

1 **3-D numerical modeling of single-lap direct shear tests of FRCM-concrete joints using a**
2 **cohesive contact damage approach**
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11 **Abstract**

12 The bond behavior of fiber reinforced cementitious matrix (FRCM) composites applied as externally
13 bonded reinforcement is the most critical concern in this type of application. FRCM-concrete joints
14 are generally reported to fail due to debonding (slippage) of the fibers from the embedding matrix.
15 However, depending on the characteristics of the composite and substrate employed, failure may also
16 occur due to detachment of the composite strip at the FRCM-support interface, interlaminar failure
17 (delamination) of the matrix, or tensile failure of the fibers. In this paper, a three-dimensional (3-D)
18 numerical model is developed to reproduce the behavior of polyparaphenylene benzo-bisoxazole
19 (PBO) FRCM-concrete joints. The numerical model accounts for the fracture mechanics mixed
20 Mode-I and Mode-II loading condition observed in single-lap direct shear tests by means of non-
21 linear cohesive contact damage laws associated with different interfaces considered in the analysis.
22 The numerical results obtained are compared with those obtained by experimental tests of PBO
23 FRCM-concrete joints. The model is capable of predicting the different failure modes, and it correctly

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24 reproduces the experimental load responses including the contribution of friction to the applied stress.

25

26 Keywords: FRCM; Fabrics/textiles; Fiber/matrix bond; Finite Element Analysis (FEA)

27

28 **Introduction**

29 Fiber reinforced cementitious matrix (FRCM) composites represent a newly-developed alternative to
30 fiber reinforced polymer (FRP) composites for strengthening existing reinforced concrete (RC) and
31 masonry elements (Triantafillou and Papanicolaou 2006; D’Ambrisi and Focacci 2011; Ombres 2012;
32 Pellegrino and D’Antino 2013; Loreto et al. 2013). FRCM composites are comprised of layers of fiber
33 nets embedded within inorganic matrix layers that are responsible for the stress transfer between fibers
34 and matrix and between the composite and the support. Compared with FRP composites, FRCM
35 composites exhibit a higher resistance to high temperatures and do not suffer of UV degradation.
36 Furthermore, inorganic matrices have unvarying workability time and can be applied to wet surfaces,
37 whereas FRP composites cannot. FRCM composites can be applied on both concrete and masonry
38 substrates and appear particularly promising for strengthening interventions on heritage buildings due
39 to their reversibility and vapor permeability (de Felice et al. 2012).

40 Different authors employed single- and double-lap direct shear tests to study the bond behavior of
41 FRCM composites observing that failure generally occurs due to debonding (slippage) of the fibers
42 from the embedding matrix (D’Ambrisi et al. 2012; D’Ambrisi et al. 2013; Sneed et al. 2014; Sneed
43 et al. 2015; de Felice et al. 2014). However, depending on the characteristics of composite, substrate,
44 and experimental set-up adopted, detachment of the composite strip at the FRCM-substrate interface
45 (D’Antino et al. 2015), interlaminar failure (delamination) of the matrix with detachment of the matrix
46 layer that covers the fibers (de Felice et al. 2014; D’Antino et al. 2015), and tensile failure of the
47 fibers inside and outside the bonded length (Carozzi and Poggi 2015) have been observed.

48 In direct shear tests, and particularly single-lap shear tests, the eccentricity between the applied load
49 and the restraint entails for the presence of a fracture mechanics Mode-I and Mode-II loading
50 condition on the composite strip. Several researchers (e.g. De Lorenzis and Zavarise 2008; Carrara
51 and Ferretti 2013) studied the debonding phenomenon in FRP-concrete joints, which typically occurs
52 within the concrete substrate (Carrara and Ferretti 2013), by taking into account the presence of Mode-
53 I and Mode-II loading conditions (Carrara and Ferretti 2013; Rabinovitch 2008; Mazzucco et al. 2012;
54 Neto et al. 2016). If a macro-scale approach is used, the Mode-I loading condition is described by the
55 relationship between the normal stress σ_{zz} (axes are defined in Figure 1) at the interface and crack
56 opening w (lifting of the composite). The Mode-II loading condition is described by the relationship
57 between the interfacial shear stress τ_{zy} and slip s (De Lorenzis and Zavarise 2008). The area under the
58 σ_{zz} - w and τ_{zy} - s curves is the Mode-I and Mode-II critical energy release rate (fracture energy),
59 respectively.

60 The bond behavior of FRCM-concrete joints has been studied employing numerical approaches that
61 assumed a pure Mode-II loading condition at the matrix-fiber interface (Carloni et al. 2015a).
62 However, the presence of a Mode-I loading condition, which was reported to be dominant for FRCM-
63 concrete joints with short bonded lengths (Sneed et al. 2014), and the occurrence of different failure
64 modes should be taken into account to fully describe the debonding process.

65 In this study, a three-dimensional (3-D) numerical model is used to describe the bond behavior of
66 FRCM-concrete joints comprised of one layer of polyparaphenylene benzo-bisoxazole (PBO) fiber
67 net embedded within two layers of a polymer-modified cement-based mortar. The numerical model
68 developed accounts for the possibility of interlaminar failure of the matrix, debonding at the matrix-
69 fiber interface, and detachment of the composite strip at the FRCM-support interface. The results
70 obtained by numerical analyses are compared with those obtained by experimental tests of PBO
71 FRCM-concrete joints.

