# **Damage localization through vibration based S <sup>2</sup>HM: a survey**

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**Abstract** Several methods proposed in literature for the localization of stiffness losses rely on the detection of irregularities in the deflected shape of the structure. This requires accurate description of the deflected shape achievable through a high spatial resolution of sensors, high quality or measures and accounting for the approximations introduced by signal processing. In the first part of this paper is reported a survey of vibration-based damage localization algorithms based on the detection of (changes of) irregularities in the deflected structural shape. Most of these methods rely on damage parameters defined in terms of the local variations of curvature due to the direct relationship of this parameter with the variations of stiffness. Due to some drawbacks related to the estimation of curvature from noisy recorded responses, other methods have been proposed to detect local variations of the deformed shape without directly computing the curvature. Also, many of the methods proposed in literature have been validated only on numerical models, due to the scarce availability of experimental data recorded on damaged structures. Recently data recorded on benchmark structures have become available giving the opportunity to verify the capability of these methods for damage localization in real-world conditions. In the last part of the paper, a method for damage localization based on the detection of localized changes in the structural deformed shapes, the Interpolation Method, is applied to two benchmark structures. The first is the UCLA Factor Building whose response to several nondestructive earthquakes has been recorded by a dense network of sensors. The second is the  $7<sup>th</sup>$ storey portion of building tested to collapse, using base inputs of increasing severities, on the USDS shaking table.

**Keywords**: vibration-based, damage, localization, shape irregularity, interpolation method

# **1 Introduction**

Vibration-based damage identification methods allow assessing structural damage states mainly induced by stiffness losses. One of the major advantages of these methods is the possibility to detect damage at a global level, using sensors not necessarily deployed close to the – unknown – location of damage. Different levels of refinement in the identification of damage are possible depending on the amount of information provided by the recorded responses. Detection, that is the identification of the existence of damage, might be possible based on a single sensor able to capture meaningful characteristics of the structural response, e.g. the natural modes more sensitive to damage. Localization requires a higher number of sensors deployed at several locations along the structure. The assessment of damage, that is the estimation of its severity, usually requires a finite element model of the structure that allows to map the responses recorded on the structure to different damage types and scenarios through the physical model of the geometry and mechanical characteristics of the real structure.

This paper is limited to response-based methods that do not make use of a physical (e.g. Finite Element) model of the structure. Model-based methods have usually a considerable computational cost, due to the need to update the model parameters through iterative optimization processes. This makes them less suitable for real-time structure damage identification. It must be said though, that when damage assessment is concerned, response-based methods fail to provide both the type and the severity of damage, whereby model-based methods become a useful option.

Several vibration-based methods that rely only on recorded responses have been proposed in literature (Pandey et al 1991, Stubbs et al 1992, Ratcliffe 1997, Zhang & Aktan 1998, Wahab & De Roeck 1999, Ho & Ewins 2000, Pai & Jin 2000, Lu et al. 2002, Parloo et al 2003, Limongelli 2003, Gentile & Messina 2003, Dutta & Talukdar 2004, Zhang et al 2013, Surace et al 2014, Corrado et al. 2015). This paper will focus on methods that perform localization of damage through the detection of irregularities in the deflected shape of the structure. Most of these methods exploit the relationship between a local loss of stiffness and the corresponding local variation of curvature. The damage feature is therefore defined in terms of this latter parameter.

One of the drawbacks related to the choice of curvature as damage feature consists in the fact that the double differentiation needed for its computation is highly sensitive to noise in recorded responses. Furthermore, a high spatial resolution, meaning a high number of sensors, and high-quality measures, that usually require more expensive sensors, are needed to obtain accurate estimates of the deformed shapes from which curvatures are computed. Due to this, in literature have been proposed methods to identify variations of curvature without explicitly computing curvatures. Some of them will be described in the next sections.

One of the main issues in the research field related to damage identification, even more challenging when damage induced by earthquakes is involved, is the validation on real structures of the algorithms proposed by researchers. The number of monitored structures in seismic prone areas is still quite low and usually, due to economic constraints, a small number of sensors is deployed on them. Beside this, many of the instrumented structures have never experienced damage during an earthquake and in some cases, even if data exist, they are not freely available for research purposes. Due to all these facts, the algorithms proposed in literature for damage identification are often verified using data simulated using number models or obtained through shaking table tests of scaled laboratory specimens.

