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AUTOMATED MODAL IDENTIFICATION OF A HISTORIC BELL-TOWER

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Abstract. *Continuous monitoring of the structural response under ambient excitation is especially suitable to Cultural Heritage structures because of the fully non-destructive and sustainable way of testing, that is performed by just measuring the dynamic response under ambient excitation and does not involve additional loads rather than those associated to normal operational conditions. Within the context of vibration-based monitoring of historic masonry structures, the paper presents the development of an automated procedure of modal parameters estimation and tracking, as well as its application in the continuous dynamic monitoring of a masonry bell-tower.*

The proposed algorithm of modal parameters estimation is based on the interpretation of the stabilization diagram associated to parametric identification methods and consists of three key steps aimed at: (1) filtering a high number of spurious poles in the stabilization diagram; (2) clustering the stabilization diagram; (3) improving the accuracy of the estimates. The developed procedure is exemplified using the data collected on the bell-tower of the church of San Gottardo in Corte in Milan. Following the practice adopted by the authors for masonry towers, a simple dynamic monitoring system is installed in the tower: the monitoring system includes two bi-axial seismometers (electro-dynamic velocity transducers), one 24-bit digitizer (6 channels, ΣΔ A/D converter, 8 Gb Ram on board for data storage) and one UMTS modem for data transfer.

After a concise presentation of the developed automated algorithm, the paper focuses on the results obtained in several months of continuous monitoring of the tower.

1 INTRODUCTION

Structural Health Monitoring (SHM) [1] is generally defined as a multi-disciplinary process involving: (a) the repeated or continuous measurement of the response of a structural system through arrays of appropriated sensors; (b) the extraction from measured data of features, which are representative of the health condition and (c) the statistical analysis of these features to detect any novelty or abnormal change in the investigated system. Among the different SHM approaches, the one based on vibration monitoring and automated operational modal analysis (OMA, i.e. the identification of modal parameters from output-only measurements) has received increasing attention in the field of Civil Engineering and many dynamic monitoring systems have been implemented in structures and infrastructures (see e.g. [2]), all over the world.

Despite the minimum impact of vibration-based SHM, that makes this methodology especially suitable to the context of Cultural Heritage (CH) preservation, the practical applications in this field are still not frequent [3-10] and generally based on the use of a limited number of sensors. The installation of a few sensors to assist the preservation and perform the SHM of CH structures has recently revealed very promising for ancient towers [4, 9]. On the other hand, even if the advances in the field of OMA have led to the developments of robust algorithms [11-13], capable of automatically identifying the modal parameters from continuously collected time-series, the availability of only a few channels of data makes not easy the processing and the accurate identification and tracking of modal parameters, especially when closely-spaced frequencies with rather similar mode shapes are met.

This paper focuses on the development of an efficient approach aimed at automatically extracting the modal parameters from continuously collected data and at providing the evolution in time of each identified resonant frequency. The modal parameters estimation is performed through an automated interpretation of stabilisation diagrams [14-15] so that it is suitable to be coupled with any parametric method involving the construction of stabilization diagrams. The modal tracking task is based on the definition of: (a) a pre-selected list of baseline modes with adaptive thresholds and (b) a dynamic list of modes associated to fixed thresholds.

After a concise description (section 2) of the developed algorithm, its application is exemplified in the paper by applying the covariance-driven Stochastic Subspace Identification [14-15] to data collected on the San Gottardo in Corte bell-tower in Milan (section 3). Subsequently, the main dynamic characteristics of the tower obtained by one year of permanent monitoring are presented and the correlation between the identified natural frequencies and the environmental parameters is discussed (sections 4 and 5).

2 THE AUTOMATED MODAL IDENTIFICATION AND TRACKING

The vibration data measured in operational conditions have been processed using the covariance-driven Stochastic Subspace Identification (SSI-Cov) [14-15] technique in the time domain. According to this approach, the discrete-time state-space model of a linear-time-invariant system under unknown excitation is considered:

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \boldsymbol{A}\,\boldsymbol{x}_k + \boldsymbol{w}_k \\ \boldsymbol{y}_k &= \boldsymbol{C}\,\boldsymbol{x}_k + \boldsymbol{v}_k \end{aligned} \tag{1}$$

where k is the generic time instant, $\boldsymbol{A}$ is the state matrix, $\boldsymbol{C}$ is the output matrix, $\boldsymbol{x}_k$ is the state vector (i.e. the vector describing the state of the system at time k), $\boldsymbol{y}_k$ collects the measured outputs and $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ represent the process noise and the output noise, respectively. When the ambient excitation (included in $\boldsymbol{w}_k$) is assumed as a white noise (i.e., its spectrum has no dominant frequency components), both $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ are modeled as white noise random processes.