72

73 **Experimental set-up and materials**

74 In this study, the results of single-lap direct shear tests on FRCC-concrete joints previously published
75 (D'Antino et al. 2014) are compared with the results obtained by a 3-D non-linear numerical model.
76 Sixty-seven of the 82 specimens reported in (D'Antino et al. 2014) were considered reliable according
77 to a criterion proposed by the authors (Carloni et al. 2015b) and were employed for the comparison,
78 while the remaining 15 were excluded. The FRCC composite was comprised of one layer of a PBO
79 bidirectional unbalanced fiber net embedded within two 4 mm thick cementitious matrix layers. The
80 composite strip was applied to one face of a concrete block with length $L=375$ mm (batch A) or $L=510$
81 mm (batch B) (Table 1). The fibers were bare outside the bonded area with a length $l=270$ mm.
82 Transversal fiber bundles, which were all on one side of the longitudinal fiber bundles in the fiber
83 net, were placed against the external matrix layer for some specimens and against the internal matrix
84 layer for some others. Details of the test set-up adopted are reported in Figure 1.

85 Concrete blocks with $L=375$ mm (batch A) were used for specimens with bonded length $\ell=100$ mm,
86 150 mm, 200 mm, 250 mm, and 330 mm, whereas concrete blocks with $L=510$ mm (batch B) were
87 used for specimens with $\ell=450$ mm. Experimental tests were conducted to obtain mean values of the
88 compressive strength f_c and tensile strength f_t of the concrete and matrix (Carloni et al. 2015b) and
89 elastic modulus E and tensile strength f_t of the PBO fibers (D'Antino et al. 2013) (Table 1).

90 Each fiber bundle was assumed to have a rectangular cross-section of width $b^*=5$ mm and thickness
91 $t^*=0.092$ mm. Poisson ratios ν , which were not measured experimentally and are needed to completely
92 define a material mechanical behavior, were assumed equal to 0.2 for the concrete and matrix (fib
93 2001) and to 0.3 for the PBO fibers (Carloni et al. 2015a).

94 The direct shear tests were carried out by increasing the loaded end slip, named global slip g , at a
95 constant rate of 0.00084 mm/s. Additional information on the testing procedure is provided in (Sneed

96 et al. 2014; Carloni et al. 2015b).

97 The peak stress σ^* is used in this paper to compare experimental and numerical results. Values of σ^*
98 are obtained with Eq. (1) when the applied load P is equal to the peak load P^* :

$$99 \quad \sigma = \frac{P}{nb^* t^*} \quad (1)$$

100 where n is the number of longitudinal fiber bundles within the composite bonded width.

101 Assuming a Mode-II fracture condition, a fracture mechanics approach was used to obtain matrix-
102 fiber interface τ_{xy} - s relationships. Only one cohesive law was determined assuming that the same
103 behavior occurs at the internal and external matrix layer-fiber interfaces (Carloni et al. 2015b). This
104 approach allowed to determine the value of the effective bond length l_{eff} , defined as the minimum
105 length needed to develop the load-carrying capacity of the interface (D'Antino et al. 2014), whose
106 average value was found to be $\bar{l}_{eff} = 260$ mm. The value of \bar{l}_{eff} was confirmed by comparing the trend
107 of the peak stress-vs-bonded length curve, which included results of the same composite obtained by
108 other authors with both single- and double-lap shear test set-ups (Sneed et al. 2015; D'Antino et al.
109 2014).

110

111 **Numerical formulation**

112 Debonding of FRCM composites applied to a concrete support was studied with a numerical model
113 developed using the commercial software Abaqus (Simulia 2016). All materials were treated as
114 homogeneous isotropic linear elastic materials and were connected through zero-thickness interfaces.
115 The assumption of material linear elasticity adopted was confirmed by the strain observed in the
116 numerical models, which was always lower than the elastic limit strain of each material. Four different
117 interfaces were considered in the FRCM-concrete joint: i) FRCM-concrete, ii) internal matrix layer-
118 fibers, iii) external matrix layer-fibers, and iv) internal-external matrix layer (Figure 2). Although

119 failure at the internal-external matrix layer interface was never observed by the authors for PBO
120 FRCM-concrete joints, such failure was observed for other FRCM composites (D'Antino et al. 2015).
121 The interfaces between the matrix and the side faces of the fiber bundles, each having an area equal
122 to 0.092 mm² per unit length, were neglected.
123 Each interface considered was modeled by means of a specific master-slave cohesive damage contact
124 interaction. The interfacial response is initially linear and, when a damage criterion is met, it degrades
125 according to a specific damage evolution law. The contact interaction linear ascending branch was
126 defined by a traction-separation model defined by the elastic constitutive matrix \mathbf{K} (Simulia 2016):

$$127 \quad \mathbf{t} = \begin{Bmatrix} t_n \\ t_s \\ t_t \end{Bmatrix} = \begin{bmatrix} K_{nn} & K_{ns} & K_{nt} \\ K_{sn} & K_{ss} & K_{st} \\ K_{tn} & K_{ts} & K_{tt} \end{bmatrix} \begin{Bmatrix} \delta_n \\ \delta_s \\ \delta_t \end{Bmatrix} = \mathbf{K}\boldsymbol{\delta} \quad (2)$$

128 where \mathbf{t} is the traction vector of normal component t_n and shear components t_s and t_t , and δ_n , δ_s , and
129 δ_t are the corresponding components of the separation vector $\boldsymbol{\delta}$. When a specific damage criterion is
130 satisfied, damage occurs and the damage variable D controls the traction vector \mathbf{t} :

$$131 \quad \bar{t}_n = \begin{cases} (1-D) \cdot t_n & \text{if } t_n > t_n^0 \\ t_n & \text{otherwise} \end{cases} \quad (3a)$$

$$132 \quad \bar{t}_i = \begin{cases} (1-D) \cdot t_i & \text{if } |t_i| > t_i^0 \\ t_i & \text{otherwise} \end{cases} \quad i=s,t \quad (3b)$$