In the last years, several vibration tests have been performed on full scale structures artificially damaged for research purposes. Data have been recorded using quite dense networks of sensors and made available to the scientific community. The last section of this paper reports the application of an algorithm for damage localization, the Interpolation Method, to two benchmark structures. The first is the UCLA Factor Building

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(see ref USGS) whose response to several earthquakes of low to medium intensity has been recorded by a dense network of sensors permanently installed. The second case study is the experimental model of a portion of a 7-storey building tested on shaking table at UCSD, through the George E. Brown Jr. Network for Earthquake Engineering Simulation (NEES) program (Mohaveni et al. 2010, 2011).

## **2 Damage features based on the detection of shape irregularity**

#### *2.1 Modal and operational shapes*

Modal and operational shapes inherently describe the geometry of a structural system, a localized reduction of stiffness produces a corresponding increase of curvature that alters the deformed structural shape. This allows to localize damage by processing the geometric changes of this shape. The irregularity induced by a stiffness loss affects the global deformed shape therefore, considering its decomposition in the frequency domain, not only the modal shapes are affected by damage but, in principle, all the operational shapes. An 'operational deflected shape' is the deflection shape of a structure subjected to harmonic excitation. If the frequency of the excitation is close to a modal one, the ODS is dominated by the corresponding mode shape; for other values of the frequency of excitation, the ODS derives from the combination of several modes. The modal or operational shapes most useful for damage identification are usually those corresponding to the highest frequency shifts induced by damage. Therefore, a low or a high mode can be both equally useful to damage identification purposes depending on the location of damage and on the sensitivity of modes to damage at that location. This means that all the available modes or of operational shapes should be included in the damage localization procedure in order to have accurate results. Several damage localization algorithms based on the detection of shape irregularities have been proposed in literature (Pandey et al 1991, Stubbs et al 1992, Ratcliffe 1997, Wahab & De Roeck 1999, Ho & Ewins 2000, Limongelli 2003, Gentile & Messina 2003, Zhang et al 2013, Surace et al 2014). Most of them rely on modal shapes, others on operational deflected shapes retrieved from Frequency Response Functions. Both families of methods present advantages and drawbacks. The estimation of modal shapes is today quite reliable thanks to the developments of the experimental and operational modal analysis techniques. The computation of operational shapes from Frequency Response Functions is more straightforward therefore more feasible to online algorithms for real-time identification of damage.



Fig. 1 Operational shapes

At resonance the effect of noise is usually lower with respect to other frequency values therefore the modal shapes are estimated with higher accuracy with respect to the generic ODS. On the other side, the number of modal shapes that can be identified in the frequency range excited by the input (forced or ambient vibrations) is always much lower than the number of operational shapes in the same frequency range. Therefore, in a certain sense, the amount of information contained in the complete set of ODS in a certain frequency range, is higher with respect to the one contained in the modal shapes. The *quality* of these information depends on the uncertainties related to the retrieval of the ODS. These can be due to e.g. noise in recorded sensors, non-linear behavior of the structure, round off in signal processing. Large set of data, obtained measuring the response of the structure to vibrations for long periods of time, can help reducing the effect of uncertainties but this is possible only if permanent monitoring systems are installed on the structure. If the acquisition of data is limited to short periods of time – as is the case for forced vibration tests – a high quality of recorded data should be sought in order to accurately identify the location of damage. In the latter case modal shapes can be more useful with respect to ODS. In reference (Limongelli 2016) a comparison between results obtained ODS or modal shapes is presented.

In the past, permanent monitoring systems where much less diffused with respect to short term monitoring and this is probably one of the reasons why most of the damage localization algorithms have been formulated in terms of modal shapes. Nowadays, with large amount of data available, the possibility to use operational shapes is becoming more and more appealing due to the lower interaction they require with an operator. This last feature makes them more feasible for the implementation in autonomous SHM systems.

The components of the deflected shapes are measured at a discrete number of points corresponding to the sensor's locations. The higher is the number of sensors and the more uniform their distribution, the higher is the spatial resolution of the deformed shapes, hence the accuracy of damage localization.

In the following section is reported an overview of some methods for damage localization based on the detection of irregularities in the deflected shapes. All methods will be generically described with reference to a generic 'deflected shape' meaning that they can be applied considering both modal and operational shapes.

#### *2.1 Shape variation due to a loss of stiffness*

The effect on the deformed shape of a localized stiffness loss can be schematically explained with reference the simple cantilever beam represented in [Fig. 2.](#page-4-0) Two configurations are represented: the reference and the damaged ones. Damage is intended as a reduction of the sectional bending stiffness in a small portion of the beam. In the circle is reported the enlarged detail of the damaged portion.