The core of the SSI-Cov [14-15] technique is the identification of the system matrices $\boldsymbol{A}$ and $\boldsymbol{C}$ from the covariance matrix of the measured output $\boldsymbol{y}_k$. Once the estimation of the system matrices has been carried out, the eigenvalue decomposition of $\boldsymbol{A}$ allows to extract the modal parameters. Since the order n of the matrix $\boldsymbol{A}$ equals twice the number of the structural modes (i.e., a system with a model order n has $n/2$ modes), usually the system matrices $\boldsymbol{A}$ and $\boldsymbol{C}$ are estimated for many model orders and the corresponding modal characteristics are calculated and reported in frequency-order plot and/or damping-order plot. Those plots are called stabilization diagrams [16] and the physical modes of the system should conceivably appear at many orders as vertical alignments of stable modes. On the other hand, it is often necessary to consider much larger values of n than what would be physically sufficient and the over-modeling results in the appearance of spurious modes associated to the noise content of the measurements. Hence, clustering techniques (i.e., approaches based on grouping modes on the basis of the consistency of their properties) have became popular tools to distinguish physical modes from spurious ones and to automate the interpretation of the stabilization diagrams [11-13].

It is further noticed that vibration-based continuous monitoring generally involves the application to each collected dataset of two tasks: the modal parameters estimation (MPE), consisting of the modal parameters identification from a single record of measured data, and the modal tracking (MT), i.e. the tracking the evolution in time of the modal parameters resulting from MPE performed on a large number of acquired datasets.

The present paper focuses on both MPE and MT tasks and a brief description of the developed algorithms is presented in the next sub-sections.

2.1 Modal parameters estimation

The MPE procedure herein presented consists of the sequential application of three steps: (a) pre-filtering; (b) clustering and (c) post-processing. The phases of pre-filtering and clustering are based on previously developed algorithms [11-13], whereas the post-processing, aimed at improving the accuracy of the modal estimates, is one distinctive aspect of the proposed procedures.

The pre-filtering is aimed at detecting the certainly spurious modes, that are removed from the stabilization diagram, through single-mode validation checks. The validation checks are based on the physical consistency of damping ratios and the complexity of mode shapes [11-13]. As usual, modes associated to negative damping ratio or high damping (i.e., damping exceeding a 10% threshold, which seems a conservative value for civil engineering structures under ambient or operational excitations) have been considered as certainly spurious [11-13]. Subsequently, the complexity degree of mode shape components has been evaluated using the Modal Phase Collinearity [12-13, 16] (MPC, which represents a measure of the correlation between the imaginary and real part of the mode shape components and tends to unity for real modes) and the Mean Phase Deviation [12-13] (MPD, which measures the phase angle of the mode shape vector in the complex plane and tends to 0 for real modes). As noted by Cabboi et al. [13], the indications given by the MPC and MPD indices are not completely consistent: consequently, threshold values can be defined for both MPC and MPD and the modes exceeding these thresholds are classified as spurious modes and filtered out.

The second step consists of a clustering procedure, ideally corresponding to the inspection of the stabilization diagram carried out by an expert user in a manual approach to identify the alignments of stable poles. Hence, the objective of clustering is to detect and group all the poles sharing the same characteristics in terms of modal parameters. In the context of OMA, the most popular way to group stable poles is to check the similarity of different poles, in terms of distances between natural frequencies and mode shapes, through the metric introduced by Magalhães et al. [11]:

$$d_{I-ref,j} = d^{f}_{I-ref,j} + d^{MAC}_{I-ref,j} = \frac{\left| f_{I-ref} - f_j \right|}{f_{I-ref}} + \left[1 - \frac{\left| \boldsymbol{\Phi}^H_{I-ref} \boldsymbol{\Phi}_j \right|^2}{(\boldsymbol{\Phi}^H_{I-ref} \boldsymbol{\Phi}_{I-ref})(\boldsymbol{\Phi}^H_j \boldsymbol{\Phi}_j)} \right] \tag{2}$$

where, as suggested in [13], each cluster I is described by reference values of frequency $f_{\text{I-ref}}$ and modal vector $\boldsymbol{\Phi}_{\text{I-ref}}$; the reference values are updated after one element is added to the cluster and are used to evaluate the similarity of any new candidate pole (described through the frequency f_{j} and the mode shape $\boldsymbol{\Phi}_{\text{j}}$). When all the poles have been considered and grouped into clusters, the clusters with a number of elements less than or equal to one third [17] of the number of elements present in the larger cluster are considered as noise modes and removed from the stabilization diagram.

As previously pointed out, the MPE is completed by a post-processing phase, aimed at improving the accuracy of the modal estimates and consisting of three steps:

a) Repetition of the clustering procedure by using the final references characterizing each cluster, so that possible inaccuracies (such as lost of poles belonging to the cluster or, conversely, outliers fallen in the cluster) occurred at the beginning of the cluster generation are amended. It should be noticed that, since the references of each cluster have been already defined, the computational cost of the second clustering is relatively limited;
b) Checking any possible replication of the same mode shape in the stabilization diagram;
c) Detection (and removal) of the outliers affecting the modal damping ratios through the application of a simple statistical tool based on the box-plot rule.

