The Gateway Location Problem: Assessing the impact of candidate site selection policies

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1. Introduction

Hazardous materials pervade our daily lives in many ways. The gasoline fueling our cars, radioactive and contaminated hospital waste, special drugs sold by pharmacists, the chemicals utilized in car body repair shops, are all examples of substances which pose little threat if properly used but can turn quite harmful to people and the environment if accidentally spilled. The most likely situation in which accidental release of hazardous materials (hazmat) can occur is during transportation. While the probability of an accident is generally very low, consequences can be extremely deleterious. Such a concern, fueled by a few fatal accidents that have occurred in the last few years, has motivated research on risk mitigation in hazardous material transportation. Much has been done on the prevention side, such as enforcing strong safety rules on material handling and packaging, as well as on the vehicles used for transportation. In the case of road transport, a critical

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step in the decision making process concerns vehicle routing, i.e., the selection of itineraries to be followed by hazmat vehicles on their trips from origin to destination. In the *unregulated scenario*, without prescribed directions, drivers are expected to select their minimum cost route among all possible origin–destination itineraries. The progressive urbanization of city outskirts has resulted, in many cases, in an incongruous mix of residential and industrial land uses. As a result, hazmat vehicle routes may cross residential areas (unless explicitly prohibited) when origins and destinations are located within mixed industrial–residential areas. Therefore, when entirely left to drivers, route selection may yield extremely risky itineraries, which generally should be avoided. At the same time, there may be alternative itineraries which are still economically acceptable to drivers but quite different with respect to risk. Drivers should be supported in this critical decision making step in order to make risk aware choices.

For some specific shipments, an ad hoc itinerary can be entirely planned and prescribed by the authority in charge of the regulation. In this case we speak of the *over regulated scenario*. However, this cannot be assumed to be common practice, since the burden of full control on route enforcement for each shipment would not be affordable. A more viable alternative to the over regulated scenario is to indirectly encouraging safer itineraries, rather than enforcing them, by exploiting the supposedly rational behavior of drivers. In such a *rule-based scenario*, the authority promulgates a set of common rules, easy to check, with which drivers must comply. Such rules, in order to be effective, must be stated by taking driver reactions into account. For this reason the problems arising in the rule-based scenario usually give rise to bilevel optimization problems, which properly model the hierarchical relationship between the decision makers. Examples are the closure of some links of the network to hazmat vehicle transit or specific pricing policies on the network links. The most significant contributions to hazmat routing are briefly discussed in Section 2.

The Gateway Location Problem (GLP) arises precisely in the framework of rule-based routing. It consists of locating a fixed number of check points (so called gateways) selected out of a set of candidate sites and assigning one such gateway to each vehicle as a compulsory crossing point along its itinerary in such a way that the sum of the risk of the minimum cost route of each vehicle from its origin to its destination via the assigned gateway is minimized.

GLP was first introduced in [2] where its efficacy as a risk mitigation tool was tested on a set of realistic instances. In [2] it was assumed that the set of candidate sites among which gateway locations are selected is randomly sampled out of the whole set of network nodes, according to a uniform probability distribution. While, for a given number of open gateways, the level of risk mitigation obviously does not decrease by enlarging the candidate site set up to considering all nodes in the network, in [2] it was also shown that a good level can be reached by way of an intelligent selection of a limited number of nodes. Such a procedure becomes a compulsory step when tackling instances related to large size urban areas, whose networks easily reach into the thousands of nodes. In such a case, the size of the candidate site set may become a distinct problem. This paper aims at providing adequate policies for this step. Section 4 is devoted to their introduction. In particular, we experimentally investigate the influence of different criteria for the selection of the ground set out of which the candidate sites will be generated according to a probability distribution; moreover, we analyze the impact of different probability distribution laws used for sampling. Indeed, previous results pointed out that this stage of the process, i.e., candidate site generation, can impact not only on the efficacy of the method, i.e., the risk level associated with the solution itineraries, but also on the efficiency of the method. In fact, [2] provides experimental evidence that a wise choice of candidate sites may reduce the number of candidate sites to be considered in order to achieve target risk mitigation thresholds. Section 5 is devoted to the experimental comparison. The proposed policies are compared against plain uniform random generation through a wide experimental campaign which is based on the results of the associated GLP run on a set of realistic instances and considers both quality and robustness of the results. In order to make the analysis less data dependent, the test bed encompasses instances characterized by three different risk functions. Conclusions are finally drawn in Section 6.

The GLP is not only the core of a new method for risk mitigation, but it also introduces a new problem in combinatorial optimization. In Section 3, after recalling the problem structure, we provide the first complexity proof of NP-hardness of GLP and highlight its relations with well known NP-hard problems.

2. Literature review

Two main research lines can be identified in the literature on risk mitigation policies to regulate the itineraries of the hazmat shipments: (i) enforcing specific itineraries to vehicles in the framework of an over regulated scenario; (ii) prescribing a set of rules that vehicle itineraries have to respect in a rule-based scenario.

Regarding the first research line, several criteria can be used to evaluate the quality of a given itinerary. Indeed, risk assessment studies propose several alternative risk measures, such as societal risk, population exposure, and incident probability. At the same time, however, carriers aim at cost reduction, usually achieved by minimizing travel time or travel distance. All these criteria are potentially conflicting and naturally lead to multi-objective flow problems as studied in [12,19]. Solution methods for hazmat routing are usually distinguished between local and global ones, as documented in [20].

Local routing is concerned with selecting a set of routes between a given origin–destination pair for the repeated shipment of a single commodity. The main issue concerns the equitable spread of risk among the population [11]. In this case, not only the total risk should be minimized, but the risk should also be distributed in the most uniform way over the whole transport region. This problem leads to the search for spatially dissimilar paths which can be addressed by various modeling approaches, e.g., by means of the Iterative Penalty Method [18], by means of the Gateway Shortest Path [15], by

p-dispersion [7], by exploiting the k-shortest paths [5], or by combining the k-shortest paths with p-dispersion [1]. Note that [15] is the first paper to introduce the notion of a *gateway path* in the context of corridor location, limited to a single origin–destination pair, in order to generate a set of spatially different alternative paths from the shortest one.

Global routing concerns many-to-many routing problems, dealing with multicommodity and multiple origin–destination routing decisions. Multiple conflicting objectives, such as cost minimization, risk minimization, and equity, are still present. However, solution approaches for multicommodity minimum cost flow problems usually tackle the single objective variant. Therefore, the aforementioned global routing problems arising in hazmat transport are usually tackled by goal programming approaches [22].

All the previously mentioned approaches return a set of itineraries to be assigned to carriers. Enforcing specific itineraries has the disadvantage of requiring the use of remote control systems, e.g., GPS for vehicles tracking [3]. Moreover, in many countries in Europe and North America, government agencies do not have the authority to dictate routes to hazmat carriers and the remote control of carriers may conflict with privacy.

An alternative research line focuses on the indirect control of hazmat routing by way of rules that carriers have to respect in a rule-based scenario. Since driver' response must be included in the models, such approaches tend to generate bilevel problems [6]. In this context, the most studied problem is the *Hazmat Network Design* (HND) problem, i.e., the interdiction of some road arcs to hazmat transit. In their seminal work, Kara and Verter [13] provide the first bilevel formulation of HND: in the upper-level problem the authority selects arcs to be closed according to risk minimization, whereas the lower-level problem models carriers' route choices that pursue cost minimization. Erkut and Alp [8] propose a greedy algorithm to heuristically select a tree subnetwork such that the sum of the risk of the unique origin–destination path of each commodity is minimized. For the general HND, Erkut and Gzara [9] propose a fast heuristic algorithm which guarantees stable solutions, i.e., solutions such that no commodity has multiple minimum cost paths with different risk values. To take into account carrier preferences, Kara and Verter in [21] introduced a single level path-based formulation, making explicit the set of paths acceptable to carriers and their preferences within such a set.

Beside arc interdiction, another rule-based risk mitigation approach exploits *toll-setting* [16]. Here, tolls are used for the first time to deter hazmat carriers from using certain roads and to channel the shipments on less-populated roads. This policy also gives rise to a bilevel problem, which can be reformulated and solved as a single-level Mixed Integer Linear Programming (MILP) problem. The authors experimentally show the effectiveness of such a technique.

Finally, in [2], a third alternative policy for indirectly regulating hazmat transport is proposed, which consists of detouring vehicles through compulsory check points. This policy gives rise to GLP. To the best of our knowledge, GLP had never been studied before nor used in the context of risk mitigation. Three mathematical formulations of GLP are proposed in [2]: a path based ILP model, a *k-median* like ILP model, and a bilevel multicommodity flow model together with its reduction to a single level MILP. All models assume that the *candidate sites set*, i.e., the set of the sites where a gateway can be installed, is known. In the present work, we address a preparatory phase of GLP concerning precisely the generation of this set.