133 where superscript 0 indicates the elastic limit values, \bar{t}_n is the traction vector damaged normal
134 component, and \bar{t}_s and \bar{t}_t are the traction vector damaged shear components. According to Eq. (3a)
135 compressive stress does not cause damage.
136 Different damage evolution laws were associated to each interface. In the case of the FRCM-concrete
137 and internal-external matrix layer interfaces, damage was defined through the exponential damage
138 evolution law:

$$139 \quad D = \int_{\bar{\delta}_0}^{\bar{\delta}_f} \frac{\bar{T} \cdot d\delta}{G_C - G_0} \quad (4)$$

140 where $\bar{\delta}_0$ and $\bar{\delta}_f$ correspond to the effective separation $\bar{\delta}$ at the onset of damage and at complete
 141 failure, respectively, \bar{T} is the *effective* traction (Simulia 2016; Mei et al. 2010), G_0 is the energy
 142 release rate at damage initiation, and G_C is the critical energy release rate (Camanho and Davila 2002):

$$143 \quad \bar{\delta} = \sqrt{\langle \delta_n \rangle^2 + \delta_s^2 + \delta_t^2} \quad (5)$$

$$144 \quad \bar{T} = \sqrt{\langle \bar{t}_n \rangle^2 + \bar{t}_s^2 + \bar{t}_t^2} \quad (6)$$

$$145 \quad G_0 = \int_0^{\bar{\delta}_0} \bar{T} d\delta \quad (7)$$

$$146 \quad G_C = \int_0^{\bar{\delta}_f} \bar{T} d\delta \quad (8)$$

147 where $\langle \cdot \rangle$ denotes the Heaviside function. The damage evolution law associated with the matrix layer-
 148 fiber interfaces was defined by specifying directly the damage variable D with respect to the non-
 149 dimensional i -th component of the displacement $\bar{\delta}_p$:

$$150 \quad \bar{\delta}_p = \bar{\delta} - \bar{\delta}_0 \quad (9)$$

151 Mode-I and Mode-II, which are described by the σ_{zz} - w and τ_{zy} - s relationships, respectively, were
 152 coupled at the FRCM-concrete and internal-external matrix interfaces adopting the expression of the
 153 coupled critical energy release rate G_F proposed by Benzeggagh and Kenane (1996) (B-K). If a Mode-
 154 III loading condition is present, the B-K critical energy release rate G_F is (Camanho and Davila 2002):

$$155 \quad G_F = G_{IF} + (G_{IIF} - G_{IF}) \left(\frac{G_{shear}}{G_T} \right)^\eta, \text{ with } G_T = G_I + G_{shear} \quad (10)$$

156 where G_{IF} and G_{IIF} are the critical energy release rates for pure Mode-I and Mode-II loading
 157 conditions, respectively, G_I , G_{II} and G_{III} are the Mode-I, Mode-II, and Mode-III energy release rates,
 158 respectively, $G_{shear} = G_{II} + G_{III}$, and η is a parameter that should be calibrated using mixed mode
 159 bending (MMB) tests at different mode ratios (Camanho and Davila 2002). According to the B-K
 160 criterion, debonding occurs when the total energy release rate G_T is equal to or greater than the critical
 161 energy release rate G_F (Krueger 2008). Equation (10) was employed without distinguishing between
 162 sliding (Mode-II) and tearing (Mode-III) modes of fracture and simply assuming G_{III} as the energy
 163 release rate associated with the Mode-II loading condition in direction x .

164 Mode-I and Mode-II loading conditions were considered uncoupled for the matrix-fiber interfaces.
 165 This assumption is justified in the case of FRCM-concrete joints where the external matrix layer
 166 contrasts the out-of-plane displacement of the fibers and consequently limits the Mode-I loading
 167 condition (D'Antino et al. 2016). A different numerical approach able to describe the coupling of
 168 Mode-I and Mode-II at the matrix-fiber interfaces is currently under development by the authors and
 169 will be used to reproduce the behavior of FRCM composites without the external matrix layer.

170 In all the contact laws adopted, damage initiates when the quadratic stress criterion expressed by Eq.
 171 (11) is satisfied:

$$172 \left(\frac{\langle \bar{t}_n \rangle}{t_n^0} \right)^2 + \left(\frac{\bar{t}_s}{t_s^0} \right)^2 + \left(\frac{\bar{t}_t}{t_t^0} \right)^2 = 1 \quad (11)$$

173

174 **Numerical model**

175 The numerical approach proposed in this paper can be employed to study the bond behavior of
 176 different FRCM composites applied to various substrates. The numerical model parameters calibrated
 177 in this study are valid only for the particular FRCM composite and concrete substrate studied. Since
 178 the behavior of FRCM composites depends on different factors, such as the matrix, fiber, and substrate

179 characteristics and their bond behavior, the parameters of the numerical approach proposed will be
180 different depending on the specific FRCM and substrate considered.

181

182 *Characteristics and Geometry*

183 Twelve different numerical models, summarized in Table 2, were used to study the bond behavior of
184 PBO FRCM-concrete joints. Models were named following the notation NDS_X_Y_(C or G or H),
185 where NDS indicates the numerical model of a direct shear specimen, X=bonded length ℓ in mm,
186 Y=bonded width b_1 in mm, C (if present) indicates that the concrete block was not modeled, G (if
187 present) indicates that the Mode-I and/or Mode-II fracture energy of the FRCM-concrete interface or
188 the matrix-fiber interfaces was increased with respect to calibrated values, and H (if present) indicates
189 that the thickness h (Figure 1) of the concrete block was reduced.