The final outputs of the MPE are the mean values of the modal estimates (mean natural frequency, mean mode shape and median modal damping ratio) belonging to each cluster. The performance of the MPE procedure is exemplified in Fig. 1, by using one dataset collected in the continuous dynamic monitoring of the San Gottardo in Corte bell-tower.

2.2 Modal tracking

After the automated MPE, the MT allows to track the evolution in time of natural frequencies and other modal parameters and is based on the choice of a baseline list of the parameters to be tracked. Although MT represents the very initial step of any OMA-based strategy of SHM, a few strategies have been developed in the literature to accurately manage the MT phase. A common approach adopted to link the currently identified modes to the baseline ones consists of checking the similarity in terms of natural frequencies and mode shapes (through the MAC index). As shown in [11], this approach might fail when the modal parameters exhibit high fluctuation, that depends on the variation of environmental and operational conditions: this issue is easily solved by manually adjusting some thresholds but requires an action from the user. A good solution to this issue was proposed in [13] and is based on the automatic definition of adaptive rejection distances, or "distance thresholds", between the current modes and the baseline ones. On the other hand, even the use of adaptive thresholds might lead to drawbacks when a limited number of sensors is available: in this case a sufficient discrimination between the different modes might not be guaranteed, especially for closely-spaced modes with relatively similar modal components.

In order to make more robust the procedure introduced in [13], the MT procedure herein presented involves the use of: (a) a static list of reference modes, associated with adaptive rejection thresholds and (b) a dynamic list of reference modes, associated to fixed thresholds.