3. GLP: a new combinatorial optimization problem

3.1. Mathematical notation

We are concerned with the problem of routing a set of vehicles away from their minimum cost routes by assigning to each vehicle a gateway as a compulsory crossing point. A given number of gateways must be located at as many sites, selected out of a set of candidate sites, and each vehicle assigned to one gateway, so that the risk of the new routes is minimized. Let us introduce some mathematical notation in order to formalize GLP as a combinatorial optimization problem.

Regarding vehicles, let $V = \{1, \dots, n\}$ be the vehicle set. For each vehicle $v \in V$, the pair (o_v, d_v) denotes its origin and destination and φ_v its demand, i.e., either the amount of the commodity transported by v or the number of shipments. Let $O = \{o_v, v \in V\}$ be the set of origins and, likewise, let $D = \{d_v, v \in V\}$ denote the destination set. Let N^{CS} be the set of the n_{CS} candidate locations where k gateways with $k < n$ and $k \ll n_{CS}$ have to be installed.

A weighted directed graph $G = (N, A)$ models the network in such a way that $N^{CS} \cup O \cup D \subseteq N$, i.e., the node set includes the candidate sites as well as the origin and the destination of each vehicle. For each arc (i, j) in the arc set $A \subseteq N \times N$, a positive cost coefficient c_{ij} and a non-negative risk coefficient r_{ij} per flow unit are given. Here, c_{ij} models the drivers utility function, such as distance, travel time, or monetary cost of travel along the arc, while r_{ij} represents a measure of the risk, i.e., the potential damage associated with transporting hazardous material along arc (i, j), and enters into the network administrator objective function. For a discussion on how to model risk functions see [10]. Note that, in some applications, cost, and more significantly, risk coefficients can be commodity dependent and thus vary according to the vehicle. However, since our discussion can be fully restated and easily adapted to such a case, hereafter we will omit the commodity index in order to keep the notation simple.

Finally, let ρ_v^c (ρ_v^r) denote the c-optimal (r-optimal) path from o_v to d_v for each $v \in V$. We will also refer to ρ_v^c as the *shortest* path and to ρ_v^r as the *safest* path for vehicle v. Let $gtw(v)$ denote the gateway assigned to vehicle v. Once location $h \in N^{CS}$ has been selected to host a gateway and the gateway at h has been assigned to vehicle v, i.e., $gtw(v) = h$, then vehicle v will travel along the shortest gateway path with respect to h, say $\rho_v(h)$. In particular, its route $\rho_v(h)$ will be made up of two paths, namely the upstream gateway path $\overline{p}_v^h$, i.e., the shortest path going from o_v to h, and the downstream gateway path

$\underline{p}_v^h$, i.e., the shortest path going from h to d_v. Indeed, each one is optimal according to the cost criterion due to the rational carrier behavior hypothesis.

GLP can be formalized as the problem of selecting a subset N^{gtw} of size k out of N^{CS} and assigning to each vehicle v one gateway $h \in N^{gtw}$ so that the sum over each vehicle of the risks of the two paths $\underline{p}_v^h$ and $\overline{p}_v^h$ is minimized. More formally, we solve:

$$\text{GLP} : \min \sum_{v \in V} \sum_{\substack{h \in N^{gtw}: \\ h = gtw(v)}} \varphi_v \left(\sum_{(i,j) \in \overline{p}_v^h} r_{ij} + \sum_{(i,j) \in \underline{p}_v^h} r_{ij} \right) \tag{1}$$

$$N^{gtw} \subseteq N^{CS} \tag{2}$$

$$|N^{gtw}| = k. \tag{3}$$

A more general version of the problem accounts for the possibility of letting a vehicle free to follow its shortest path, in case no shortest gateway path decreases its risk level. In such a case the vehicle is said to be *exempted*. Exemption can be easily cast in the former framework by letting the summation over h vary in the broader set $h \in N^{gtw} \cup \{o_v\}$ rather than only in N^{gtw}. Indeed, if $h = gtw(v)$ belongs to ρ_v^c, then its shortest gateway path $\overline{p}_v^h \cup \underline{p}_v^h$ coincides to ρ_v^c. The above feature can be implemented by letting each o_v act as a gateway for vehicle v without increasing the cardinality of N^{gtw}. Exemption is quite important, since it guarantees that the risk value associated with any optimal solution of the GLP will never increase the risk level of the unregulated scenario in which each vehicle travels along ρ_v^c.

Note that GLP is a hierarchical decision problem since expressions $\overline{p}_v^h$ and $\underline{p}_v^h$ hide a nested level of optimization. In fact, the minimum risk solution has to be searched for in the rational "reaction set" of the drivers. If the reaction set is not a singleton, i.e., the shortest gateway path going through $gtw(v)$ is not unique for at least one driver, we can distinguish two cases. In the first case, drivers behave in a collaborative way, i.e., they choose the minimum risk path among their shortest gateway paths; in the second case, drivers behave in an adversarial way, i.e., they choose the maximum risk path among their shortest gateway paths. Note that the drivers' adversarial behavior is not due to a real aversion to the administrator but to the simple fact that drivers are not aware of the risks associated with their itineraries. In a similar way, suppose that the administrator has multiple minimum risk alternatives regarding the gateways to be opened and their assignment to drivers, and, in addition, he knows the costs of the vehicle itineraries. When the administrator selects the cheapest solution among those of minimum risk, we say that the administrator is *cost aware*. All the MILP models presented in [2] can be generalized to encompass the above mentioned variants, by suitably perturbing the cost (risk) coefficients of the drivers (administrator) objective function. The two options for the follower and those for the leader can be combined giving rise to four variants. In this paper we focus on the adversarial cost aware variant, which represents the most challenging case.

Finally, note that the solutions provided by GLP could not be obtained neither by solving a minimum risk multicommodity flow nor a goal programming problem where risk is minimized first and cost comes as a second objective. Cost minimization does not play the role of a lower priority objective function but it is introduced in the model with the sole purpose of representing driver reaction and mathematically characterizing the so called reaction set, i.e., the set of shortest gateway paths.

3.2. Computational complexity

GLP shares a few common features with location problems. Indeed, it can be shown to be a strongly NP-hard problem itself due to a reduction from the *k-median* problem as described in the following. Recall that the *k-median* problem is defined as follows: given a complete undirected graph $G = (V, E)$, with $V = \{v_1, \ldots, v_n\}$, and a distance function $g : E \to \mathcal{R}^+$, the *k-median* problem consists of finding a subset $V' \subseteq V$ such that $|V'| = k$ and $\sum_{u \in V} \min_{v \in V'} g(u, v)$ is minimized.

Theorem 1. *The GLP is strongly NP-hard.*

Proof. Suppose that an instance of the *k-median* problem is given, we build the following GLP instance on a tripartite complete graph $\tilde{G} = (N, A)$.

We consider a set of $n = |V|$ commodities, each associated with the origin–destination pair (o_i, d_i) for $i = 1, \ldots, n$. Let $O = \{o_i : i = 1, \ldots, n\}$ and $D = \{d_i : i = 1, \ldots, n\}$. The set of candidate gateways is given by $N^{CS} = \{h_1, \ldots, h_n\}$. N is the union of O, D, and N^{CS}. Each node $o_i, d_i, h_i \in N$ is associated with vertex $v_i \in V$. The set of arcs is defined as $A = A' \cup A''$, where $A' = \{(o_i, h_j) : o_i \in O, h_j \in N^{CS}\}$ and $A'' = \{(h_j, d_i) : h_j \in N^{CS}, d_i \in D\}$ (see Fig. 1). For each arc $(o_i, h_j) \in A'$ and $(h_j, d_i) \in A''$ we set the cost equal to $M - g(v_i, v_j)$ where $M = \max_{[v_p, v_q] \in E} g(v_p, v_q) + \varepsilon$ and ε is any small positive constant, and the risk equal to $\frac{1}{2} g(v_i, v_j)$. In this way, the cost coefficients are positive, the risk coefficients are non-negative as required in GLP, and the administrator's objective function and the drivers objective functions become negatively correlated.

Firstly, observe that for each commodity every origin–destination path is made of two consecutive arcs belonging to A' and A'', respectively. Moreover, note that the authority has no advantage in exempting any commodity because in such a case such a commodity $\hat{\imath}$ would choose the path $o_{\hat{\imath}}, h_{\hat{\jmath}}, d_{\hat{\imath}}$ where $h_{\hat{\jmath}}$ is $\arg\max_{h_j \in N^{CS}} g(v_{\hat{\imath}}, h_j)$, i.e., $\hat{\imath}$ would choose the riskiest path. Therefore, the reaction set of the follower is a singleton and this GLP instance is well posed.