190 For six models, indicated as “Condition E” in Table 2, the concrete block and FRCM composite were
191 modeled with the geometry and constrains according to the experimental test set-up employed by the
192 authors (D’Antino et al. 2016). For these models, the parameters needed to define the behavior of a
193 specific contact interface (see section *Numerical formulation*) were calibrated according to the
194 experimental results or assumed based on the literature (section *Loading condition and parameters*
195 *employed*). For five models, indicated as “Condition 1, 2, 3, 4, or 5” in Table 2, the parameters needed
196 to define the behavior of a specific contact interface and/or the specimen geometry were varied to
197 investigate the effect of the load eccentricity and of the different interfaces, as described in the next
198 sections. The effect of a high FRCM-concrete bond capacity was investigated for all bonded lengths
199 ℓ considered by assuming that the Mode-I and Mode-II fracture energies of the FRCM-concrete
200 interface were equal to those corresponding to debonding in a thin layer of concrete. Since the results
201 obtained were not affected by this assumption except for $\ell=100$ mm, only model NDS_100_60_G
202 (Condition 1) is reported in this paper. For model NDS_330_60_H (Condition 2) the thickness h of

203 the concrete block was reduced to 1 mm, and it was fully constrained at the face opposite to the face
204 where the FRCM composite was applied. For models NDS_330_60_C_1 (Condition 3) and
205 NDS_330_60_C_2 (Condition 4), the concrete block was omitted, and the composite strip was fully
206 constrained at the internal matrix layer bottom face. For model NDS_330_60_G (Condition 5), the
207 Mode-II fracture energy of the internal matrix layer- and external-matrix layer fiber interfaces was
208 increased relative to those of the other numerical models to investigate the effect of a high matrix-
209 fiber bond capacity (Table 2).

210 The FRCM-concrete joints tested experimentally were modeled considering FRCM composite strips
211 with bonded widths $b_1=60$ mm and different bonded lengths ℓ (see Table 2). In addition to numerical
212 models that reproduce the actual geometry of the specimens tested, an FRCM-concrete joint
213 comprised of a concrete block with $L=1000$ mm and an FRCM strip with $b_1 = 60$ mm and $\ell = 955$
214 mm, which was not tested experimentally, was also modeled to investigate the behavior of an FRCM-
215 concrete joint with a long bonded length (model NDS_955_60, Condition 6, Table 2). The geometry,
216 constraints modeled, and properties of the contact interfaces of model NDS_955_60 were consistent
217 with those of the experimental test set-up.

218 The length of the bare longitudinal PBO fiber bundles beyond the composite loaded end was kept
219 equal to 270 mm in each model. Further details on the specimen geometry are reported in Figure 3.

220 The concrete blocks were constrained by a hinge along the x -direction at the bottom edge, back face
221 of the blocks (Figure 3). The concrete top face was constrained by a series of springs acting in the y -
222 and z -directions located along the top face's axis of symmetry parallel to x (Figure 3). Since the
223 geometric and mechanical properties of the experimental set-ups employed were known, the stiffness
224 of each spring was computed by assuming rigid rotation of the concrete block, which is equivalent to
225 assuming that the concrete block deformation is negligible (Carloni et al. 2015b), and imposing the
226 equilibrium condition to the system (D'Antino et al. 2016). For the case of the concrete block with

227 $L=1000$ mm (model NDS_955_60), which was not tested experimentally, values of the spring
228 stiffness were obtained assuming the same cross-sectional dimensions as the frames used for concrete
229 blocks with $L=375$ mm and $L=510$ mm (D'Antino et al. 2016).

230 The FRCM composite was bonded to the concrete block through the FRCM bonded area, whereas
231 the individual matrix layers, whose thickness was 4 mm, were connected to each other through the
232 openings between the bundles of the fiber net. Transversal fiber bundles were assumed to be placed
233 against the external matrix layer according to the majority of experimental tests (D'Antino et al. 2014;
234 Carloni et al. 2015b). The contact areas of the different interfaces considered are shown in Figure 2,
235 whereas the interfaces (i, ii, iii, iv) considered for each model are indicated in Table 2.

236 Each component of the models (concrete block, matrix layers, fiber bundles) was implemented using
237 8-node brick elements. The mesh sensitivity of the model was investigated by varying the mesh size
238 and comparing the solutions obtained. Except for models NDS_100_60 and NDS_100_60_G, for
239 which the mesh size of concrete block elements in proximity to the FRCM strip was reduced to
240 accurately describe the FRCM-concrete interaction (Figure 3), square concrete block elements with
241 approximate edge dimensions equal to 17 mm (Table 3) were adopted. For model NDS_330_60_H,
242 three mesh elements were present in the thickness (z -direction) of the 1 mm concrete block. The mesh
243 size adopted, reported in Table 3 together with the resulting number of elements per direction, allowed
244 for reducing the computational time without affecting the accuracy of the results.

245

246 ***Loading condition and parameters employed***

247 The numerical analyses were carried out in displacement control, adopting the large deformation
248 formulation, by enforcing displacements v (Figure 3) in y -direction using the arc-length method with
249 automatic determination of the step amplitude.

250 The mechanical properties of concrete, mortar matrix, and PBO fibers reported in Table 1 were used

251 to define the parameters needed by the numerical models. In model NDS_330_60_H, the elastic
252 modulus of the 1 mm thick concrete block was set equal to 10^6 MPa to simulate a rigid support. For
253 model NDS_955_60, which was not tested experimentally, the mechanical characteristics of concrete
254 blocks with length $L=375$ mm (i.e. batch A) were assumed.

255 Since the fracture properties of the FRCM-concrete interface could not be investigated
256 experimentally, analytical models available in the literature were used to obtain reference values of
257 the Mode-I and Mode-II fracture energies. For the concrete employed in this study, the analytical
258 models proposed by fib Bulletin 70 (2013) provided a value of the Mode-I fracture energy between
259 $G_{Ic}=0.070$ N/mm and $G_{Ic}=0.147$ N/mm. Karihaloo et al. (1994) reported, as a reference value, a
260 concrete fracture energy of $G_{Ic}=0.05$ N/mm, whereas Hoover and Bazant (2013), who investigated
261 the size effect on the fracture properties of notched concrete beams, reported a value of the concrete
262 Mode-I fracture energy of approximately $G_{Ic}=0.1$ N/mm. The characteristics of the concrete mixture
263 used to cast the concrete blocks with $L=375$ mm employed in this study are comparable with those of
264 Hoover and Bazant (2013), who employed a concrete mixture with a compressive strength of 46.53
265 MPa (at 31 days) and maximum aggregate diameter of 10 mm.