<span id="page-4-0"></span>As clearly shown by the comparison inside the circle, a sharp variation of the deflected shape occurs at the location of damage whilst, at all the other sections, the two de-

flected shapes can be almost perfectly superimposed with a simple vertical shift. Therefore a feature able to describe the shape of the beam and to detect the local difference between two shapes is needed to identify the correct location of damage. It has been shown that for 'beam-like' structures like buildings or bridges a very good approximation of the deformed shape can be obtained through cubic spline functions (Limongelli 2003, 2005). Comparison of shapes is a very important topic widely studied in several different fields. One technique, commonly used for example in the field of Computer vision, is to define the shape of a curve through its curvature. A major reason for this is that small variations of shapes are hardly detectable through the comparison of the shapes themselves (Salawu and Williams 1994, Khan et al, 1999, Huth et al, 2005) while they sensibly affect curvatures. Furthermore, curvatures are invariant to rigid transformations that is independent of the spatial translation or rotation of the curve.

Finally, for the detection of stiffness losses, curvature are the most straightforward feature being directly linked to the bending stiffness.

For a simple Eulero-Bernoulli beam, the curvature and the flexural stiffness are related by the expression:

$$
v'' = \frac{M}{EI} \tag{1}
$$

where  $v^{\dagger}$ , *M* and *EI* are respectively the curvature, the bending moment and the bending stiffness of a same section. Damage entailing a reduction of the bending stiffness at one location of the beam determines an increase in the magnitude of curvature in the same section. Therefore, changes in the curvature can be used to detect and locate damage.

A totally different and quite challenging point is the accurate curvature estimation from the available data. The relationship between the curvature 1/*r* and the second derivative of a function  $v(x)$  is the following:

$$
\frac{1}{r} = \frac{|v''(x)|}{|1 + v'(x)|^2} \tag{2}
$$

If the function  $v(x)$  represents a deflected shape (as is the case herein),  $v'(x)$  is the rotation. For small values of the rotations, the square of the rotation can be neglected with respect to unity and it can be assumed that the curvature  $1/r$  is equal to the modulus of the second derivative of the function  $v''(x)$ . A discontinuity in the curvature

 $(1/r)$  in this case has a direct effect on the second derivative of the function. In real applications, data from accelerometers or displacement transducers provide few components of the deflected shape of the structure that is a discrete representation of a continuous function. Therefore, the computation of curvature requires the use of a numerical algorithm, such as for example the central difference approximation to the second derivative. The curvature  $v_i$ <sup>"</sup> at the *i*-th location is given as a function of the

values of the function at the neighboring locations:

<span id="page-5-0"></span>
$$
v_i = \frac{v_{i+1} - 2v_i + v_{i-1}}{h^2}
$$
 (3)

Where  $v_i$  is the component of the deflected shape at location  $i$  and  $h$  is the distance between the locations where structural responses are measured, that is the distance between the sensors.

Two practical drawbacks affect the computation of the curvature for discrete functions. The first is related to the spatial resolution of the available deflected shapes. This depends on the location and number of sensors that is usually limited due to economic constraints therefore is related to the 'quantity' of available data. Some authors (Stubbs et al. 1992) proposed to tackle the problem by interpolating the deflected shape using a smooth function such as a cubic spline. The interpolation allows to refine the location of damage in the identified region but does not introduce new 'information' therefore is not able to reduce the uncertainty about the correct damaged portion of the structure. The second drawback is related to the '*quality*' of available data: estimators of curvature, like the one reported in equation [\(3\)](#page-5-0), are very sensitive to the noise introduced by the acquisition process (Chance et al., 1994; Gentile and Messina, 1997). Noise is further amplified by the double differentiation which, behaving as high-pass filters in the Fourier-domain, can mask the damage hampering its correct localization. A common approach to tackle this problem is to smooth the data - prior or after to curvature calculation - but this might as well remove the effect of damage, thus rendering the data less informative for damage identification purposes. A more effective approach is to apply methods that detect variations in the curvature profile without a direct estimation of this parameter. A number of these will be described in one of the following sections.

## **3 Damage localization**

This section reports a short, not exhaustive, description of a number of methods proposed in literature to localize stiffness losses through the detection of variations of local curvature. Many of them require the direct estimation of this parameter whereas other methods use feature that do not require the direct computation of curvature to detect its changes.

All the methods can be applied to a generic 'shape' be it a modal or an operational one. However, each of the method was originally proposed for one of the two families, usually for modal shapes. In the following reference will be made to the original version.

