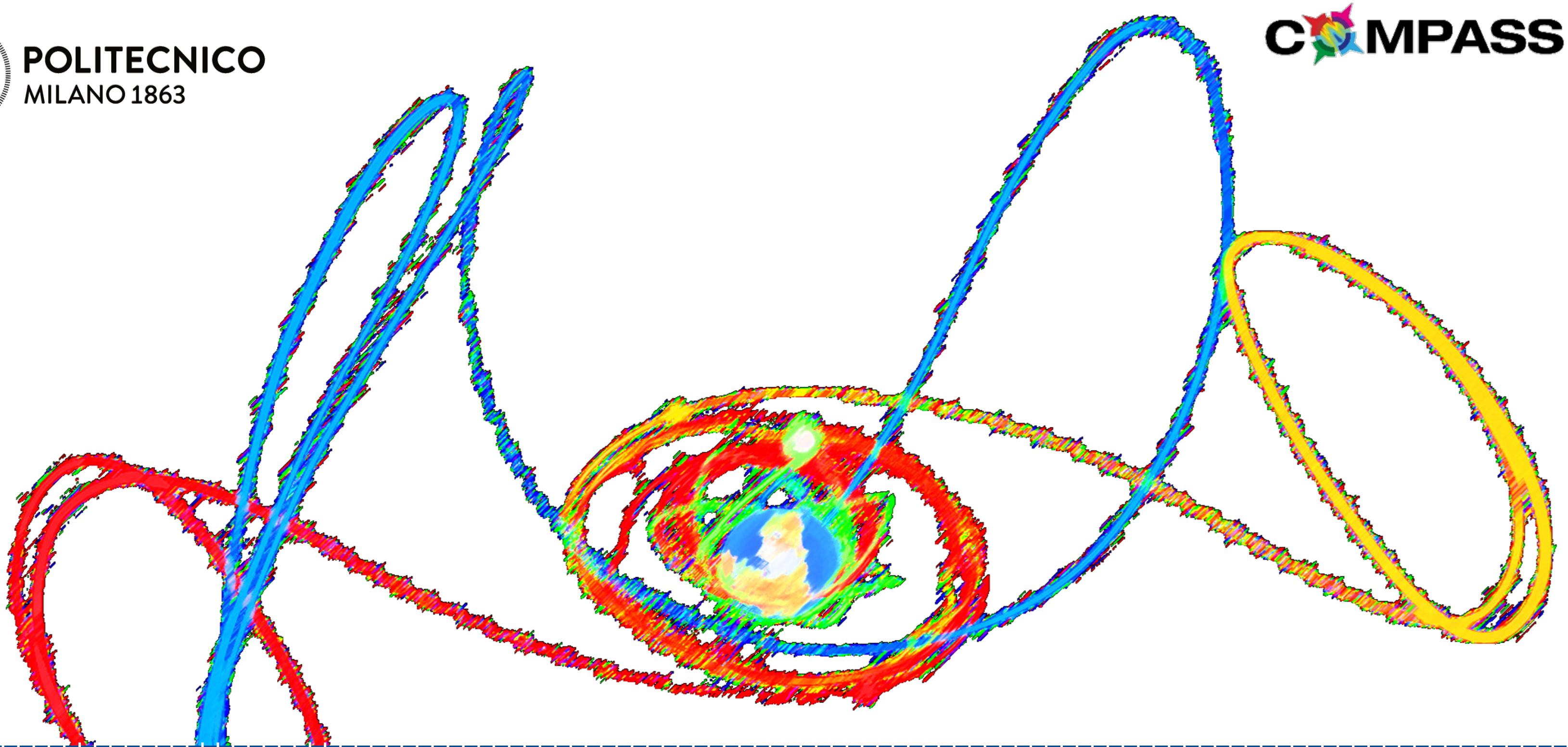


Third body effect in PlanODyn

Luni-solar perturbations for missions design

Camilla Colombo, Politecnico di Milano

KePaSSA2017 - 25-27 July 2017, ESA/ESTEC

INTRODUCTION

Introduction

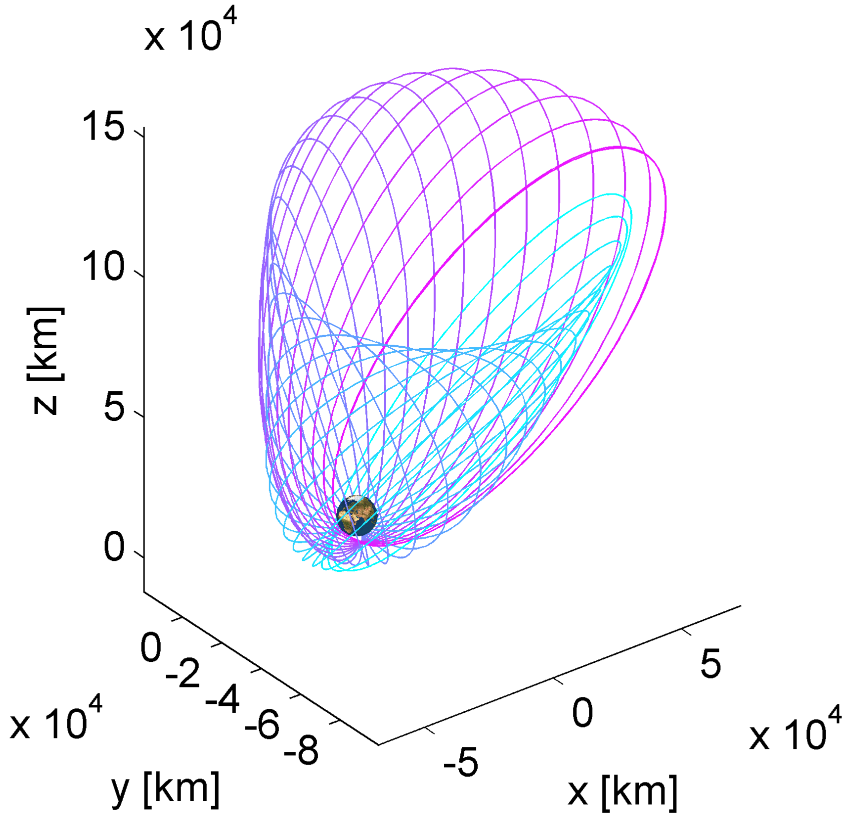
Luni-solar perturbations

…Fascinating interaction between third body luni-solar perturbation and

- Earth's oblateness
- Solar radiation pressure
- Tesseral harmonics

…Perfect example on how we can leverage the natural dynamical effect trough manoeuvres to obtain free long-term effect on the orbit:

- Frozen orbits
- End-of-life Earth re-entry
- End-of life graveyard orbit injection



Introduction

Outline

- Dynamical model in PlanODyn
- Analytical interpretation
- Engineering the perturbation effects
- Current work

DYNAMICAL MODEL

Dynamical model

PlanODyn suite

Space Debris Evolution, Collision risk, and Mitigation
FP7/EU Marie Curie grant 302270

COMPASS, ERC “Control for orbit manoeuvring through perturbations for supplication to space systems”

End-Of-Life Disposal Concepts for Lagrange-Point, Highly Elliptical Orbit missions, **ESA GSP**

GEO disposal in “Revolutionary Design of Spacecraft through Holistic Integration of Future Technologies”
ReDSHIFT, H2020

Dynamical model

Orbit propagation based on averaged dynamics

For conservative orbit perturbation effects

Disturbing potential function

$$R = R_{\mathrm{SRP}} + R_{\mathrm{zonal}} + R_{3-\mathrm{Sun}} + R_{3-\mathrm{Moon}}$$

Planetary equations in Lagrange form

$$\frac{d\boldsymbol{\alpha}}{dt} = f\left(\boldsymbol{\alpha}, \frac{\partial R}{\partial \boldsymbol{\alpha}}\right) \qquad \boldsymbol{\alpha} = \begin{bmatrix} a & e & i & \Omega & \omega & M \end{bmatrix}^T$$

Average over one orbit revolution of the spacecraft around the primary planet

$$\bar{R} = \bar{R}_{\mathrm{SRP}} + \bar{R}_{\mathrm{zonal}} + \bar{R}_{3-\mathrm{Sun}} + \bar{R}_{3-\mathrm{Moon}}$$

$$\frac{d\bar{\boldsymbol{\alpha}}}{dt} = f\left(\bar{\boldsymbol{\alpha}}, \frac{\partial \bar{R}}{\partial \bar{\boldsymbol{\alpha}}}\right)$$

Single average

Average over the revolution of the perturbing body around the primary planet

$$\bar{\bar{R}} = \bar{\bar{R}}_{\mathrm{SRP}} + \bar{R}_{\mathrm{zonal}} + \bar{\bar{R}}_{3-\mathrm{Sun}} + \bar{\bar{R}}_{3-\mathrm{Moon}}$$

$$\frac{d\bar{\bar{\boldsymbol{\alpha}}}}{dt} = f\left(\bar{\bar{\boldsymbol{\alpha}}}, \frac{\partial \bar{\bar{R}}}{\partial \bar{\bar{\boldsymbol{\alpha}}}}\right)$$

Double average

Dynamical model

Perturbation model

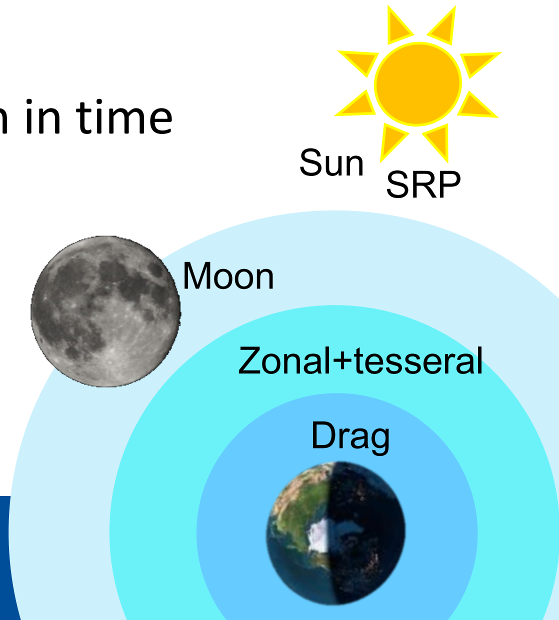
Perturbations in planet centred dynamics

- Atmospheric drag (smooth exponential model)
- Zonal harmonics of the Earth's gravity potential, J_2^2
- Selected tesseral terms (e.g., J_{22} for GEO)
- Solar radiation pressure (with eclipses)
- Third body perturbation of the Sun and the Moon

Ephemerides options

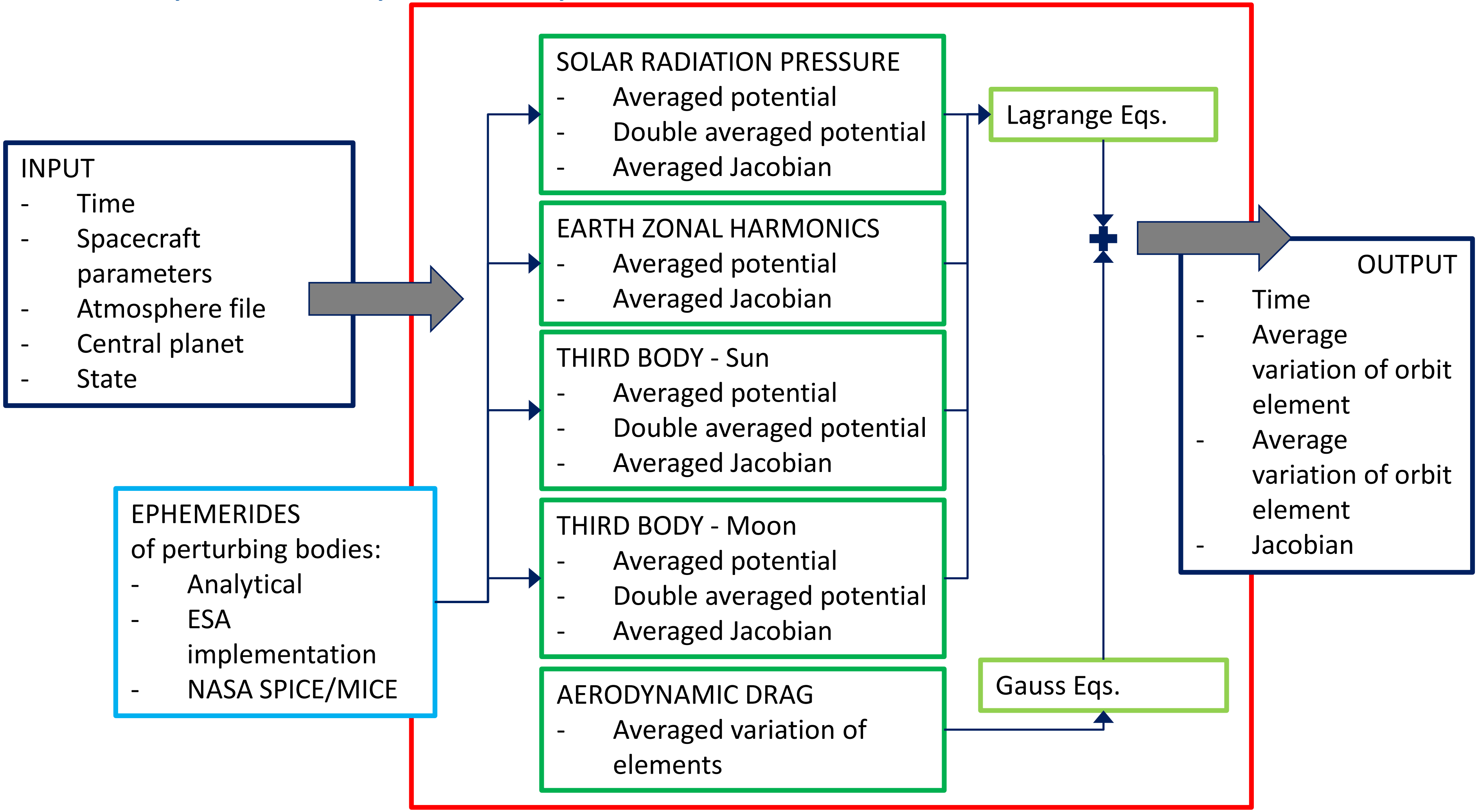
- Analytical approximation based on polynomial expansion in time
- Numerical ephemerides through the NASA SPICE toolkit
- Numerical ephemerides from an ESA implementation