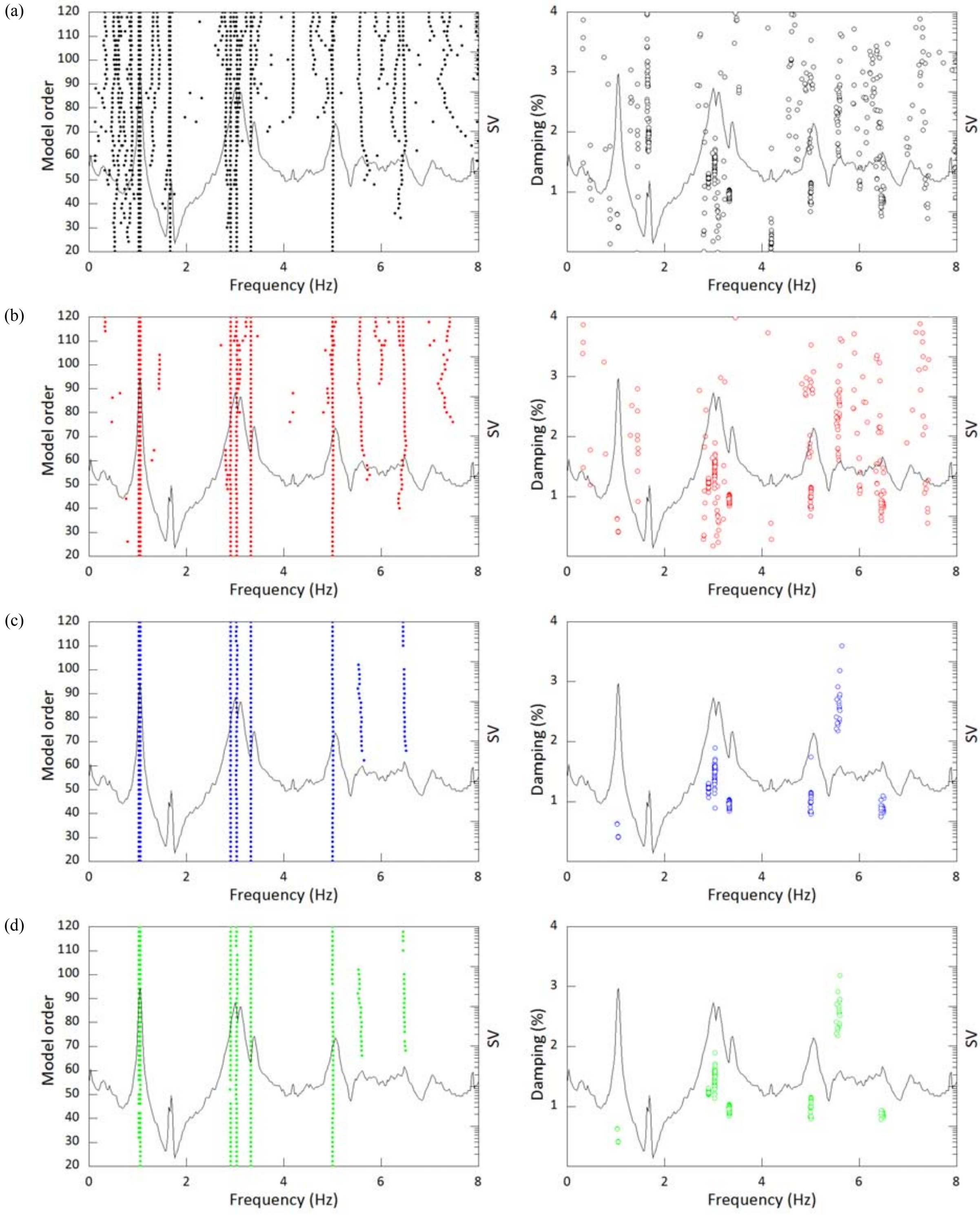

Figure 1: Stabilization diagrams (San Gottardo bell-tower): (a) resulting from SSI-Cov; (b) after the pre-filtering; (c) after the clustering; (d) after the post-processing to improve the estimate accuracy (final results).

Two adaptive thresholds are defined, and denoted to as $d^{f}_{i,max}$ and $d^{MAC}_{i,max}$, to represent the rejection distance from the reference of *i*-th natural frequency and *i*-th mode shape, respectively. Since the storage of a minimum number *N* of structural modes is needed to define the adaptive thresholds, these features are fully active after an initial training period, whereas during the training period pre-selected values of $d^{f}_{i,max}$ and $d^{MAC}_{i,max}$ are assumed. After the *i*-th mode has been tracked N times, the adaptive thresholds are defined (and subsequently updated by using the last available N values) according to the following:

$$d^{\mathrm{f}}_{i,\mathbf{max}} = \sqrt{\mathrm{std}(d^{\mathrm{f}}_{i-ref,j})} \qquad d^{\mathrm{MAC}}_{i,\mathbf{max}} = \sqrt{\mathrm{std}(d^{\mathrm{MAC}}_{i-ref,j})} \tag{3}$$

where $\mathrm{std}(d^{\mathrm{f}}_{i\text{-}ref,j})$ and $\mathrm{std}(d^{\mathrm{MAC}}_{i\text{-}ref,j})$ are the standard deviations of the distance − computed in terms of natural frequency and MAC, respectively − between the j-th estimate (j=1, …, N) of the i-th mode and its preselected reference.

It is worth noting that the tracking regions defined by the adaptive thresholds (3) could overlap for closely-spaced modes with relatively similar modal components so that an inaccurate association might arise. To avoid this possibility, the association between each candidate mode and the static list of references is completed when a positive assessment, within preselected thresholds d^{f}_{dyn} and d^{MAC}_{dyn}, is obtained also by comparing the candidate mode with the last available linked mode. Therefore, the series of linked modes preceding the current one has the role of dynamic list of reference modes and progressively changes as the MT process runs.

Following this strategy, the developed MT procedure complies with three important aspects: 1) accounting for the changes of modal parameters induced by environmental and operational conditions, 2) producing an accurate evolution of the structural modes even in case of closely-spaced modes with similar mode shapes, and 3) conceivably reducing the number of outliers during the tracking process.

It is worth mentioning that the application of the tracking procedure to the tower investigated in this paper provided the following indications:

a) A training period of 15 days turned out to be sufficient for the definition of the adaptive thresholds (3), with the maximum value N of the estimates used to define those thresholds being equal to 360 (since each dataset is 1-hour long);
b) The thresholds d^{f}_{dyn} and d^{MAC}_{dyn} (associated to the dynamic list of reference modes) are set equal to 0.02 and 0.90, respectively.

3 DESCRIPTION OF THE SAN GOTTARDO BELL-TOWER

The church of San Gottardo in Corte (Milan, Italy), placed in the close neighborhood of the Milan Cathedral, was completed in 1336 and originally it was a chapel attached to the residence of the Duke of Milan. The building was initially dedicated to the Virgin, to whom the Milanese were very devoted at the construction period, and subsequently to San Gottardo.

The church construction included a slender bell-tower (Fig. 2) and the architect responsible for the tower was Francesco Pegorari, as it is testified by a stone at the base of the building. The tower exhibits a fairly good structural condition and the last reported restoration works date back to 1887 and 1925 (and were committed to the architects Beltrami and Calzecchi-Onesti, respectively).

The bell-tower (Fig. 2) is about 54.0 m high and consists of a stone masonry square basement of 12.0 m, an octagonal portion in solid brick masonry until the height of 41.0 m and a high cusp. It should be noticed that, although the tower is permanently instrumented since November 2016, a comprehensive geometric survey of the structure is not yet available.

4 DYNAMIC BEHAVIOUR OF THE TOWER AND MONITORING SYSTEM

The monitoring system installed in the San Gottardo tower includes two bi-axial seismometers (electro-dynamic velocity transducers), one 24-bit digitizer (6 channels, ΣΔ A/D converter, 8 Gb Ram on board for data storage) and one UMTS modem for data transfer.

