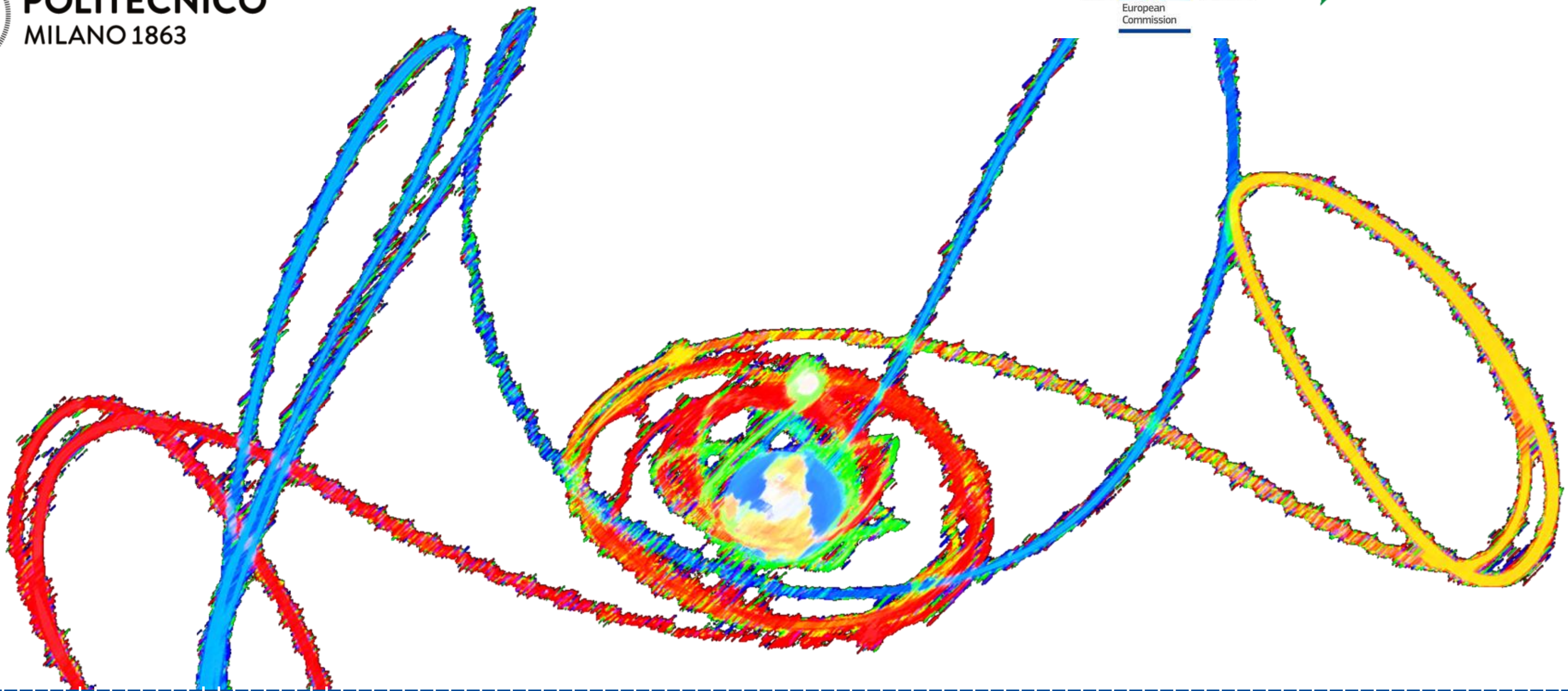


Orbit manoeuvring enhancing natural perturbations

Camilla Colombo

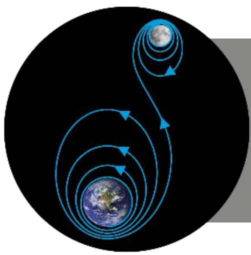
VII CELMEC 2017, San Martino al Cimino, 7 September 2017

INTRODUCTION

Introduction

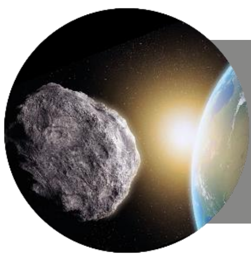
Services, technologies, science, space exploration	ORBIT PERTURBATIONS Traditional approach: counteract perturbations	APPROACH **leverage** and **control** perturbations

SPACE TRANSFER



Reach, control operational orbit

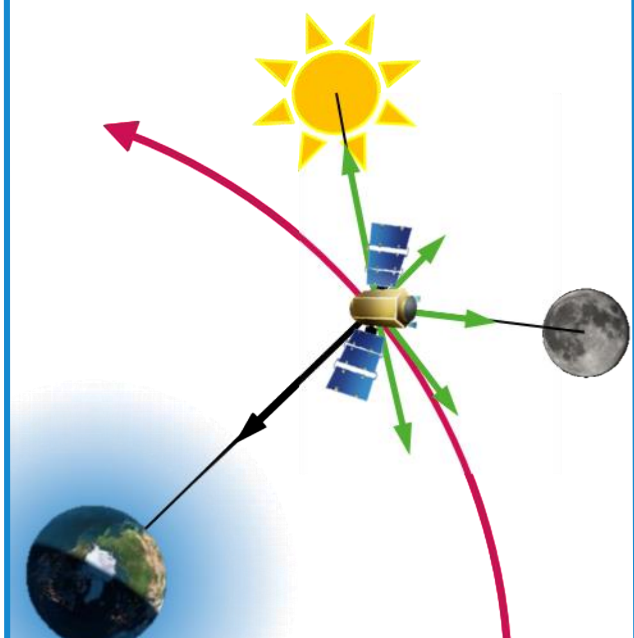
SPACE SITUATION AWARENESS



Asteroids. planetary protection

Space debris

- Complex orbital dynamics
- Increase fuel requirements for orbit control



Reduce extremely high space mission costs especially for small satellites

Create new opportunities for exploration, exploitation and planetary protection

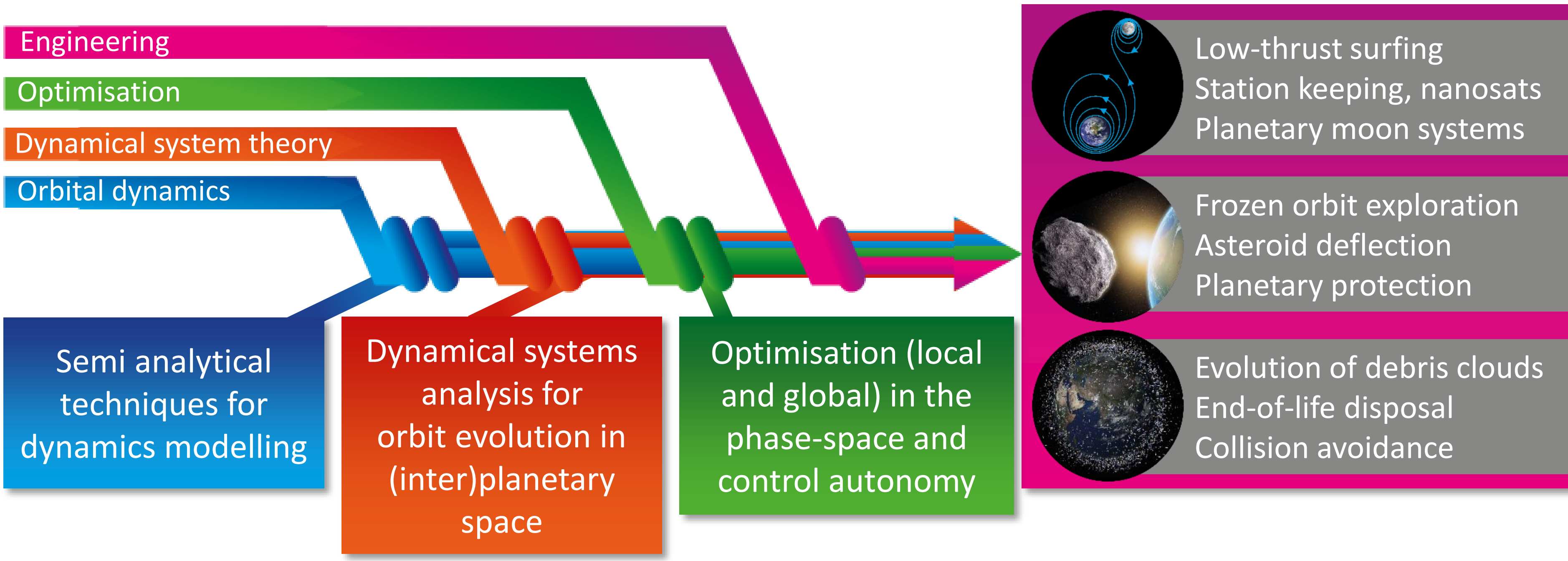
Mitigate space debris

Develop autonomous techniques for orbit manoeuvring and control by surfing through orbit perturbations

Introduction

Methodology

Control for Orbit Manoeuvring through Perturbations for Application to Space Systems



DYNAMICAL MODEL

Dynamical model

Orbit propagation based on averaged dynamics

For conservative orbit perturbation effects

Disturbing potential function

$$R = R_{\text{SRP}} + R_{\text{zonal}} + R_{3-\text{Sun}} + R_{3-\text{Moon}}$$

Planetary equations in Lagrange form

$$\frac{d\boldsymbol{\alpha}}{dt} = f\left(\boldsymbol{\alpha}, \frac{\partial R}{\partial \boldsymbol{\alpha}}\right) \qquad \boldsymbol{\alpha} = \begin{bmatrix} a & e & i & \Omega & \omega & M \end{bmatrix}^T$$

Average over one orbit revolution of the spacecraft around the primary planet

$$\bar{R} = \bar{R}_{\text{SRP}} + \bar{R}_{\text{zonal}} + \bar{R}_{3-\text{Sun}} + \bar{R}_{3-\text{Moon}}$$

$$\frac{d\bar{\boldsymbol{\alpha}}}{dt} = f\left(\bar{\boldsymbol{\alpha}}, \frac{\partial \bar{R}}{\partial \bar{\boldsymbol{\alpha}}}\right)$$

Single average

Average over the revolution of the perturbing body around the primary planet

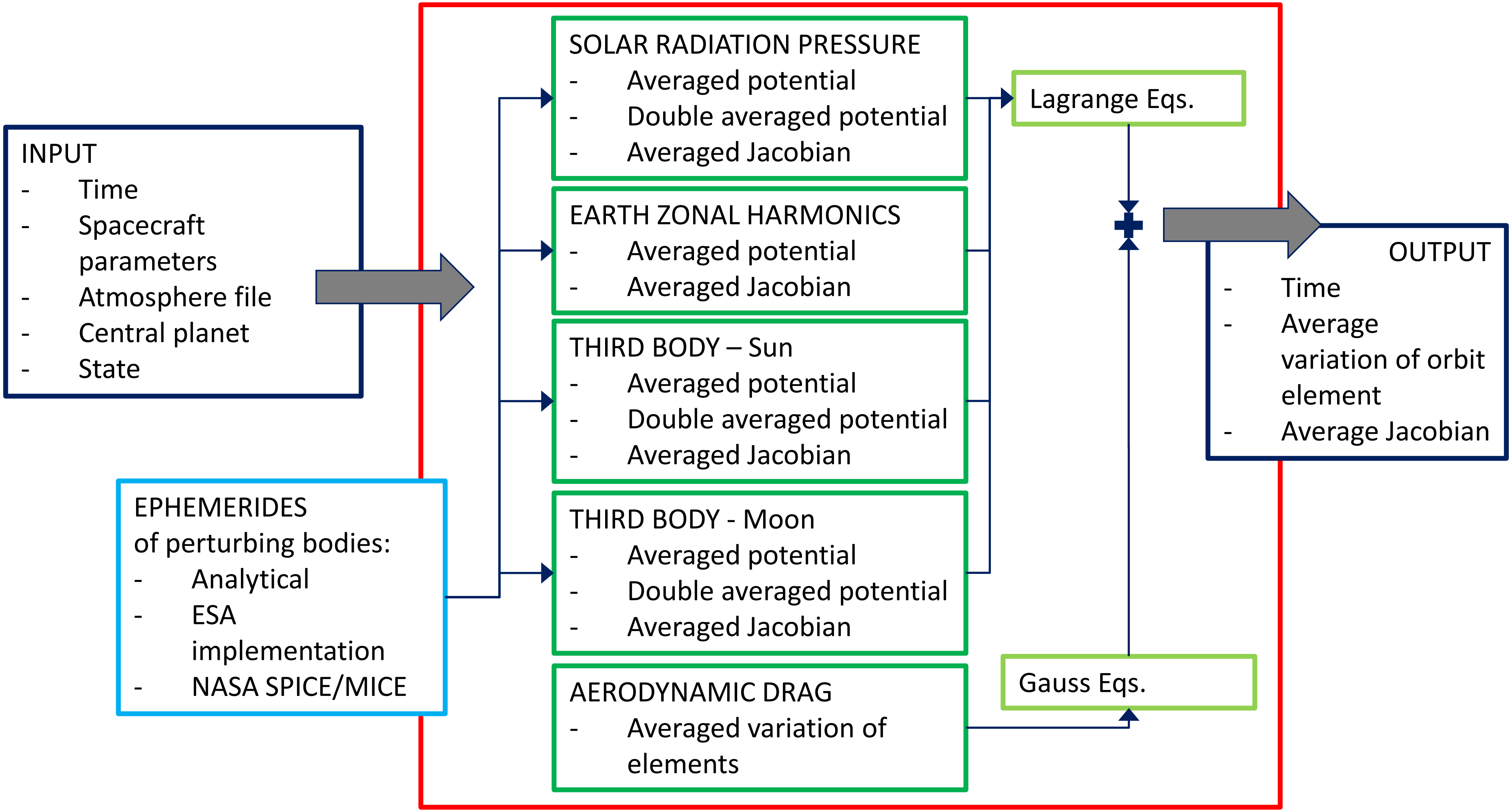
$$\bar{\bar{R}} = \bar{\bar{R}}_{\text{SRP}} + \bar{R}_{\text{zonal}} + \bar{\bar{R}}_{3-\text{Sun}} + \bar{\bar{R}}_{3-\text{Moon}}$$

$$\frac{d\bar{\bar{\boldsymbol{\alpha}}}}{dt} = f\left(\bar{\bar{\boldsymbol{\alpha}}}, \frac{\partial \bar{\bar{R}}}{\partial \bar{\bar{\boldsymbol{\alpha}}}}\right)$$

Double average

Dynamical model

PlanODyn: Planetary Orbital Dynamics



► *"Planetary Orbital Dynamics Suite for Long Term Propagation in Perturbed Environment," ICATT, ESA/ESOC, 2016.*

Dynamical model

Perturbation model

Perturbations in planet centred dynamics

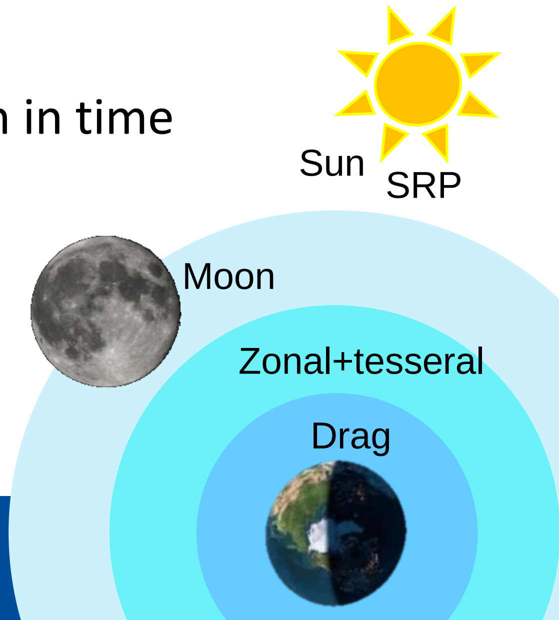
- Atmospheric drag (smooth exponential model)
- Zonal harmonics of the Earth's gravity potential, J_2^2
- Solar radiation pressure
- Third body perturbation of the Sun
- Third body perturbation of the Moon

► *King-Hele, D., Theory of Satellite Orbits in an Atmosphere, Butterworths, London, 1964.*

Ephemerides options

- Analytical approximation based on polynomial expansion in time
- Numerical ephemerides through the NASA SPICE toolkit
- Numerical ephemerides from an implementation by ESA

Orbital elements in Earth centred equatorial J2000 frame



Third body luni-solar effect

DYNAMICAL MODEL

Dynamical model

Third body potential

$$R_{3B}(r, r') = \frac{\mu'}{r'}\left(\left(1 - 2\frac{r}{r'}\cos\psi + \left(\frac{r}{r'}\right)^2\right)^{-1/2} - \frac{r}{r'}\cos\psi\right)$$