Orbital elements in Earth centred equatorial J2000 frame



Dynamical model

PlanODyn: Planetary Orbital Dynamics



► *"Planetary Orbital Dynamics Suite for Long Term Propagation in Perturbed Environment," ICATT, ESA/ESOC, 2016.*

Dynamical model

Third body potential

$$R_{3B}(r,r') = \frac{\mu'}{r'}\left(\left(1-2\frac{r}{r'}\cos\psi+\left(\frac{r}{r'}\right)^2\right)^{-1/2} - \frac{r}{r'}\cos\psi\right)$$

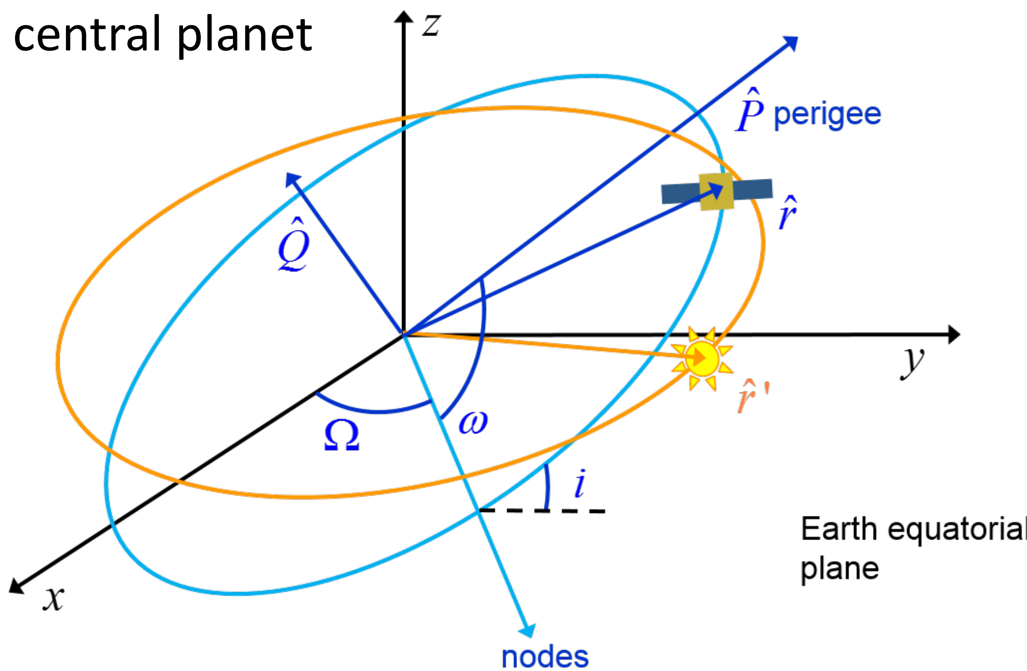
μ' gravitational coefficient of the third body

$\mathbf{r}'$ position vector of third body w.r.t. central planet

$\mathbf{r}$ position vector of satellite

ψ angle between satellite $\mathbf{r}$ and third body $\mathbf{r}'$

$$\cos\psi = \frac{\mathbf{r}\mathbf{r}'}{rr'}$$



Dynamical model

Third body potential

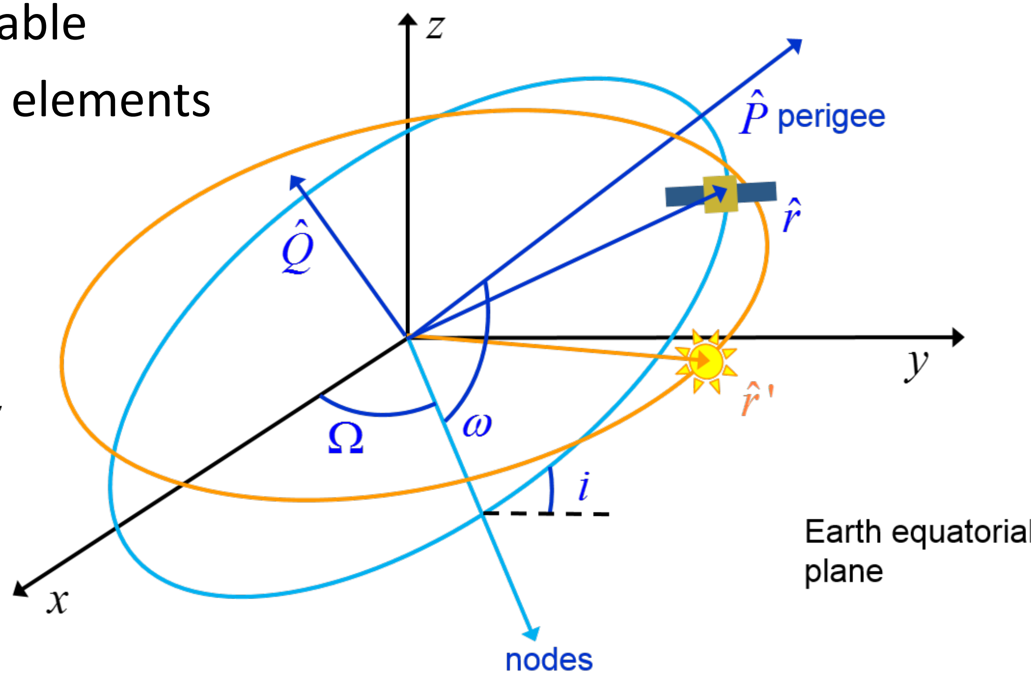
Third body potential in terms of:

- Ratio between orbit semi-major axis and distance of the third body $\delta = \frac{a}{r'}$
- Orientation of orbit eccentricity vector with respect to third body $A = \hat{P} \cdot \hat{r}'$
- Orientation of semi-latus rectum vector with respect to third body $B = \hat{Q} \cdot \hat{r}'$
- Eccentric anomaly as angular variable
- Composition of rotation in orbital elements

$$\hat{P} = R_3(\Omega) R_1(i) R_3(\omega) \cdot [1 \quad 0 \quad 0]^T$$

$$\hat{Q} = R_3(\Omega) R_1(i) R_3(\omega + \pi/2) \cdot [1 \quad 0 \quad 0]^T$$

$$\hat{r}' = R_3(\Omega') R_1(i') R_3(\omega' + f') \cdot [1 \quad 0 \quad 0]^T$$



Dynamical model

Third body potential

hp: the spacecraft is far enough from the perturbing body

μ' gravitational coefficient of the third body

$\mathbf{r}'$ position vector of third body

E eccentric anomaly

Series expansion around $\delta = 0$

$$R_{3B}(r, r') = \frac{\mu'}{r'} \sum_{k=2}^{\infty} \delta^k F_k(A, B, e, E)$$

Average over one orbit revolution

$$\overline{R}_{3B}(r, r') = \frac{\mu'}{r'} \sum_{k=2}^{\infty} \delta^k \overline{F}_k(A, B, e)$$

$$\overline{F}_k(A, B, e) = \frac{1}{2\pi} \int_{-\pi}^{\pi} F_k(A, B, e, E) \underbrace{(1 - e\cos E)dE}_{dM}$$

Partial derivatives to be included in Lagrange equations

$$A(\Omega, i, \omega, \Omega', i', u')$$

$$B(\Omega, i, \omega, \Omega', i', u')$$

$$\overline{F}_k(A, B, e)$$

→

$$\frac{\partial \overline{F}_k}{\partial \Omega} = \frac{\partial \overline{F}_k}{\partial A}\frac{\partial A}{\partial \Omega} + \frac{\partial \overline{F}_k}{\partial B}\frac{\partial B}{\partial \Omega}$$

$$\frac{\partial \overline{F}_k}{\partial i} = \frac{\partial \overline{F}_k}{\partial A}\frac{\partial A}{\partial i} + \frac{\partial \overline{F}_k}{\partial B}\frac{\partial B}{\partial i}$$

$$\frac{\partial \overline{F}_k}{\partial \omega} = \frac{\partial \overline{F}_k}{\partial A}\frac{\partial A}{\partial \omega} + \frac{\partial \overline{F}_k}{\partial B}\frac{\partial B}{\partial \omega}$$

$$\frac{\partial \overline{F}_k}{\partial a} = \frac{k}{a} F_k$$

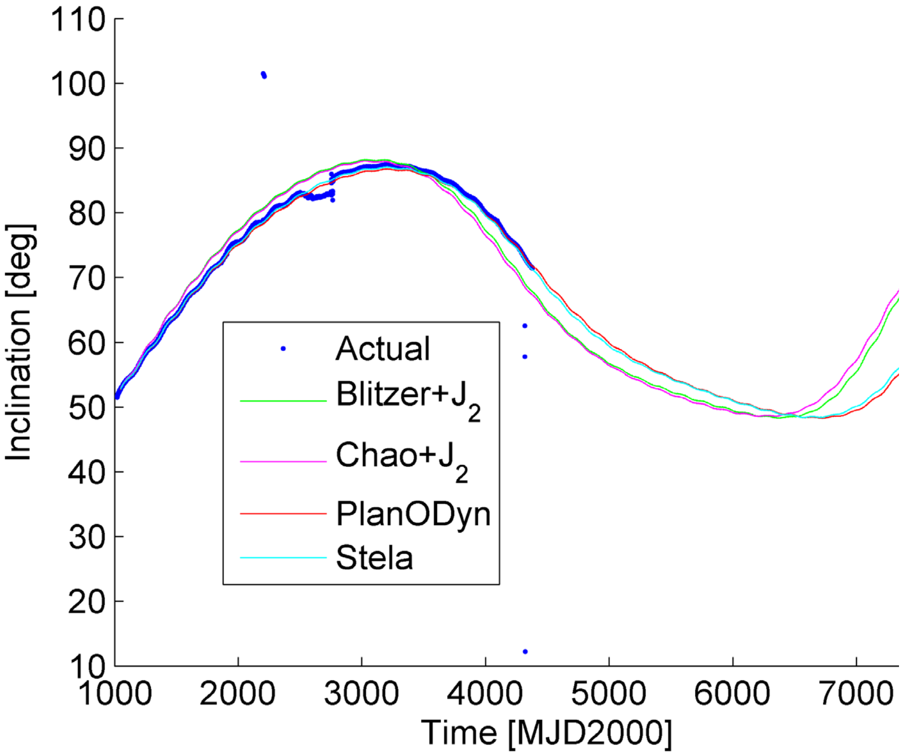
$$\frac{\partial \overline{F}_k}{\partial e}$$

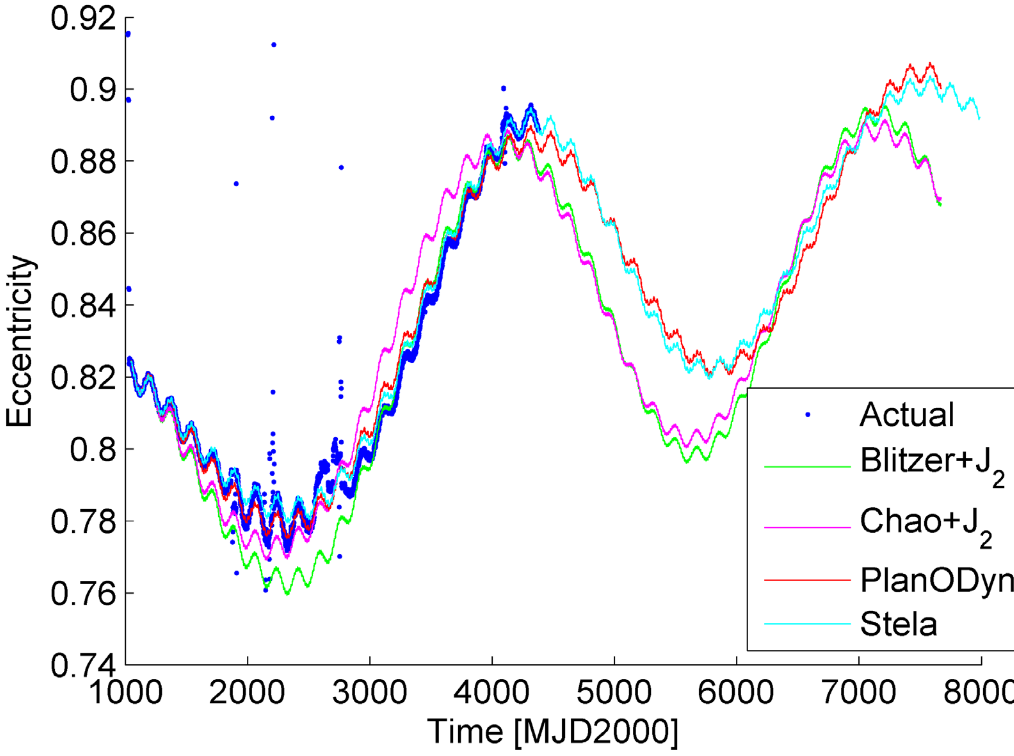
► *Kaufman and Dasenbrock, NASA report, 1979*