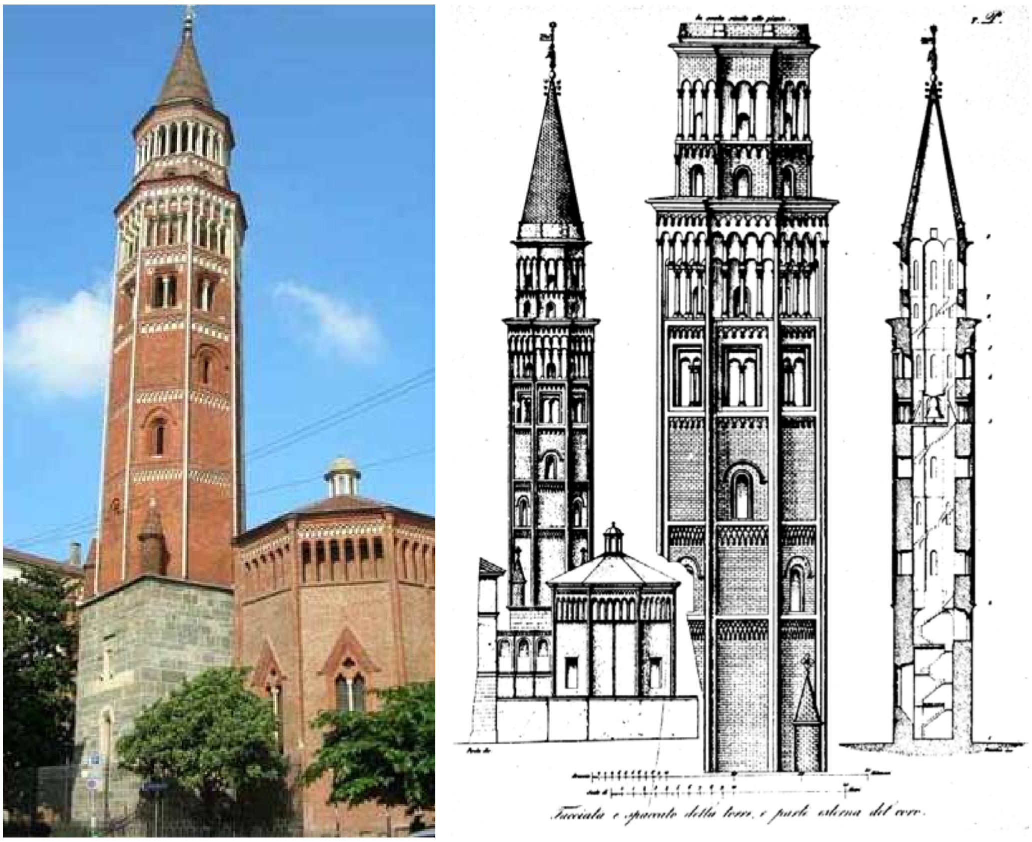

Figure 2: Bell-tower of the church of San Gottardo in Corte, Milan: view and drawings of the building.

Figure 3: Measurement devices installed in the San Gottardo bell-tower.

Since the tower exhibits only one wooden floor, at about 32.0 m, corresponding to the level of the bell chamber, the measurement devices (Fig. 3) were all installed at that level.

The automated modal identification was performed using time windows of 3600s (corresponding to more than 3500 times the fundamental period of the tower), in order to comply with the widely agreed recommendation of using an appropriate duration of the acquired time window to obtain accurate estimates from output-only data. The sampling frequency was 100 Hz, which is much higher than that required for the investigated structure, as the significant frequency content of signals is below 12 Hz. Hence, low pass filtering and decimation were applied to the data before the use of the identification tools. In more details, after low-pass filtering the data through a 7th order Butterworth filter with cut-off frequency of 12.5 Hz, the velocity time series were down-sampled from 100 Hz to 25 Hz.