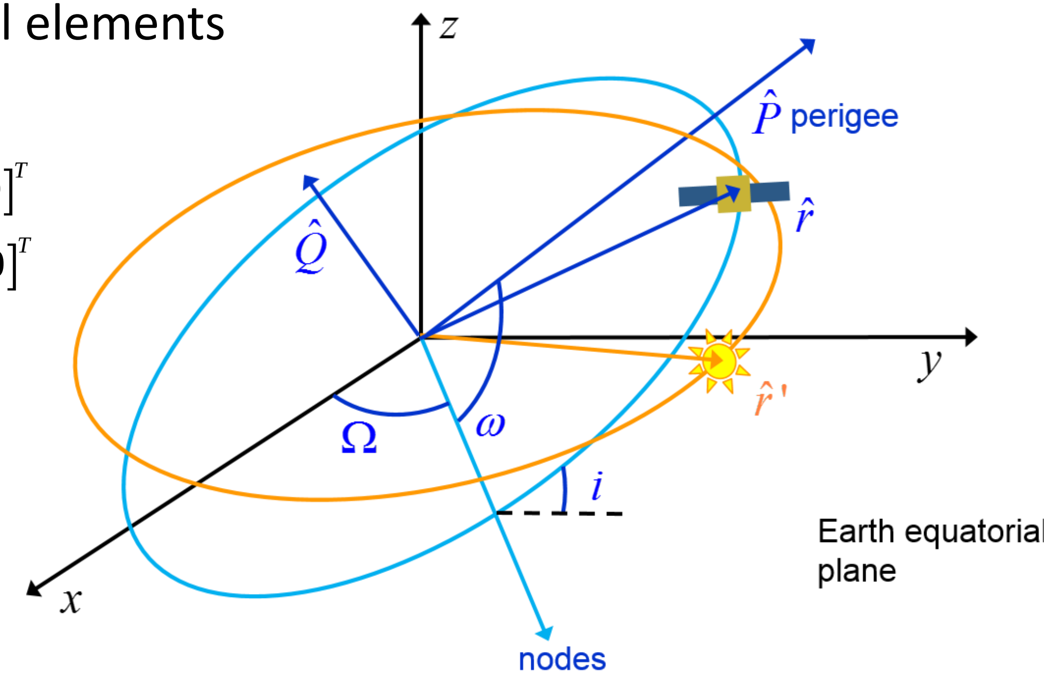
Third body potential in terms of:

- Ratio between orbit semi-major axis and distance of the third body $\delta = \frac{a}{r'}$
- Orientation of orbit eccentricity vector with respect to third body $A = \hat{P} \cdot \hat{r}'$
- Orientation of semi-latus rectum vector with respect to third body $B = \hat{Q} \cdot \hat{r}'$
- Composition of rotation in orbital elements

$$\hat{P} = R_3(\Omega) R_1(i) R_3(\omega) \cdot [1 \quad 0 \quad 0]^T$$

$$\hat{Q} = R_3(\Omega) R_1(i) R_3(\omega + \pi/2) \cdot [1 \quad 0 \quad 0]^T$$

$$\hat{r}' = R_3(\Omega') R_1(i') R_3(\omega' + f') \cdot [1 \quad 0 \quad 0]^T$$



μ' gravitational coefficient of the third body

$\mathbf{r}'$ position vector of third body

$\mathbf{r}$ position vector of satellite

ψ angle between satellite and third body

Dynamical model

Third body potential

Series expansion around $\delta = 0$

$$R_{3B}(r,r') = \frac{\mu'}{r'} \sum_{k=2}^{\infty} \delta^k F_k(A,B,e,E)$$

μ' gravitational coefficient of the third body

$\mathbf{r}'$ position vector of third body

E eccentric anomaly

Average over one orbit revolution

$$\bar{R}_{3B}(r,r') = \frac{\mu'}{r'} \sum_{k=2}^{\infty} \delta^k \bar{F}_k(A,B,e)$$

$$\bar{F}_k(A,B,e) = \frac{1}{2\pi} \int_{-\pi}^{\pi} F_k(A,B,e,E) \underbrace{(1 - e\cos E)dE}_{dM}$$

Partial derivatives for Lagrange equations

$$A(\Omega,i,\omega,\Omega',i',u')$$

$$B(\Omega,i,\omega,\Omega',i',u')$$

$$\bar{F}_k(A,B,e)$$

$$\frac{\partial \bar{F}_k}{\partial \Omega} = \frac{\partial \bar{F}_k}{\partial A}\frac{\partial A}{\partial \Omega} + \frac{\partial \bar{F}_k}{\partial B}\frac{\partial B}{\partial \Omega}$$

$$\frac{\partial \bar{F}_k}{\partial i} = \frac{\partial \bar{F}_k}{\partial A}\frac{\partial A}{\partial i} + \frac{\partial \bar{F}_k}{\partial B}\frac{\partial B}{\partial i}$$

$$\frac{\partial \bar{F}_k}{\partial \omega} = \frac{\partial \bar{F}_k}{\partial A}\frac{\partial A}{\partial \omega} + \frac{\partial \bar{F}_k}{\partial B}\frac{\partial B}{\partial \omega}$$

$$\frac{\partial \bar{F}_k}{\partial a} = \frac{k}{a} F_k$$

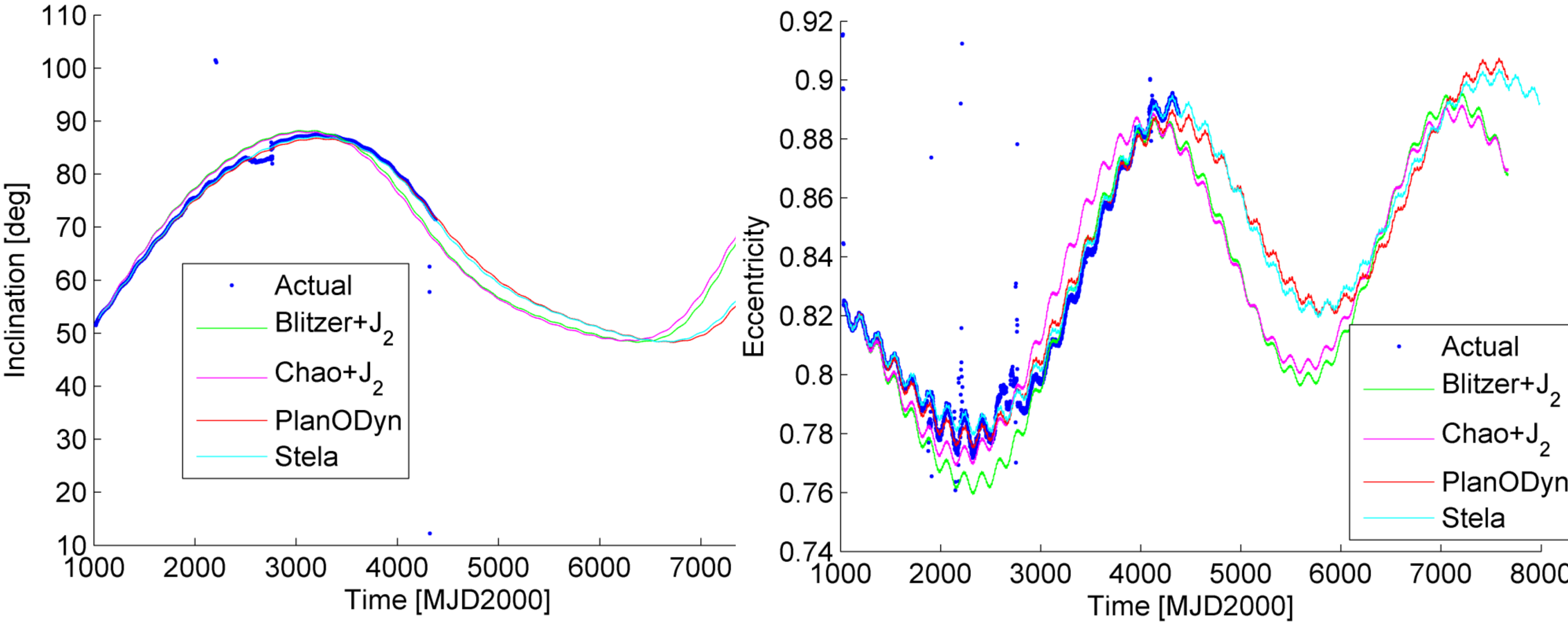
$$\frac{\partial \bar{F}_k}{\partial e}$$

► *Kaufman and Dasenbrock, NASA report, 1979*

Dynamical model

Order of the luni-solar potential expansion

Third-body perturbing potential of the Moon at least up to the fourth order of the power expansion



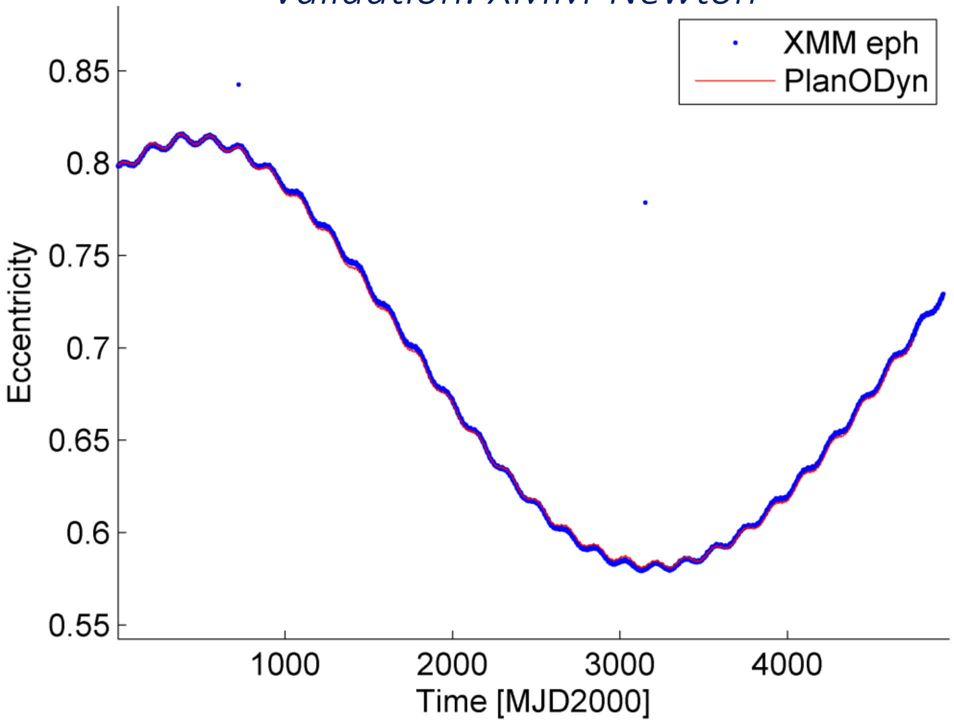
► *Blitzer L., Handbook of Orbital Perturbations, Astronautics, 1970*
► *Chao-Chun G. C., Applied Orbit Perturbation and Maintenance, 2005*

Dynamical model

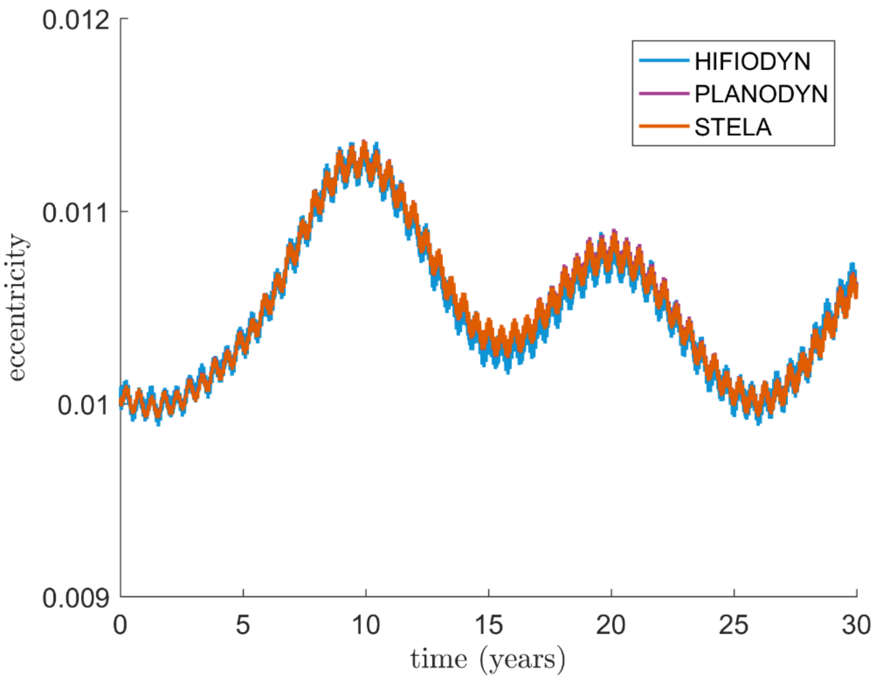
Validation

- HEO/GTO with real ephemerides/TLE data
- LEO with high fidelity non-averaged models
- GEO with high fidelity non-averaged models

Validation: XMM-Newton



Validation: GEO spacecraft [by Gkolias]



Analytical interpretation

Third-body double averaged potential

Double averaging over one orbit revolution of the s/c and one orbit evolution of the perturbing body (either Sun or Moon) around the Earth

$$\bar{\bar{R}}_{3B}(r,r')=\frac{\mu'}{r'}\sum_{k=2}^{\infty}\delta^k\bar{\bar{F}}_k(e,i,\Omega,\omega,i')$$

Same approach as El'yasberg (and Kozai, Lidov) with some changes:

- Avoid simplification that Moon and Sun orbit on the same plane (very important for precise orbit evolution)
- Facilitate the introduction of the effect of the zonal harmonics

$$\bar{\bar{F}}_k(e,i,\Delta\Omega,\omega,i')=\frac{1}{2\pi}\int_0^{2\pi}\bar{F}_k\big(A(\Omega,i,\omega,\Omega',i',\omega'+f'),B(\Omega,i,\omega,\Omega',i',\omega'+f'),e\big)df'$$

► *Kozai, Secular Perturbations of Asteroids with High Inclination and Eccentricity, 1962*
► *El'yasberg, Introduction to the theory of flight of artificial Earth satellites - translated, 1967*
► *Lidov, Planetary Space Science, Vol. 9, 1961*

Analytical interpretation

Third body Lidov-Kozai theory

- Delaunay's transformation
- Time-independent Hamiltonian

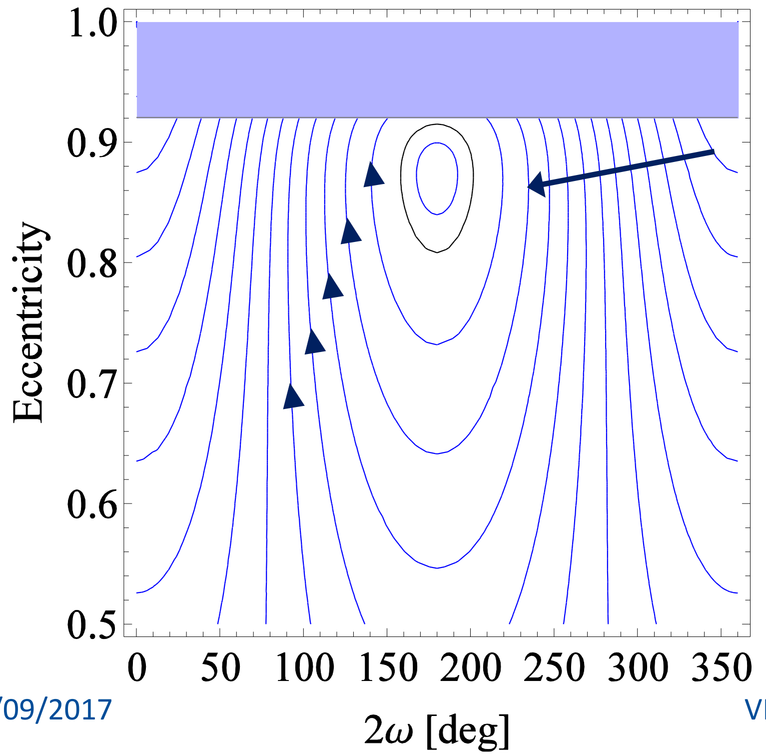
$$W\left(\frac{a}{a'},\Theta,e,2\omega\right)=\text{cost} \qquad \Theta=\left(1-e^2\right)\cos i^2$$

▶ *Kozai, Secular Perturbations of Asteroids with High Inclination and Eccentricity, 1962*

- Double averaged potential
- Rotating reference system

$$\bar{\bar{F}}_{3\mathrm{Bsys},2}\left(e,\omega,i\right)=\frac{1}{32}\left(\left(2+3e^2\right)\left(1+3\cos\left(2i\right)\right)+30e^2\cos\left(2\omega\right)\sin^2 i\right)$$

▶ *El'yasberg, Introduction to the theory of flight of artificial Earth satellites - translated, 1967*