Dynamical model

Order of the luni-solar potential expansion

Third-body perturbing potential of the Moon at least up to the fourth order of the power expansion





► *Blitzer L., Handbook of Orbital Perturbations, Astronautics, 1970*
► *Chao-Chun G. C., Applied Orbit Perturbation and Maintenance, 2005*

Dynamical model

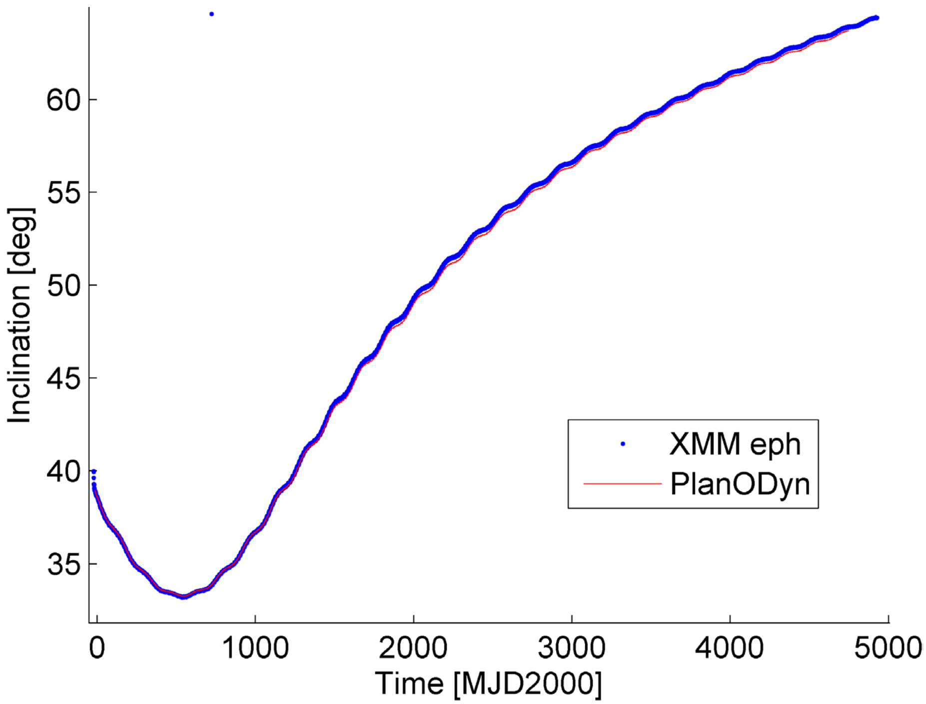
Validation: XMM Newton trajectory

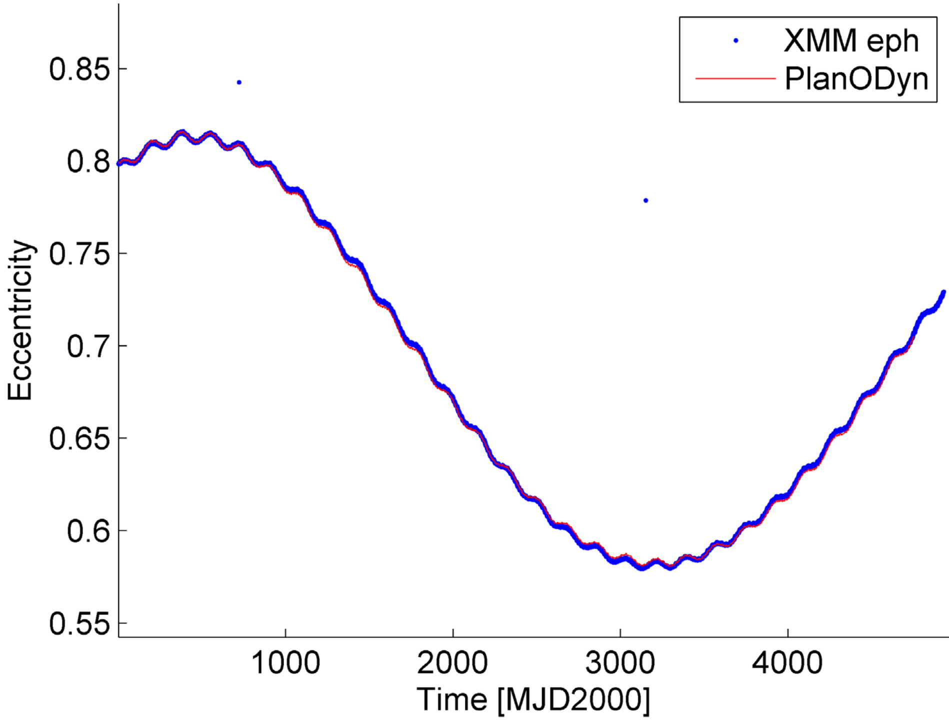
Propagation time: 1999/12/15 to 2013/01/01

Initial Keplerian elements from ESA on 1999/12/15 at 15:00:

a = 67045 km, e = 0.7951, i = 0.67988 rad, Ω = 4.1192 rad, ω = 0.99259 rad

System: Earth centred, equatorial J2000



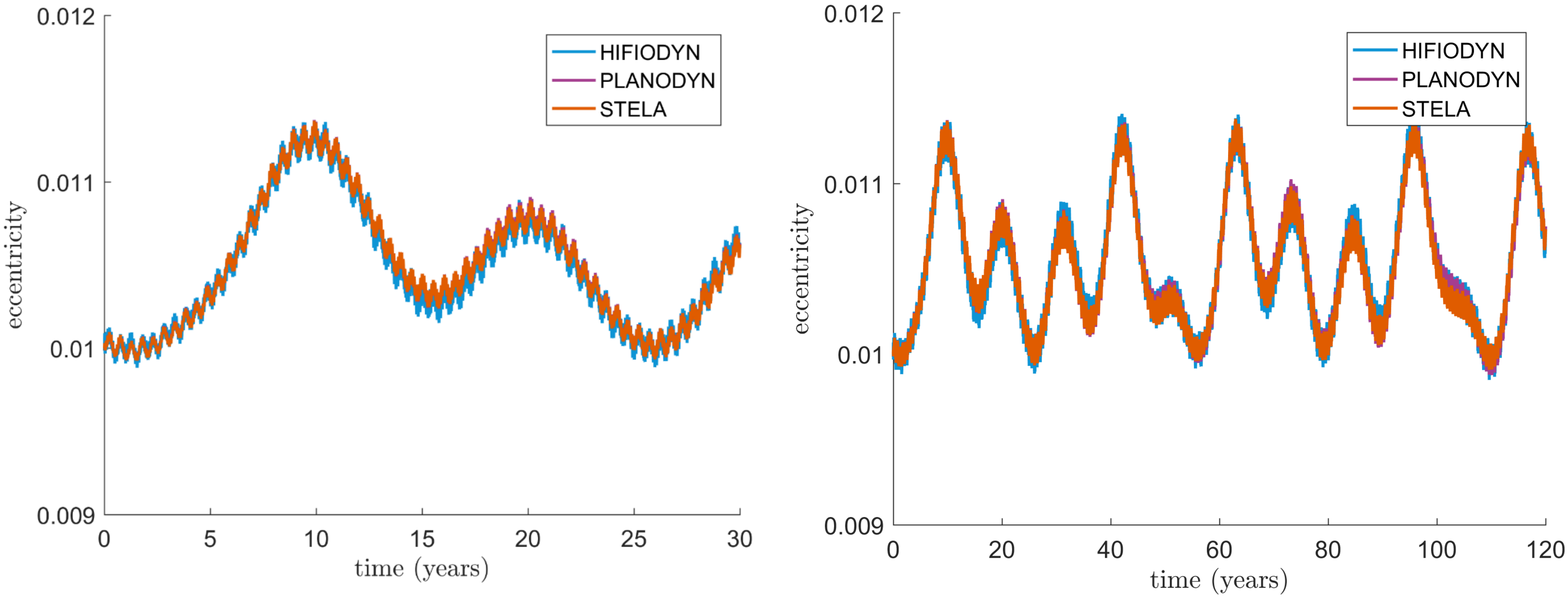


Dynamical model

Validation: GEO orbit

Comparison of PlanODyn with STELA (CNES) and with HiFiODyn (full dynamical model) for a typical GEO orbit.

System: Earth centred, equatorial J2000



► *Gkolias and Colombo, KePASSA 2017*

ANALYTICAL INTERPRETATION

Analytical interpretation

Third-body double averaged potential

Double averaging over one orbit revolution of the s/c and one orbit evolution of the perturbing body (either Sun or Moon) around the Earth

$$\bar{\bar{R}}_{3B}(r,r') = \frac{\mu'}{r'}\sum_{k=2}^{\infty}\delta^k \bar{\bar{F}}_k(e,i,\Delta\Omega,\omega,i')$$

$$\bar{\bar{F}}_k(e,i,\Delta\Omega,\omega,i') = \frac{1}{2\pi}\int_0^{2\pi}\bar{F}_k\left(A\left(\Omega,i,\omega,\Omega',i',\omega'+f'\right),B\left(\Omega,i,\omega,\Omega',i',\omega'+f'\right),e\right)df'$$

Earth’s centred equatorial reference system.

Same approach as El’yasberg (and Kozai) with some improvements:

- Avoid simplification that Moon and Sun orbit on the same plane (very important for precise orbit evolution)
- Facilitate the introduction of the effect of the zonal harmonics

► *Kozai, Secular Perturbations of Asteroids with High Inclination and Eccentricity, 1962*
► *El’yasberg, Introduction to the theory of flight of artificial Earth satellites - translated, 1967*

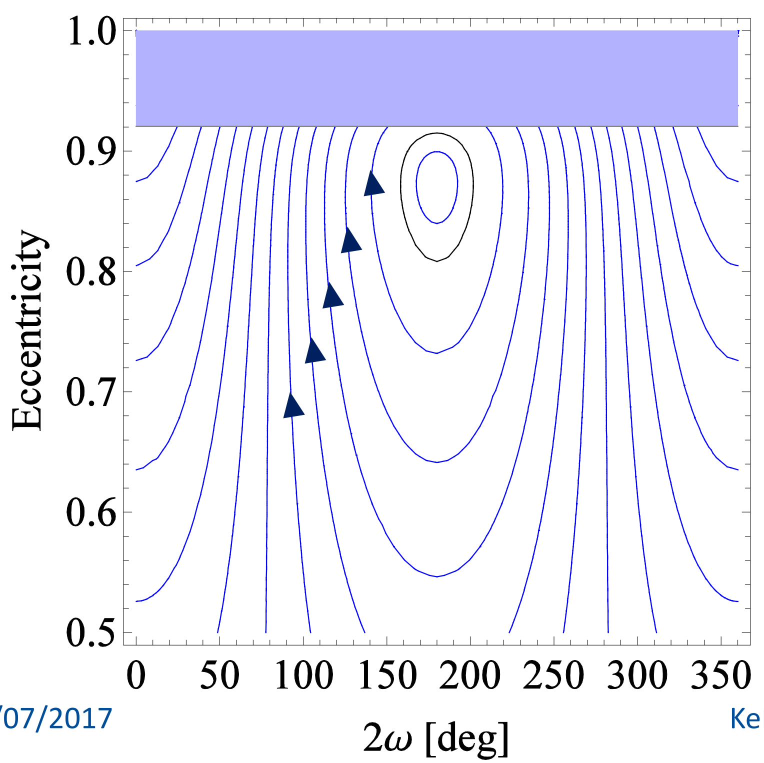
Analytical interpretation

Third body Kozai theory

- Delaunay's transformation
- Time-independent Hamiltonian

$$W\left(\frac{a}{a'},\Theta,e,2\omega\right)=\text{cost} \qquad \Theta=\left(1-e^2\right)\cos i^2$$