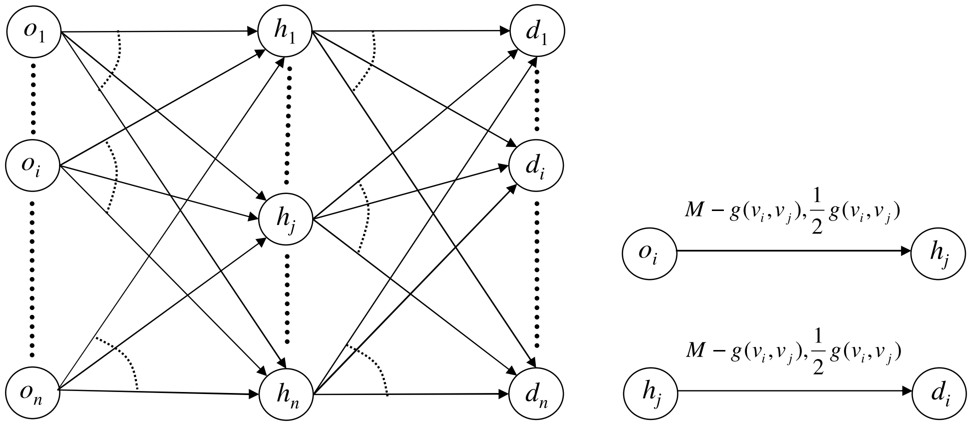

Fig. 1. GLP instance associated with the *k-median* one (the two values depicted on the arcs are the cost and the risk, respectively).

It is easy to see that any feasible solution to GLP where just k gateways can be opened corresponds to a feasible solution to the *k-median* problem with same value. Indeed, let $h_{i_1}, \ldots, h_{i_k}$ be the open gateways, then the corresponding solution to the *k-median* problem is given by $V' = \{v_{i_1}, \ldots, v_{i_k}\}$. Likewise, the risk of the GLP feasible solution is $\sum_{i=1}^{n} \min_{h \in N^{CS}} (\frac{1}{2} g(o_i, h) + \frac{1}{2} g(h, d_i)) = \sum_{u \in V} \min_{v \in V'} g(u, v)$, which is also the value of the corresponding *k-median* feasible solution. Therefore, from the optimal solution to the GLP on graph $\tilde{G} = (N, A)$, we can obtain the optimal solution to the *k-median* problem for the original instance. ■

Corollary 1. *Given any constant $\epsilon > 0$, the problem of approximating the GLP within an ϵ relative factor is NP-hard.*

Proof. The proof follows immediately from the fact that the supposition holds for the *k-median* problem (Lemma 2 of [14]) and the reduction from *k-median* to GLP used in the proof of Theorem 1 is a *gap-preserving reduction* since the GLP optimal value for the transformed instance coincides with the *k-median* optimal value for the original instance. ■

4. Ground sets and generation policies

In this section we introduce 16 alternative policies for candidate site generation obtained by properly combining a criterion for defining the ground set with a probability distribution law for sampling the current ground set. For any such policy, we also propose a deterministic version based on the solution of a combinatorial optimization problem. Random sampling according to uniform distribution on the whole set of the nodes of the network, as in [2], is the benchmark policy.

All policies have been inspired by the fact that the minimum risk level is achieved when each vehicle $v \in V$ travels along its minimum risk path ρ_v^r. In an ideal scenario, there would exist an internal node h of ρ_v^r such that $\rho_v(h) = \rho_v^r$, i.e., path ρ_v^r is made of a shortest path from o_v to h and one from h to d_v, respectively. In such a case, selecting h as a gateway and assigning h to v would route v on ρ_v^r. At the same time, however, no node on the shortest path ρ_v^c, if selected as a gateway and assigned to v, would contribute to risk mitigation since the vehicle would follow ρ_v^c as it does in the unregulated scenario where no risk mitigation policy is enforced. Therefore, from the perspective of a single vehicle v, there is no gain in adding the nodes of path ρ_v^c to the ground set. We further elaborate on these basic concepts to build up several criteria upon which a node qualifies to be part of the ground set. Criteria and sampling distribution laws are introduced in Sections 4.1 and 4.2.

4.1. Ground sets

One or more *target* paths are considered for each vehicle, i.e., paths such that, if followed by the driver as origin–destination itineraries, would decrease the risk level with respect to the unregulated scenario. Let Θ_v denote the set of the target paths of vehicle v (potentially a singleton) and let $\mathcal{N}(\Theta_v)$ be the set of their internal nodes, i.e., the union of the nodes of the paths in Θ_v minus o_v and d_v. The main target path for each vehicle v is the safest path ρ_v^r, while its shortest path ρ_v^c provides the highest risk level admitted. In between these two extremes lies the Pareto frontier of the so called *efficient* paths, i.e., those representing the best compromises between the objective functions of the two stakeholders involved, namely the drivers and the network authority. Therefore, each efficient path can be considered as a target path. Unfortunately, there is no guarantee that a vehicle gets actually routed on a target path just by setting $gtw(v)$ equal to an internal node of that path. On the contrary, for each elementary shortest gateway path, there is a value of p sufficiently large to ensure that this path belongs to $\Gamma_v^r(p)$, i.e., the set of the p safest (less risky) loopless paths. In particular, $\Gamma_v^r(p)$ provides a set of target paths as long as the risk of the p-th path is below the risk of path ρ_v^c, denoted by $r_{\rho_v^c}$.

Each criterion C_i, $i = 1, \ldots, 4$, identifies a ground set $N_i^{GS} \subseteq N$ out of which the selection probability of a node is null. Each C_i induces a different set of target paths Θ_v for each vehicle. We say that a vehicle v *sponsors* the nodes in $\mathcal{N}(\Theta_v)$.

A node belongs to the ground set according to the selected criterion only if it is sponsored by at least one vehicle. In this study, the following criteria have been considered:

- The first criterion C_1 aims at routing each vehicle on its safest path by using the internal nodes in ρ_v^r as potential gateways. Formally, $\Theta_v = \{\rho_v^r\}$ and $N_1^{GS} = \bigcup_{v \in V} \mathcal{N}(\rho_v^r)$. In a cost aware variant, if ρ_v^r is not unique, the shortest one would be selected.
- The second criterion C_2 aims at getting rid of useless nodes, such as those belonging to the shortest path of a vehicle. A vehicle sponsors a node $i \in \mathcal{N}(\rho_v^r)$ only if $i \notin \mathcal{N}(\rho_v^c)$. Therefore, $N_2^{GS} = \bigcup_{v \in V}(\mathcal{N}(\rho_v^r) \backslash \mathcal{N}(\rho_v^c))$. As $N_2^{GS} \subseteq N_1^{GS}$, its cardinality may become a critical issue and it should be checked to ensure it is large enough to allow for sampling subsets of the size required in case of partial collinearity between risk and cost.
- The third criterion C_3 aims at attracting the vehicles into safe paths which are not too expensive with respect to the drivers cost function. Here, we trade risk optimality for cost awareness. Let $\Gamma_v^{r,c}$ be the set of paths on the Pareto optimal frontier of the bicriteria shortest path minimizing both risk and cost for vehicle v. According to C_3, the set Θ_v coincides with $\Gamma_v^{r,c}$ and $N_3^{GS} = \bigcup_{v \in V} \mathcal{N}(\Gamma_v^{r,c})$.
- According to the fourth criterion C_4, the target paths are not necessarily optimal with respect to risk: we relax risk optimality to enlarge the set of target paths with respect to C_1 and C_2, hoping to capture target paths similar to a shortest gateway path. In this case, Θ_v is given by $\Gamma_v^r(p)$ and N_4^{GS} is made of the internal nodes of all the paths which are risk suboptimal up to a threshold for at least one vehicle: $r_{\rho_v^c}$ is the straightforward threshold of risk deterioration and indirectly provides a bound on p. Any such target path would contribute to risk mitigation if it could be enforced as a shortest gateway path for the associated vehicle.

Each criterion can be properly combined with a distribution law to sample its ground set: several options are possible. We investigate the following three probability distributions: (i) a uniform probability distribution over the ground set; (ii) a distribution which considers how many vehicles sponsor each node; (iii) a distribution similar to the previous one that also takes into account the vehicle's demand. In case of several target paths for each vehicle, alternatives are possible.

4.2. Probability mass functions

Formally, given a ground set N^{GS}, we introduce a probability mass function as a function $\Phi: N^{GS} \mapsto [0, 1]$ so that $\Phi(i) = \phi_i$, i.e., the probability of selecting a generic node $i \in N$, is null outside of N^{GS} and the pair (Φ, N^{GS}) forms a probability space, i.e., $\sum_{i \in N^{GS}} \phi_i = 1$ and $0 \le \phi_i \le 1$. In order to enforce such properties, ϕ_i is defined as the ratio of a weight $\omega_i \ge 0$ and the sum of such weights over all nodes in N^{GS}. We propose five different implementations of function Φ which differ regarding the computation of the node weights $\{\omega_i\}$.

The basic version Φ_1 is the uniform probability function, i.e., $\omega_i = 1$ and $\phi_i = \frac{1}{|N^{GS}|}$, so that each node in the ground set gets the same chance of being sampled.

A more refined version Φ_2 distinguishes among the nodes in the ground set according to how many target paths each node belongs to. Let a_{vi} be the number of paths in Θ_v having i as an internal node. Then, $\omega_i = \sum_{v \in V} a_{vi}$.