Initial condition in *a, e, i, ω* defines a contour line in phase space

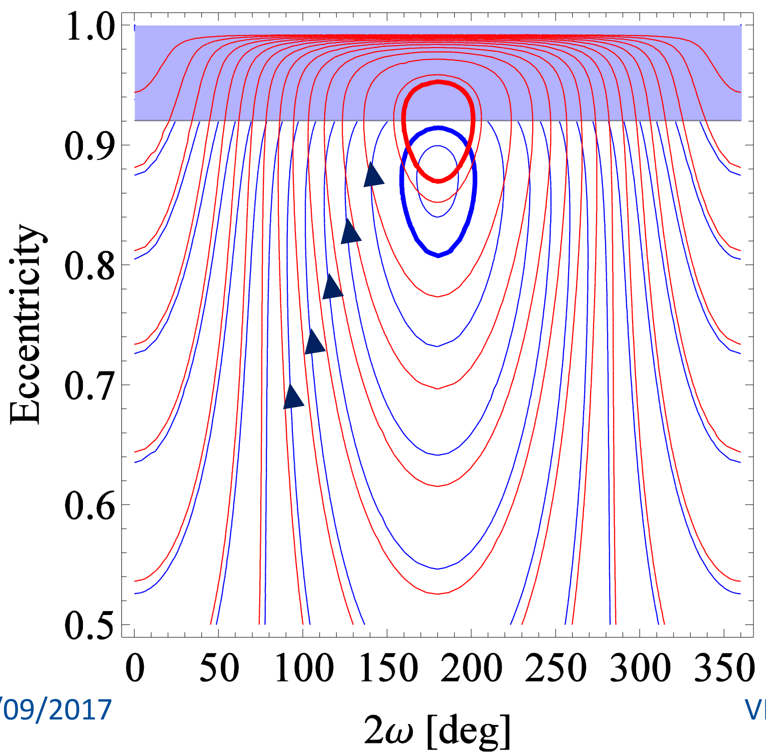
Analytical interpretation

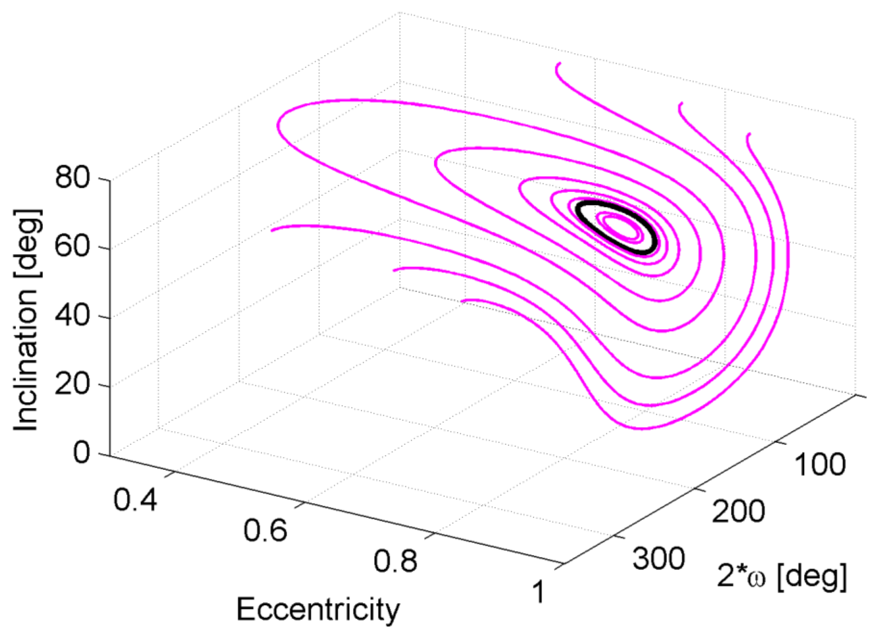
Third body Lidov-Kozai theory

$$W\left(\frac{a}{a'}, \Theta, e, 2\omega\right) = \text{cost} \qquad \Theta = \left(1 - e^2\right)\cos i^2$$

a/a' increases

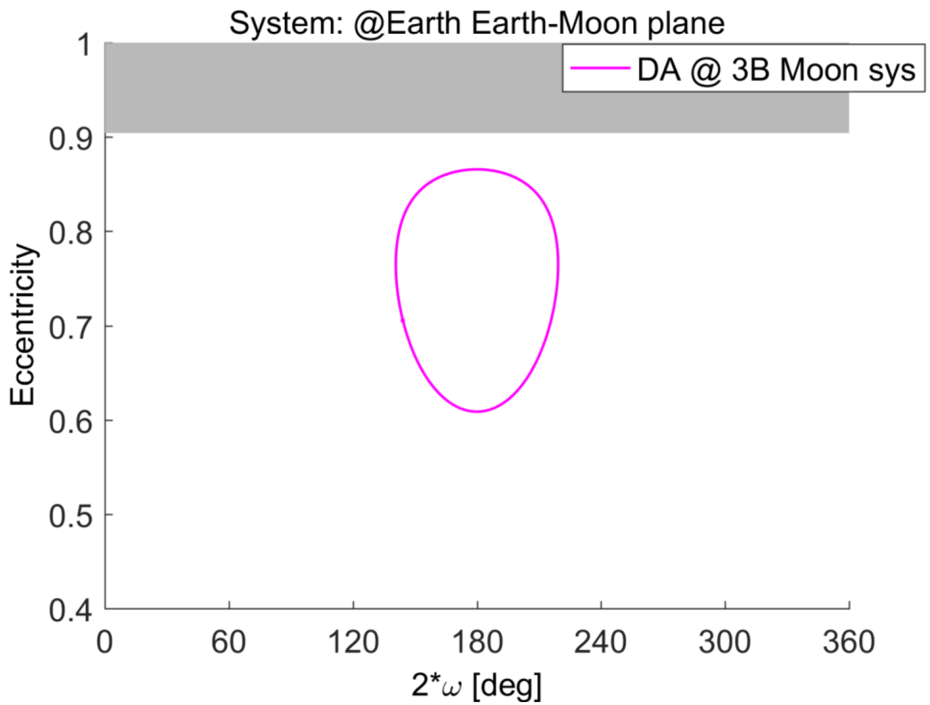
Θ decreases

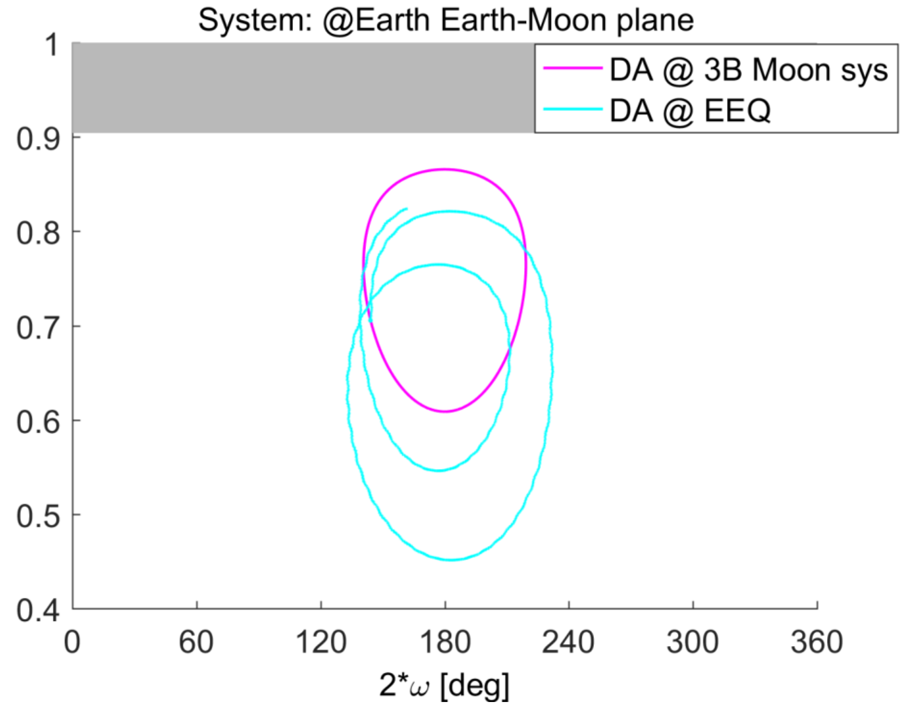




Analytical interpretation

Third-body double averaged potential





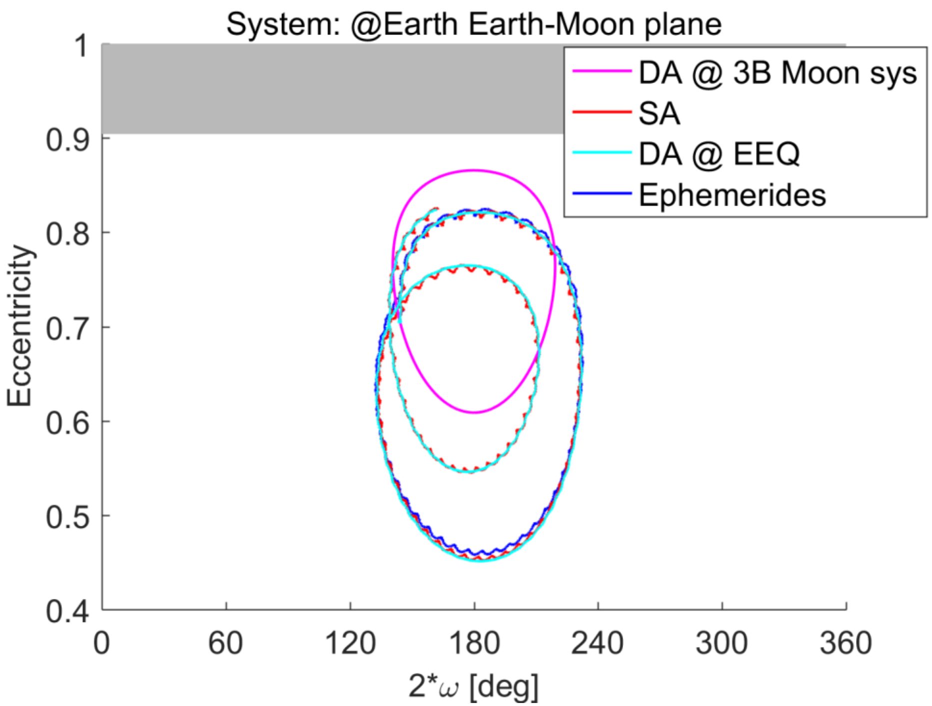
Kozai, El'yasberg: $\bar{\bar{F}}_{3\text{Bsys},2}\left(e,\omega,i\right)$ → $\bar{\bar{F}}_{k}\left(e,i,\Delta\Omega,\omega,i'\right)$

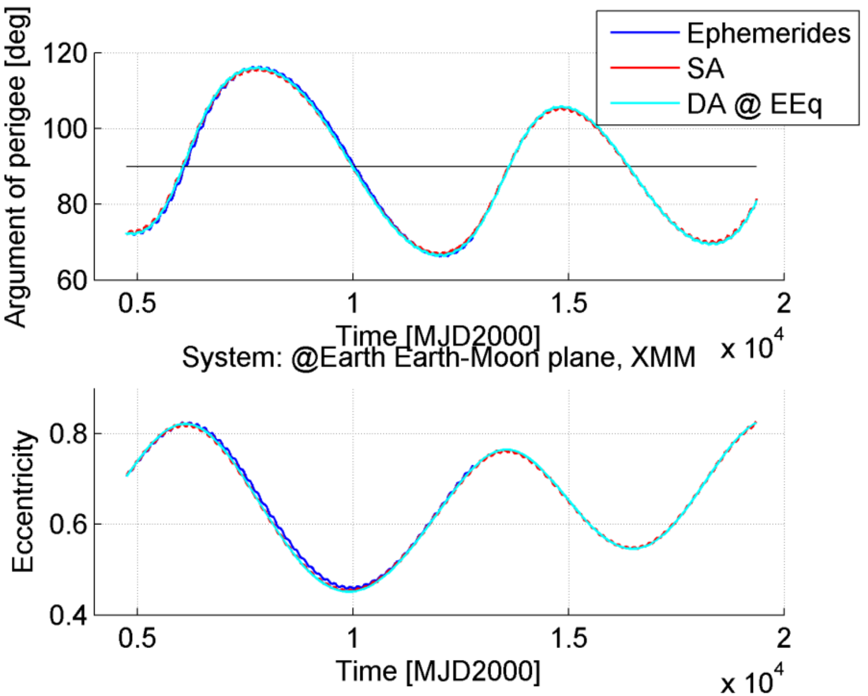
Reference frame:

- x-y plane lays on the Moon orbital plane
- z-axis in the direction of the Moon angular momentum

Analytical interpretation

Third-body double averaged potential



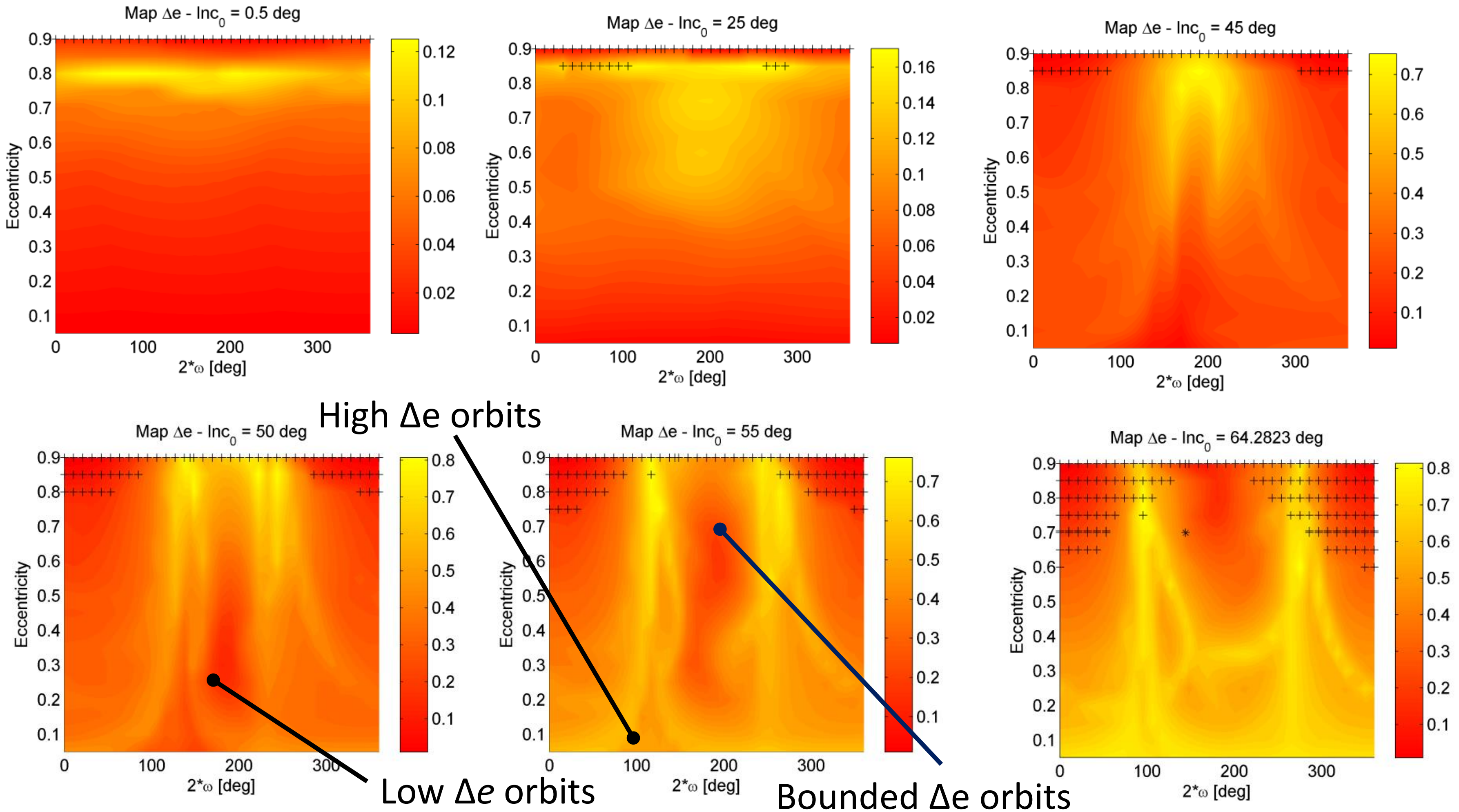


Non autonomous loops in the e-ω phase space

Dynamical maps

Long-term orbit evolution

Luni-solar + zonal Δ*e* maps: Semi-major axis equal to 67045.39 km (XMM Newton's orbit)



► *Colombo, 2015 "Long-Term Evolution of Highly-Elliptical Orbits: Luni-Solar Perturbation Effects for Stability and Re-Entry," 25th AAS/AIAA Space Flight Mechanics Meeting, 2015*

Solar radiation pressure

DYNAMICAL MODEL

Dynamical model

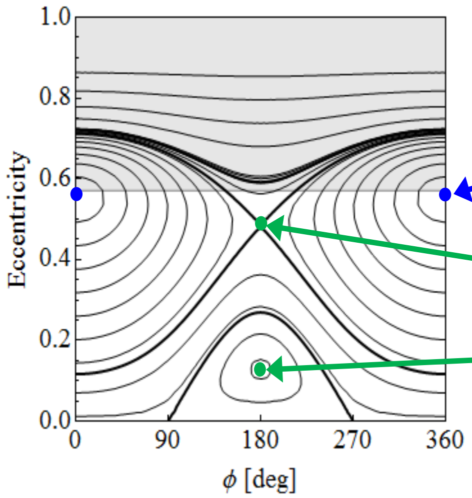
Solar radiation pressure and Earth oblateness