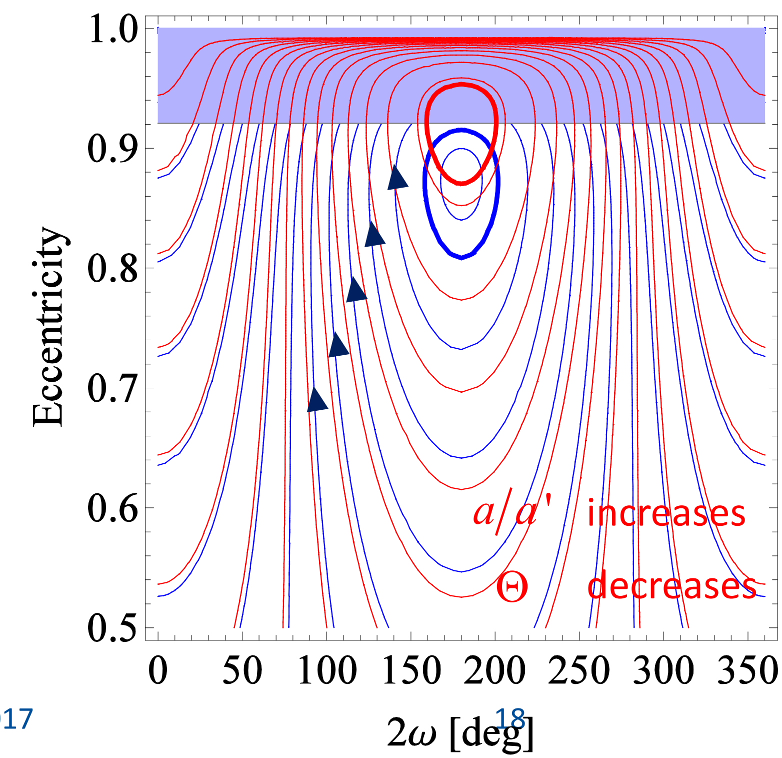
▶ *Kozai, Secular Perturbations of Asteroids with High Inclination and Eccentricity, 1962*



- Double averaged potential
- Rotating reference system

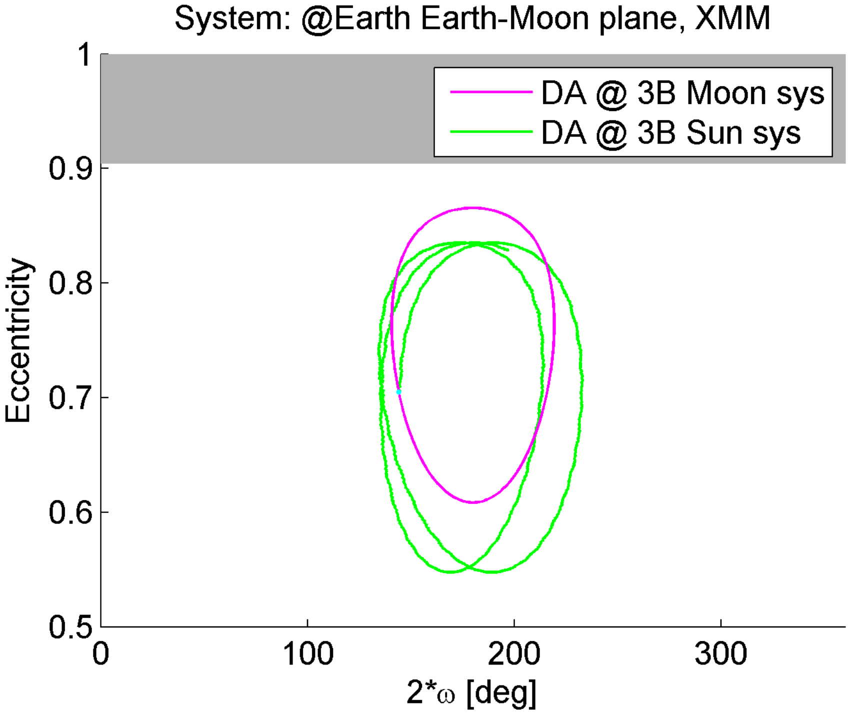
$$\overline{\overline{F}}_{3\text{Bsys},2}\left(e,\omega,i\right)=\frac{1}{32}\left(\left(2+3e^2\right)\left(1+3\cos\left(2i\right)\right)+30e^2\cos\left(2\omega\right)\sin^2 i\right)$$

▶ *El'yasberg, Introduction to the theory of flight of artificial Earth satellites - translated, 1967*



Analytical interpretation

Third-body double averaged potential



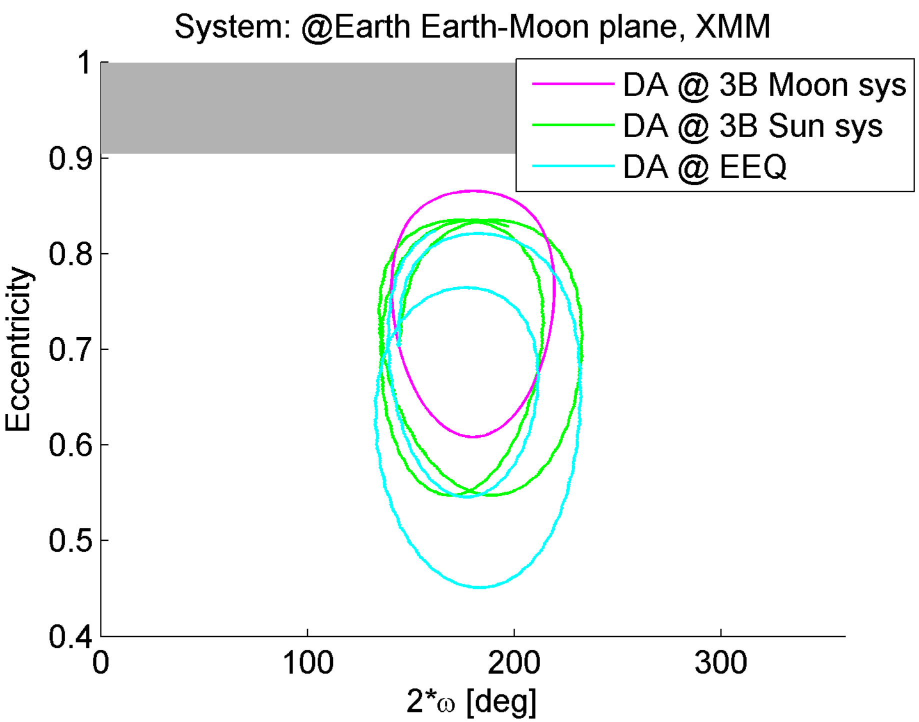
Reference system for figure:

- x-y plane lays on the Moon orbital plane
- z-axis in the direction of the Moon angular momentum

Kozai, El'yasberg: $\bar{\bar{F}}_{3Bsys,2}(e,\omega,i)$

Analytical interpretation

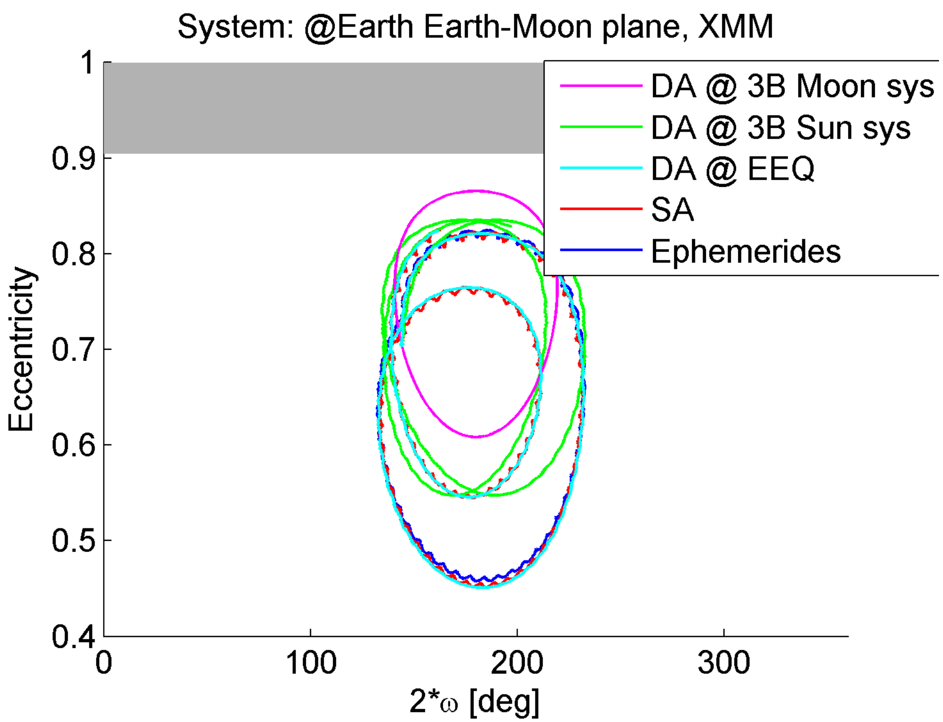
Third-body double averaged potential

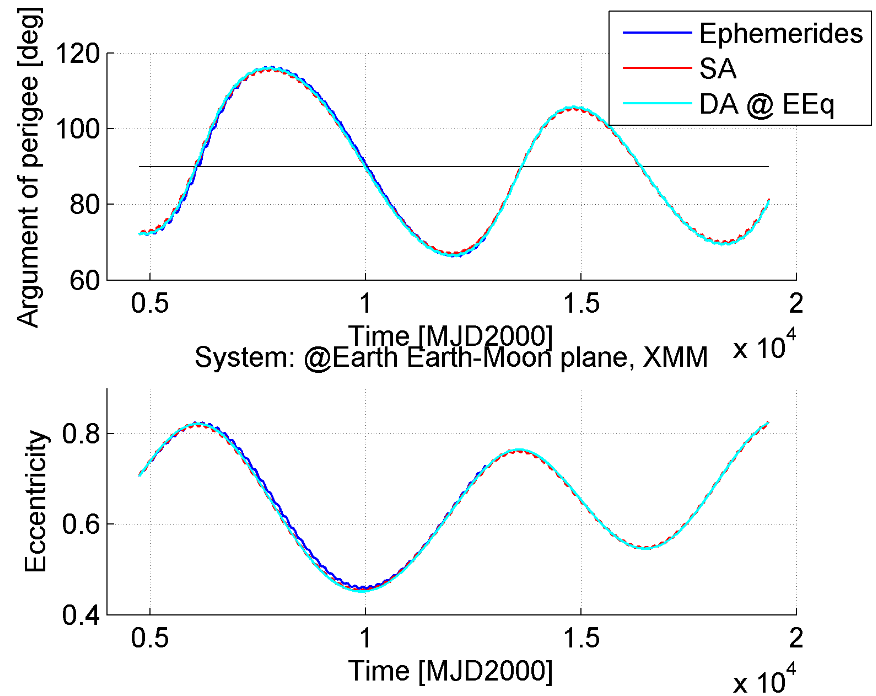


Kozai, El'yasberg: $\bar{\bar{F}}_{3\mathrm{Bsys},2}(e,\omega,i)$ → $\bar{\bar{F}}_k(e,i,\Delta\Omega,\omega,i')$

Analytical interpretation

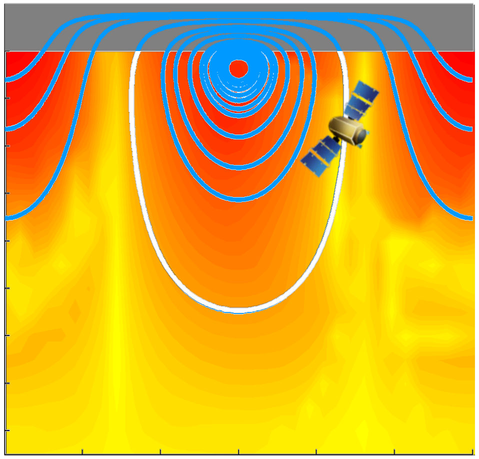
Third-body double averaged potential





Kozai, El'yasberg: $\bar{\bar{F}}_{3\text{Bsys},2}(e,\omega,i)$ $\bar{\bar{F}}_k(e,i,\Delta\Omega,\omega,i')$

Non autonomous loops in the e-ω phase space!