The third probability mass function Φ_3 also considers the influence of vehicle demands. In this case, $\omega_i = \sum_{v \in V} \varphi_v a_{vi}$.

As to the fourth and the fifth cases, we focus on vehicles. Let $b_{vi} = 1$ if and only if vehicle v sponsors node i (i.e. $a_{vi} \ge 1$), $b_{vi} = 0$ otherwise. According to Φ_4, a node's probability is proportional to the number of its sponsor vehicles: $\omega_i = \sum_{v \in V} b_{vi}$.

According to Φ_5, the previous quantity is weighted by the vehicle's demand so that $\omega_i = \sum_{v \in V} \varphi_v b_{vi}$.

4.3. Building the set of candidate sites: sampling and covering

Here we combine the ground set criteria introduced in 4.1 with the probability laws introduced in 4.2 in order to generate the different node subsets that we use as candidate sites in our instances. Note that criteria C_1 and C_2 can be combined only with probability mass function Φ_1, Φ_2 and Φ_3, while criteria C_3 and C_4, which potentially admit more than one target path for each vehicle, can be combined with all the five probability mass functions, thus yielding a total of 16 combinations.

Two alternative approaches are considered. The first one is based on random sampling and it is made up of the following steps: select a criterion, build the corresponding ground set, carry out the sampling according to any proper probability distribution among those introduced in 4.2. Each sample is obtained by sampling without replacement individual nodes out of the selected N^{GS} according to the selected function Φ until we reach the required size. According to this scheme, our benchmark policy can be recasted as the pair (C_0, Φ_1), where criterion C_0 simply sets N^{GS} equal to the whole set of nodes N.

The second one consists of a deterministic procedure based on the solution of a combinatorial optimization problem, where the input is a ground set whose nodes are weighted by the ϕ_i coefficients of the distribution law. The procedure combines heuristic features with an exact solution method. Indeed the criteria and the weights of distribution functions come from qualitative observations and are used to define the coefficients of an Integer Linear Programming (ILP) model which will be solved exactly. Since with sampling we are not able to guarantee that as many vehicles as possible have at least one sponsored node selected as a candidate site, a covering model with cardinality constraint whose objective function is to maximize such a number can thus be used. A lower priority objective can be added based on the node probabilities ϕ_i, which aims at maximizing their sum over the selected nodes. The hierarchy between the two objective functions is implemented by using a suitable big M constant. Formally, we solve the following ILP model:

$$P^{cover}: \min \; M \sum_{v \in V} s_v + \sum_{i \in N^{GS}} (1 - \phi_i)\, x_i \tag{4}$$

$$\sum_{i \in N^{GS}} x_i = n_{CS} \tag{5}$$

$$\sum_{i \in N^{GS}} b_{vi}\, x_i + s_v \geq 1 \quad \forall v \in V \tag{6}$$

$$x_i \in \{0, 1\} \quad \forall i \in N^{GS} \tag{7}$$

$$s_v \in \{0, 1\} \quad \forall v \in V. \tag{8}$$

Variables x_i are the node selection variables. Coefficients b_{vi}, as previously introduced, are equal to 1 if node i is sponsored by v. Two types of constraints are present: a cardinality constraint (5), requiring a selection of exactly n_{CS} nodes out of N^{GS}; a covering constraint (6) for each vehicle v, requiring that either at least one node among those sponsored by v is selected or the corresponding slack variable s_v is set to 1. Indeed, it may not be possible to cover all vehicles with only n_{CS} nodes.

In addition, an alternative function can be used as the lower priority objective of P^{cover} that maximizes the product of the probabilities of the selected nodes, i.e., $\max \Pi_{i \in N^{GS}} \phi_i^{x_i}$. Actually, for each subset of N^{GS} made of n_{CS} nodes, $\Pi_{i \in N^{GS}} \phi_i^{x_i}$ models the probability of selecting such nodes according to the sampling procedure described above. Such an objective function can be easily recast in linear form by minimizing the negative log-likelihood, i.e., $\min - \sum_{i \in N^{GS}} \ln(\phi_i) x_i$. Therefore, the resulting model can still be solved by an ILP solver.

In the next section, previously mentioned policies are experimentally compared. First, the candidate site sets N^{CS} are computed by the random sampling procedure as well as by the deterministic approach according to each of the 16 aforementioned policies. In the deterministic approach, both the sum and the product based functions are tested as lower priority objectives in the objective function of model P^{cover}. Then, each candidate site set N^{CS} is given as input to the GLP (1)–(3) which is solved by running the *k-median* based ILP model introduced in [2]. The resulting risk values are used to compare the candidate site generation policies.

5. Experimental campaign

The aim of the experimental campaign is to provide numerical evidence for the following conjectures concerning the impact of information guided policies for candidate site generation:

- The stable behavior observed for the random generation case is confirmed for the presented ad hoc policies, so that it is possible to capture most of the risk mitigation potential of the whole gateway based strategy by way of a limited number of open gateways. The minimum number of open gateways required to reach almost the maximum achievable risk reduction can be computed for each risk function when using information guided policies for candidate site generation. Although these values vary according to the specific risk function, they are very close to each other.
- For such a fixed number k of open gateways, the same level of risk mitigation that is achieved, on average, by a given number of randomly generated candidate sites can also be reached, on average, by way of an information guided selection of a much smaller subset of nodes.
- Conversely, for a fixed size of the candidate site set, an information guided policy reaches on the average a lower risk threshold than a purely random selection policy.
- Different kinds of information have different levels of effectiveness in selecting promising sets of candidate sites, that is, containing k nodes with which low risk level solutions are associated.
- Adding proper information to the selection process may also improve the robustness of the solution approach. This may be the key point when the percentage of nodes in N^{CS} needs to be lowered to tackle larger, real sized network instances without compromising performance.

5.1. Data set

The experimental campaign was carried out on the same set of instances used in our benchmark [2]. These, in turn, had been derived from published studies in the field of hazardous material transportation. Instances were built on the data set described in [9], i.e., an undirected graph with 105 nodes and 134 arcs as an abstraction of the road network of Ravenna (Italy). For each arc, a positive cost coefficient is given while three different risk functions are considered, yielding three different networks. Each network has been transformed into a directed network in standard way. In all cases, costs and risks are not collinear. For all three risk functions, transport risk on the arc is computed by multiplying a population density figure by the frequency of hazmat release in case of an accident. Regarding population density, the following data is used in [9] for devising the risk functions: population density on the arc (persons/meter); population density in the proximity of

Table 1
k_{stab} related values.

Risk measure	Min k_{stab}	Max k_{stab}	Avg k_{stab}	Var k_{stab}	k^*
Aggregate	2	5	4.41	0.424	5
Around-arc	2	5	2.74	0.326	3
On-arc	2	4	3.60	0.324	4

the arc (within a given bandwidth); location and population density at population concentration points (schools, hospitals, theaters, commercial centers, etc.) within 500 m of the arc. On the basis of this information, three different risk measures are proposed in [9], namely, *on-arc*, *around-arc*, *aggregate*. More precisely, the aggregate risk function was first used in [8]. A few comments can be made. According to the aggregate risk measure, which depends on the presence of congregation points near the arc, a considerable percentage of arcs have null risk, providing a patchy risk distribution on the network. On the other hand, the other two risk measures have no null risk value arc, although risk coefficients span a wide range of values, varying from almost zero to $5 \cdot 10^6$ for the on-arc measure and an order higher for the around-arc. The on-arc risk measure tends to be weakly correlated to distance, so that, for most carriers, there is not much difference between the cheapest and the safest path and thus the gain in applying risk mitigation policies is quite limited. On the contrary, the around-arc risk measure is uncorrelated to distance, so that risk mitigation policies can really impact population safety. While we do not question which risk function is the most appropriate, we believe that a risk mitigation policy should be as robust as possible with respect to how risk is measured; therefore, we test our information guided policies on all of the three risk measures proposed in the hazmat transport literature. Finally, travel demand data is also taken from [9], i.e., a set $|V| = 35$ of commodities along with their origin–destination pairs and shipment requests.

For the required size of N^{CS}, we generate 10 samples of candidate site nodes for each policy based on random sampling and we compute one candidate site set by way of the deterministic procedure. For each such set N^{CS}, the associated GLP is solved according to the adversarial scenario regarding driver behavior and to the cost aware scenario regarding the network administrator. In practice, this means that in case of multiple shortest paths, the riskiest one is selected, while, if the same risk level is achieved with a lesser cost, such a solution is preferred. All instances have been solved using Cplex 12.1 on a AMD Athlon (tm) 64×2 Dual Core Processor 4200 + (CPU MHz 2211.186). Running times are negligible and are in the order of few milliseconds. The Pareto frontier of the efficient paths with respect to risk and cost was computed by an implementation of the algorithm described in [4], while criterion C_4 exploited the code described in [17] which was kindly provided by the authors.