266 The analytical models proposed in the literature provide values of the Mode-I fracture energy needed
267 to open a unit crack in a thin layer of concrete. For the FRCM-concrete joints considered, when failure
268 occurred at the FRCM-concrete interface it was observed to be characterized by detachment of the
269 composite strip without damage of the substrate (Sneed et al. 2014; D'Antino et al. 2015b). Therefore,
270 the Mode-I fracture energy associated to the FRCM-concrete interface should be lower than
271 corresponding Mode-I fracture energy of concrete reported in the literature. Starting from $G_{Ic}=0.07$
272 N/mm, which is the average of values reported in the literature (fib 2013; Karihaloo et al. 1994;
273 Hoover and Bazant 2013), and assuming a ratio between Mode-II and Mode-I fracture energies equal
274 to 4 (De Lorenzis and Zavarise 2008) and a stress normal to the crack surface equal to zero for a crack

275 opening of approximately 0.1 mm (Karihaloo et al. 1993), the Mode-I fracture energy of the FRCM-
 276 concrete interface, G_{Ia} , was calibrated to reach failure by detachment of the composite strip for the
 277 numerical model with bonded length $\ell = 100$ mm, as observed experimentally. The components of the
 278 elastic stiffness matrix \mathbf{K} , assumed equal for the Mode-I and Mode-II loading conditions, were
 279 calibrated to obtain, using the exponential damage evolution law (see section *Numerical formulation*),
 280 a shape of the $\sigma_{zz}-w$ and $\tau_{zy}-s$ relationships similar to those obtained experimentally for the Mode-II
 281 loading condition of the matrix-fiber interface and were found equal to 16 N/mm^3 (Figure 4). The
 282 fracture energies $G_{Ia}=0.013 \text{ N/mm}$ and $G_{IIa}=0.052 \text{ N/mm}$ were obtained. As a first attempt the B-K
 283 exponent η was assumed equal to 2 as found for woven composites comprised of brittle resin
 284 (Benzeggagh and Kenane 1996). It should be noted that the fracture energy value found is valid only
 285 for the specific matrix and concrete support employed.

286 The effect of a high FRCM-concrete bond capacity was investigated in model NDS_100_60_G by
 287 assuming that the Mode-I and Mode-II fracture energies of the FRCM-concrete interface were equal
 288 to those corresponding to debonding in a thin layer of concrete, as in the case of FRP-concrete joints.
 289 In order to evaluate the FRCM-concrete interface Mode-II fracture energy the analytical model
 290 proposed in CNR-DT 200 R1/2013 (CNR 2013) was adopted:

$$291 \quad G_{IIc} = \frac{k_b \cdot k_G}{FC} \sqrt{f_{ctm} \cdot f_{cm}} \quad (12)$$

$$292 \quad k_b = \sqrt{\frac{2 - b_f/b}{1 + b_f/b}} \geq 1 \quad (13)$$

293 where b_f and b are the width of the composite strip and concrete cross-section, respectively, k_G is a
 294 corrective factor that depends on the type of composite used (pre- or in-situ-impregnated), FC is a
 295 factor depending on the level of knowledge of the strengthened element, and f_{ctm} and f_{cm} are the mean
 296 concrete tensile and compressive strength, respectively. It should be noted that Eq. (12) does not

297 correctly take into account the effect of the width ratio b_f/b , expressed by k_b . If applied to the Mode-
298 II fracture energy equation, k_b should be raised to the power of 2, so that it would have the correct
299 magnitude when the fracture energy [Eq. (12)] is used to compute the load-carrying capacity (for
300 reference see (fib 2001; Chen and Teng 2001). Furthermore, coefficient k_b should not be employed in
301 calculating G_{IIc} that, since it is a fracture parameter, should not depend on geometrical characteristics.
302 For the FRCM-concrete joints studied in this paper, assuming $b_f=b_1$ and $k_G=0.07$, which is the average
303 value of k_G for pre- and in-situ-impregnated FRP composites, Eq. (12) provided $G_{IIc}=0.85$ N/mm.
304 The coupled Mode-I fracture energy of the FRCM-concrete interface was set equal to $G_{Ic}=G_{IIc}/4$, as
305 assumed for FRP-concrete joints (De Lorenzis and Zavarise 2008).
306 The fracture properties of the matrix employed, which were not investigated experimentally, could
307 not be directly validated by the load responses available, as in the case of FRCM-concrete interface
308 properties. Considering that the mean compressive and tensile strengths of the mortar employed in
309 this study were relatively high (Table 1), a Mode-I fracture energy of $G_{Im}=0.05$ N/mm was assumed
310 for the internal-external matrix layer interface (fib 2013; Karihaloo et al. 1994). Applying the same
311 ratio $G_{IIIm}/G_{Im}=4$ assumed in the case of the FRCM-concrete interface, the matrix Mode-II fracture
312 energy was assumed equal to $G_{IIIm}=0.20$ N/mm. As in the case of the FRCM-concrete interface, the
313 components of the elastic stiffness matrix \mathbf{K} were assumed equal for the Mode-I and Mode-II loading
314 conditions and, considering a stress normal to the crack surface equal to zero for a crack opening of
315 approximately 0.1 mm (Karihaloo et al. 1993), were found equal to 50 N/mm³ (Figure 4). The
316 coupling of Mode-I and Mode-II loading conditions was obtained by assuming the B-K criterion with
317 the exponent $\eta=2$.
318 A parametric study was conducted to investigate the validity of the ratio $G_{Ic}/G_{IIc}=4$ and $G_{IIIm}/G_{Im}=4$
319 made for the FRCM-concrete and internal-external matrix layer interfaces, respectively. The results
320 suggest that varying this ratio has a limited influence on the behavior of the specific composite and