Design of disposal manoeuvres

ENGINEERING PERTURBATION EFFECTS

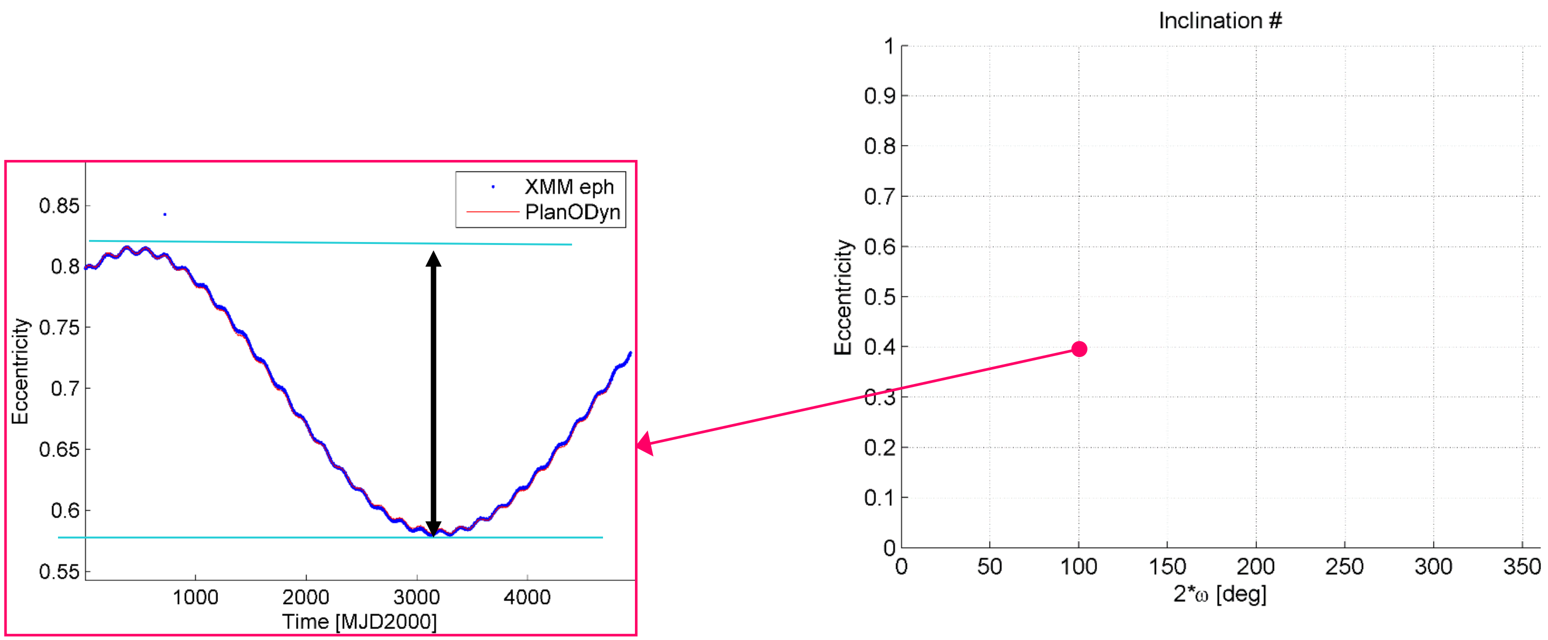
Dynamical maps

Long-term orbit evolution

1. Grid in inclination, eccentricity and ω (Moon plane reference system)
2. Propagation over ±30 years with PlanODyn
3. Evaluate $\Delta e = e_{max} - e_{min}$

$$e_{max} = \max_t e(t) \quad t \in \left[-\Delta t_{graveyard} \quad +\Delta t_{graveyard}\right]$$

$$e_{min} = \min_t e(t) \quad t \in \left[-\Delta t_{graveyard} \quad +\Delta t_{graveyard}\right]$$



Dynamical maps

Long-term orbit evolution

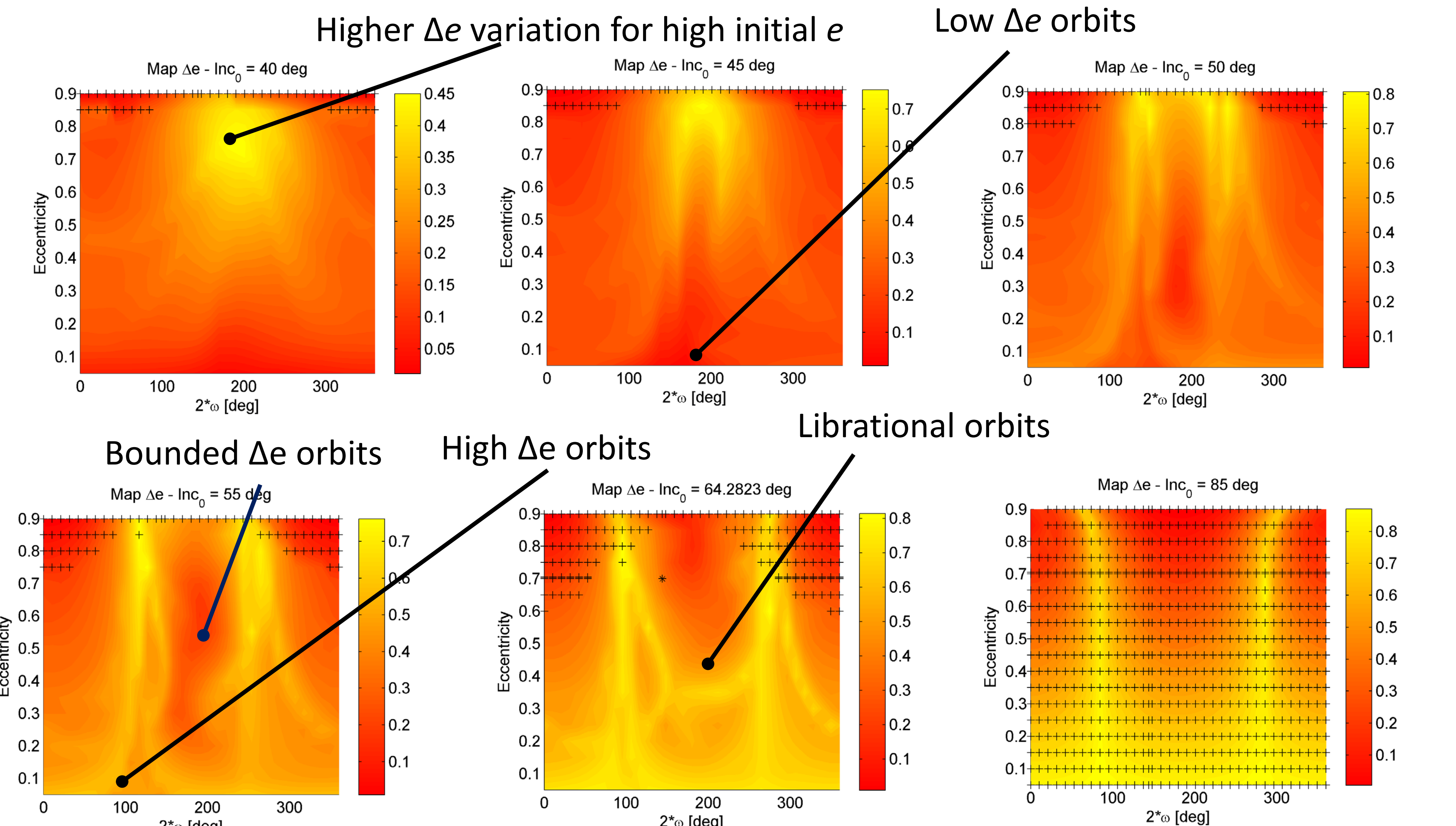
Luni-solar + zonal Δe maps

- Semi-major axis equal to 67045.39 km (XMM Newton's orbit)
- Different values of initial inclination with respect to the orbiting plane of the Moon
- Here: fixed t_0 and fixed Ω_0 to analyse one loop in the phase space but different Ω_0 can be taken into account with $2\omega + \Omega_0$

► *"Long-Term Evolution of Highly-Elliptical Orbits: Luni-Solar Perturbation Effects for Stability and Re-Entry," 25th AAS/AIAA Space Flight Mechanics Meeting, 2015*

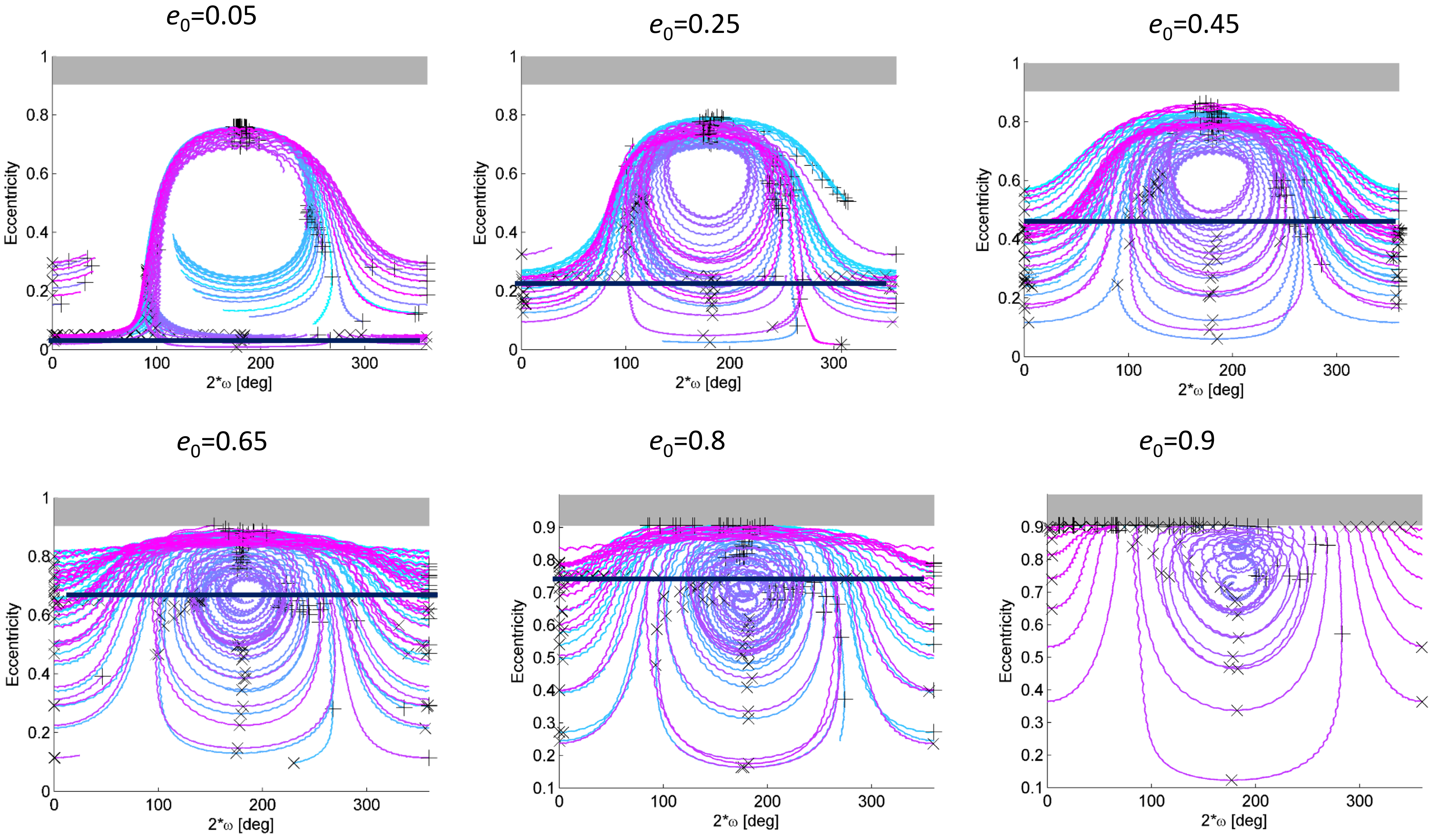
Dynamical maps

Long-term orbit evolution



Dynamical maps

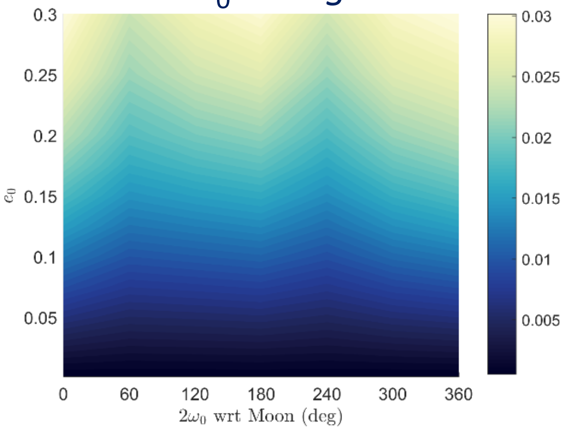
Long-term orbit evolution - Initial inclination 64.28 degrees