### *3.1. Methods based on curvature*

Pandey et al. 1991 showed that the absolute change in *modal curvature* can be an efficient indicator stiffness losses. The damage index is defined at each location *i* as the sum of the curvature variation between a reference (*U*) and possibly damaged (*D*) configuration, over all the identified modal shapes:

$$
\Delta c_i = \sum_{k=1}^{n_{modes}} \left| \phi_{k,i,D} - \phi_{k,i,U} \right| \tag{4}
$$

Stubbs et al (1992) proposed to use the variation of the *modal strain energy* stored in each portion (sub-element) of a beam as a damage feature. The basic idea is that damage does not change the fractional modal strain energy  $F_{k,i}$  that is the ratio between the strain energy stored in the *i*-th element and total strain energy for the *k*-th mode. Based on this assumption, the ratio between the bending stiffness in the damaged and undamaged states can be written as a function of the modal curvature and, summing over all the modes, the following expression is obtained for the damage index**:**

$$
\beta_{i} = \frac{\sum_{k=1}^{n_{model}} (EI)^{U}_{i,k}}{\sum_{k=1}^{n_{model}} (EI)^{D}_{i,k}} = \sum_{k=1}^{n_{model}} \left[ \frac{\left(\phi_{k,i}^{*}\right)_{D}^{2}}{\sum_{i=1}^{L} \left(\phi_{k,i}^{*}\right)_{D}^{2}} \right] / \sum_{k=1}^{n_{model}} \left[ \frac{\left(\phi_{k,i}^{*}\right)_{U}^{2}}{\sum_{i=1}^{L} \left(\phi_{k,i}^{*}\right)_{U}^{2}} \right]
$$
(5)

 $\beta_i$  is defined as the sum, over all identified modes, of the elements bending stiffness in the damaged and in the reference configurations. The damage index  $\beta_i$  assumes values higher than 1 at a damaged location.

Another index was proposed by Zhang and Aktan in 1998 based on the curvature of the *Uniform Load Surface* (ULS). The ULS is the deflection vector due to a unit load applied at each of the *p* DOF (uniform load). The deflection *f*ij at location *i* due to a unit load at location *j* is approximated by the sum of the contributions of the identified *nmodes*, the deflection *f*<sup>i</sup> due to a uniform unit load distribution is the sum of the contributions due to the single unit loads.

$$
f_{i,j} \cong \sum_{k=1}^{n_{modes}} \frac{\phi_i^k \phi_j^k}{\omega_k^2} \qquad f_i \cong \sum_{k=1}^{n_{modes}} \frac{\phi_i^k \sum_{j=1}^p \phi_j^k}{\omega_k^2} \qquad (706)
$$

The flexibility is sensitive to the number of modes, to the load location and to the boundary conditions. The curvature of the deflection  $f_i$  can be obtained through a central difference approximation as in equation [\(3\)](#page-5-0) and the damage index at each location is defined as the variation of curvature of the deflection between the damaged and the reference states:

$$
f_i = f_{i,D} - f_{i,U} \tag{8}
$$

Due to the low values of flexibility close to supports, this index may not allow a correct localization when damage is located at these regions due the masking effect of numerical errors (Yan and Golinval, 2006).

All the previous methods require the computation of the variation of curvature from signals recorded in the reference and in the - possibly - damaged state.

The knowledge of the reference condition apparently is not needed by the *Gapped smoothing method* proposed by (Ratcliffe 1997). A smooth cubic polynomial function is used to interpolate the curvature of the modal shape and the damage index at the *i*th location is calculated as:

<span id="page-7-0"></span>
$$
\delta_i = \left[ \left( p_0 + p_1 x_i + p_2 x_i^2 + p_3 x_i^3 \right) - C_i \right]^2 \tag{9}
$$

 $C_i$  is the curvature computed at location  $i$  from recorded responses and the coefficients  $p_0$ ;  $p_1$ ;  $p_2$ , and  $p_3$  are determined interpolating the values of curvature  $C_{i-2}$   $C_{i-1}$ ,  $C_{i+1}$ ,  $C_{i+2}$  and skipping  $C_i$ . The locations corresponding to the maximum values of the damage index are assumed as the damaged ones. The idea underlying the method is that the error between the curvature calculated from data and the value interpolated through a smooth (polynomial) function, is higher at locations with an irregularity. Therefore, the damaged location is identified as the one corresponding to the maximum value of the interpolation error. Even if the computation of the curvature in the reference condition is not explicitly required by this method, the underlying assumption is that a discontinuity in the curvature profile is necessarily related to damage. This is not necessarily true since a discontinuity in the curvature profile may be due to an intrinsic irregular distribution of stiffness: due for example to a change of the transversal section along the axis of a beam, or to the local change of the vertical bearing elements along the height of a building. This may lead to false alarms in the identification of damage locations. Damage is a change with respect to a reference conditions therefore its identification requires the comparison of the current state with a reference one. The

assumption that an irregularity in the deformed profile corresponds to a damage, implies that in the reference state the deformed shape was regular, meaning that in the original configuration there was a regular distribution of stiffness. If the stiffness distribution is already irregular at the 'birth' of the structure or generally in the reference state, the simple detection of an irregularity in the deflected shapes does not necessarily correspond to a damage. In this cases equation [\(9\)](#page-7-0) might give false indications of damaged locations.