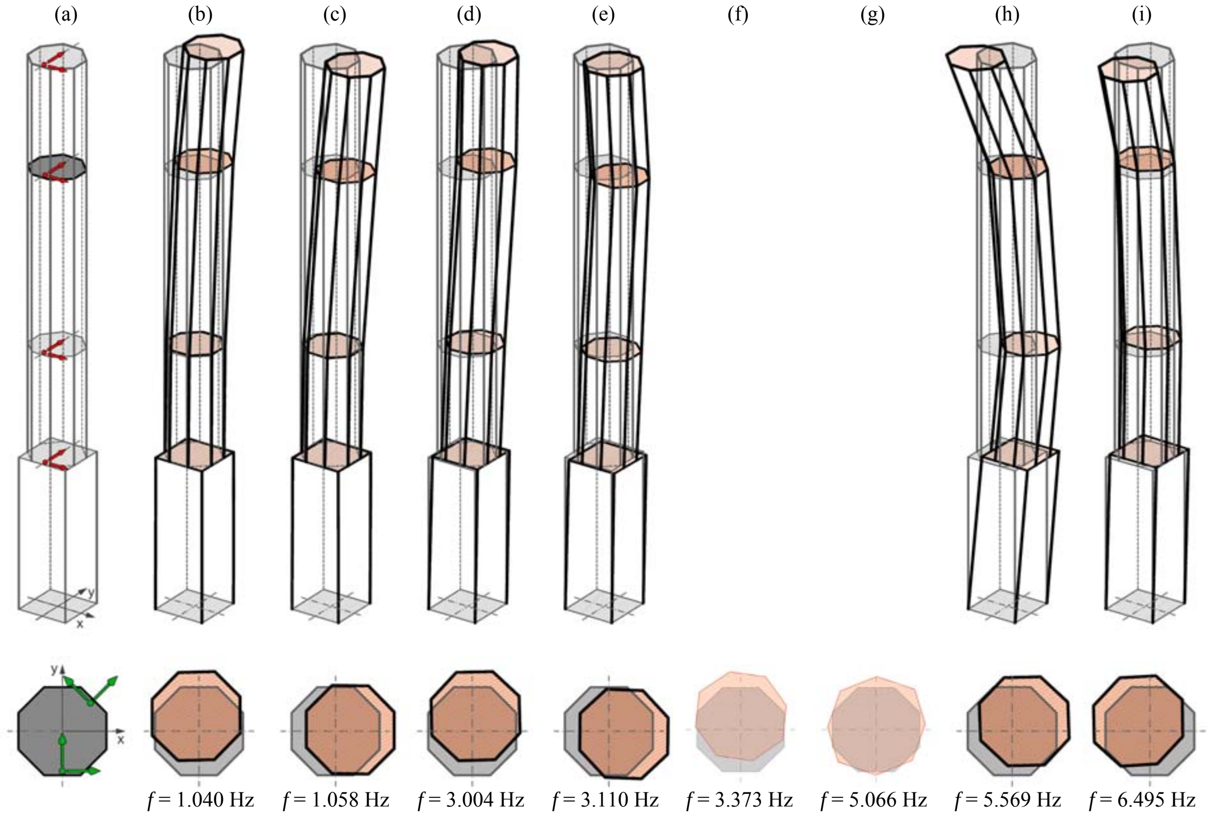

Figure 4: San Gottardo bell-tower: (a) Sensor layout in the test of 21/03/2017 (red arrows) and in the permanent monitoring (green arrows); (b)-(i) Automatically identified vibration modes (Y-axis: N-S direction).

Figure 1 refers to one 1-hour dataset recorded on 21/03/2017 and shows the typical cleaning action exerted by the various steps of the proposed MPE procedure on the stabilization diagrams. In more details: (a) Fig. 1a shows the results initially obtained applying the SSI-Cov method; (b) Figs. 1b and 1c show the performance of the pre-filtering (i.e., the check on damping ratio and mode shape complexity) and the clustering steps, respectively; (c) Fig. 1d contains the final alignments of stable poles corresponding to physical modes. It is worth noting that all plots in Fig. 1 also show the first Singular Value (SV) line of the spectral matrix, which is the mode indication function adopted in the Frequency Domain Decomposition (FDD) method [18].

The inspection of Fig. 1d highlights that:

- Within the investigated frequency range (0-8 Hz), the alignments of the stable poles in the stabilization diagram provide a clear indication of 8 tower modes and 6 of those alignments of stable poles correspond to well defined local maxima in the first SV line of the FDD procedure;
- The last 2 modes, although weakly excited, are clearly identified as the model order increases;
- As it is common for historic towers [4, 6, 8-10], the resonant frequencies of the two lower modes are closely spaced.

The modal parameters resulting from the estimates summarized in Fig. 1d and the test performed at the same hour using 8 high-sensitivity accelerometers are illustrated in Fig. 4. The inspection of Fig. 4 allows the following comments: (a) the first couple of modes (Figs. 4b and 4c) involves bending in two orthogonal N-S and E-W planes of the octagonal cantilever, without significant participation of the square basement; (b) the higher couples of bending modes (Figs. 4d-e and 4h-i) occur in orthogonal planes, which are different from N-S and E-

W, and does involve appreciable motion of the basement; (c) modes 5 and 6 (Figs. 4f and 4g) involve coupling between bending and torsion (and consequently the complete representation of those mode shape cannot be uniquely defined from the adopted sensor layout, even introducing the assumption of "stiff diaphragm", and is not shown in Fig. 4).

5 MONITORING RESULTS

This section summarizes the main results of the dynamic monitoring for a period of 14 months, from the beginning of November 2016 to the end of December 2018. It should be noticed that the monitoring system described in section 4 does not include any sensor to measure the environmental parameters since temperature and humidity data were available from the neighboring weather station *Osservatorio di Brera*.