5.2. Computational results

Hereafter, the following notation is used. ΔR denotes the marginal improvement of the risk mitigation level (i.e., the marginal risk reduction when opening an additional gateway divided by the risk achieved for $k = 1$). Likewise, ΔC denotes the corresponding marginal cost. k_{stab} denotes the minimum value of k such that ΔR becomes negligible (below 0.5%) and remains such from k_{stab} onward. For each pair (C, Φ), RobIndex denotes how many samples over 10 reach a risk mitigation value that is very close to the best one achievable (within 2%).

As a first step, we investigated the stability of k when using ad hoc policies for candidate site generation. In particular, we wanted to verify if the marginal improvement of the risk mitigation level for increasing values of k tends to 0, as we had observed in [2] for the benchmark policy. Given a risk function, for each ad hoc policy, for each of the 10 samples, and for different sizes of the candidate site set, we solved GLP with k ranging from 1 to 10 and computed the respective value of k_{stab}; Table 1 reports the minimum, the maximum, the average, and the variance of these values. On the basis of this data, we proposed a value for k for each risk function, that we denote by k^*, to be used in the experimental campaign as the number of gateways to be opened; these values are reported in the last column of Table 1, i.e., $k^* = 5$ for the aggregate risk measure, $k^* = 3$ for the around-arc risk measure, and $k^* = 4$ for the on-arc risk measure.

Tables 2–4, pertain to the aggregate, on-arc, and around-arc risk measure, respectively, and report data on risk reduction computed on the previous set of instances for the proposed values of $k^* \pm 1$. Such data supports our statement concerning the stability of k when using ad hoc policies and confirms our choice regarding the k^* values. In particular, the first column of each table reports the percentage marginal improvement in risk reduction with respect to the case $k = 1$ when comparing the values $k^* - 1$ to k^*, while the second column reports the values when increasing k's value from k^* to $k^* + 1$. It can be noted that, for all risk measures, the average value is by and large below the 0.5% threshold required for stability when going from k^* to $k^* + 1$ (second column), whereas it goes beyond this threshold when going from $k^* - 1$ to k^* (first column). Furthermore, when increasing the value of k^* (second column) in all tables, the maximum increase is always below 1%, the minimum is 0, and, notably, variance is at most 0.02. This data supports the idea that the gain in further adding another open gateway to the k^* open ones is very limited.

Having set $k = k^*$, let us analyze the data provided in Tables 5–10 related to the random sampling based approach. Here, the average of the 10 instances corresponding to the 10 samples is considered for all configurations, which is to say, for each pair (C, Φ) and for each considered size of the candidate site set, namely 20%, 25%, and 30% of the nodes. The risk function values are expressed as the percentage gap from the values of a *reference solution* obtained by solving the GLP with 100% of the network nodes as candidate sites ($N^{CS} = N$) and using $k = k^*$ for each risk measure. The columns report, from left to

Table 2
Stability analysis on risk function aggregate.

	k^* vs $k^* - 1$	$k^* + 1$ vs k^*
Min %ΔR	0.00	0.00
Max %ΔR	0.73	0.34
Avg %ΔR	0.57	0.23
Var %ΔR	0.04	0.01

Table 3
Stability analysis on risk function on-arc.

	k^* vs $k^* - 1$	$k^* + 1$ vs k^*
Min %ΔR	0.05	0.00
Max %ΔR	1.01	0.33
Avg %ΔR	0.67	0.20
Var %ΔR	0.06	0.01

Table 4
Stability analysis on risk function around-arc.

	k^* vs $k^* - 1$	$k^* + 1$ vs k^*
Min %ΔR	0.37	0.00
Max %ΔR	1.71	0.92
Avg %ΔR	0.88	0.41
Var %ΔR	0.07	0.02

right, the policy name expressed as criterion-Φ-percentage, the average, the variance, the minimum and the maximum on the 10 instances; column (*RobIndex*) reports how many cases out of 10 achieve a value within 2% from the reference solution value and we intend it as a robustness indicator; finally, the last column reports information on the cost of the solutions, i.e., the average percentage increase with respect to the cost of the reference solution. The top of Tables 5, 7 and 9 reports the values obtained for the benchmark policy, which is referred to as the pair (C_0, Φ_1).

For both the aggregate and the around-arc risk measures, every information guided policy improves, on average, upon the benchmark policy (C_0, Φ_1) for the same size of N^{CS}. Regarding the on-arc risk measure, any policy based on criteria C_1 and C_2 improves upon (C_0, Φ_1) for any percentage. On the contrary, policies (C_3, Φ_1) and (C_4, Φ_1) are worse for the 20% case, while (C_4, Φ_4) and (C_4, Φ_5) are worse than (C_0, Φ_1) for any percentage.

It should be said that, for the on-arc risk measure, there is little room for risk reduction probably due to its weak correlation to the cost function. Therefore, it is very challenging to improve upon the unregulated scenario.

It appears that, for any criteria, function Φ_1 is too naive, which also means that the information about how many paths to which a node belongs as well as the demand of the associated vehicles play a role, especially when the size of the ground set is much larger than n_{CS}. Criterion C_3 deserves some further comment. All associated policies have a high variance and the performance of each policy differs according to the risk measure. This suggests that, while the intuition of using the paths on the Pareto frontier as a reference should not be discarded as a whole, the node selection procedure has to be refined. Intuitively, too many paths have been considered as target paths, including the shortest paths, leading us towards the unregulated scenario concerning risk. At the same time, however, not enough significant cost decreases can be correlated to this criteria to support its use in the present form. A similar observation holds for criterion C_4. We believe that this is due to the current value $p = 5$ probably being too large for sparse networks. For this reason C_4 is disregarded from any further analysis. Finally, it is evident that policy (C_2, Φ_3) not only is the one with the best performance but also the more robust with respect to risk measures. Concerning the size of the candidate sites, results suggest that the 30% percentage value captures the maximum level of risk mitigation that is achievable using the GLP method. Indeed, for most of the policies, the risk level of the best solution is the same as the one achieved when the candidate sites comprises the whole set of nodes. Thus, 30% represents a satisfactory trade-off between the candidate sites size and the solution quality. For this reason results are reported for the 30% percentage only from now on.

For each risk measure, Figs. 2–4 show the boxplot of the 10 solution values for percentage value 30% and $k = k^*$. The median is the bolded row within the box which divides the second from the third percentile; whiskers span the first and the fourth percentiles; single points denote outliers ($*$), i.e., values that are between 1.5 and 3 box lengths from either end of the box, and extremes ($\circ$), i.e., values that are more than 3 box lengths from either end of the box.

Regarding the deterministic variant of the candidate site generation procedure, where N^{CS} is computed by solving P^{cover}, in Tables 11–13 we report the numerical results for both the sum based and the product based lower priority objective functions. These are denoted by the sum and product symbol $\sum$ and $\prod$, respectively. It can be noted that $\sum$ and $\prod$ yield basically the same results; indeed, they are identical for the aggregate risk function. Concerning the around-arc and the on-arc risk functions, differences arise only for the policies involving the Φ_1 probability mass function, where all nodes in the ground set have the same coefficient in the lower priority objective function. A possible explanation is that several optimal

Table 5
Statistic information on risk function aggregate for criteria C_1, C_2, and C_3.

Policy + %Gtw	Avg %ΔR	Var %ΔR	Min %ΔR	Max %ΔR	RobIndex	Avg %ΔC
C0-Φ1-20%	17.19	254.39	0.75	49.32	2	0.28
C0-Φ1-25%	8.15	93.09	0.06	25.37	5	3.71
C0-Φ1-30%	6.47	73.79	0.05	18.97	6	5.57
C1-Φ1-20%	6.27	70.27	0.09	19.21	6	3.24
C1-Φ1-25%	4.32	51.59	0.00	17.81	7	2.03
C1-Φ1-30%	3.52	50.95	0.00	17.33	8	3.94
C1-Φ2-20%	3.66	49.39	0.00	17.19	8	2.10
C1-Φ2-25%	3.68	50.01	0.00	17.39	8	0.88
C1-Φ2-30%	2.37	29.17	0.00	16.79	8	2.80
C1-Φ3-20%	8.53	66.75	0.02	23.50	3	−0.73
C1-Φ3-25%	3.50	52.37	0.00	22.81	7	0.04
C1-Φ3-30%	0.61	3.63	0.00	6.04	9	0.09
C2-Φ1-20%	7.29	75.75	0.34	17.61	6	−0.36
C2-Φ1-25%	5.39	67.71	0.00	17.77	7	0.40
C2-Φ1-30%	1.91	29.57	0.00	17.37	9	0.45
C2-Φ2-20%	10.01	109.63	0.00	23.47	5	−1.11
C2-Φ2-25%	10.50	77.75	0.02	17.61	4	−1.46
C2-Φ2-30%	5.15	64.44	0.00	16.79	7	0.98
C2-Φ3-20%	7.77	72.19	0.00	23.21	4	−1.86
C2-Φ3-25%	2.06	8.43	0.00	6.38	7	−1.90
C2-Φ3-30%	0.01	0.00	0.00	0.02	10	−0.56
C3-Φ1-20%	11.28	370.06	0.09	61.11	5	0.90
C3-Φ1-25%	13.41	60.52	0.09	18.97	2	−1.33
C3-Φ1-30%	5.93	84.16	0.05	21.13	7	2.68
C3-Φ2-20%	5.94	69.34	0.02	23.49	5	−1.08
C3-Φ2-25%	5.50	70.79	0.00	23.35	6	0.78
C3-Φ2-30%	4.95	50.47	0.02	18.02	6	1.12
C3-Φ3-20%	5.84	49.00	0.03	23.21	4	−0.78
C3-Φ3-25%	5.73	44.39	0.03	17.93	4	−1.50
C3-Φ3-30%	2.27	8.13	0.00	6.06	6	−1.18
C3-Φ4-20%	12.38	136.35	0.06	24.91	4	−2.79
C3-Φ4-25%	3.82	29.61	0.02	17.61	5	0.67
C3-Φ4-30%	4.27	51.02	0.02	17.79	7	−1.37
C3-Φ5-20%	5.36	38.73	0.49	21.38	3	−6.25
C3-Φ5-25%	14.59	70.40	0.47	21.62	1	−4.68
C3-Φ5-30%	6.53	73.96	0.00	20.98	5	−3.91