Beside this last drawback, inherent in methods that do not compare the inspection to the reference value of the damage feature, all the damage identification algorithms described so far, need the direct identification of the curvature values and this may introduce large uncertainties in case of noisy signals. To overcome this problem, methods have been developed to detect curvature changes without directly estimating curvature. Some of them are described in the next section.

## *3.2. Methods based on the indirect detection of curvature changes*

Several proposals have been formulated for robust estimators of curvature from noisy signals. Most of them have been applied in the field of image processing (e.g. Page et. al. 2002) but a final solution of the problem is still a research topic.

A completely different approach consist in the use of methods able to detect changes in curvature without actually computing the curvature itself.

Methods based or the use of *wavelet functions* treat the deflected shape as a signal in the spatial domain, and use the wavelet transform to detect the signal irregularity caused by damage. The use of these functions for damage detection purposes has been investigated by several authors. In the paper by Fan and Qiao, 2011 a comprehensive survey is given. Herein reference is made to the work of Gentile and Messina, 1997 focused on the detection of cracks simulated through a local reduction of the elastic modulus inducing a local loss of stiffness. In this paper an interesting and clear physical interpretation of the damage localization capability of wavelets is proposed. It can be proved that these functions, if properly chosen, are a good approximation of the derivatives of the deflected shape.

Specifically, if a wavelet function has *m* vanishing moments, the following equation holds:

<span id="page-8-0"></span>
$$
\lim_{s \to 0} \frac{Y(x;s)}{s^{m+1/2}} = K \frac{d^m y(x)}{dx^m}
$$
\n(10)

where  $y(x)$  is the recorded signal,  $Y(x; s)$  is the signal transformed using the wavelet

and *s* is a real positive number called 'dilation parameter' of the wavelet function. A specific derivative can be approached through a wavelet transform, by choosing appropriately the number of vanishing moments of a Gaussian wavelet (Gentile and Messina, 2003). As equation [\(10\)](#page-8-0) shows, the signal transformed using the wavelet is proportional to the *m*-th derivative of the function: for *m*=2 the transformed signal is proportional to the curvature. Therefore a discontinuity in the proper wavelet transform corresponds to a discontinuity in the corresponding derivative that can be used to identify the location of damage. In their paper Gentile and Messina apply this technique to the detection of open cracks that are modelled through a local reduction of the elastic modulus.

A second approach is based on the use of *smooth functions* to interpolate the deflected shapes. The Interpolation method (Limongelli 2010) is based on the use of a cubic spline to interpolate the deflected shapes retrieved from recorded responses. Interpolation is performed at the *i*-th location considering all the measured components of the deflected shapes  $v_1, v_2, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n$  except  $v_i$ . The interpolation error is defined as follows:

$$
E_{i} = \left| \left( c_{0,i} + c_{1,i} \left( x_{i} - x_{i-1} \right) + c_{2,i} \left( x_{i} - x_{i-1} \right)^{2} + c_{3,i} \left( x_{i} - x_{i-1} \right)^{3} \right) - v_{i} \right| = \left| \hat{v}_{i} - v_{i} \right| \tag{11}
$$

The coefficients  $c_{0,i}$ ,  $c_{1,i}$ ,  $c_{2,i}$  and  $c_{3,i}$  of the cubic spline function are computed imposing interpolation and continuity conditions at all the instrumented locations. More details on the interpolation procedure can be found in reference (Limongelli 2003). Due to the so-called 'Gibbs phenomenon for splines, a sharp increase of the interpolation error occurs at the locations with a curvature discontinuities and this can be used to detect the damaged location as the one where the highest value of the interpolation error is found.