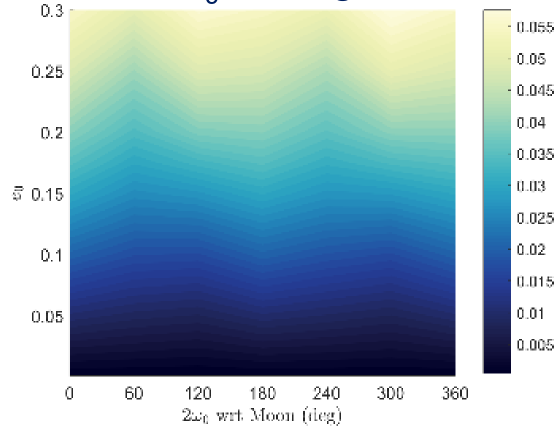
Dynamical maps

Long-term orbit evolution

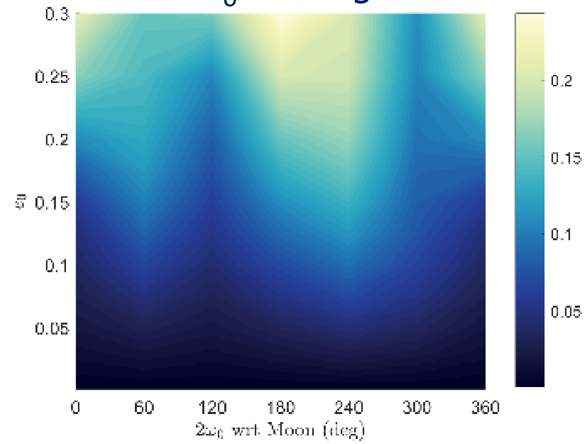
i_0=0 deg



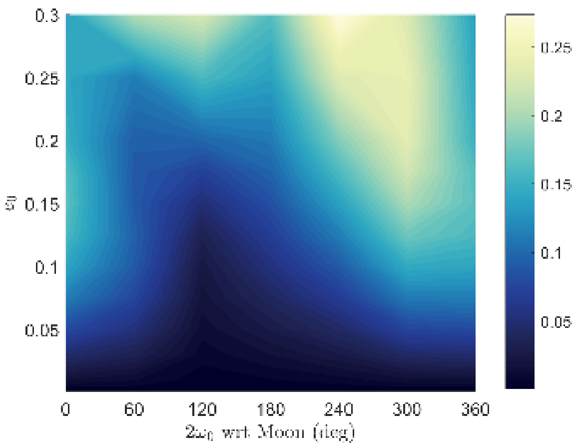
i_0=25 deg



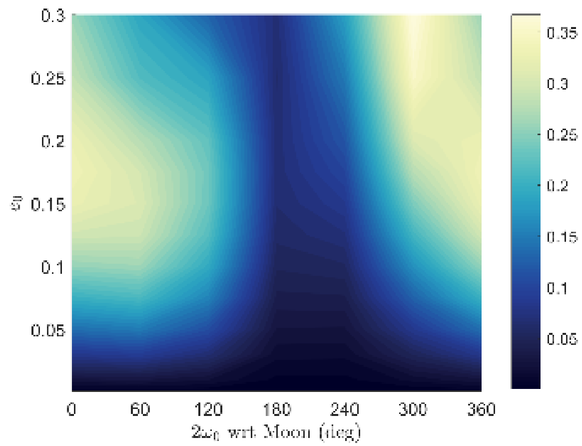
i_0=55 deg



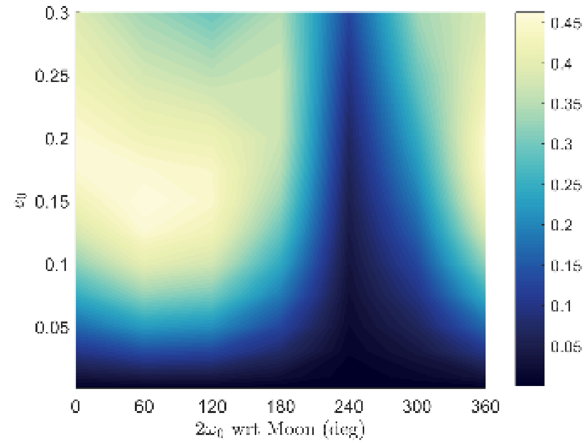
i_0=60 deg



i_0=65 deg



i_0=70 deg



► *Gkolias and Colombo, KePASSA 2017*

Engineering the perturbation effects

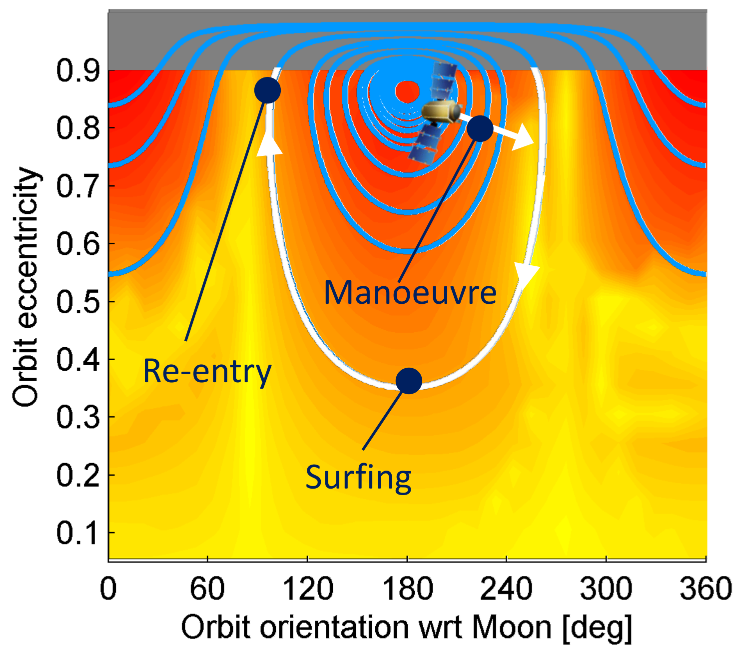
Design disposal manoeuvre in the phase space

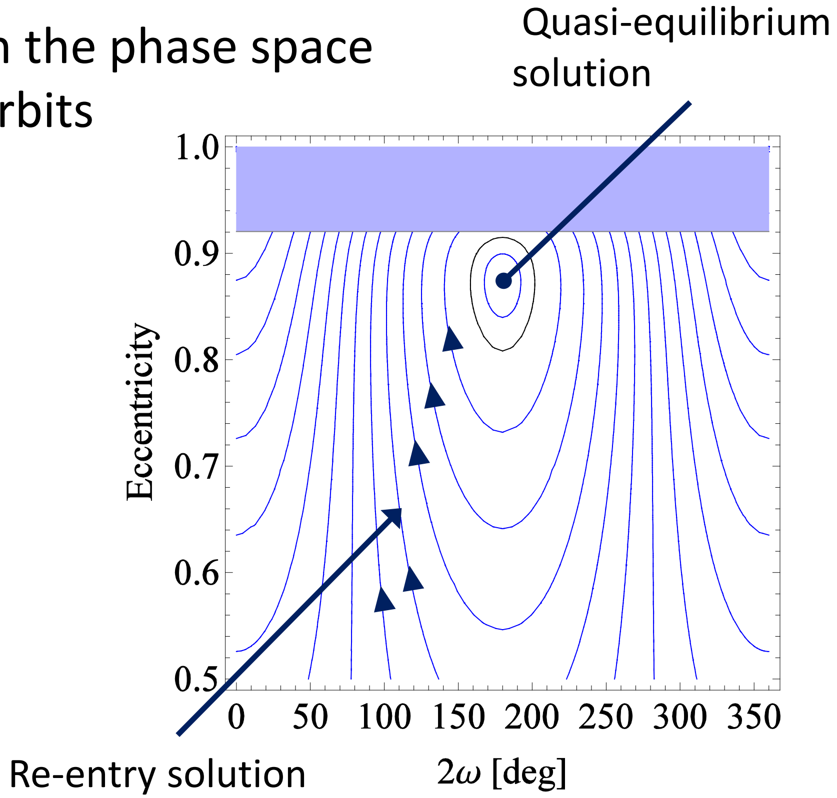
Design manoeuvre in the phase space

- Re-entry transfer on trajectories in the phase space to reach $e_{crit} = 1 - \left(R_{Earth} + h_{p,\,drag}\right)/a$

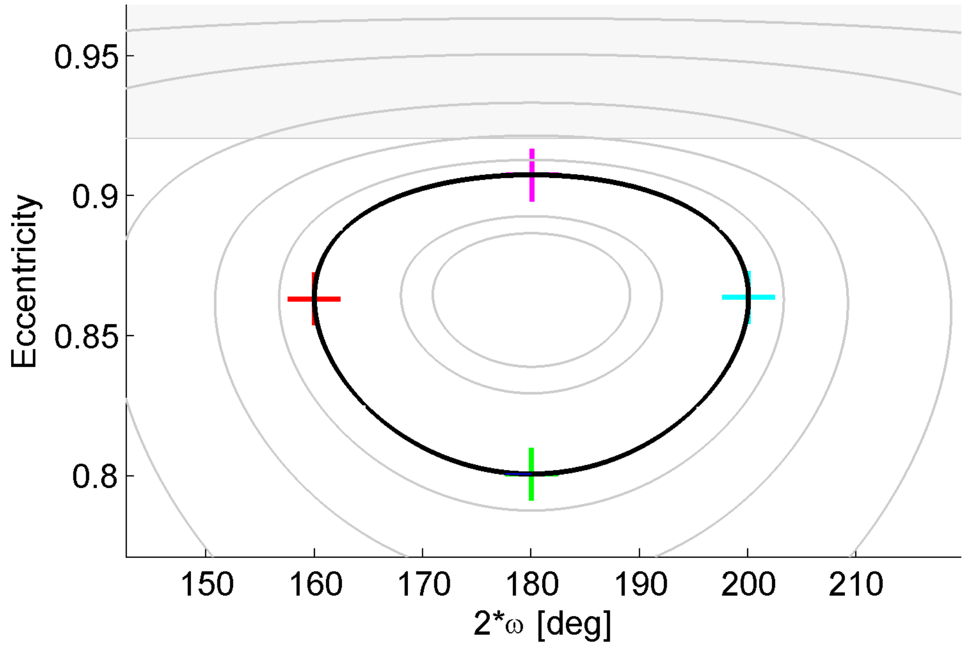
  Maximum Δe exploitable for re-entry or free orbit change
- Graveyard: transfer to quasi-stable point in the phase space

  Bounded Δe for graveyard disposal orbits



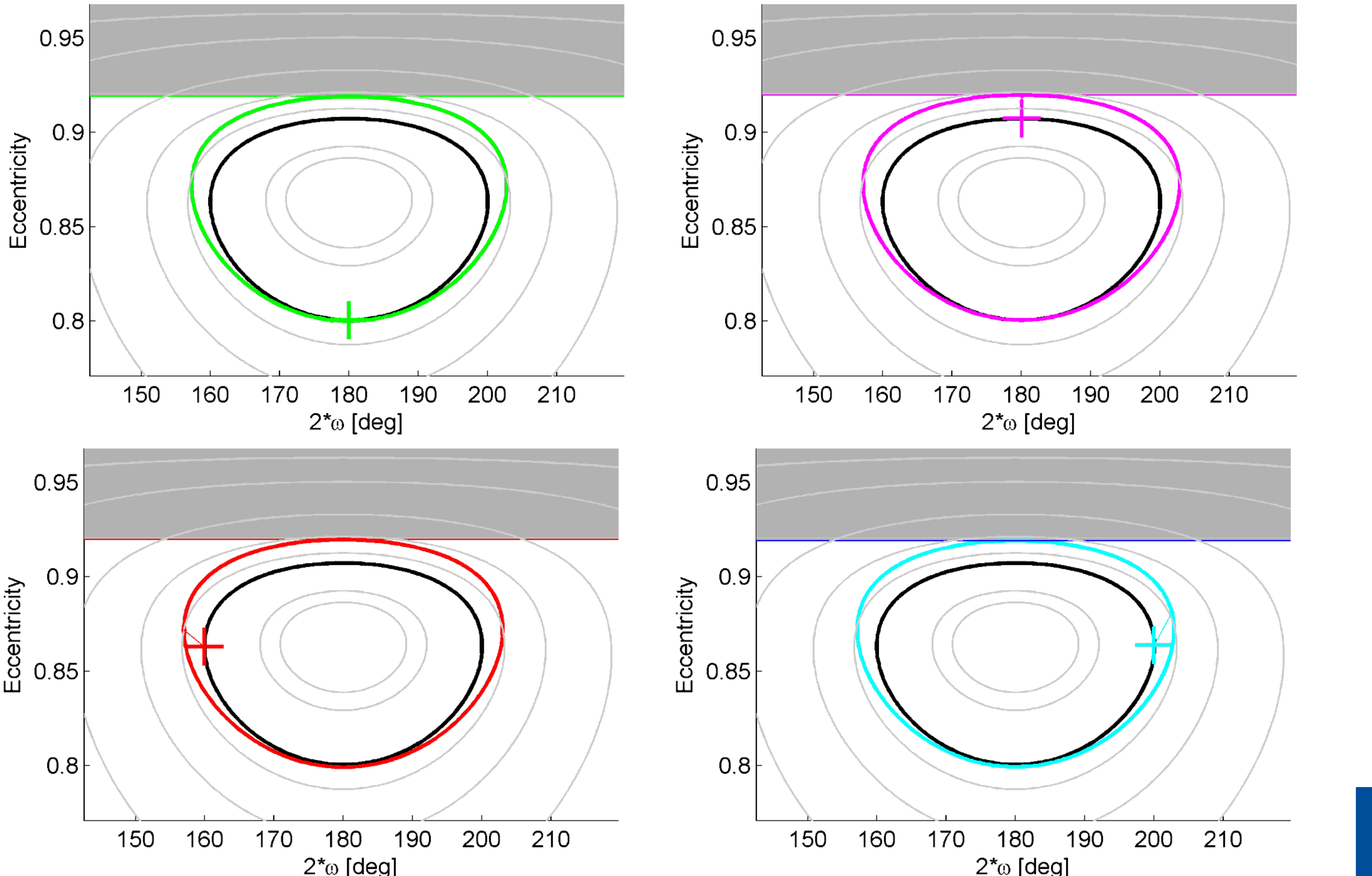


Preliminary analysis Earth re-entry



- Optimisation $\min_{\{\Delta v, \delta, \beta, f\}} \Delta v \qquad C : \max\left[e(t)\right] = e_{\text{crit}}$
- Multi-start method plus local constrained optimisation based on gradient
- Gauss planetary eqs. for finite differences to compute change in orbital elements
- Orbit evolution computed with double average eqs.