As previously stated, the automated MPE procedure was applied to each hourly collected dataset after appropriate filtering and decimation of the time series. In the application of the SSI-Cov technique, the time-lag parameter was set equal to 90 and the data was fitted using stochastic subspace models of order n varying between 20 and 120. The MT procedure has been finalized to track the evolution in time of 8 structural modes (Fig. 4).

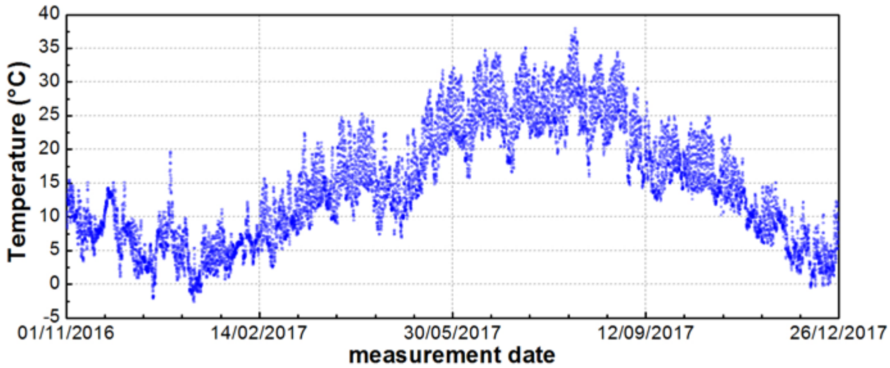

Figure 5: Time evolution of the outdoor temperature measured from 01/11/2016 to 25/12/2017

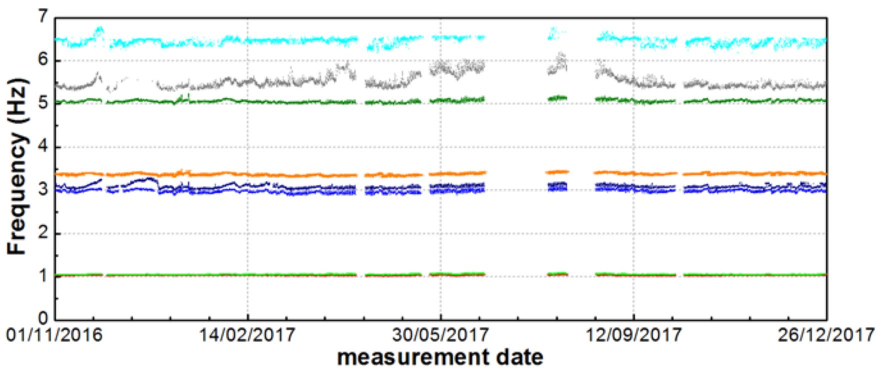

Figure 6: Time evolution of the automatically identified natural frequencies from 01/11/2016 to 25/12/2017

Figure 5 presents the evolution of the air temperature during a period of about 14 months, from 01/11/2016 to 25/12/2017, and shows that the temperature changed between −2°C and +38°C, with significant daily variations in sunny days. Automated identification of the modal frequencies from the datasets collected in the same period provided the frequency evolution shown in Fig. 6, whereas the corresponding statistics of natural frequencies are summarized in Table 1. This table includes the mean value (f_{ave}), the standard deviation (σ_f) and the extreme values (f_{min}, f_{max}) of each modal frequency.

Mode	f_{ave} (Hz)	σ_f (Hz)	f_{min} (Hz)	f_{max} (Hz)
1	1.039	0.008	1.015	1.078
2	1.055	0.009	1.034	1.097
3	2.976	0.036	2.871	3.079
4	3.090	0.047	2.982	3.291
5	3.371	0.028	3.294	3.483
6	5.059	0.032	4.961	5.226
7	5.510	0.155	5.252	6.182
8	6.450	0.117	6.177	6.779

Table 1: Statistics of the natural frequencies automatically identified from 01/11/2016 to 25/12/2017.