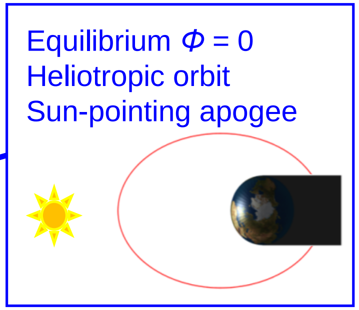
Solar radiation pressure and Earth oblateness (single averaged)

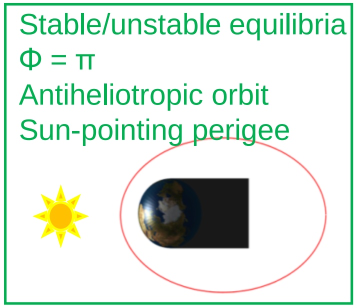
$$\bar{R}_{SRP} = C(a, A/m)\, n a^2 e \Big(\cos\omega\big(\cos\Omega\cos\lambda_{Sun} + \sin\Omega\sin\lambda_{Sun}\cos\varepsilon\big) + \sin\omega\big(\cos\Omega\cos i\sin\lambda_{Sun}\cos\varepsilon + \sin i\sin\lambda_{Sun}\sin\varepsilon - \sin\Omega\cos i\cos\lambda_{Sun}\big)\Big)$$

$$C(a, A/m) = \frac{3}{2} a_{SRP} \frac{a^2}{\mu_{Earth}} \frac{n(a)}{n_{Sun}}$$

$$\bar{R}_{J_2} = W(a, J_2) \frac{n a^2}{6} \frac{3\cos^2 i - 1}{\left(1 - e^2\right)^{3/2}}$$



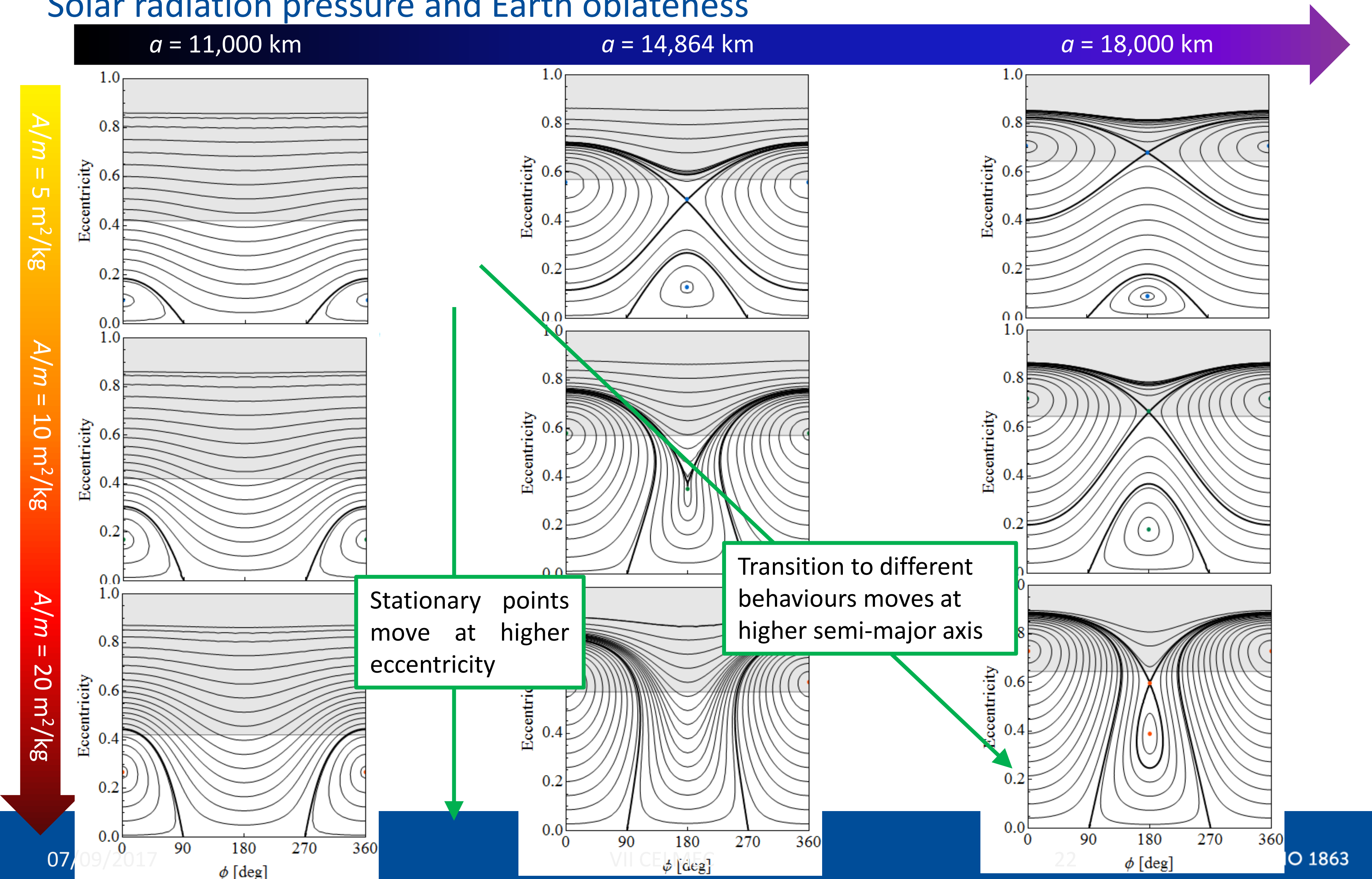




► Krivov, A. V., Sokolov, L. L. and Dikarev, V. V., "Dynamics of Mars-Orbiting Dust: Effects of Light Pressure and Planetary Oblateness," Celestial Mechanics and Dynamical Astronomy, Vol. 63, No. 3, 1995, pp. 313-339. doi: 10.1007/bf00692293

Dynamical model

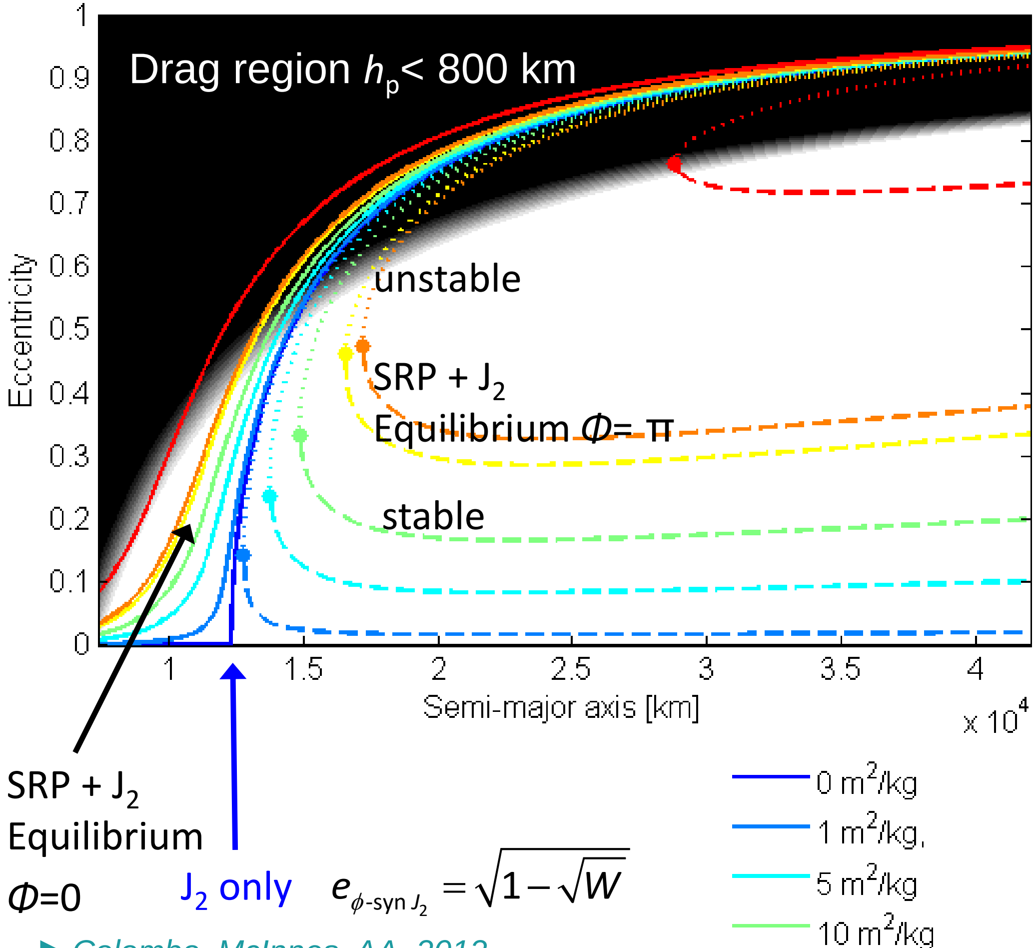
Solar radiation pressure and Earth oblateness



Dynamical model

Solar radiation pressure and Earth oblateness

Planar case

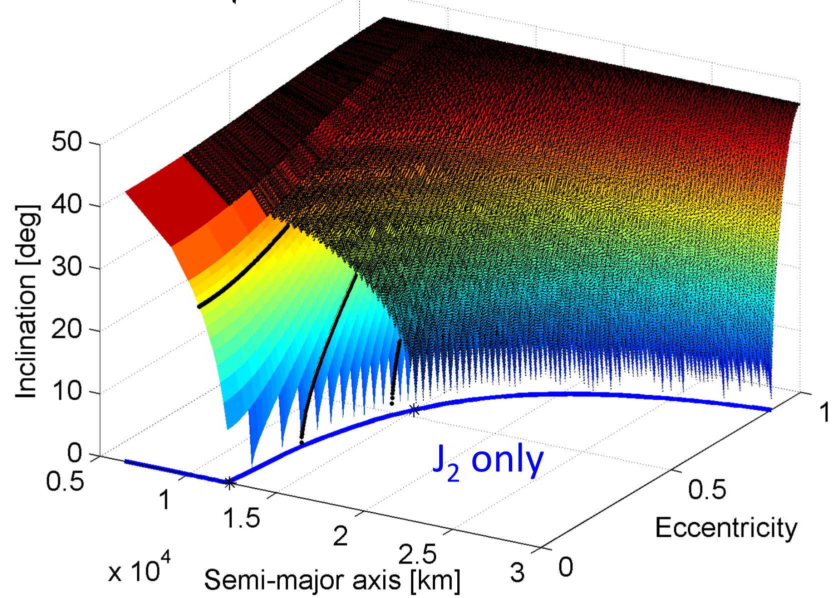


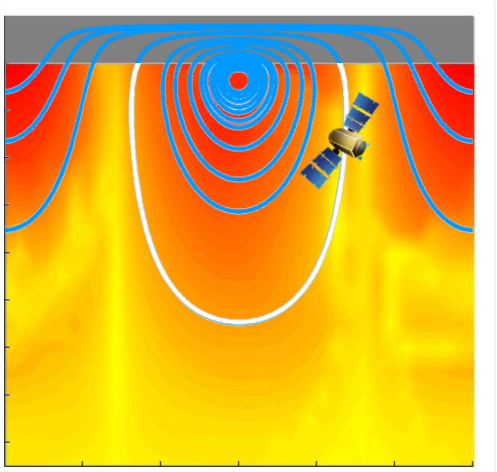
$$e_{\phi\text{-syn}\,J_2} = \sqrt{1-\sqrt{W}}$$

► *Colombo, McInnes, AA, 2012.*

Inclined case

$$e_{\phi-\text{syn}\,J_2} = \sqrt{1-\frac{1}{2}\sqrt{W\left(3-4\cos i+5\cos\left(2i\right)\right)}}$$



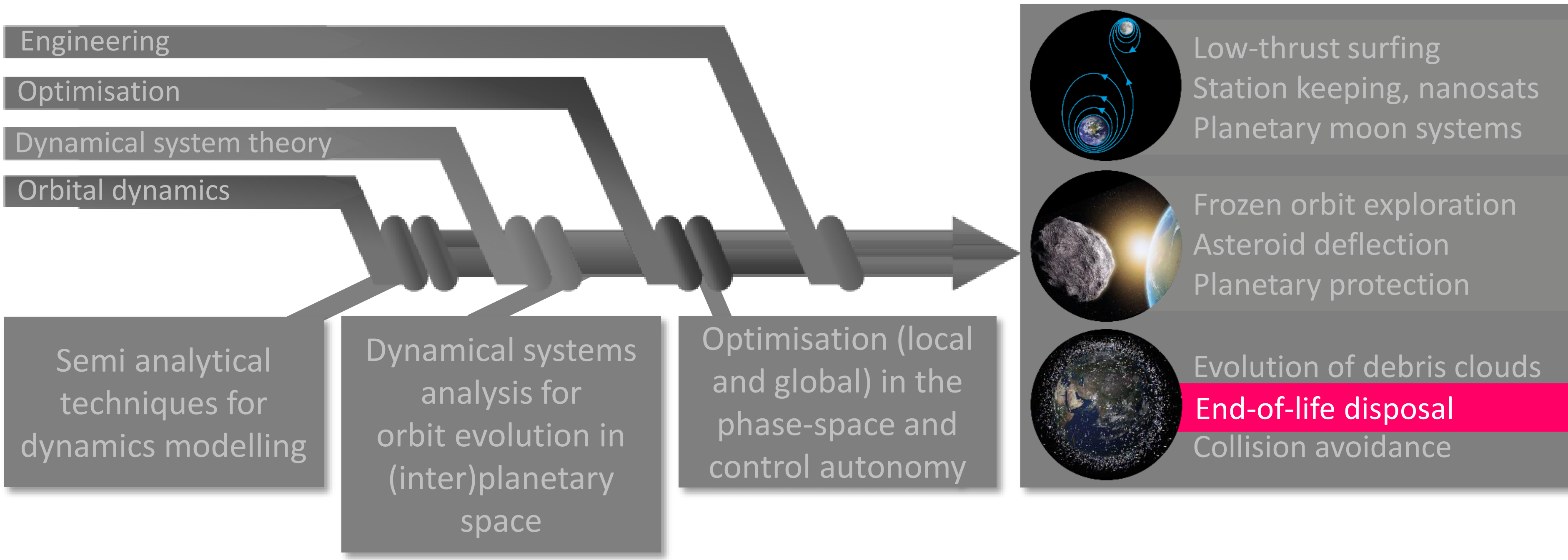


Design of disposal manoeuvres

CONTROLLING THE PERTURBATION EFFECTS

Application

Space debris



Engineering the perturbation effects

Design disposal manoeuvre in the phase space

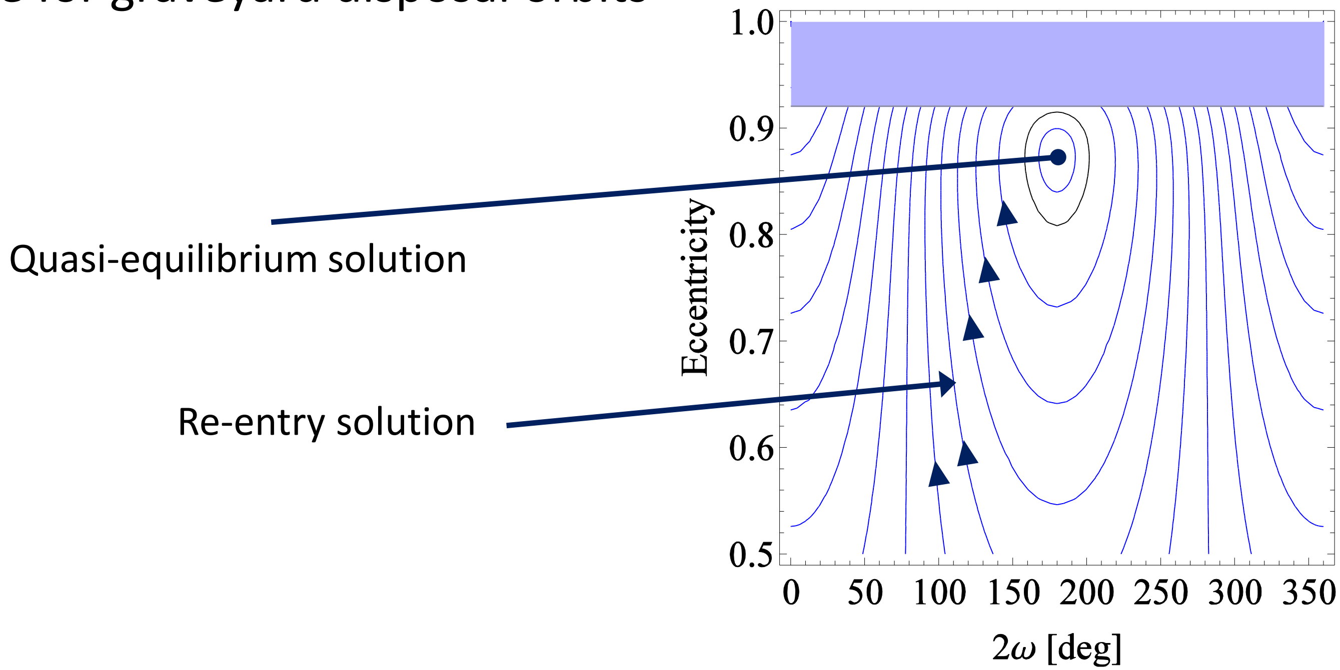
Design manoeuvre in the phase space

- Re-entry transfer on trajectories in the phase space to reach $e_{crit} = 1 - \left(R_{Earth} + h_{p,\,drag}\right)/a$