Preliminary analysis Earth re-entry



Engineering the perturbation effects

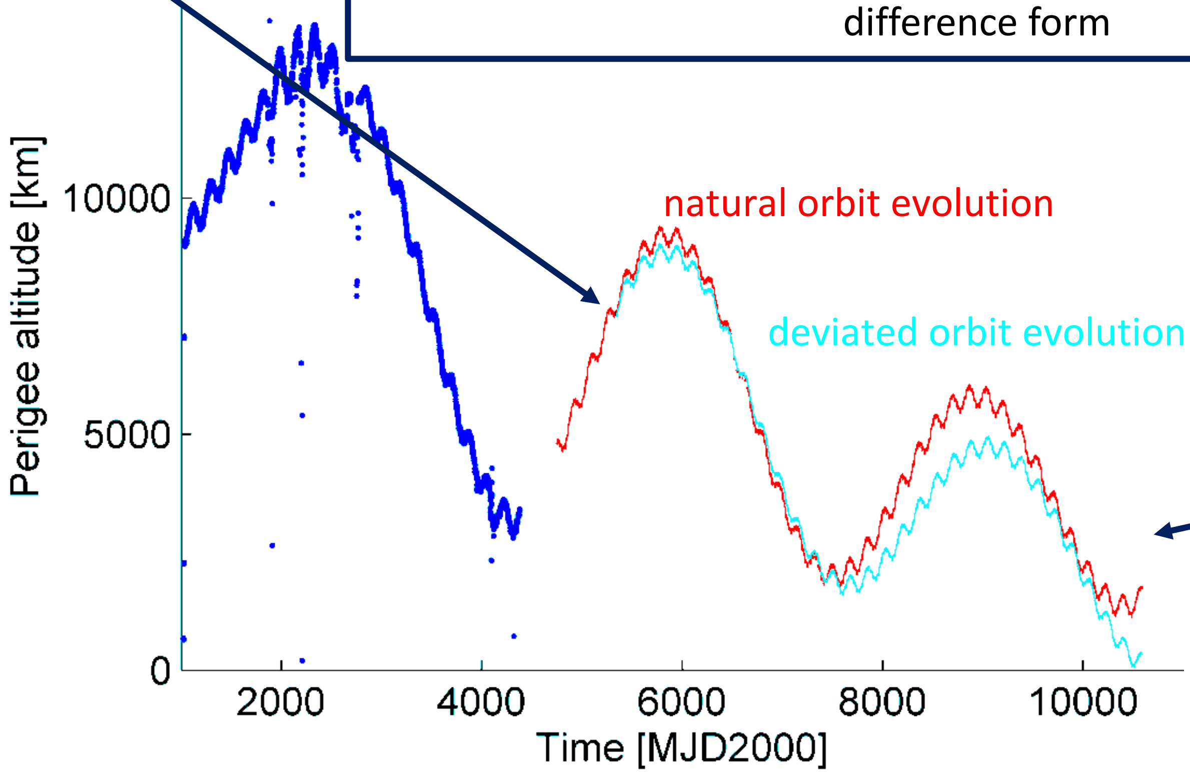
Design disposal manoeuvre in the phase space

$$\Delta \mathbf{v} = \Delta v \begin{bmatrix} \cos\alpha \cos\beta \\ \sin\alpha \cos\beta \\ \sin\beta \end{bmatrix} \Rightarrow \Delta kep = G\left(kep\left(t_m\right), f_m, \Delta \mathbf{v}\right) \Rightarrow kep_d = kep\left(t_m\right) + \Delta kep$$

Gauss' planetary equations in finite-difference form

Deviated condition

Single manoeuvre



Minimum perigee within selected time interval for disposal

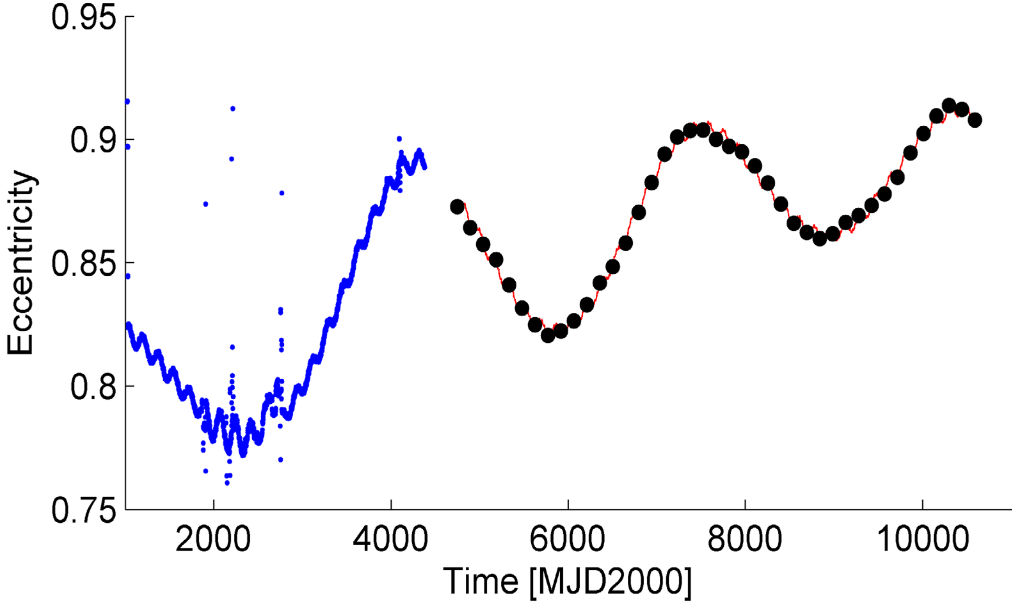
$$h_{p,\min} = \min_{t \in \Delta t_{\text{disposal}}} h_p\left(t\right)$$

► *Colombo, Letizia, Alessi, Landgraf, 24th AAS/AIAA 2014*

Applications

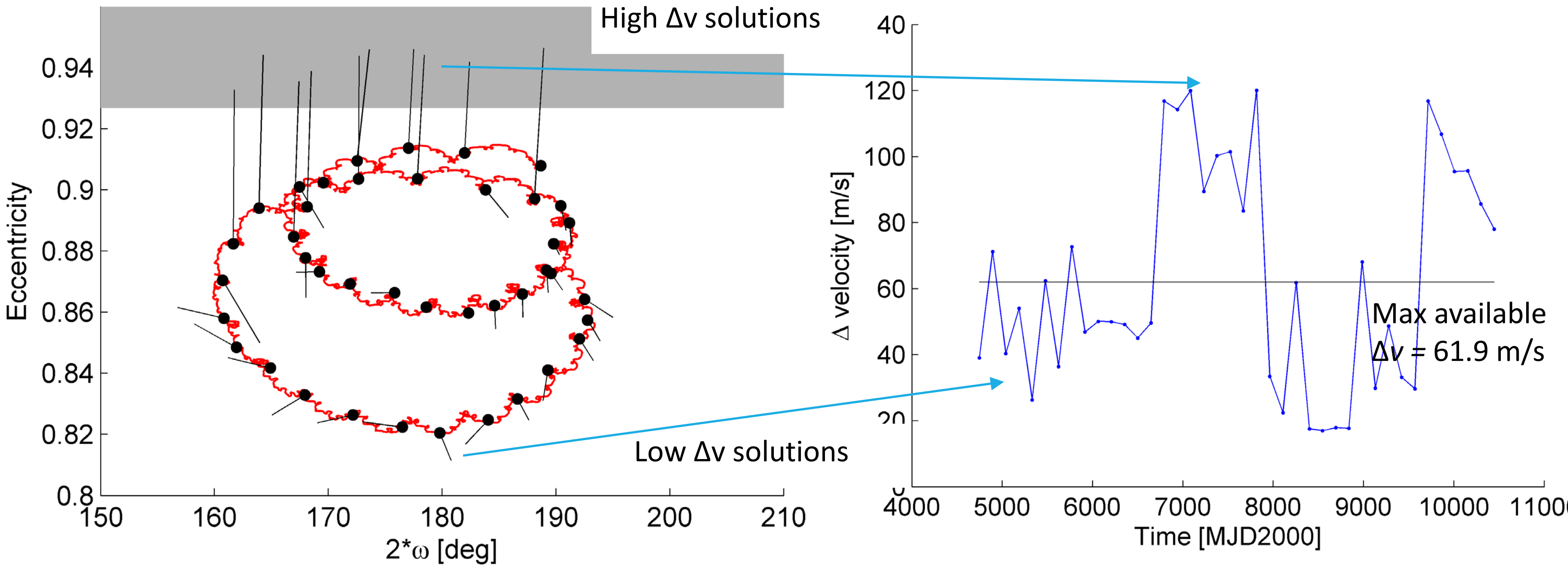
Design disposal manoeuvre in the phase space

- Only 5 Keplerian elements are propagated: a, e, i, Ω, ω
- Optimal true anomaly f_M where the manoeuvre is applied is selected through optimisation
- Dynamics of the mean/true anomaly is much faster than the evolution
- Single manoeuvre considered at different dates within a wide disposal window [2013/01/01 to 2029/01/01]