The curvature discontinuity affects both modal and operational shapes. In order to enhance the value of the interpolation error at the damaged location with respect to all the others, the sum of the interpolation error computed for all the *nshapes* shapes - modal or operational - is considered:

$$
E_{i} = \sqrt{\sum_{k=1}^{n_{shape}} E_{i}^{2}} = \sqrt{\sum_{k=1}^{n_{shape}} |\hat{\nu}_{k,i} - \nu_{k,i}|^{2}}
$$
(12)

Changes of interpolation error between two different states (reference and potentially damaged) highlight the onset of a curvature discontinuity therefore the following difference  $\delta E_i$  is assumed as the damage feature at location *i*:

$$
\delta E_i = E_i^D - E_i^U \tag{13}
$$

This definition of the damage feature allows to overcome the shortcoming intrinsic in the Gapped smoothing method and related to the assumption of a regular distribution of stiffness in the reference configuration.

A further improvement of the Interpolation Method, currently under investigation, is related to its possible use for the estimation of damage severity beyond for damage localization. A linear relationship exist and has been proven (Limongelli 2018) between the value of the interpolation error at the damaged location and the curvature discontinuity. Currently the application of this relationship to identify the severity of real damage scenarios is being studied.

## **4 Damage indices and thresholds**

In real world conditions, due to several sources of variability influencing recorded responses the damage features described in the previous section can exhibit changes even if no damage occurs or, viceversa, they can exhibit no change when damage exist. This leads to false or missing detection of damage. This problem could be tackled through statistical analyses if the distributions of the damage feature are known. This is possible if large set of data are available to identify these disributions.

Assuming this is the case, in [Fig. 3](#page-10-0) the distribution of the damage feature in the reference configuration  $f_{i,ref}$  at one instrumented location is compared to the distribution in the inspection (damaged) state  $f_{l,isp}$ . In the figure is assumed that the damage feature increases with damage (e.g. the modal period increases with the loss of stiffness). The comparison of the two distributions allows to investigate the onset of damage.



Fig. 3. Distributions in the undamaged ( $f_{l,ref}$ ) and in the damaged ( $f_{l,isp}$ ) states

<span id="page-10-0"></span>If one, or both distributions cannot be determined reference can be done to the distribution of values of the damage feature computed at all the instrumented locations (Stubbs 1995, Limongelli 2010, Domaneschi 2013). The values of the sample mean and standard deviation are obtained from the available set of damage features at all the instrumented location. In order to differentiate the intact from the damage states a threshold value has to be defined. This can be done in terms of an '*accepted probability of false alarms*' in the undamaged configuration. Assuming a normal distribution the threshold can be computed as (Limongelli 2010):

$$
I_T = M_I + v \cdot \sigma_I \tag{14}
$$

where  $M_I$  and  $\sigma_I$  are respectively the sample mean and the sample standard deviation of the sample population of the damage index in the reference state. The value  $\nu$ is defined in the standard normal distribution as the  $\alpha$ -percentile that defines the accepted probability of false alarm.

The classification of a certain location *i* as "damaged" is carried out basing on the comparison of the current value of the damage feature  $I_i$  with the threshold at the same location *Ii,T*.

if  $I_i - I_{i,T} > 0$ damage at location  $i$ 

if  $I_i - I_{i,T} < 0$ no damage at location  $i$ 

## **5 Case studies**

In the following is reported the application of the Interpolation Method, recalled in section 3.2) to the case of two multistory building under seismic excitation. In the first case, the UCLA Factor Building, responses recorded during or after a severe earthquake, able to damage the building, are not available. For this reason, the responses recorded by the monitoring system have been used to calibrate a finite element model and damage has been simulated reducing the stiffness of several elements of the model. In the second case, for the  $7<sup>th</sup>$  storey building responses have been recorded in several damage states of increasing severity ad used to verify the capability of the Interpolation Method to correctly localize damage.

# **6 The UCLA Factor Building**

The UCLA Factor building (see [Fig. 4](#page-11-0)) is a 17-story moment-resisting steel frame structure consisting of two stories below grade and 15 above grade. The building houses laboratories, faculty offices, administrative offices, the School of Nursing, School of Medicine, auditoriums, and classrooms. The building is permanently instrumented with an embedded 72-channel accelerometer array recording both ambient vibrations of the building and motions from local earthquakes. The sensors array is composed by four horizontal channels per floor: two in North-South direction and two in East-West direction. The two floors below grade are also equipped with two vertical channels. The array continuously records ambient vibrations as well as motions from local earthquakes. More details on the Factor Building and on the recording network can be found on reference (USGS).