  Maximum Δe exploitable for re-entry or free orbit change

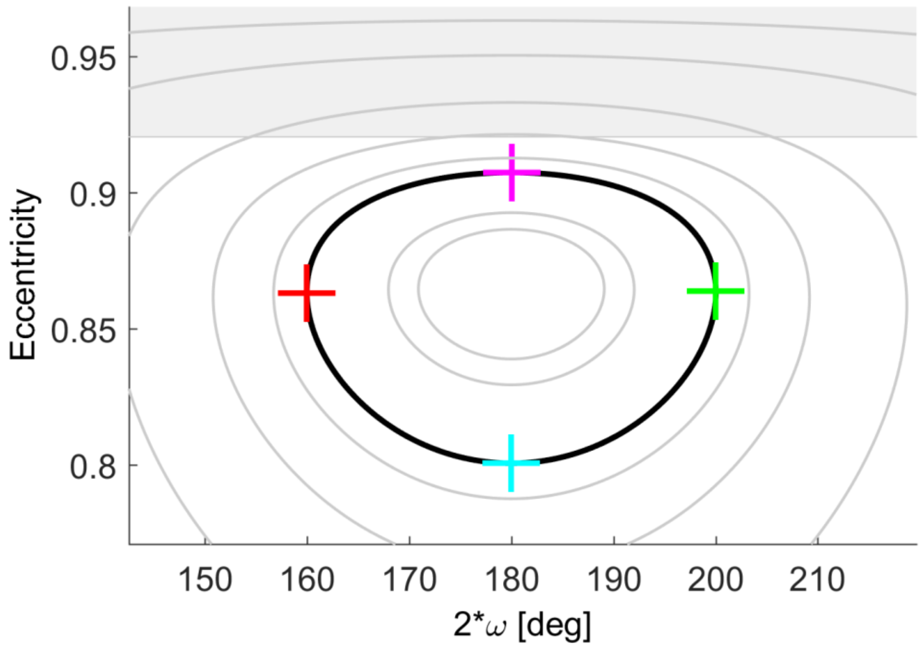
- Graveyard: transfer to quasi-stable point in the phase space

  Bounded Δe for graveyard disposal orbits



Engineering the perturbation effects

Preliminary analysis Earth re-entry

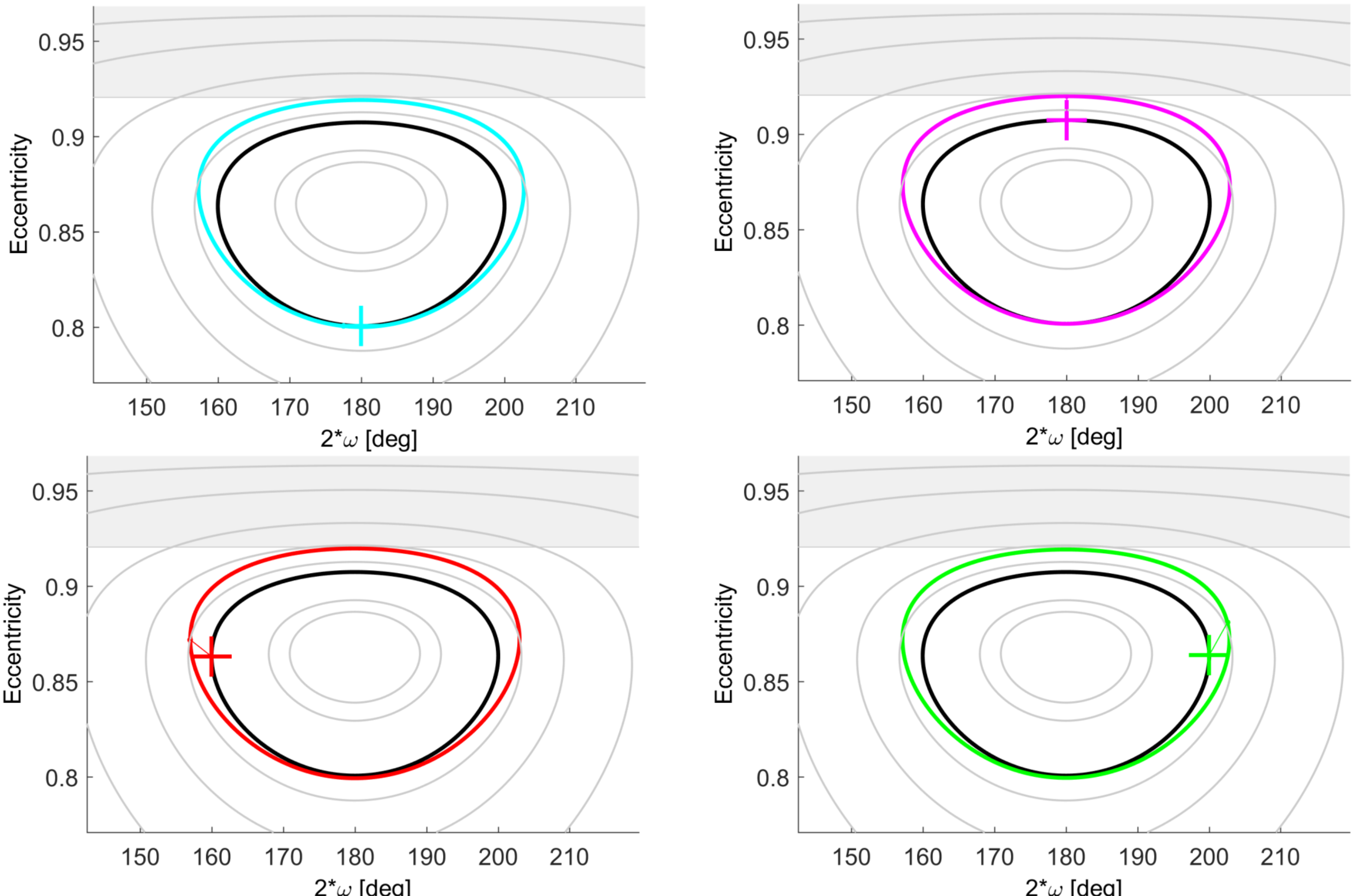


Disposal design without use of pre-calculated maps

- Gauss planetary eqs. compute change in orbital elements due to manoeuvre
- Orbit evolution computed with double average eqs.
- Multi-start method + local constrained optimisation $\min_{\{\Delta v,\delta,\beta,f\}} \Delta v \quad s.t.\ \max\left[e(t)\right] = e_{crit}$

Engineering the perturbation effects

Preliminary analysis Earth re-entry



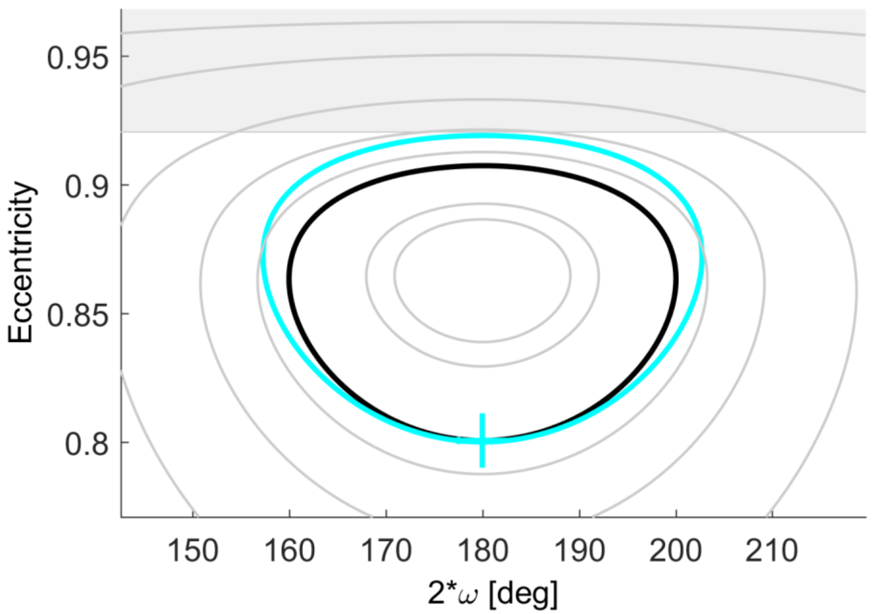
Engineering the perturbation effects

Simplified vs accurate model

Preliminary mission design

Moon effect only

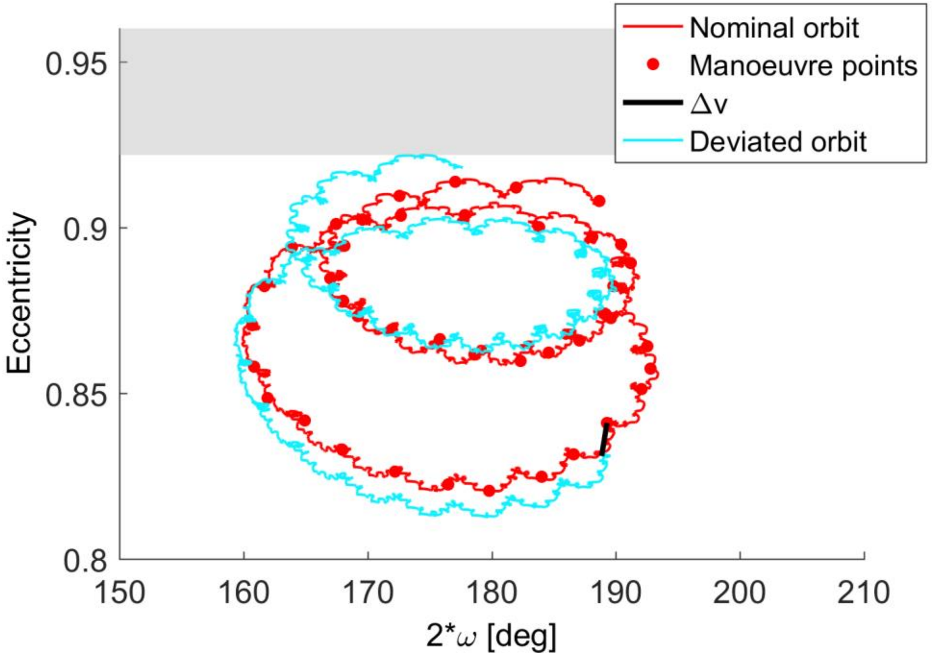
<u>Double averaged</u> potential + gradient based optimisation

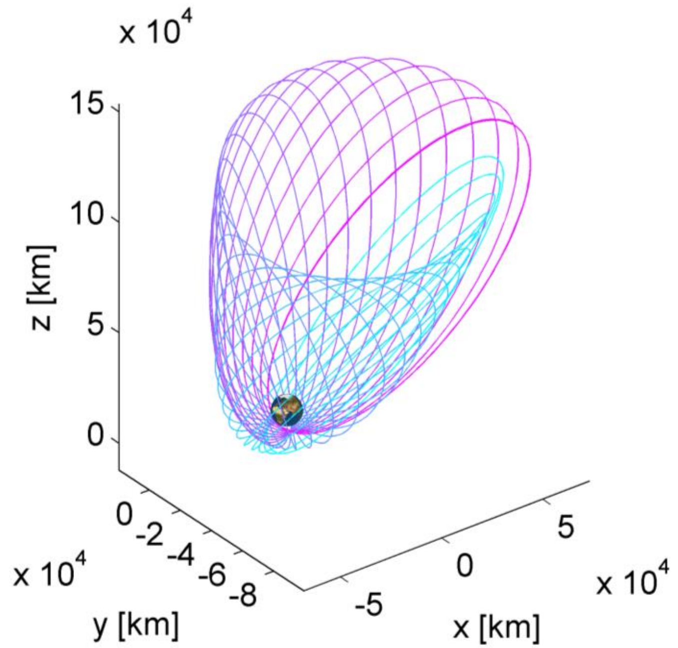


Optimised solution

Moon + Sun + J_2

<u>Single averaged</u> dynamics + global optimisation





Design of disposal manoeuvres for INTEGRAL

APPLICATION

Integral: gamma-ray observatory
ESA’s Integral observatory is able to detect gamma-ray bursts, the most energetic phenomena in the Universe

INTEGRAL mission disposal

Design disposal manoeuvre in the phase space

Single manoeuvre

$$\Delta \mathbf{v} = \Delta v \begin{bmatrix} \cos\alpha \cos\beta \\ \sin\alpha \cos\beta \\ \sin\beta \end{bmatrix}$$

⇨

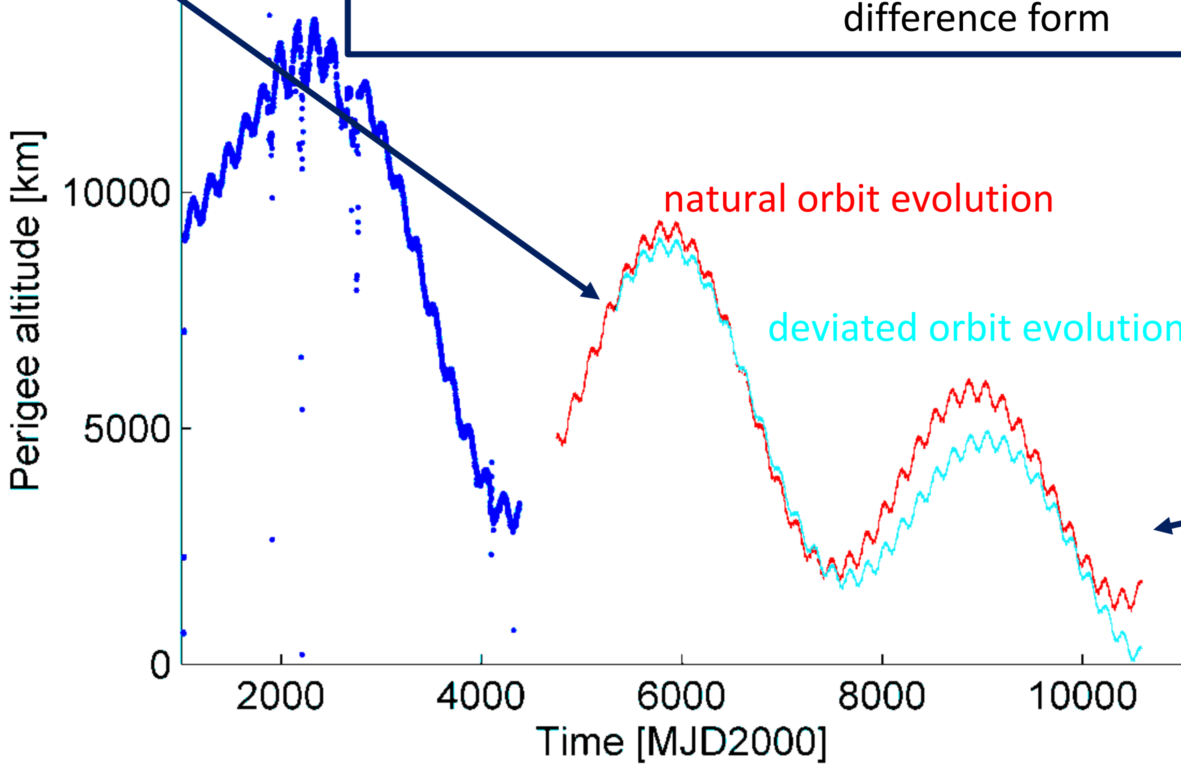
$$\Delta kep = G\left(kep\left(t_m\right), f_m, \Delta \mathbf{v}\right)$$

Gauss' planetary equations in finite-difference form

⇨

$$kep_d = kep\left(t_m\right) + \Delta kep$$

Deviated condition



Minimum perigee within selected time interval for disposal

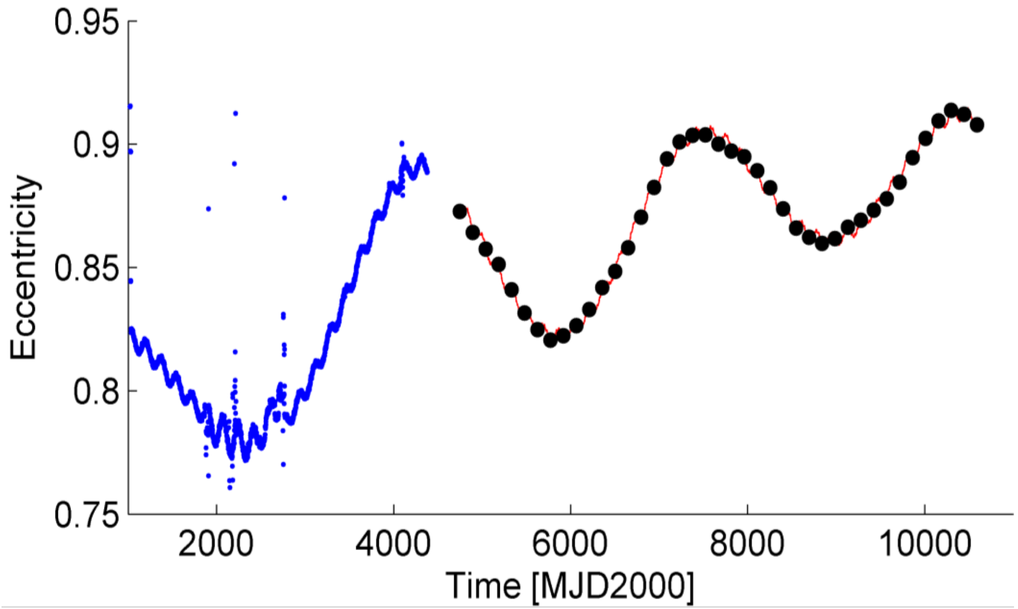
$$h_{p,\min} = \min_{t \in \Delta t_{\text{disposal}}} h_p\left(t\right)$$

