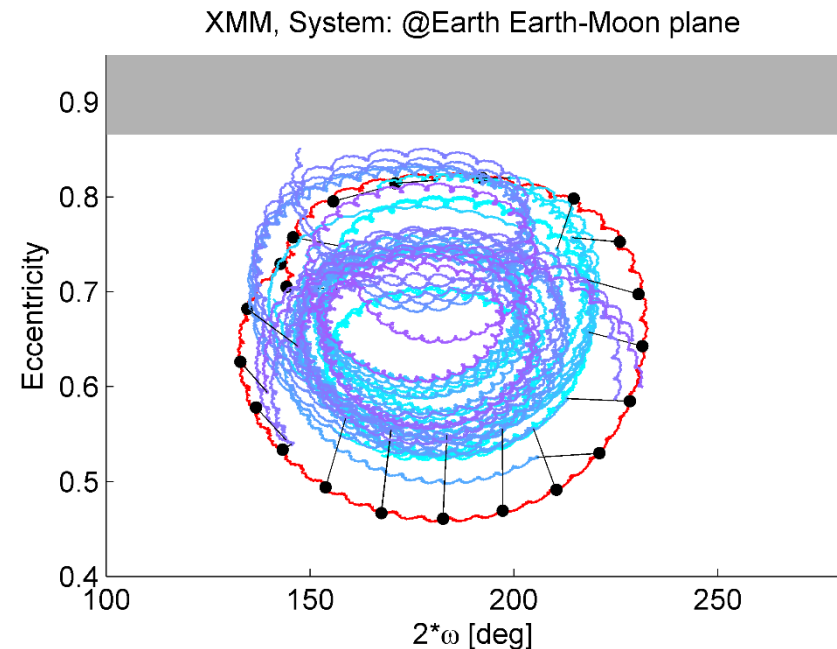
## Re-entry

Max  $\Delta e$ , min  $h_p$  orbits



## Graveyard

Limited  $\Delta e$ ,  $\Delta i$  orbits

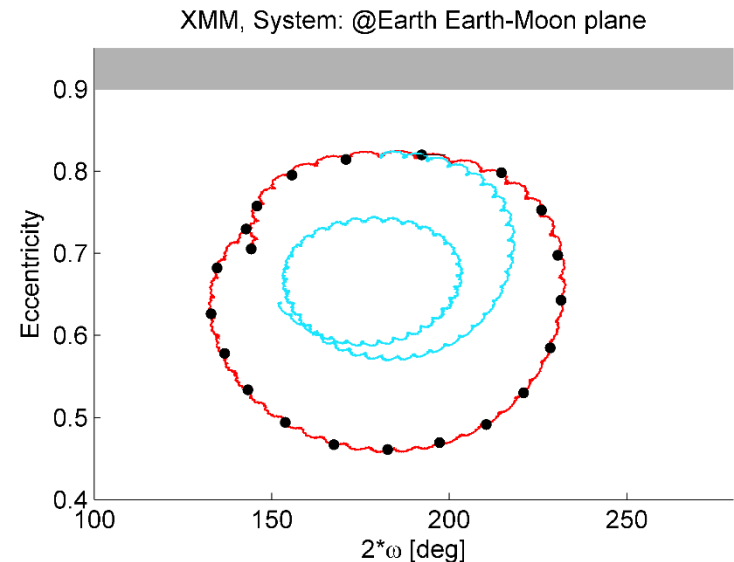




## Graveyard disposal

- Time window for starting the disposal manoeuvre [2013/01/01 to 2035/01/01].
- Max  $\Delta v$  available for the manoeuvre 81 m/s

Example graveyard maneuver performed on 20/04/2016





# CONCLUSIONS

- Effect of luni-solar perturbations and the Earth's oblateness on the stability of highly elliptical orbits
- Natural orbital dynamics can be exploited and enhanced
- INTEGRAL is the demonstration in Space!



## Schedule for revolution 1799

(this list is also available in csv-format, click [here](#) to download)

Rev	Start time (UTC)	End time (UTC)	Exp. time (s)	Target	Ra (J2000)	Dec (J2000)	Pattern	PI	Proposal	Observation	N
1799	2017-03-30 10:11:09	2017-03-30 13:52:56	12600	Gal. Bulge region	17:45:36.00	-28:56:00.0	HEX	Erik Kuulkers	<a href="#">1420001</a>	1420001 / 0009	P
1799	2017-03-30 14:11:52	2017-03-30 14:45:12	2000	Galactic Center	17:36:47.26	-31:25:52.3	5x5 Seg	Joern Wilms	<a href="#">1420009</a>	1420009 / 0001	
1799	2017-03-30 15:06:03	2017-03-31 05:48:09	50000	Galactic Center	17:35:00.58	-32:37:41.9	5x5 Seg	Joern Wilms	<a href="#">1420009</a>	1420009 / 0005	
1799	2017-03-31 06:08:30	2017-03-31 09:50:16	12600	Galaxy (l=0, b=0)	17:41:53.52	-29:13:22.8	HEX	Rashid Sunyaev	<a href="#">1420021</a>	1420021 / 0009	
1799	2017-03-31 10:50:17	2017-03-31 11:52:13	3600	Galaxy (l=0, b=-30)	19:58:20.40	-40:46:37.2	HEX	Rashid Sunyaev	<a href="#">1420021</a>	1420021 / 0010	
1799	2017-03-31 12:27:37	2017-03-31 15:05:31	9000	Galaxy (l=0, b=-30)	19:58:20.40	-40:46:37.2	HEX	Rashid Sunyaev	<a href="#">1420021</a>	1420021 / 0010	
1799	2017-03-31 15:33:09	2017-03-31 19:14:56	12600	Galaxy (l=0, b=0)	17:47:59.52	-30:08:27.6	HEX	Rashid Sunyaev	<a href="#">1420021</a>	1420021 / 0011	
1799	2017-03-31 19:42:29	2017-03-31 23:24:15	12600	Galaxy (l=0, b=-30)	20:06:37.68	-41:09:50.4	HEX	Rashid Sunyaev	<a href="#">1420021</a>	1420021 / 0012	