<span id="page-11-0"></span>Fig. 4: The Factor Building (a) East face; (b) Sensors location (from ref. USGS) In reference (Limongelli, 2014) the Interpolation Method has been applied to the Factor Building. Specifically, data recorded on the building during several seismic events recorded in 2004 have been used to retrieve the probability distributions of the Interpolation Error at all the stories of the building. Results show that a lognormal distribution fits correctly the statistical variation of the damage feature, the Interpolation

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Error, in the undamaged configuration. At the time being, only responses recorded during events that did not severely damage the Factor Building are available therefore, in order to test the performance of the IM, a numerical model of the building has been used to simulate several damage scenarios. A reduction of the storey stiffness was simulated in the numerical model by removing a number of columns at one or more storeys. A selection of results is reported in [Fig. 5](#page-12-0) for damage scenarios corresponding to 2 damage columns at 2 or 4 storeys in the transversal  $(x)$  or in the longitudinal  $(y)$ direction of the building. The name of the scenario describes the location of the damaged columns: for example Dx\_04\_12\_2c means 2 damaged columns (2c) along the *x* (transversal) direction, located at storeys 4 and 12.In the figures the blue bars indicates the correct location of damage. The black curve joins the values of the Interpolation Damage Index (IDI) at all the storeys. A value equal to  $v=2$  corresponding to probability of false alarms of about 2% has been assumed in all these cases. For all scenarios the IDI attains the highest values at the damaged locations, allowing a correct localization.



<span id="page-12-0"></span>Fig. 5. Results for two and four damaged columns per story along respectively the transversal (*x*) and the longitudinal (*y*) direction

In [Fig. 6](#page-13-0) are reported the values of the IDI at all the stories of the building for different severities of damage. The value of the damage index increases with damage showing the direct correlation, mentioned in section 3.2 with the loss of stiffness.



<span id="page-13-0"></span>Fig. 6: Increase of damage index with the severity of damage (from Limongelli 2014)

# **7 The 7 th storey portion of building at UCSD**

This structure, that represents a slice of a full-scale reinforced concrete shear wall building, is 20 metres in height and 275 tons in weight. It consists of a main shear wall (web wall), a back wall perpendicular to the main wall (flange wall) for transversal stability, a concrete slab at each floor level, an auxiliary post-tensioned column to provide torsional stability, and four gravity columns to transfer the weight of the slabs to the shake table. [Fig. 7](#page-13-1) shows the test building on the shake table (NEES)



Fig. 7. Test structure (NEES)

<span id="page-13-1"></span>The tests on the specimen were performed at the UCSD-NEES shake table located at the Englekirk Structural Engineering Center, 15 km east of the main campus of the University of California–San Diego (UCSD). The building was progressively damaged through several historical seismic motions ( $M_W$  from 6.6 to 6.7) reproduced on the shake-table. Before and between the seismic shake-table tests, the building was subjected to long-duration (8 min) ambient vibration tests and to long-duration (3 min) low-amplitude white-noise (WN) excitation tests. The test structure was instrumented with a dense array of sensors including accelerometers, strain gauges, potentiometers, and linear variable displacement transducers (LVDTs). Herein data from 8 longitudinal acceleration channels (located at each floor level) were used to apply the Interpolation Evolution Method (Iacovino, 2018). This is an extended version of the IM that can be applied using the nonlinear responses recorded during a damaging event. The IEM is based on the idea of repeating the application of the IM at each time instant during the strong motion therefore retrieving the location of the damage at each time instant. At the end of the motion, the histogram of the damage locations throughout the entire shaking is obtained and the damage location can be selected, for example, as that corresponding to the highest frequency of detection.

#### *1.1.1 Damage scenarios*

As described in Panagiotou et al. (2011) and Moaveni et al. (2010, 2011), after the 4 seismic tests, damage was concentrated at the first and second storeys. Flexural cracks occurred at the base and at the first story. During the last test EQ4 a large split crack appeared and extended up to one-third of the height of the second level. [Fig. 8](#page-14-0) shows the structural damage at the bottom of the structure and at the first storey after at the end of the tests.



<span id="page-14-0"></span>Fig. 8. (a) extent of flexure-shear cracking in the first story at the bottom corner of the first story of the web wall during EQ4; (c) splitting crack due to lap-splice failure at the bottom of the second story of the web wall on the west side after EQ4 (Panagiotou et a. 2011)

Results obtained from the application of the IEM to the  $7<sup>th</sup>$  story specimen are shown in Figure 17 for the different excitations. In all cases, the correct location of damage at storeys 1 and 2 is found except for the case of EQ3 where damage is found between storeys 2 and 3. This circumstance is due to the interpolation process performed for the computation of the damage feature that, as discussed in reference (Limongelli 2010), may somehow 'spread' the effect of damage to the locations nearby the damaged one.