Table 6
Statistic information on risk function aggregate for criterion C_4.

Policy + %Gtw	Avg %ΔR	Var %ΔR	Min %ΔR	Max %ΔR	RobIndex	Avg %ΔC
C4-Φ1-20%	8.92	99.97	0.34	25.79	4	2.66
C4-Φ1-25%	5.66	49.31	0.04	18.42	5	1.19
C4-Φ1-30%	4.20	49.76	0.04	17.39	7	2.49
C4-Φ2-20%	6.29	72.41	0.02	23.96	5	−1.35
C4-Φ2-25%	6.33	89.23	0.00	24.01	6	−1.33
C4-Φ2-30%	2.43	28.93	0.00	16.79	8	2.62
C4-Φ3-20%	3.18	9.49	0.02	6.86	5	−1.21
C4-Φ3-25%	4.55	49.11	0.00	17.39	6	−1.33
C4-Φ3-30%	1.66	7.28	0.00	6.49	7	3.76
C4-Φ4-20%	5.90	46.62	0.09	17.47	5	1.69
C4-Φ4-25%	0.27	0.08	0.02	0.78	10	4.06
C4-Φ4-30%	0.17	0.03	0.02	0.47	10	5.31
C4-Φ5-20%	4.90	24.60	0.09	16.77	3	0.69
C4-Φ5-25%	2.86	9.13	0.02	6.49	5	2.29
C4-Φ5-30%	0.57	1.69	0.02	4.23	9	1.94

solutions to P^{cover} are present whenever a *cover*, i.e., a subset of nodes in the ground set such that each vehicle sponsors at least one node in the subset, is made of less than the required number of nodes, i.e., n_{CS}. In this case, the set N^{CS} can be built by completing this cover in more than one way. The selection of the additional nodes required to reach the requested size is thus guided by the lower objective function according to the ϕ_i coefficients. Therefore, in case of Φ_1, all solutions to P^{cover} obtained by completing a cover have the same objective function value. Moreover, due to the cardinality constraint (5), the

Table 7
Statistic information on risk function around-arc for criteria C_1, C_2, and C_3.

Policy + %Gtw	Avg %ΔR	Var %ΔR	Min %ΔR	Max %ΔR	RobIndex	Avg %ΔC
C0-Φ0-20%	36.64	979.24	3.10	104.27	0	2.78
C0-Φ0-25%	20.88	450.99	0.48	53.14	1	3.30
C0-Φ0-30%	12.12	155.68	0.01	43.40	1	2.17
C1-Φ1-20%	7.41	216.77	0.49	48.57	2	−0.39
C1-Φ1-25%	8.07	250.94	0.00	52.50	3	−0.04
C1-Φ1-30%	6.22	197.03	0.00	46.00	3	−0.24
C1-Φ2-20%	0.92	1.01	0.00	2.58	7	−0.66
C1-Φ2-25%	0.22	0.08	0.00	0.60	10	−0.42
C1-Φ2-30%	0.05	0.02	0.00	0.49	10	−0.10
C1-Φ3-20%	0.57	0.96	0.00	2.58	8	−0.41
C1-Φ3-25%	0.05	0.02	0.00	0.48	10	−0.11
C1-Φ3-30%	0.00	0.00	0.00	0.00	10	0.00
C2-Φ1-20%	1.09	0.97	0.00	2.23	6	−0.63
C2-Φ1-25%	1.52	4.51	0.00	6.63	6	−0.62
C2-Φ1-30%	0.54	1.04	0.00	2.67	8	−0.32
C2-Φ2-20%	0.73	0.64	0.00	2.20	8	−0.74
C2-Φ2-25%	0.49	0.83	0.00	2.20	8	−0.21
C2-Φ2-30%	0.22	0.48	0.00	2.20	9	−0.05
C2-Φ3-20%	0.00	0.00	0.00	0.00	10	0.00
C2-Φ3-25%	0.00	0.00	0.00	0.00	10	0.00
C2-Φ3-30%	0.00	0.00	0.00	0.00	10	0.00
C3-Φ1-20%	21.82	1860.64	0.56	136.40	1	−0.41
C3-Φ1-25%	16.69	528.97	0.02	56.59	2	1.61
C3-Φ1-30%	2.61	5.36	0.00	6.64	3	−0.94
C3-Φ2-20%	6.20	208.04	0.02	46.86	5	−0.30
C3-Φ2-25%	1.57	4.56	0.00	6.64	6	−0.76
C3-Φ2-30%	0.70	0.98	0.00	2.80	8	−0.58
C3-Φ3-20%	11.89	417.64	0.02	51.15	5	0.77
C3-Φ3-25%	17.27	543.84	0.49	53.79	3	2.65
C3-Φ3-30%	1.73	1.23	0.00	2.80	3	−0.73
C3-Φ4-20%	6.77	196.84	0.48	46.38	3	−0.45
C3-Φ4-25%	1.99	3.89	0.00	6.63	4	−0.86
C3-Φ4-30%	7.07	257.38	0.00	52.12	5	0.49
C3-Φ5-20%	2.72	7.52	0.00	7.96	4	−1.28
C3-Φ5-25%	8.17	218.02	0.00	49.05	4	−0.66
C3-Φ5-30%	1.71	1.37	0.00	2.68	4	−1.15

Table 8
Statistic information on risk function around-arc for criterion C_4.

Policy + %Gtw	Avg %ΔR	Var %ΔR	Min %ΔR	Max %ΔR	RobIndex	Avg %ΔC
C4-Φ1-20%	7.97	41.93	0.50	22.56	1	−0.46
C4-Φ1-25%	11.33	285.85	0.48	56.59	2	2.14
C4-Φ1-30%	8.51	190.15	0.48	46.48	3	0.38
C4-Φ2-20%	6.44	206.39	0.01	46.85	5	−0.28
C4-Φ2-25%	1.60	1.12	0.00	2.62	4	−0.84
C4-Φ2-30%	0.58	0.98	0.00	2.59	8	−0.41
C4-Φ3-20%	3.41	12.27	0.00	7.11	5	−1.06
C4-Φ3-25%	0.82	1.26	0.00	2.79	7	−0.42
C4-Φ3-30%	2.11	3.86	0.00	6.64	3	−0.73
C4-Φ4-20%	8.43	213.71	0.00	49.05	2	0.11
C4-Φ4-25%	2.45	3.11	0.00	6.63	2	−0.82
C4-Φ4-30%	1.12	5.44	0.00	7.48	8	−0.51
C4-Φ5-20%	3.72	21.34	0.01	14.66	4	−0.22
C4-Φ5-25%	4.35	16.67	0.00	10.30	4	−1.24
C4-Φ5-30%	6.54	20.07	2.20	14.66	0	−0.29

$\sum$ and the $\prod$ objective functions share the same set of optimal solutions. As a consequence, the returned optimal solution is chosen arbitrarily by the ILP solver in this set. However, different optimal solutions provide different N^{CS} sets which, in turn, may yield different GLP solutions with different risk values.

As mentioned, $\sum$ and $\prod$ yield dissimilar results for the risk functions on-arc and around-arc but not for the aggregate risk function, which is the only one with zero-risk arcs. We suppose that in such a case, target paths of different vehicles tend

Table 9
Statistic information on risk function on-arc for criteria C_1, C_2, and C_3.