► *Colombo, Letizia, Alessi, Landgraf, 24th AAS/AIAA 2014*

INTEGRAL mission disposal

Design disposal manoeuvre in the phase space

- Single manoeuvre at possible dates within disposal window [2013/01/01 to 2029/01/01]
- Only 5 mean elements (slow variables) are propagated: *a, e, i, Ω, ω*
- Optimal true anomaly f_M (and timing) where the manoeuvre is applied is selected through optimisation



Optimisation with genetic algorithms

$$x = [\Delta v \quad \alpha \quad \beta \quad f]$$

Objective function

$$J = \max(h_{p,\min} - h_{p,\text{target}}, 0)^2 + w \cdot \Delta v$$

INTEGRAL mission disposal

Family 1

re-entry in 2028, Δ*v* between 27 and 73 m/s

Family 2

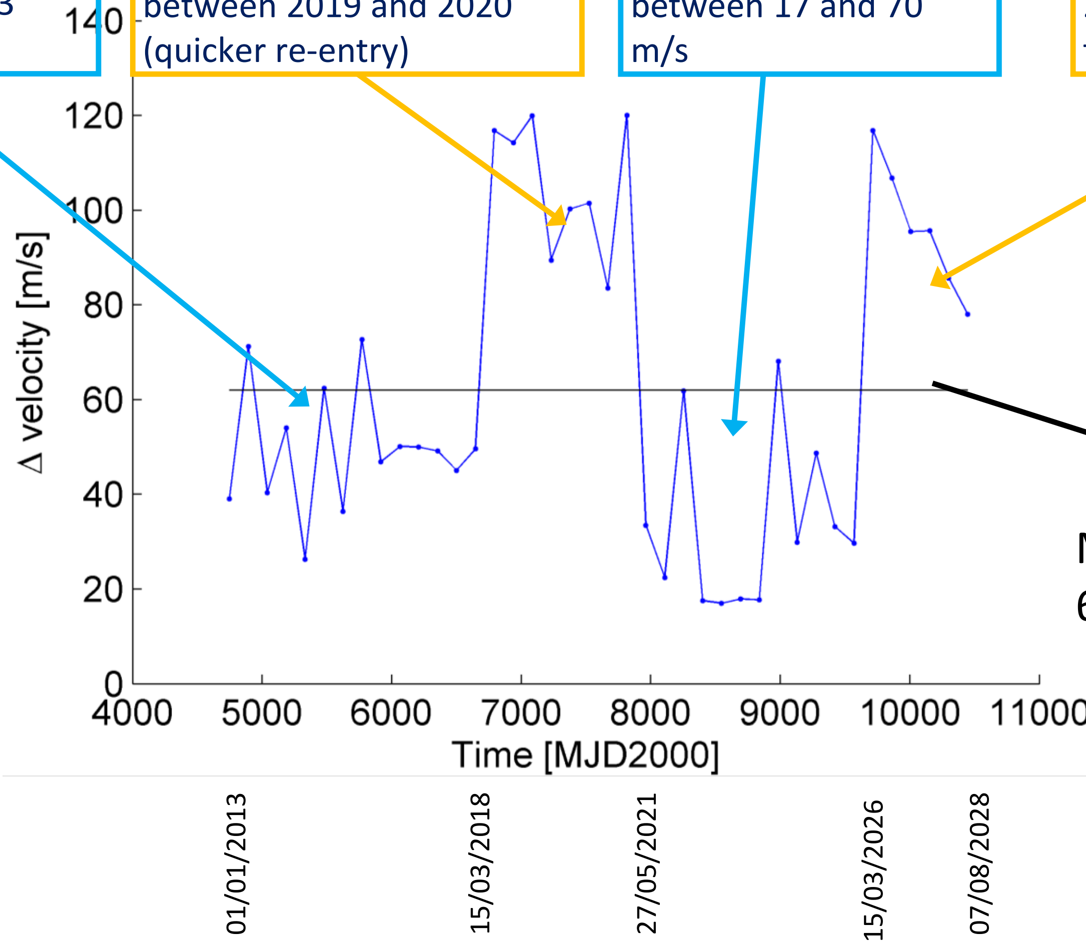
reach minimum perigee between 2019 and 2020 (quicker re-entry)

Family 3

re-entry in 2028, Δ*v* between 17 and 70 m/s

Family 4

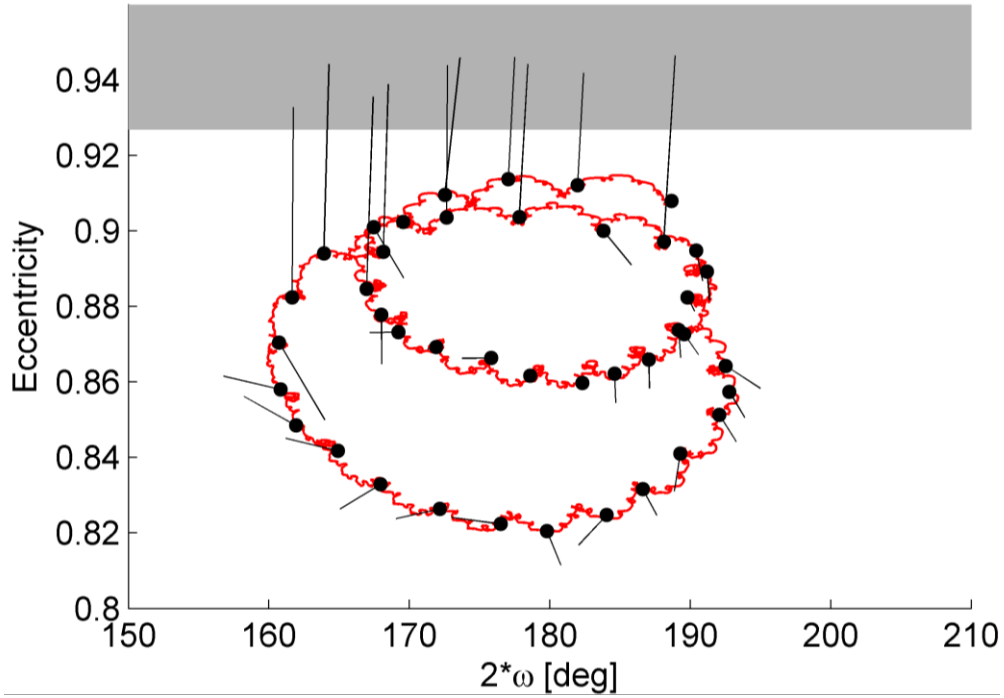
have a quick re-entry in 2028 but requires higher Δ*v* than family 3



Maximum available Δ*v* 61.9 m/s in 2013/01/01

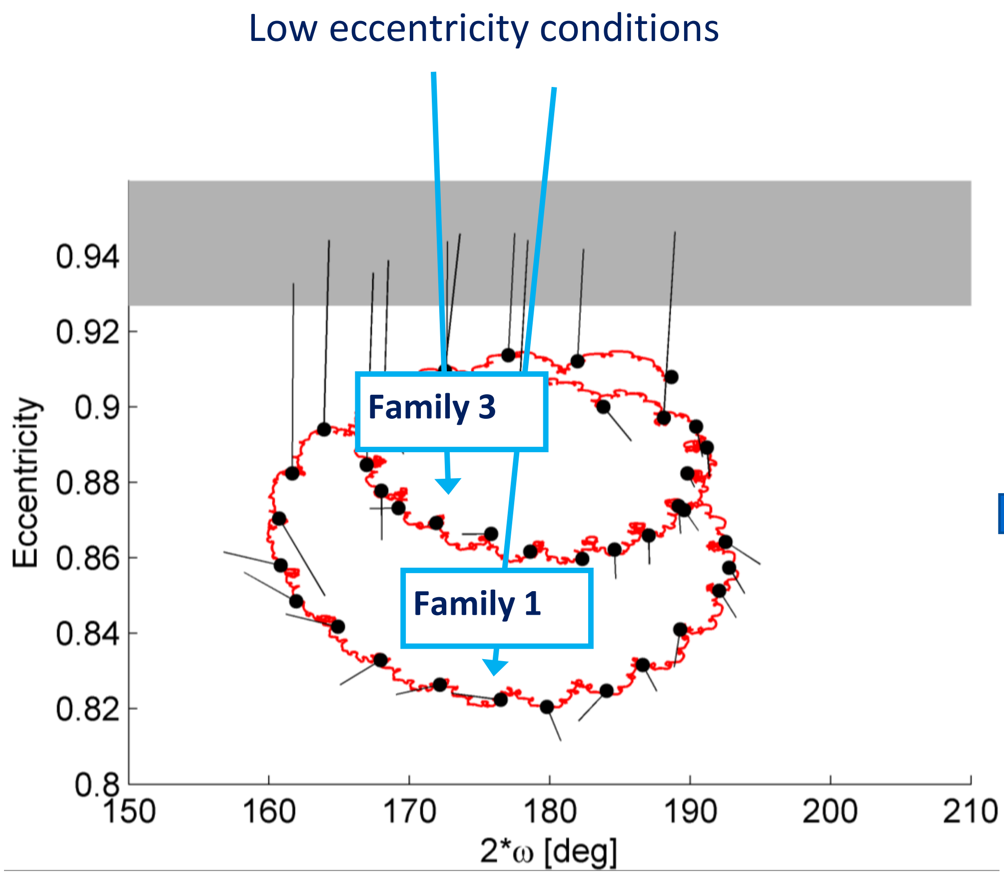
INTEGRAL mission disposal

Results



INTEGRAL mission disposal

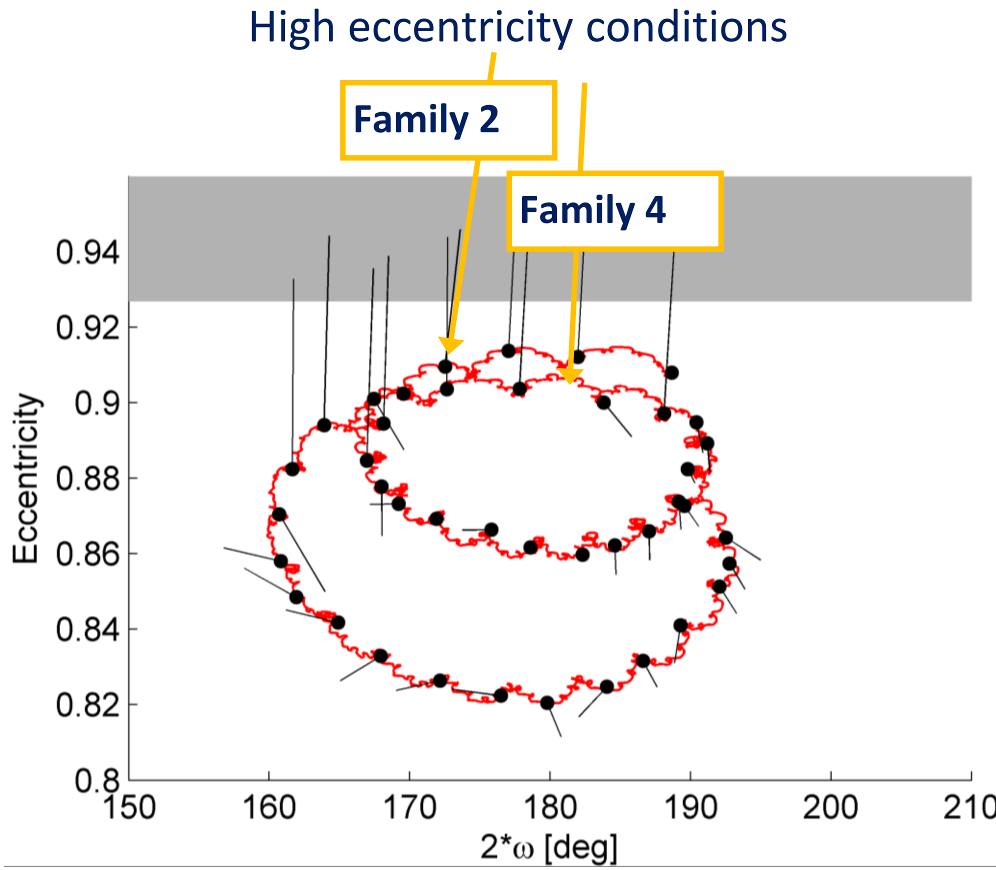
Results



The re-entry manoeuvre aims at further decreasing the eccentricity and changing the inclination so that a “better” Lidov-Kozai (Moon+Sun+J_2) loop is reached.

INTEGRAL mission disposal

Results

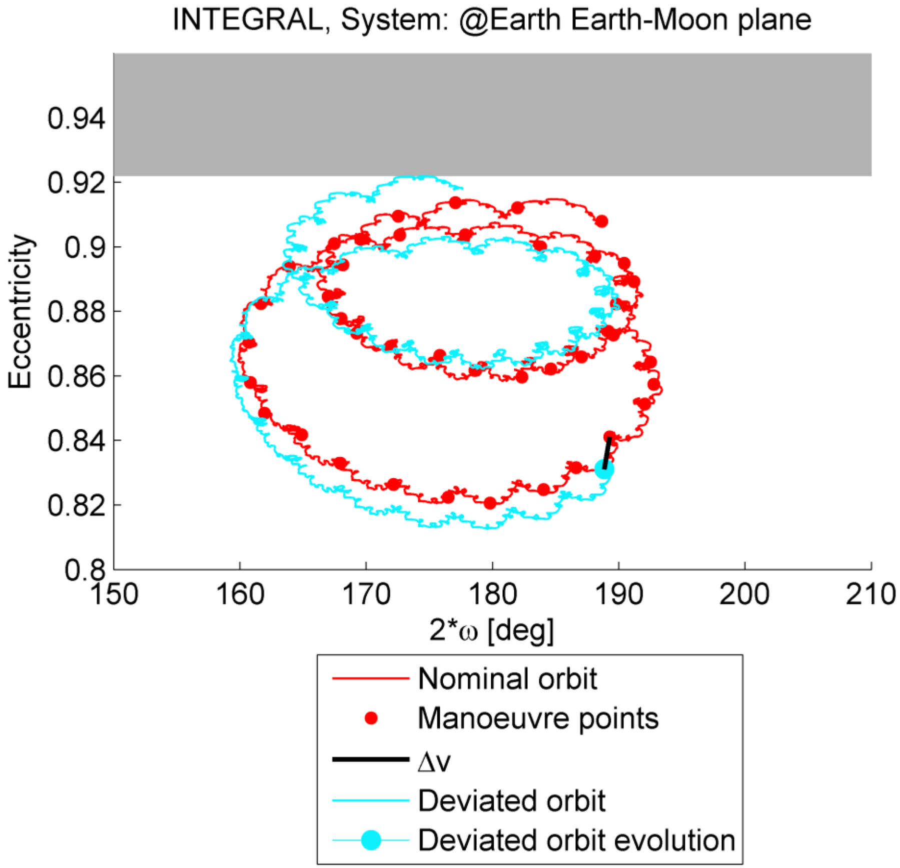


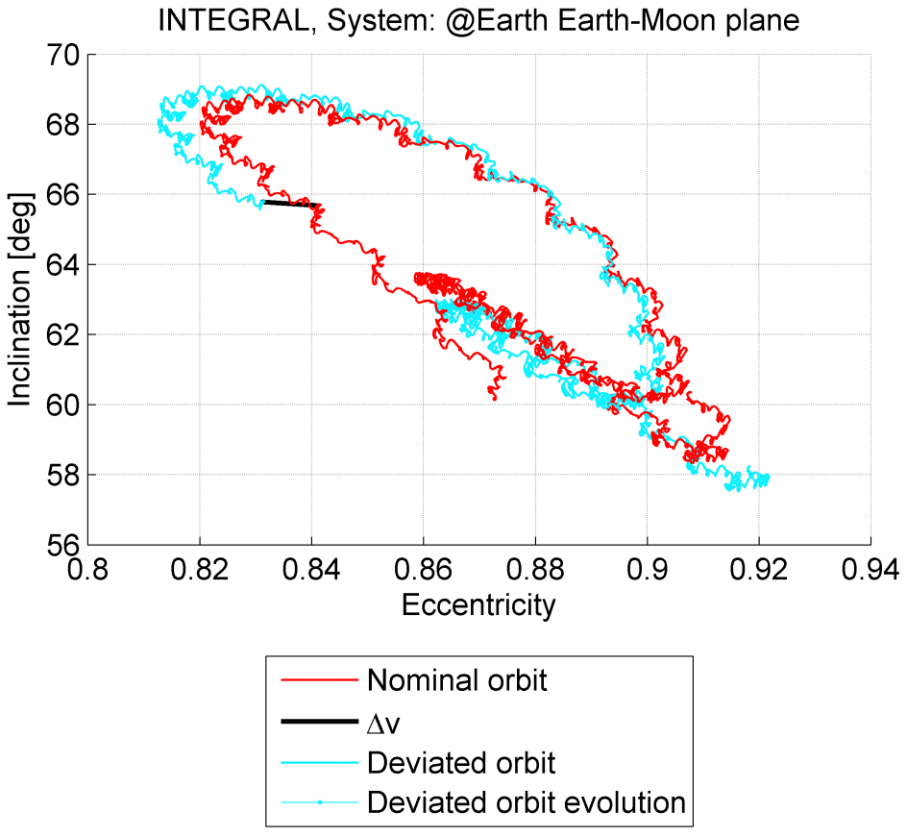
The re-entry manoeuvre aims at further increasing the eccentricity to decrease the energy but no perturbation is exploited.