321 substrate considered. This can be attributed to the limited effect of the Mode-I condition on the
322 behavior of FRCM-concrete joints with the external layer of matrix (D'Antino et al. 2016).
323 For the internal matrix layer- and external matrix layer-fiber interfaces, the damage variable was
324 specified for discrete values of the displacement $\bar{\delta}_p$ to reproduce the τ_{zy} - s relationships obtained
325 experimentally (Carloni et al. 2015b). The area under the τ_{zy} - s curve between $s=0$ and $s=\bar{s}_f=1.57$
326 mm, where $s=\bar{s}_f$ is the slip corresponding to complete matrix-fiber debonding measured
327 experimentally (Carloni et al. 2015b), is $G_{If}=0.481$ N/mm. As a first attempt, the τ_{zy} - s curves
328 associated with the internal matrix layer- and external matrix layer-fiber interfaces were assumed
329 equal (Carloni et al. 2015b). The σ_{zz} - w relationship, which was not measured experimentally, was
330 obtained by assuming the same shape of the τ_{zy} - s relationships and a Mode-I fracture energy of
331 $G_{If}=0.04$ N/mm (Figure 4). This assumption is justified by the limited effect of the Mode-I condition,
332 which did not affect significantly the behavior of the matrix-fiber interface. The elastic stiffness
333 obtained experimentally for the Mode-II loading condition, equal to 9.85 N/mm³, was assumed also
334 for the Mode-I loading condition.
335 For model NDS_330_60_G the value of G_{If} was increased to a value at which model NDS_330_60_G
336 failed due to detachment of the composite strip instead of debonding at the matrix-fiber interface. The
337 increased value of $G_{If}=0.843$ N/mm was obtained by increasing the shear stress corresponding to the
338 onset of damage $\tau_{zy}^0=0.3$ MPa to $\tau_{zy}^0=1.5$ MPa. Values of stresses σ_{zz}^0 and τ_{zy}^0 corresponding to the
339 onset of damage are reported in Figure 4 for each interface considered.

340

341 **Results and discussion**

342 ***Models with bonded length $\ell=330$ mm and bonded width $b_1=60$ mm***

343 The numerical applied stress σ -global slip g curves of specimens with bonded length $\ell=330$ mm and

344 bonded width $b_1=60$ mm were compared with corresponding envelopes of the experimental curves
345 obtained from specimens with the same bonded dimensions (Figure 5). The envelopes in Figure 5a
346 were obtained considering specimens with $\ell=330$ mm and $b_1=60$ mm. Some specimens, indicated in
347 Figure 5a with dashed lines, reported a load response different from those of other specimens with
348 the same characteristics and were not considered in the envelopes. Specimens that were considered
349 in the envelopes are shown in Figure 5a with light grey lines.

350 The scatter observed between the experimental curves depicted in Figure 5a is due to the quasi-brittle
351 nature of the matrix-fiber interface and can be observed for different FRCM composites (Carozzi and
352 Poggi 2015; D'Ambrisi et al. 2012; de Felice et al. 2012). However, the accuracy of numerical models
353 that employ a contact damage interface to reproduce the behavior of FRCM-concrete joints was
354 studied in Carloni et al. (2015a), where a pure Mode-II loading condition at the matrix-fiber interface
355 was assumed and a good agreement between experimental and numerical results was observed.

356 The load response obtained by model NDS_330_60 resembles the idealized load response put forward
357 by the authors (D'Antino et al. 2014). The numerical σ - g curve shows an initial linear behavior, which
358 corresponds to the linear elastic region of the τ_{zy} - s and σ_{zz} - w relationships of the different interfaces.

359 As the global slip increases, the normal stress σ_{zz} associated with all interfaces considered remains
360 lower than the corresponding elastic limit, whereas the shear stress at the matrix-fiber interfaces
361 attains the elastic limit, which triggers the onset of damage at the matrix-fiber interface causing the
362 load response to become non-linear. When the energy release rate associated with the matrix-fiber
363 interfaces is equal to the corresponding Mode-II fracture energy, the stress transfer zone (STZ)
364 (D'Antino et al. 2015) is fully developed, and debonding at the matrix-fiber interface occurs. For
365 bonded lengths $\ell > l_{eff}$, as in the case of model NDS_330_60, the applied stress further increases after
366 the onset of debonding due to the contribution of friction (interlocking) along the debonded matrix-
367 fiber interface as the STZ translates toward the free end. In the experimental response, after reaching

368 the peak stress σ^* , the applied stress decreases with increasing global slip until the fibers are
369 completely debonded from the matrix and a constant stress σ_f due to friction is present. The post-
370 peak softening curve, which was experimentally obtained by increasing the global slip at a constant
371 rate (section *Experimental set-up and materials*), could not be observed in the numerical analyses that
372 were carried out by controlling the displacement at the bare fiber end. It should be noted that the
373 softening branch of the applied load curve does not play a key role in the behavior of FRCM
374 strengthened elements, for example beams, which do not show a softening behavior (D'Ambrisi and
375 Focacci 2011). In order to verify if the interface shear stress-slip relationship can successfully capture
376 the experimental response, it is important that the numerical applied load curve of direct-shear tests
377 reproduce the experimental response only up to the peak load, i.e. without considering the softening
378 branch. In fact, the interface shear stress-slip relationship is fully exploited when the peak load is
379 reached. The numerical approach adopted was effective in reproducing the overall theoretical
380 behavior of FRCM-concrete joints, i.e. including the snap-back. The snap-back phenomenon
381 observed in the numerical models, which could be obtained by employing the arc-length method, was
382 observed in experimental tests of FRP-concrete joints carried out by controlling the slip between
383 fibers and concrete at the loaded and free ends (Carrara et al. 2011).