Policy + %Gtw	Avg %ΔR	Var %ΔR	Min %ΔR	Max %ΔR	RobIndex	Avg %ΔC
C0-Φ1-20%	1.98	0.83	0.27	3.55	5	−3.05
C0-Φ1-25%	2.30	0.52	1.65	4.06	4	−2.03
C0-Φ1-30%	1.87	1.35	0.26	4.16	6	−2.32
C1-Φ1-20%	0.90	0.85	0.00	1.87	10	−1.71
C1-Φ1-25%	0.90	0.74	0.00	1.95	10	−1.44
C1-Φ1-30%	0.48	0.95	0.00	2.84	9	0.12
C1-Φ2-20%	0.71	0.77	0.00	1.95	10	−1.64
C1-Φ2-25%	0.04	0.01	0.00	0.26	10	0.08
C1-Φ2-30%	0.64	0.69	0.00	1.67	10	−1.74
C1-Φ3-20%	0.49	0.46	0.00	1.64	10	0.50
C1-Φ3-25%	0.25	0.28	0.00	1.26	10	0.84
C1-Φ3-30%	0.00	0.00	0.00	0.00	10	0.00
C2-Φ1-20%	1.15	0.60	0.00	1.68	10	−1.17
C2-Φ1-25%	1.50	0.20	0.27	1.86	10	−2.26
C2-Φ1-30%	0.17	0.25	0.00	1.59	10	−0.44
C2-Φ2-20%	0.36	0.45	0.00	1.67	10	−0.29
C2-Φ2-25%	0.05	0.01	0.00	0.26	10	0.07
C2-Φ2-30%	0.32	0.45	0.00	1.59	10	−0.91
C2-Φ3-20%	0.17	0.16	0.00	1.29	10	0.34
C2-Φ3-25%	0.02	0.00	0.00	0.04	10	−0.08
C2-Φ3-30%	0.01	0.00	0.00	0.04	10	0.01
C3-Φ1-20%	2.18	0.81	0.30	3.43	4	−2.21
C3-Φ1-25%	1.67	0.47	0.03	2.84	9	−3.04
C3-Φ1-30%	0.97	1.20	0.00	3.06	9	−2.28
C3-Φ2-20%	1.66	0.72	0.03	3.04	8	−0.31
C3-Φ2-25%	1.18	1.33	0.00	2.98	8	−0.35
C3-Φ2-30%	1.06	1.52	0.00	3.03	8	−1.60
C3-Φ3-20%	1.17	1.80	0.00	2.98	7	−1.23
C3-Φ3-25%	0.98	1.34	0.00	2.84	8	0.33
C3-Φ3-30%	1.21	0.64	0.00	2.84	9	1.44
C3-Φ4-20%	1.43	0.98	0.06	2.91	8	−1.58
C3-Φ4-25%	0.94	0.93	0.00	2.84	9	−0.27
C3-Φ4-30%	0.30	0.35	0.00	1.59	10	0.01
C3-Φ5-20%	1.09	0.97	0.00	2.84	9	−2.37
C3-Φ5-25%	1.00	0.90	0.00	2.84	9	0.89
C3-Φ5-30%	0.27	0.32	0.00	1.46	10	0.28

Table 10
Statistic information on risk function on-arc for criterion C_4.

Policy + %Gtw	Avg %ΔR	Var %ΔR	Min %ΔR	Max %ΔR	RobIndex	Avg %ΔC
C4-Φ1-20%	2.21	1.83	0.07	4.62	7	−2.90
C4-Φ1-25%	1.21	1.14	0.03	2.97	9	−2.57
C4-Φ1-30%	1.65	2.48	0.02	4.54	7	−1.85
C4-Φ2-20%	1.48	1.50	0.02	3.33	7	−0.34
C4-Φ2-25%	0.82	0.72	0.00	2.02	9	−0.38
C4-Φ2-30%	0.64	0.65	0.00	1.68	10	−0.83
C4-Φ3-20%	1.22	0.81	0.00	2.68	9	−0.21
C4-Φ3-25%	0.66	0.47	0.00	1.49	10	1.49
C4-Φ3-30%	0.94	0.86	0.00	2.84	9	2.11
C4-Φ4-20%	2.43	1.11	1.46	4.28	5	−1.30
C4-Φ4-25%	2.32	0.86	1.26	3.43	5	−2.67
C4-Φ4-30%	1.79	0.48	1.26	3.26	8	1.09
C4-Φ5-20%	2.04	2.28	1.26	6.13	8	−0.32
C4-Φ5-25%	2.31	1.13	1.46	4.57	6	−1.11
C4-Φ5-30%	2.20	0.76	1.25	3.25	5	−0.25

to have few nodes in common and, consequently, covers with few nodes tend to be quite rare. Therefore, we can conclude that, whatever the criterion, a policy that does not discriminate among the nodes in the ground set, i.e., using the uniform probability mass function, is not robust.

These results suggest that the deterministic approach provides an effective tool for candidate site generation. In fact, for all policies and risk measures, it provides solutions as good as the average of the solution values over the 10 instances

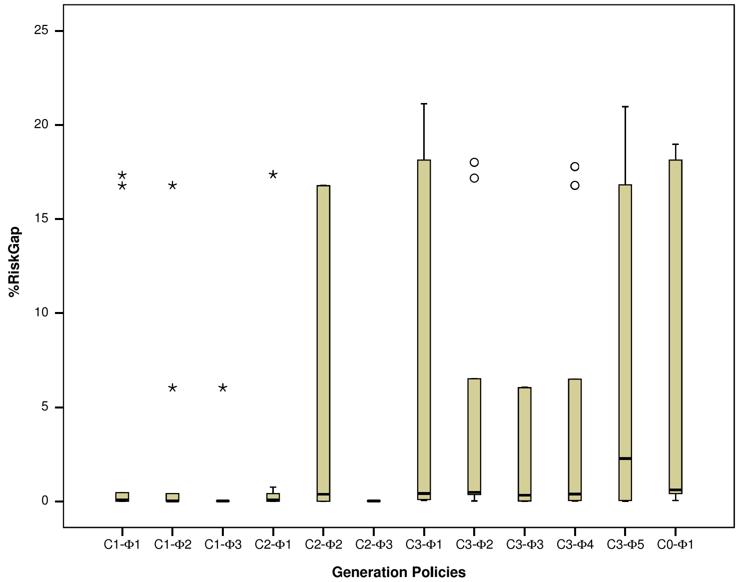

Fig. 2. BoxPlot representation of the percentage risk reductions for risk measure aggregate, $k = k^*$, and $n_{CS} = 30\%|N|$.

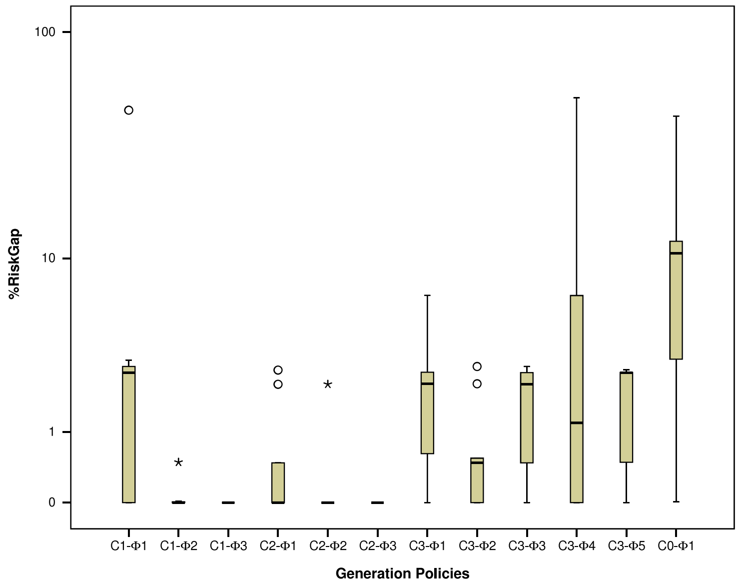

Fig. 3. BoxPlot representation of the percentage risk reductions for risk measure around-arc, $k = k^*$, and $n_{CS} = 30\%|N|$.

computed by the random sampling procedure (reported in column Avg%ΔR), while, for policy (C_2, Φ_3), the deterministic approach achieves the best solution values (reported in column Min %ΔR).

The GLP effectiveness has already been experimentally proved in [2] and it is not a target of this paper. However, it is worth providing additional information in order to rank GLP solutions in the range between the two extremes provided by the risk level achieved in the over regulated scenario and the one achieved in the unregulated scenario, denoted by $R(over\ regulated)$ and $R(unregulated)$, respectively. In the former each vehicle is forced by the authority to follow the minimum risk path, while in the latter each vehicle follows its minimum cost path. These two values provide a lower and an upper bound to risk level and are commonly used in the literature to evaluate the risk reduction achieved by any risk mitigation policy [9]. Due to the exemption mechanism, the risk of our solution is guaranteed to belong to this interval. Table 14 reports the risk of our reference solution R(GLP) obtained by solving GLP with $N^{CS} = N$ and k^* open gateways for each risk measure. This data indirectly allows to evaluate the quality of the information guided policy solutions reported in Tables 5–10. At a glance, consider that the percentage gap between the minimum and the maximum risk, i.e., $(R(unregulated) - R(over\ regulated))/R(over\ regulated)$, is 1669.14%, 257.13%, and 13.61%, for the aggregate, around-arc, and on-arc risk measure, respectively. By making $R(over\ regulated)$ 0 and $R(unregulated)$ 100, our reference solution ranks

Table 11
Results of the deterministic approach for risk measure aggregate—30%.