INTEGRAL mission

Re-entry manoeuvre

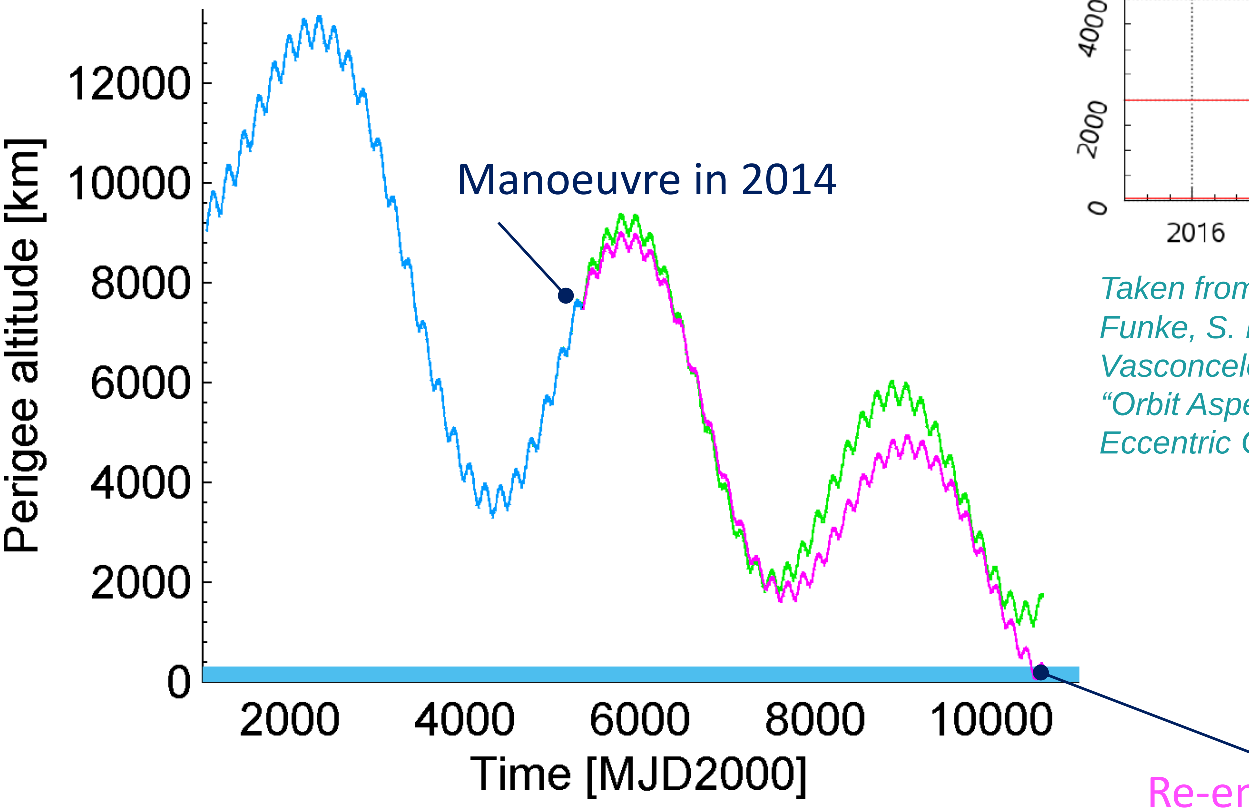
Example: manoeuvre performed on 08/08/2014

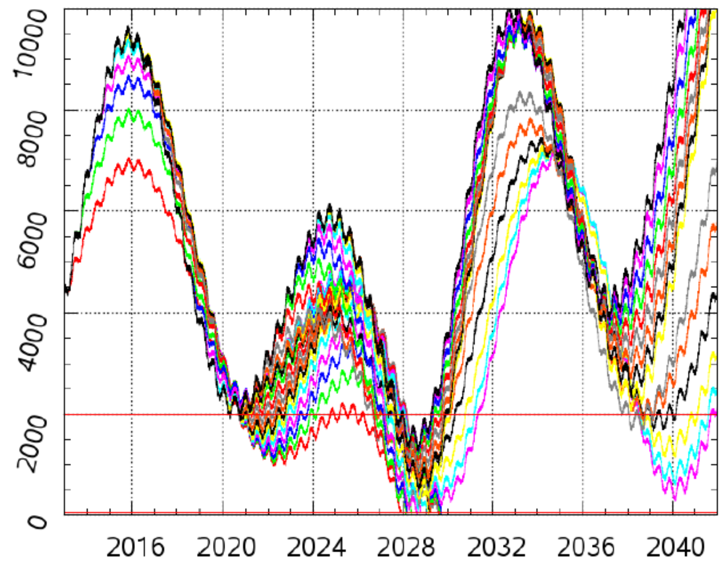




INTEGRAL mission

Re-entry manoeuvre





Taken from ESA: K. Merz, H. Krag, S. Lemmens, Q. Funke, S. Böttger, D. Sieg, G. Ziegler, A. Vasconcelos, B. Sousa, H.-J. Volpp, R. Southworth, "Orbit Aspects of End-Of-Life Disposal from Highly Eccentric Orbits", ISSFD

End-of-life disposal with by solar sail

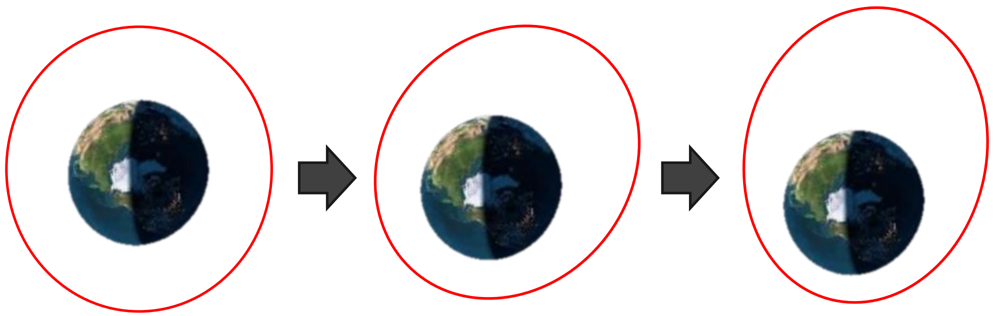
APPLICATION

End-of-life disposal by solar sail

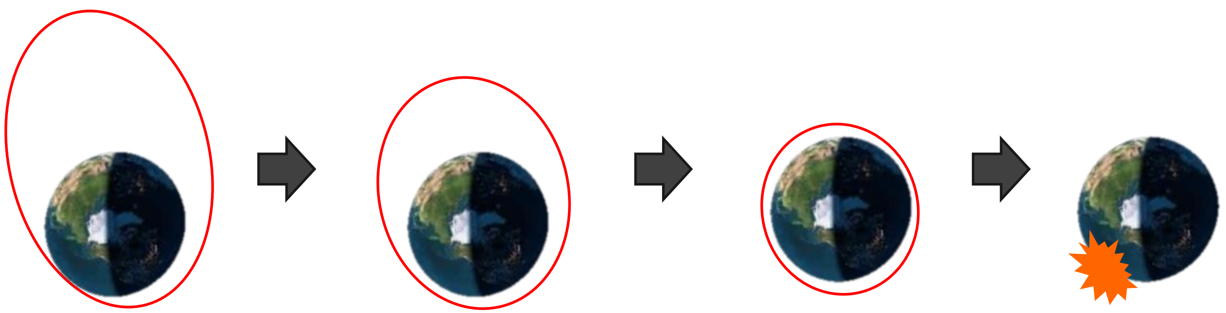
Passive outward elliptical deorbiting

Deploy area-increasing device to augment effect of solar radiation pressure

Phase 1: Passive eccentricity increase due to SRP from initial circular orbit (until reach critical eccentricity in drag region)



Phase 2: Deorbit augmented through drag

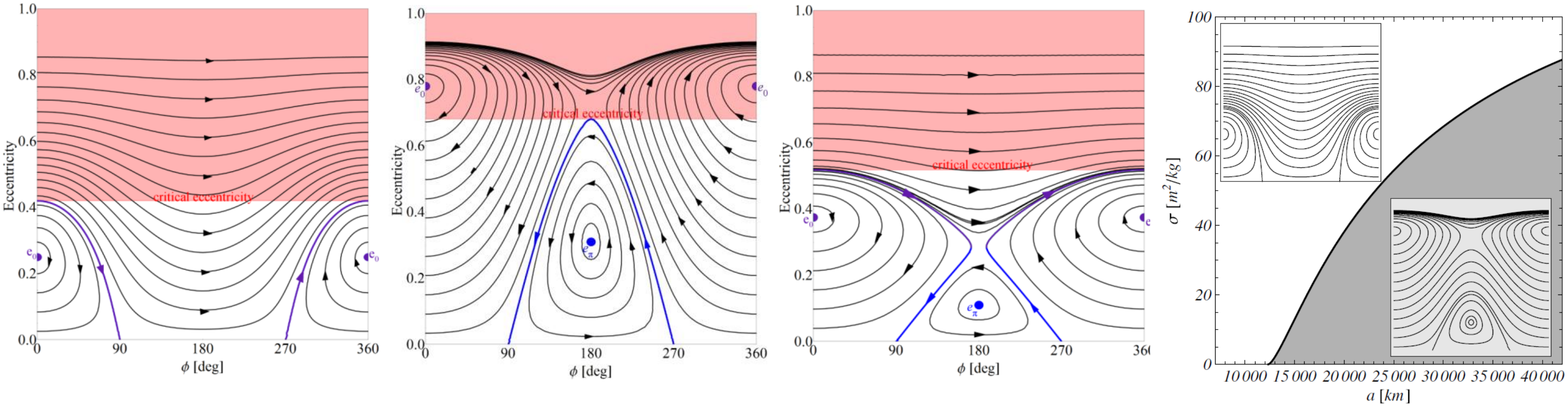


► *Lücking et al. "A Passive Satellite Deorbiting Strategy for MEO using Solar Radiation Pressure and the J2 Effect", Acta Astronautica, 2012.*

End-of-life disposal by solar sail

Passive outward elliptical deorbiting

a = 11,000 *a* = 13,250 *a* = 20,000

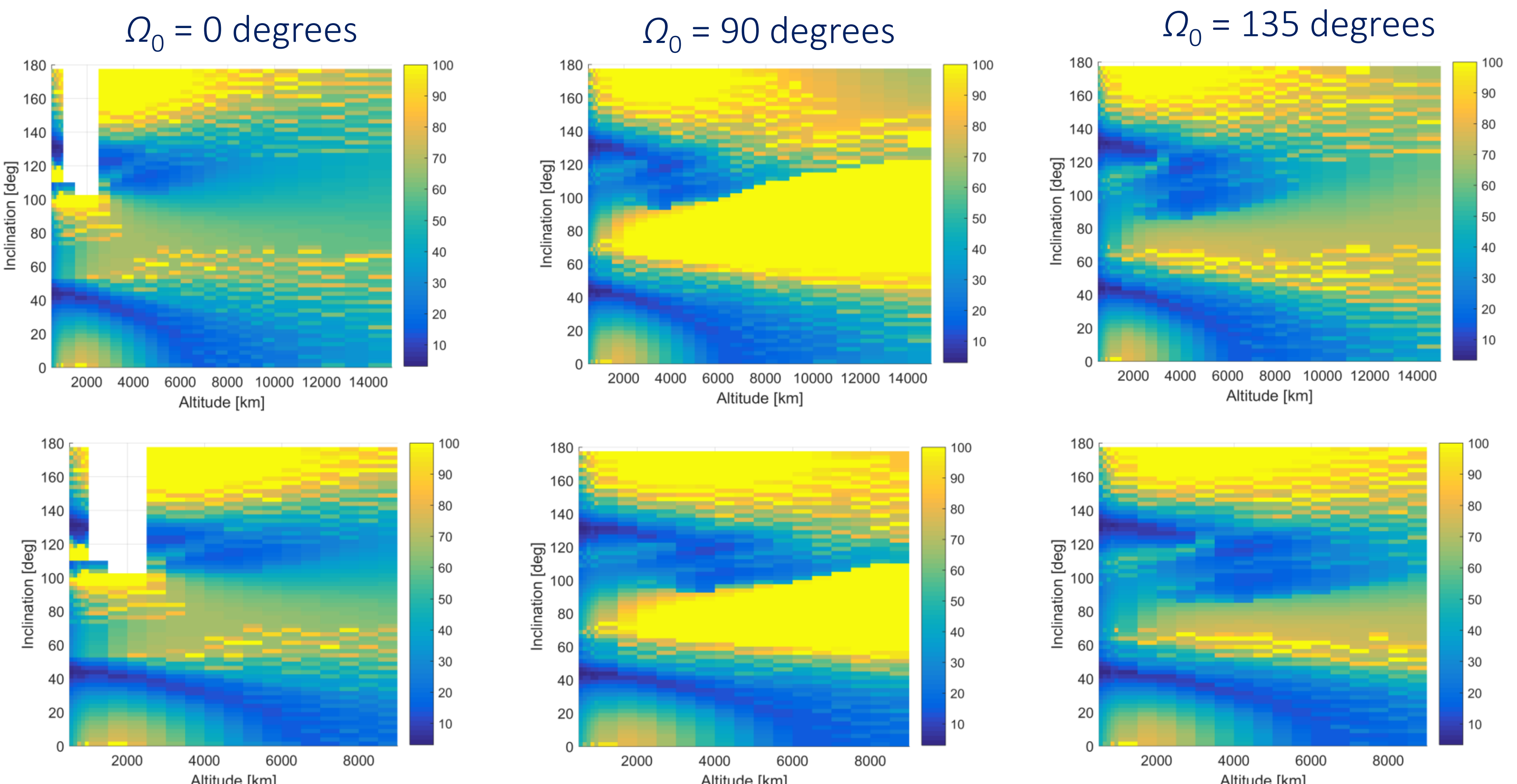


- Sun-perigee angle governs increase or decrease of the eccentricity
- Starting from a circular orbit, the effect of solar radiation pressure is to naturally increase the eccentricity until a maximum value. The sail area-to-mass is chosen so that, the maximum eccentricity attained during the orbit evolution is equal to the critical eccentricity

► *Lücking et al. "A Passive Satellite Deorbiting Strategy for MEO using Solar Radiation Pressure and the J2 Effect", Acta Astronautica, 2012.*

End-of-life disposal by solar sail

Passive outward elliptical deorbiting



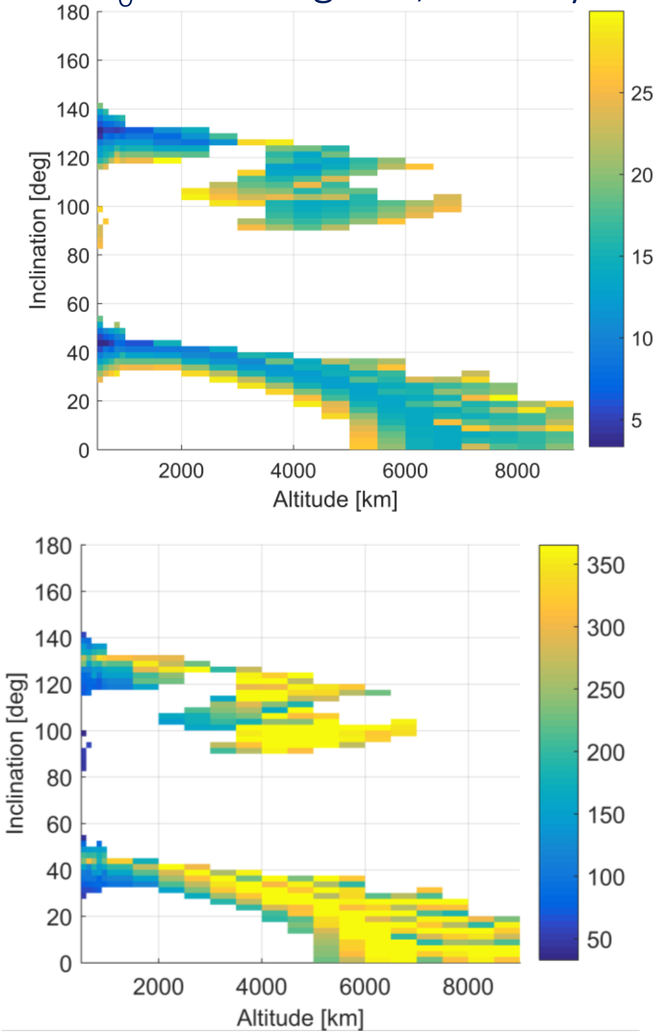
Sail requirements is low in correspondence of the SRP+J_2 resonances
(e.g. see Alessi at al, 2017, IAC, Schettino et al 2017, IAC)