384 The numerical load responses show that damage affects only the Mode-II matrix-fiber contact
385 interactions, whereas the numerical shear stress at the FRCM-concrete and internal-external matrix
386 interfaces and peeling stress at all interfaces considered were lower than the corresponding elastic
387 limit during the entire analysis.

388 The load response obtained by model NDS_330_60_H, where the concrete block thickness was equal
389 to 1 mm, resembles the load response obtained by model NDS_330_60. This confirms that the effect
390 of the eccentricity between the load and constraints of single-lap direct shear tests is not significant
391 for FRCM-concrete joints with a (relatively) long bonded length and where the external layer of

392 matrix contrasts the occurrence of out-of-plane displacement of the fibers (D'Antino et al. 2016).
393 Models NDS_330_60_C_1 and 2, in which the concrete block was omitted, also show a load response
394 very similar to the load response of model NDS_330_60. Model NDS_330_60 provided a peak stress
395 $\sigma^*=2002$ MPa, whereas models NDS_330_60_H, NDS_330_60_C_1, and NDS_330_60_C_2
396 provided values of σ^* equal to 2005 MPa, 2009 MPa, and 2012 MPa, respectively. The slight
397 differences between peak stress values can be explained by the different loading conditions and
398 different contact interfaces of the models. The eccentricity between the point where displacement v is
399 enforced and the concrete block constraints of model NDS_330_60 entails for a higher Mode-I
400 loading condition with respect to models where the concrete block was either omitted or its thickness
401 was reduced to 1 mm, which entails for a reduced eccentricity.

402 As discussed in section *Loading condition and parameters employed*, for model NDS_330_60_G the
403 Mode-II fracture energy at the internal matrix layer- and external matrix layer-fiber interfaces was
404 increased with respect to that obtained experimentally to simulate the effect of a higher matrix-fiber
405 bond capacity. Model NDS_330_60_G failed due to detachment of the composite strip at the FRCM-
406 concrete interface at a peak load 18% lower than the peak load obtained by model NDS_330_60,
407 where debonding occurred at the matrix-fiber interface (Figure 5a). It should be noted that failure
408 occurs at a specific interface depending not only on the corresponding fracture energy value but also
409 on the value corresponding to the onset of damage. Assuming the matrix-fiber Mode-I and Mode-II
410 fracture energies employed for model NDS_330_60_G, if the Mode-I and Mode-II fracture energies
411 associated with the FRCM-concrete interface would be calibrated such that failure would occur within
412 the concrete substrate, and not by detachment as observed in model NDS_330_60_G, the
413 corresponding peak stress would be higher than that obtained for model NDS_330_60_G. The results
414 obtained show that the matrix bond properties with both the fibers and the substrate should be
415 considered, and that improving the matrix characteristics may not lead to a better overall bond

416 performance of the joint.

417

418 *Effect of bonded length ℓ*

419 The numerical approach adopted allowed for studying the effect of different bonded lengths on the
420 load response of PBO FRCM-concrete joints. Results of numerical models with the same bonded
421 width $b_1=60$ mm and seven different bonded lengths ℓ (Table 2) were analyzed. Figure 6a shows the
422 numerical σ - g curves obtained for numerical models with different bonded lengths ℓ . FRCM-concrete
423 joint models with the bonded length equal to or longer than 150 mm failed due to debonding of the
424 fibers from the embedding matrix, which is consistent with the experimental results. In addition,
425 model NDS_100_60_G (Condition 1) presented in section *Loading condition and parameters*
426 *employed*, which failed due to debonding at the matrix-fiber interfaces, was included in Figure 6a for
427 comparison with model NDS_100_60. According to the fracture mechanics approach proposed by
428 the authors, the debonding stress σ_{deb} of the FRCM-concrete joint is (Carloni et al. 2015b):

$$429 \quad \sigma_{deb} = \frac{P_{deb}}{nb^*t^*} = \sqrt{\frac{4EG_{If}}{t^*}} \quad (14)$$

430 where E is the fiber elastic modulus. The numerical debonding stress σ_{deb} , which is attained when the
431 matrix-fiber slip at the loaded end is equal to 1.57 mm for models with bonded length ℓ greater than
432 250 mm (Figure 6a), is approximately equal to the theoretical value obtained by Eq. (14), although it
433 increases slightly with increasing bonded length (see models NDS_450_60 and NDS_955_60 in Table
434 2). The authors postulate that the variation of σ_{deb} with ℓ is due to the energy contributions associated
435 with the FRCM-concrete and internal-external matrix interfaces, which increase with increasing
436 bonded length.

437 The variation of the peak stress σ^* with the bonded length ℓ is shown in Figure 6b. Figure 6b also
438 reports the average peak stress obtained by averaging σ^* of the experimental tests with the same

439 bonded length and width considered. The numerical peak stress values are in good agreement with
440 corresponding experimental tests for each bonded length adopted except for joints with $\ell=150$ mm,
441 where the numerical models overestimated the experimental average peak stress by approximately
442 20%. This discrepancy may be due to limits of the numerical approach adopted in describing the
443 matrix-fiber stress-transfer mechanism for short bonded lengths and to the limited number of
444 experimental tests carried out with $\ell=150$ mm. The scatter between the experimental results obtained
445 affects the comparison with corresponding numerical results shown in Figure 6b. However, the good
446 agreement observed, particularly for specimens with bonded length equal to 200 mm and 250 mm
447 (for which experimental results were not used to calibrate the numerical models), confirms the
448 effectiveness of the numerical approach proposed in studying FRCM composites.

449 When $\ell > l_{eff}$, the peak stress σ^* increases with increasing bonded length due to the contribution of
450 friction. The contribution of friction to the applied stress is clearly recognizable from the linear
451 increasing branch after the debonding stress is attained (Figure 6a and b) and from the constant applied
452 stress, at the end of the analyses (Figure 6a).