Policy	$\sum$		$\prod$	
	$\%\Delta R$	$\%\Delta C$	$\%\Delta R$	$\%\Delta C$
C1-Φ1	0.00	0.30	0.00	0.30
C1-Φ2	0.02	−1.41	0.02	−1.41
C1-Φ3	0.02	−1.10	0.02	−1.10
C2-Φ1	0.34	−0.04	0.34	−0.04
C2-Φ2	0.00	−0.01	0.00	−0.01
C2-Φ3	0.00	−0.01	0.00	−0.01
C3-Φ1	0.88	6.45	0.88	6.45
C3-Φ2	0.02	−3.47	0.02	−3.47
C3-Φ3	6.04	−3.76	6.04	−3.76
C3-Φ4	6.04	−3.75	6.04	−3.75
C3-Φ5	6.04	−3.71	6.04	−3.71
C4-Φ1	28.50	−5.18	28.50	−5.18
C4-Φ2	0.02	4.73	0.02	4.73
C4-Φ3	0.02	−1.11	0.02	−1.11
C4-Φ4	0.02	−1.10	0.02	−1.10
C4-Φ5	6.04	−3.46	6.04	−3.46

Table 12
Results of the deterministic approach for risk measure around-arc—30%.

Policy	$\sum$		$\prod$	
	$\%\Delta R$	$\%\Delta C$	$\%\Delta R$	$\%\Delta C$
C1-Φ1	46.48	4.61	0.49	−1.04
C1-Φ2	0.00	0.00	0.00	0.00
C1-Φ3	0.00	0.00	0.00	0.00
C2-Φ1	0.00	0.00	0.00	0.00
C2-Φ2	0.00	0.00	0.00	0.00
C2-Φ3	0.00	0.00	0.00	0.00
C3-Φ1	0.64	−1.01	8.08	−1.82
C3-Φ2	0.01	0.00	0.01	0.00
C3-Φ3	46.38	5.51	46.38	5.51
C3-Φ4	0.00	0.00	0.00	0.00
C3-Φ5	11.73	−0.76	11.73	−0.76
C4-Φ1	74.92	1.30	21.79	11.71
C4-Φ2	0.00	0.00	0.00	0.00
C4-Φ3	0.00	0.00	0.00	0.00
C4-Φ4	0.00	0.00	0.00	0.00
C4-Φ5	7.11	−2.54	7.11	−2.54

Table 13
Results of the deterministic approach for risk measure on-arc—30%.

Policy	$\sum$		$\prod$	
	$\%\Delta R$	$\%\Delta C$	$\%\Delta R$	$\%\Delta C$
C1-Φ1	2.00	−4.55	2.01	−4.52
C1-Φ2	1.25	4.18	1.24	4.18
C1-Φ3	1.25	4.18	1.25	4.18
C2-Φ1	1.74	−3.84	1.93	−4.62
C2-Φ2	0.00	0.00	0.00	0.00
C2-Φ3	0.00	0.00	0.00	0.00
C3-Φ1	2.10	−4.30	2.01	−4.52
C3-Φ2	1.25	4.18	1.25	4.18
C3-Φ3	1.25	4.18	1.25	4.18
C3-Φ4	1.25	4.18	1.25	4.18
C3-Φ5	1.25	4.18	1.25	4.18
C4-Φ1	4.20	−5.62	4.20	−5.62
C4-Φ2	1.25	4.18	1.25	4.18
C4-Φ3	1.25	4.18	1.25	4.18
C4-Φ4	1.51	4.04	1.51	4.04
C4-Φ5	1.51	4.04	1.51	4.04

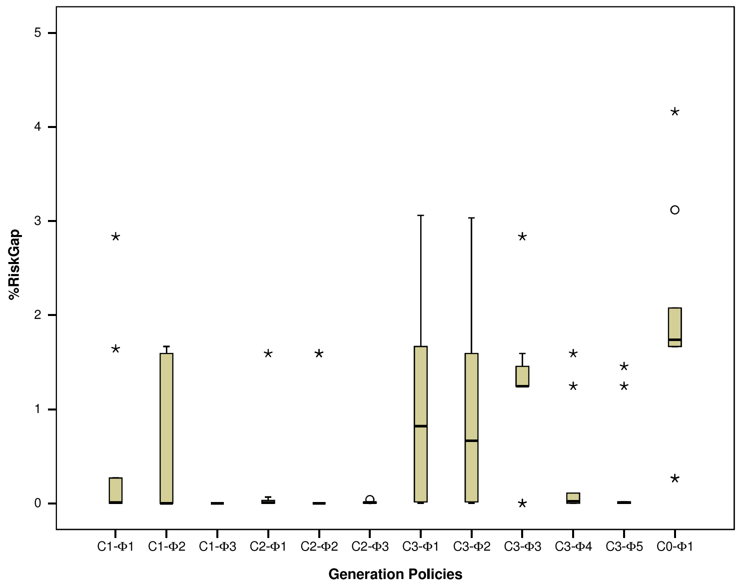

Fig. 4. BoxPlot representation of the percentage risk reductions for risk measure on-arc, $k = k^*$, and $n_{CS} = 30\%|N|$.

Table 14
Risk range width and GLP efficacy.

Risk measure	*R* (*unregulated*)	*R* (*GLP*)	*R* (*over regulated*)
Aggregate	2208,839,655	396,152,220	124,854,028
Around-arc	7229,256,314	2079,803,990	2024,247,704
On-arc	567,773,424	514,788,176	499,767,899

at 13.02%, 1.07%, and 22.09%, for the aggregate, around-arc, and on-arc risk measure, respectively. It can be noted that when the window is large, there is room for improvement and indeed we achieve large risk reductions. However, when the unregulated solution is not very different from the over regulated one, our method is still able to reach almost 80% of the achievable risk reduction.

In summation, we can affirm that policy (C_2, Φ_3) is the best performing and the most robust with respect to size of the candidate site set and risk measure. In fact, a considerable gain is achieved with respect to the benchmark policy and the variance on the 10 instances is small. Enlarging the set of target paths to efficient paths or to risk suboptimal paths increases the variance of the results quality. We believe that these ideas deserve more investigation and that these new criteria must be strengthened by acquiring more information in order to guarantee that we trade what we relax on the risk side for a real improvement on the cost side.

6. Conclusions and open issues

This paper is concerned with the first step to solving the Gateway Location Problem (GLP), a new problem in combinatorial optimization arising in the framework of rule-based policies for risk mitigation in hazardous material transportation. GLP consists of installing k gateways at as many locations selected out of a set of candidate sites, N^{CS}, and assigning to each vehicle one of such gateways as a compulsory check point along its route from origin to destination. Previous experience, in which N^{CS} was randomly sampled on the whole set of the network nodes, has shown that the features of N^{CS} may influence the performance of the whole approach. Here, different policies for building N^{CS} have been proposed and tested. Each policy is made of two phases, the generation of a ground set N^{GS} and a distribution law for sampling N^{CS} out of N^{GS}. Furthermore, for each policy, the deterministic variant has also been studied in which the probabilities of each node in N^{GS} being sampled are used as weights in a covering-like, ILP problem. Such policies have been tested on realistic instances taken from the literature of hazmat transit.

First of all, for all policies, a limited number of open gateways is sufficient to provide a good level of risk mitigation. Furthermore, the results provide computational evidence that some of the information guided policies are quite effective in selecting a good quality subset of nodes, i.e., a subset containing k nodes with which low risk solutions are associated. When compared to the purely random generation policy, such information guided policies allow us to achieve, on average, the same level of risk mitigation with less candidate sites, or a lower level of risk with the same number of candidate sites. These policies prove to be more robust in the sense of reducing the variance of risk level mitigation achieved over the generated samples. If willing to get rid of any random component in the whole process, one can resort to the deterministic variant for the N^{CS} generation, which also proved to be effective.

The same behavior for the three risk functions has been observed in the majority of the cases, adding more generality to our findings. Different levels of risk reduction have been achieved for each risk function. However, the range width of the risk levels associated with the solutions of the over regulated scenario and the unregulated scenario varies remarkably according to the risk function, and this may explain the previous behavior.

We can conclude that this initial step towards a GLP solution, i.e., candidate site generation, deserves as much attention as solving GLP itself, in order to exploit all the potentials of GLP as a risk mitigation policy. Information guided policies, which reduce the size of the candidate site set required to achieve most of the risk mitigation potential of GLP, have been presented. Basically, the tools so far developed for risk mitigation by GLP are all risk driven. At the same time, however, the issue of cost control seems to be open and requires ad hoc strategies. Indeed, the issue of a controlled trade-off between risk and cost within GLP is currently under investigation.

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