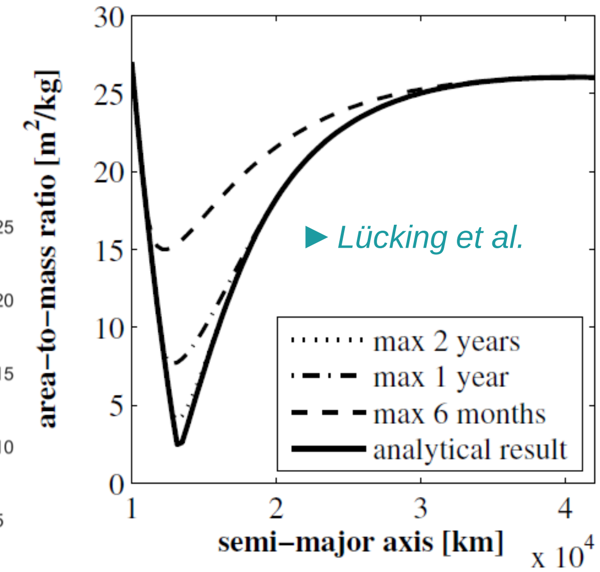
End-of-life disposal by solar sail

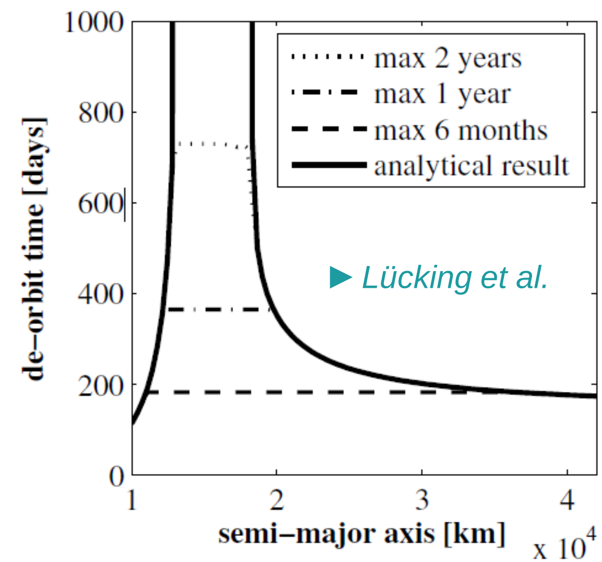
Passive outward elliptical deorbiting

Ω_0 = 135 degrees, max 1 year

max 5 years



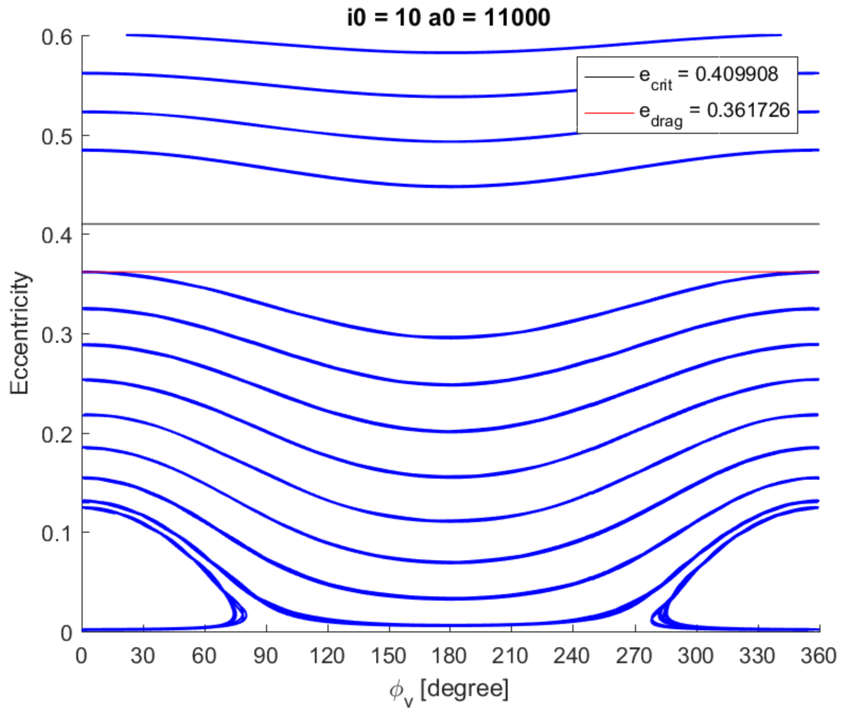




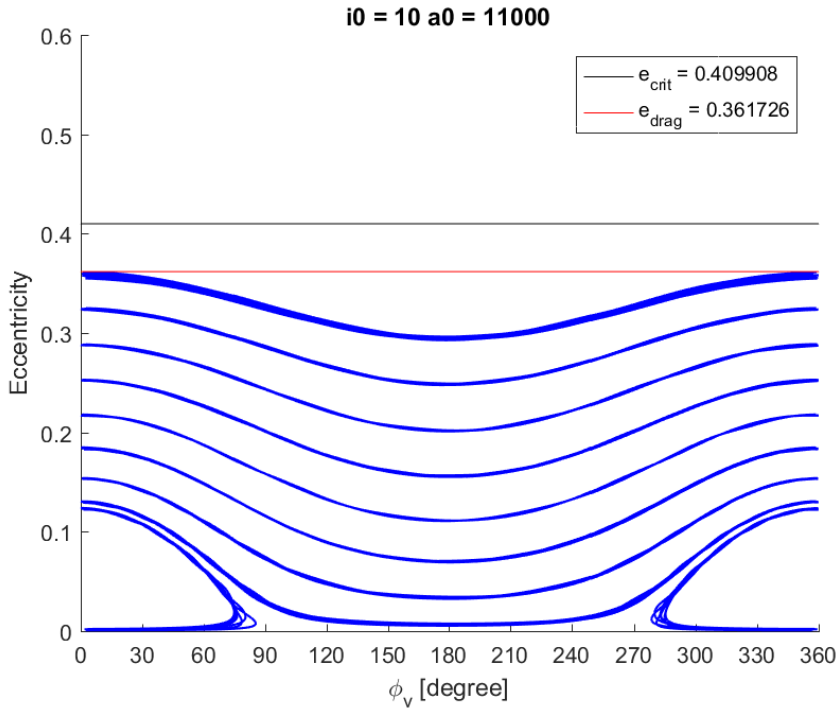
End-of-life disposal by solar sail

Drag-SRP-J_2 interaction

Propagation over 45 years without drag



Propagation over 45 years with drag

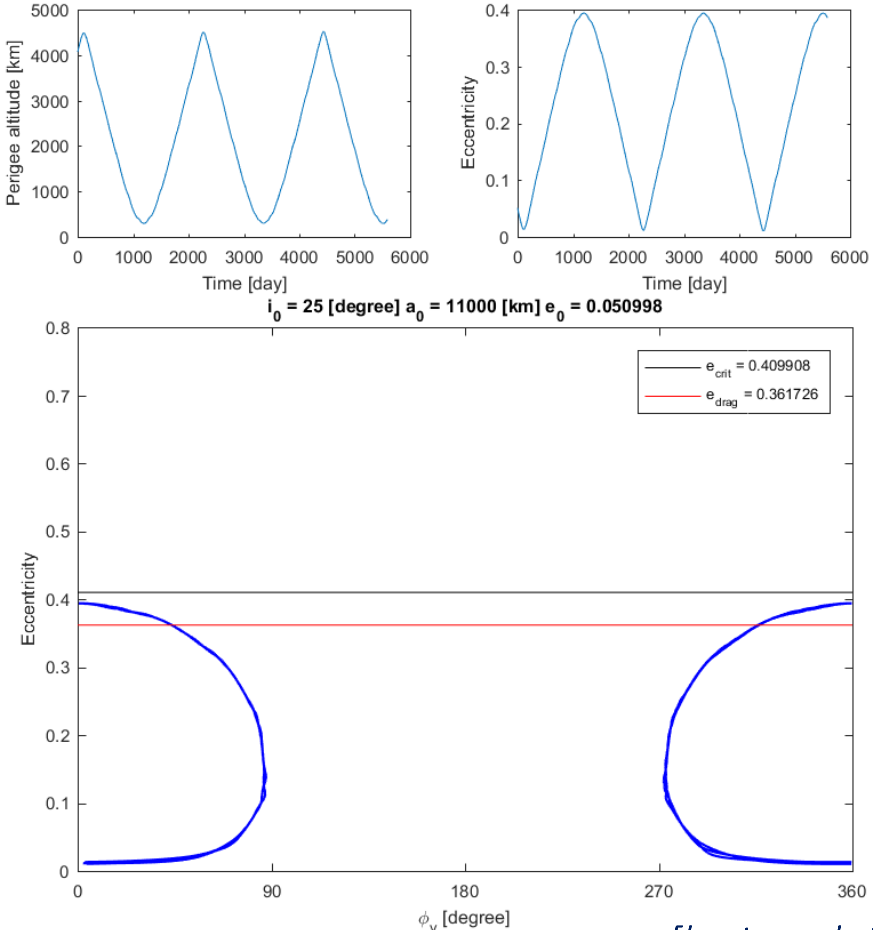


[by Langlois d'Estaintot]

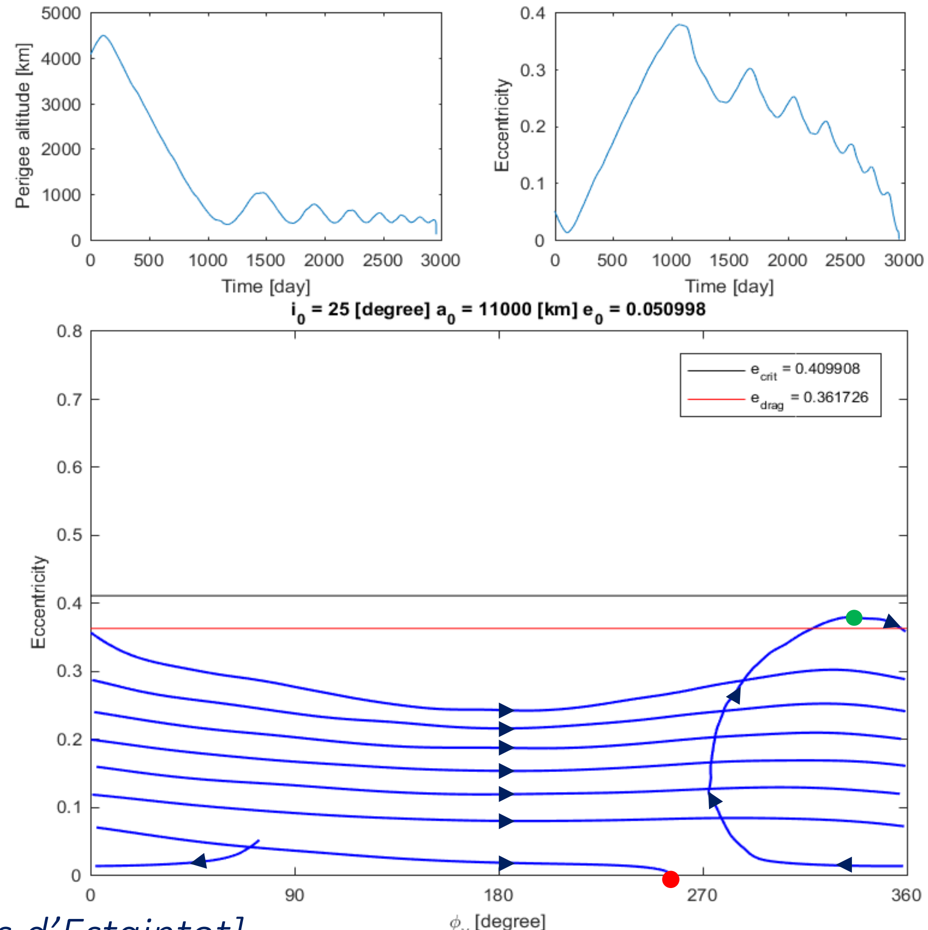
End-of-life disposal by solar sail

Drag-SRP-J2 interaction

Without drag – an example



With drag – an example



[by Langlois d'Estaintot]

CONCLUSIONS

Conclusions

Effect orbit perturbations can be exploited and enhanced…

We have already demonstration in Space

Schedule for revolution 1859

(this list is also available in csv-format, click here to download)

Rev	Start time (UTC)	End time (UTC)	Exp. time (s)	Target	Ra (J2000)	Dec (J2000)	Pattern	PI	Proposal	Observation	Notes
1859	2017-09-05 20:56:07	2017-09-06 01:07:59	13860	SAS Cal	17:20:00.00	+00:00:00.0	SAS pointings	Public	8860350	8860350 / 0001	Public
1859	2017-09-06 01:29:30	2017-09-06 16:11:19	50000	Galactic Center	17:46:16.46	-29:53:15.0	5x5 Seq	Joern Wilms	1420009	1420009 / 0009	
1859	2017-09-06 17:14:59	2017-09-06 18:13:19	3500	OMC FF #32	14:33:36.00	-16:24:00.0	Staring	Public	8860351	8860351 / 0001	Public
1859	2017-09-06 18:14:49	2017-09-06 18:28:49	840	OMC FF #32	14:33:36.00	-16:24:00.0	Custom 3x3 raster	Public	8860351	8860351 / 0002	Public
1859	2017-09-06 18:31:08	2017-09-06 18:45:08	840	OMC FF #32	14:33:36.00	-16:24:00.0	Custom 3x3 raster	Public	8860351	8860351 / 0002	Public
1859	2017-09-06 18:47:27	2017-09-06 19:01:27	840	OMC FF #32	14:33:36.00	-16:24:00.0	Custom 3x3 raster	Public	8860351	8860351 / 0002	Public
1859	2017-09-06 19:03:46	2017-09-06 19:17:46	840	OMC FF #32	14:33:36.00	-16:24:00.0	Custom 3x3 raster	Public	8860351	8860351 / 0002	Public
1859	2017-09-06 19:20:05	2017-09-06 19:34:05	840	OMC FF #32	14:33:36.00	-16:24:00.0	Custom 3x3 raster	Public	8860351	8860351 / 0002	Public
1859	2017-09-06 19:36:24	2017-09-06 19:50:24	840	OMC FF #32	14:33:36.00	-16:24:00.0	Custom 3x3 raster	Public	8860351	8860351 / 0002	Public
1859	2017-09-06 19:52:43	2017-09-06 20:06:43	840	OMC FF #32	14:33:36.00	-16:24:00.0	Custom 3x3 raster	Public	8860351	8860351 / 0002	Public
1859	2017-09-06 20:09:02	2017-09-06 20:23:02	840	OMC FF #32	14:33:36.00	-16:24:00.0	Custom 3x3 raster	Public	8860351	8860351 / 0002	Public
1859	2017-09-06 20:25:21	2017-09-06 20:39:21	840	OMC FF #32	14:33:36.00	-16:24:00.0	Custom 3x3 raster	Public	8860351	8860351 / 0002	Public
1859	2017-09-06 20:41:40	2017-09-06 21:15:00	2000	OMC FF #32	14:33:36.00	-16:24:00.0	Staring	Public	8860351	8860351 / 0003	Public
1859	2017-09-06 22:17:30	2017-09-06 23:19:26	3600	Gal. Bulge region	17:45:36.00	-28:56:00.0	HEX	Erik Kuulkers	1420001	1420001 / 0020	Public
1859	2017-09-06 23:50:11	2017-09-07 00:52:07	3600	Gal. Bulge region	17:45:36.00	-28:56:00.0	HEX	Erik Kuulkers	1420001	1420001 / 0020	Public
1859	2017-09-06 23:50:11	2017-09-07 00:52:07	3600	Gal. Bulge region	17:45:36.00	-28:56:00.0	HEX	Erik Kuulkers	1420001	1420001 / 0020	Public
1859	2017-09-07 01:17:25	2017-09-07 02:51:17	5400	Gal. Bulge region	17:45:36.00	-28:56:00.0	HEX	Erik Kuulkers	1420001	1420001 / 0020	Public
1859	2017-09-07 03:09:46	2017-09-08 01:36:21	78232	Galactic Center	17:45:40.04	-29:00:28.2	5x5 Seq	Sergei Grebenev	1420031	1420031 / 0001	

Acknowledgments

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Postdoc positions Open at Politecnico di Milano

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