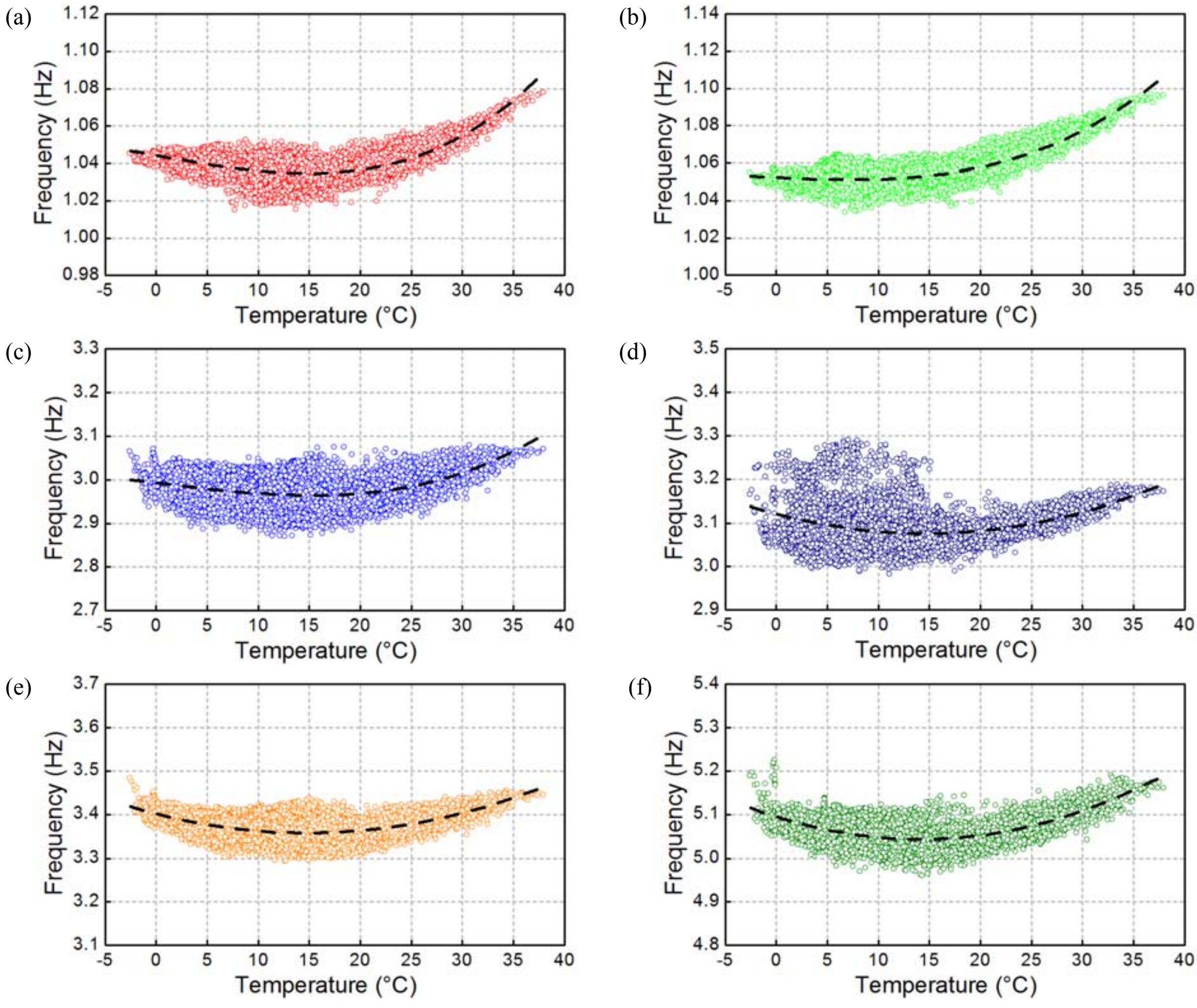

Figure 7: Correlation between automatically identified frequencies and measured temperature: (a) Mode 1; (b) Mode 2; (c) Mode 3; (d) Mode 4; (e) Mode 5; (f) Mode 6.

It should be noticed from Table 1 that the standard deviations: (a) are very low (0.008-0.009 Hz) for the lower 2 modes; (b) range between 0.028 Hz and 0.047 Hz for the subsequent 4 modes and (c) and became larger than 0.1 Hz for the higher modes.

Inspecting the evolution in time of temperature (Fig. 5) and modal frequencies (Fig. 6) clearly suggests that the frequency increase with increased temperature in the hot season [4, 6, 8-10]. On the other hand, the correlation between resonant frequencies and temperature (Fig. 7) reveals a more complex and non linear dependence on temperature. The non-linear frequency-temperature correlation, exemplified in Fig. 7 for modes 1-6, turns out to be characterized by the increase of modal frequencies with increased temperature, when $T \geq 15$°C (in agreement with a trend that has been commonly observed on masonry towers [4, 6, 8-10]), but also by the increase of natural frequencies with decreased temperature, when $T \leq 10$°C.

This kind of dependence on temperature seems to be quite distinctive and deserves further investigation, in order to better understand the possible effects exerted by the two construction phases (i.e., the square basement in stone masonry and the octagonal cantilever in solid brick masonry) and the internal wooden spiral staircase (that might act as a sort of inner confinement) and to remove or minimize the effects of changing environment on natural frequencies [4, 6, 8-10].

6 CONCLUSIONS

An automated modal identification procedure, based on parametric methods and the construction of stabilization diagrams, has been presented. The performance of the proposed algorithm has been demonstrated using real data continuously collected through the dynamic monitoring system installed in the bell-tower of the church of San Gottardo in Corte (Milan, Italy).

The application of the proposed algorithms allowed to accurately identifying and tracking the evolution in time of 8 normal modes of the tower and to highlight a distinctive dependence of the automatically identified natural frequencies on temperature and environmental parameters.

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