INTEGRAL REVOLUTION

1799

INTEGRAL REVOLUTION

1809

INTEGRAL CURRENT TARGET

Galactic Center

INTEGRAL CURRENT TARGET

Slewing...

## ReDSHIFT – H2020

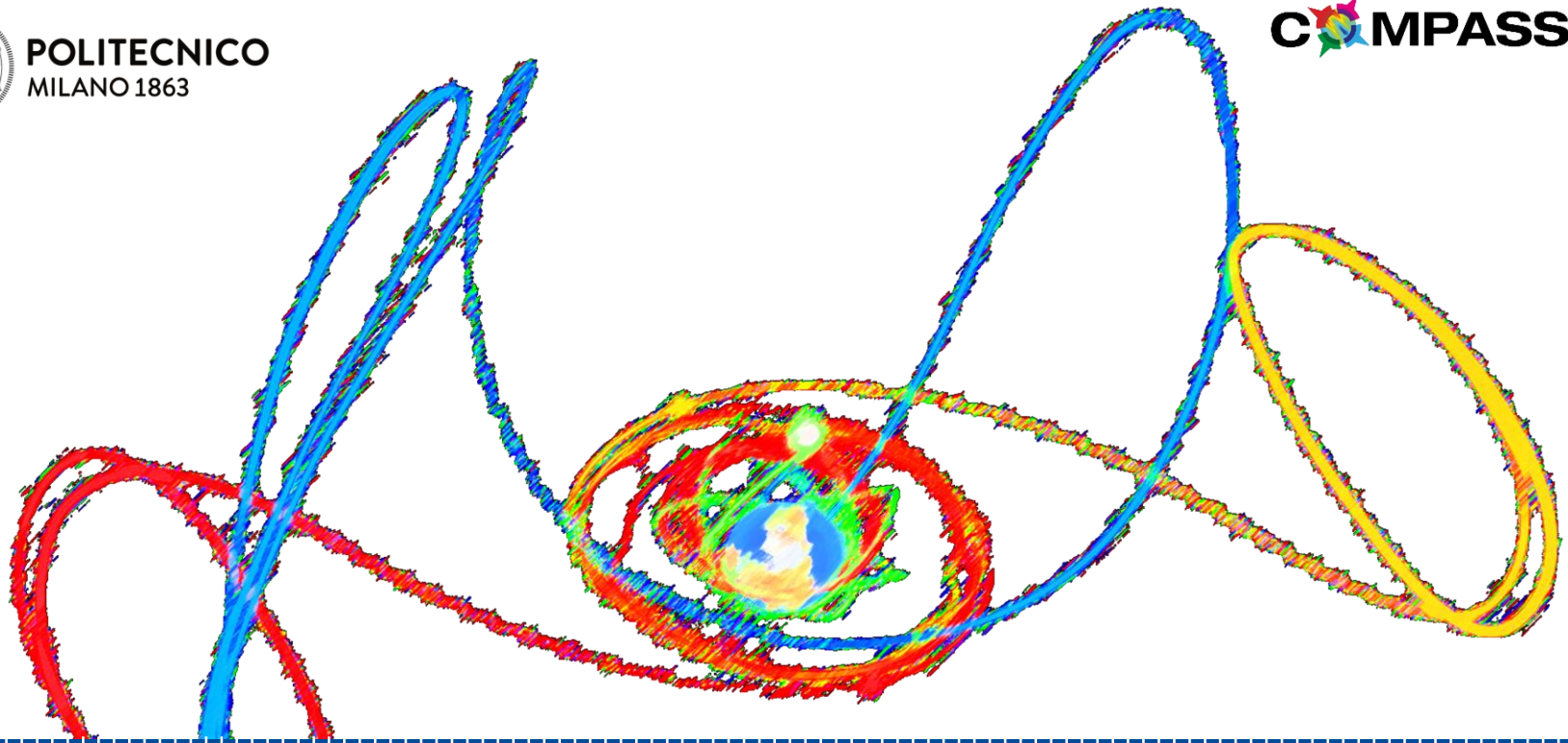
- Compute maps for the whole LEO to GEO environment
- Stability maps
- Disposal design for current and future s/c
- Online software
- See effect on global debris population

## COMPASS- ERC

- Develop automatic optimiser that exploit natural dynamics
- Study planetary protection requirements/uncertainties
- To be applied to debris cloud fragmentation, re-entry prediction, multi-moon mission, highly perturbed orbits (e.g. around asteroids)



POLITECNICO  
MILANO 1863



# Luni-solar perturbations for missions design in highly elliptical orbits

Camilla Colombo

Barcelona Mathematical Days, 28 April 2017