453

454 **Conclusions**

455 Three-dimensional non-linear numerical models were used in this paper to study the behavior of PBO
456 FRCM-concrete joints. The numerical approach assumed linear constitutive laws for the materials
457 and non-linear cohesive contact damage laws for the different interfaces considered. Results obtained
458 by the numerical models are in good agreement with corresponding experimental results. Based on
459 the findings of this paper the following conclusions can be drawn:

- 460 • The numerical model developed in this study is capable of reproducing different failure modes
461 observed in direct shear tests of FRCM concrete joints, namely debonding of the fibers from the
462 embedding matrix, interlaminar failure within the matrix, or detachment of the composite strip.

463 The model can potentially be used to investigate the influence of different parameters and
464 reproduce the load response of different FRCM systems.

465 • The numerical load response correctly reproduces the idealized load response put forward by the
466 authors. When $\ell > l_{eff}$, the contribution of friction to the applied stress is clearly recognizable from
467 the linear increasing branch after the debonding stress is attained and from the constant applied
468 stress at the end of the analyses. The softening observed in the post peak descending branch in the
469 experimental tests is not observed in the numerical load response.

470 • Varying the properties of the matrix does not necessarily improve the FRCM-concrete joint
471 response. Increasing the Mode-II fracture energy of the matrix-fiber interfaces with respect to that
472 obtained experimentally caused premature detachment of the FRCM composite at a load lower
473 than the peak load obtained for the same FRCM-concrete joint with the matrix-fiber Mode-II
474 fracture energy obtained experimentally.

475

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566 Figure Captions

567
568 **Fig. 1.** a) Photo of specimen DS_450_60_S_1 (Carloni et al. 2015b; D’Antino et al. 2014). b) Front
569 and c) side view of the experimental set-up illustrating the equivalent constraints assumed.

570
571 **Fig. 2.** Contact areas the different interfaces considered. a) Front view and b) back view.

572
573 **Fig. 3.** Geometry (in mm) of numerical model NDS_100_60.

574
575 **Fig. 4.** Internal-external matrix layer, FRCM-concrete, and matrix-fiber interface: a) σ_{zz} - w and b) τ_{zy} -
576 s relationships adopted

577
578 **Fig. 5.** a) Comparison between applied load-global slip curves of numerical models and experimental
579 curves envelope of specimens with bonded length and width $\ell=330$ mm and $b_1=60$ mm, respectively.
580 b) Enlarged detail of peak stress of curves reported in Figure 4a.

581
582 **Fig. 6.** a) Applied stress σ -global slip g curves of numerical models with different bonded length. b)
583 Applied peak stress σ^* versus bonded length ℓ .

584 **Table 1.** Mechanical properties of the materials.

Material	f_c [MPa]	f_t [MPa]	E [GPa]	ν [-]	Material	f_c [MPa]	f_t [MPa]	E [GPa]	ν [-]
Concrete batch A	42.5	3.4*	34.0**	0.2	Concrete batch B	33.5	3.0*	31.6**	0.2
Matrix	28.4	3.5*	7.0 ⁺	0.2	PBO fibers	-	3014	206.0	0.3

585 *Splitting tensile strength; **Obtained from the mean compressive strength according to (CEN

586 2004); ⁺Reported by the manufacturer (Ruredil 2009).

587 **Table 2.** Characteristics of the numerical models.

Name	Cnd ⁺	Interface				σ_{deb} [MPa]	σ^* [MPa]	Name	Cnd ⁺	Interface				σ_{deb} [MPa]	σ^* [MPa]
		i	ii	iii	iv					i	ii	iii	iv		
NDS_100_60	E	✓	✓	✓	✓	1053	1053	NDS_330_60_H ²	2	✓	✓	✓	✓	1934	2005
NDS_100_60_G	1	✓ ¹	✓	✓	✓	1330	1330	NDS_330_60_C_1	3	-	✓	✓	✓	1935	2009
NDS_150_60	E	✓	✓	✓	✓	1666	1666	NDS_330_60_C_2	4	-	✓	✓	-	1935	2012
NDS_200_60	E	✓	✓	✓	✓	1815	1815	NDS_330_60_G	5	✓	✓ ¹	✓ ¹	✓	1645	1645
NDS_250_60	E	✓	✓	✓	✓	1901	1901	NDS_450_60	E	✓	✓	✓	✓	1935	2139
NDS_330_60	E	✓	✓	✓	✓	1933	2002	NDS_955_60	6	✓	✓	✓	✓	2073	2793

588 ⁺Condition. ¹The Mode-I and/or Mode-II fracture energies were increased relative to those of the
589 other numerical models. ²Thickness h of the concrete block was 1 mm. i) FRCM-concrete. ii)
590 internal matrix layer-fiber. iii) external matrix layer-fiber. iv) internal-external matrix layer.

591 **Table 3.** Approximate size of mesh element adopted and resulting number of elements.

Component	Size (number) x - direction [mm]	Size (number) y - direction [mm]	Size (number) z - direction [mm]
Concrete block	1.90 (31)	2.50 (40)	1.00 (3)
interface			
Concrete block	17.00 (7)	17.00 (8)	17.00 (22 ¹ , 29 ²)
Internal matrix layer	1.00 (46)	3.00 (32 ³)	1.00 (4)
External matrix layer	1.00 (40)	5.00 (55 ³)	1.00 (4)
Bonded fiber bundles	1.00 (5)	2.50 (6 ³)	0.03 (3)
Bare fiber bundles	1.00 (5)	6.00 (2.5 ³)	0.03 (3)

592 ¹Concrete block with length 375 mm. ²Concrete block with length 510 mm. ³Total number of
593 elements (within the fiber bundle cross-section) per mm in y -direction.

594