Applications

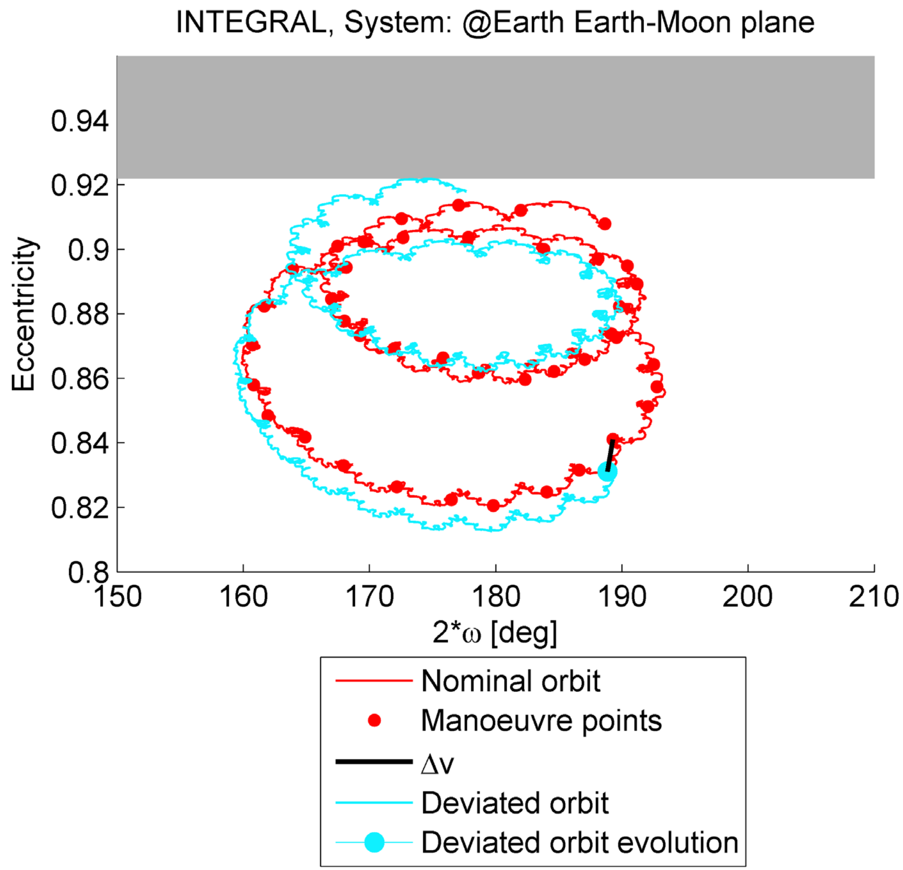
INTEGRAL re-entry

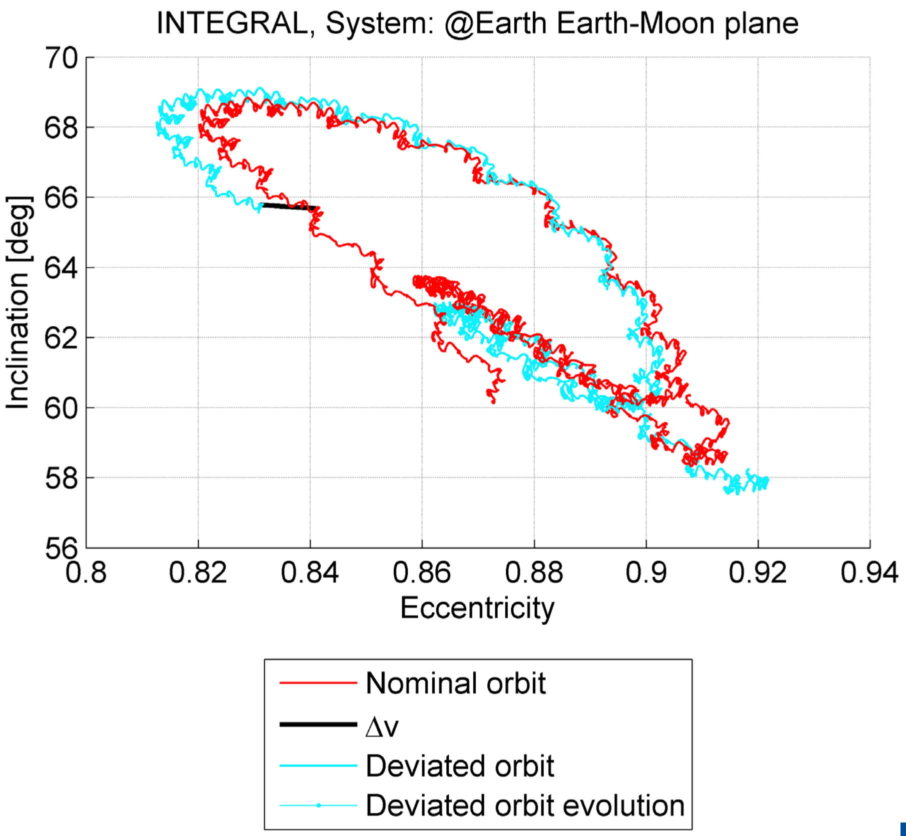


Applications

INTEGRAL re-entry

Example: manoeuvre performed on 08/08/2014





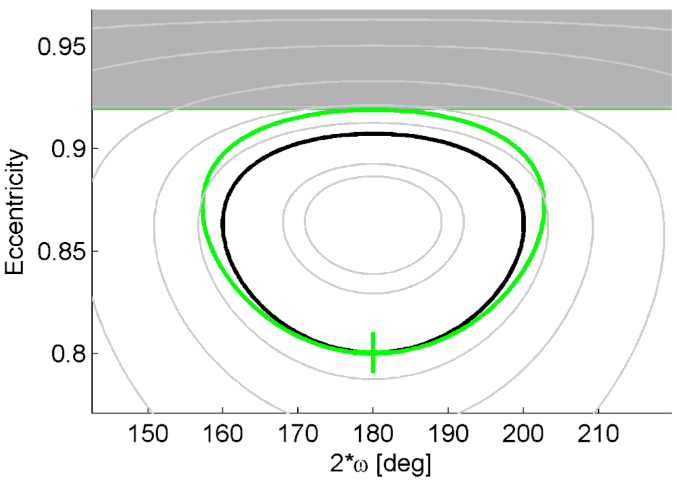
Applications

Re-entry manoeuvre

Preliminary mission design

Moon effect only

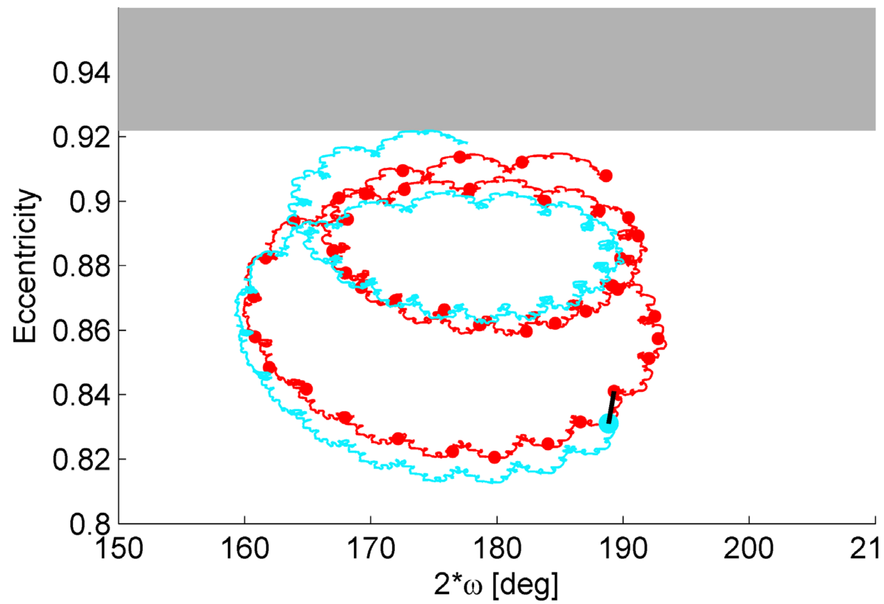
Double averaged potential



Optimised solution

Moon + Sun + J2:

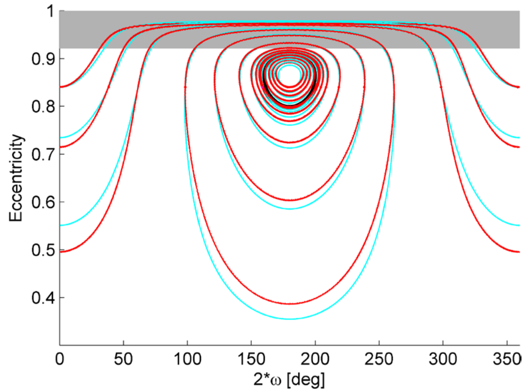
Single averaged dynamics + global optimisation

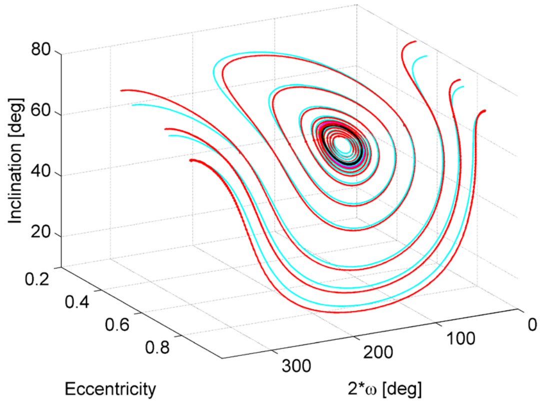


Extending the applicability

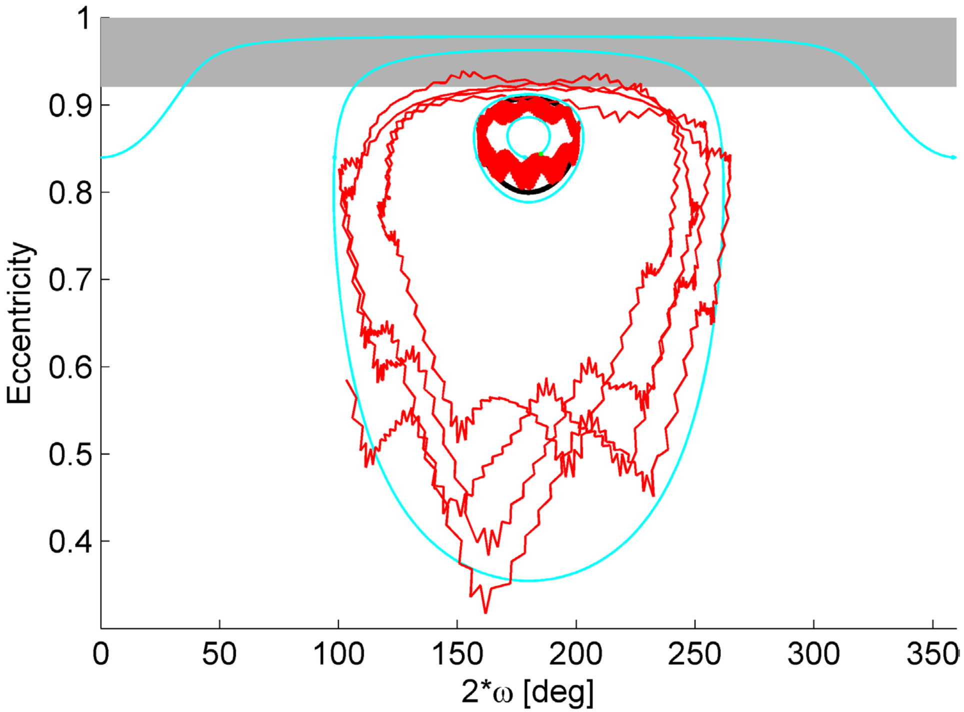
Region of validity

a = 87,736 km





a = 200,000 km



Extending the applicability

Proposed method

Under development

$$\overline{R}_{3B}(r,r')=\frac{\mu'}{r'}\left(\sum_{k=2}^{\infty}\delta^k\overline{F}_{k,0}(A,B,e)+\sum_{k=2}^{5}\delta^k\overline{F}_{k,1}(A,B,e)\frac{n'}{n}\right)$$

Term due to the motion of the perturbing body during averaging

$$A\simeq A_0+\frac{\partial A}{\partial f'}\frac{n'}{n}(M-M_0)=A_0+\frac{1}{n'}\hat{\mathbf{P}}\frac{d\mathbf{r'}}{dt}\frac{n'}{n}(M-M_0)$$

For very high *a* is not enough to assume that orbital elements remain constant during average so need to include coupling between short period fluctuation of orbital elements with the short periodic part of the disturbing function

CONCLUSIONS

Conclusions

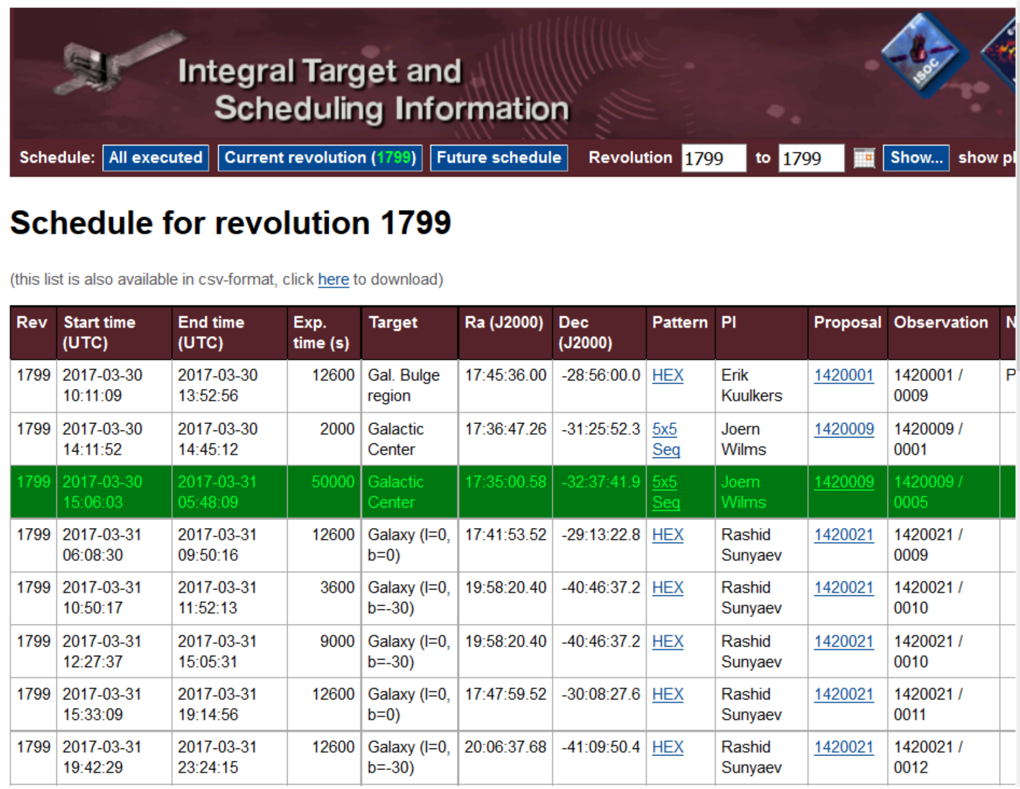
- Effect of luni-solar perturbations and the Earth's oblateness on the stability of MEO, GEO and HEOs
- Natural orbital dynamics can be exploited and enhanced
- INTEGRAL is the demonstration in Space!

INTEGRAL REVOLUTION

1799

INTEGRAL CURRENT TARGET

Galactic Center



Schedule for revolution 1799

(this list is also available in csv-format, click here to download)

Rev	Start time (UTC)	End time (UTC)	Exp. time (s)	Target	Ra (J2000)	Dec (J2000)	Pattern	PI	Proposal	Observation	N
1799	2017-03-30 10:11:09	2017-03-30 13:52:56	12600	Gal. Bulge region	17:45:36.00	-28:56:00.0	HEX	Erik Kuulkers	1420001	1420001 / 0009	P
1799	2017-03-30 14:11:52	2017-03-30 14:45:12	2000	Galactic Center	17:36:47.26	-31:25:52.3	5x5 Seq	Joern Wilms	1420009	1420009 / 0001	
1799	2017-03-30 15:06:03	2017-03-31 05:48:09	50000	Galactic Center	17:35:00.58	-32:37:41.9	5x5 Seq	Joern Wilms	1420009	1420009 / 0005	
1799	2017-03-31 06:08:30	2017-03-31 09:50:16	12600	Galaxy (l=0, b=0)	17:41:53.52	-29:13:22.8	HEX	Rashid Sunyaev	1420021	1420021 / 0009	
1799	2017-03-31 10:50:17	2017-03-31 11:52:13	3600	Galaxy (l=0, b=-30)	19:58:20.40	-40:46:37.2	HEX	Rashid Sunyaev	1420021	1420021 / 0010	
1799	2017-03-31 12:27:37	2017-03-31 15:05:31	9000	Galaxy (l=0, b=-30)	19:58:20.40	-40:46:37.2	HEX	Rashid Sunyaev	1420021	1420021 / 0010	
1799	2017-03-31 15:33:09	2017-03-31 19:14:56	12600	Galaxy (l=0, b=0)	17:47:59.52	-30:08:27.6	HEX	Rashid Sunyaev	1420021	1420021 / 0011	
1799	2017-03-31 19:42:29	2017-03-31 23:24:15	12600	Galaxy (l=0, b=-30)	20:06:37.68	-41:09:50.4	HEX	Rashid Sunyaev	1420021	1420021 / 0012	

POLITECNICO
MILANO 1863

This project has received funding from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation programme (grant agreement No 679086 – COMPASS)

Third body effect in PlanODyn
Luni-solar perturbations for missions design

Camilla Colombo, Politecnico di Milano
camilla.colombo@polimi.it