Figure 1. 7<sup>th</sup> story shear wall building at UCSD. Histograms of the identified damaged storey: (a) EQ1, (b) EQ2, (c) EQ3, (d) EQ4.

# **8 Conclusions**

In this paper, response-based methods for damage localization in structures under seismic excitation are presented. Damage is intended as a loss of stiffness. Thanks to the low computational efforts they require, these methods can be more feasible for online real-time damage identification purposes with respect to model-based approaches. A large majority of these method are based on the detection of localized variations of curvature performed though a direct computation of this parameter or using techniques based on interpolation or on wavelet functions to identify losses of regularity in the deformed shape. A short survey of these methods is reported in the paper.

In order to tackle the problem of false or missing alarms – caused by the several sources of uncertainties that affect the identification of damage features from responses to vibrations (sensor's noise, signal processing assumptions and truncation, low spatial resolution) - the distribution of the damage feature should be retrieved and a threshold values should be fixed to distinguish damaged from undamaged states.

Response-based methods are usually not able to identify the type and severity of damage. This usually requires the use of a physical model. However, several damage features obtained though response- based approaches exhibit a strong correlation with the severity of damage and this appears promising for the definition of a damage feature requiring a low computational effort – so that it can be efficiently used for on-line damage identification – but enabling a more refined description of damage through the estimation of its severity beyond its existence and location.

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#### **References**

- Chance, J., Thomlinson, G.R., Worden, K., (1994). A simplified approach to the numerical and experimental modelling of the dynamics of a cracked beam. In: 12th International Modal Analysis Conference, Honolulu, USA, pp. 778–785.
- Corrado N., Gherlone M., Surace C., Hensman J., Durrande N., (2015). Damage localisation in delaminated composite plates using a Gaussian process approach, Meccanica, 50 (10), 2537- 2546.
- Dilena M., Morassi A., (2011). Dynamic testing of a damaged bridge, *Mechanical Systems and Signal Processing* 25 (2011) 1485–1507.
- Domaneschi M., Limongelli M.P., Martinelli L. (2013). "Multi-Site Damage Localization in a Suspension Bridge via Aftershock Monitoring*", International Journal of Earthquake Engineering*, **3**, pp. 56-72.
- Domaneschi M., Limongelli M.P., Martinelli L. (2012). "Damage detection in a suspension bridge model using the Interpolation Damage Detection Method". *Bridge Maintenance, Safety, Management, Resilience and Sustainability – Biondini & Frangopol (Eds)* © 2012 Taylor & Francis Group, London, ISBN 978-0-415-62124-3.
- Dutta, A.; Talukdar, S. (2004). Damage detection in bridges using accurate modal parameters. *Finite Elements in Analysis and Design*, 40.3: 287-304.
- Fan W. and Qiao P., (2011). "Vibration-based Damage Identification Methods: A Review and Comparative Study", *Structural Health Monitoring*, 10(1), pp 83-29
- Gentile, A., Messina, A., (2003). On the continuous wavelet transforms applied to discrete vibrational data for detecting open cracks in damaged beams. *Int. Journ of Solid and Structures*, Volume 40, pp. 295-315.
- Ho, Y. K.; Ewins, D. J. (2000). On the structural damage identi\_cation with mode shapes. In: *International Conference on System Identification and Structural Health Monitoring*.. p. 677-686.
- Huth, O., Feltrin, G., Maeck, J., Kilic, N. and Motavalli, M. (2005). Damage identification using modal data: Experiences on a prestressed concrete bridge*. Journal of Structural Engineering-ASCE*, 131, 1898–1910.
- Khan, A.Z., Stanbridge, A.B. and Ewins, D.J. (1999). Detecting damage in vibrating structures with a scanning LDV. *Optics and Lasers in Engineering*, 32, 583–592.
- Iacovino C., Ditommaso R. , Limongelli M.P., Ponzo F.C. (2018). The Interpolation Evolution Method for damage localization in structures under seismic excitation. *Earthquake Engineering and Structural Dynamics*. [Vol. 47\(10\)](https://onlinelibrary.wiley.com/toc/10969845/2018/47/10) p. 2117-2136. DOI: 10.1002/eqe.3062.
- Limongelli, M.P., (2003). "Optimal location of sensors for reconstruction of seismic responses through spline function interpolation". *Earthquake Engineering and Structural Dynamics,*  vol. 32, 1055-1074.
- Limongelli M.P., (2005). "Performance evaluation of instrumented buildings" *ISET Journal of Earthquake Technology*, 42, pp. 47-62.
- Limongelli M.P. (2010). "Frequency response function interpolation for damage detection under changing environment". *Mechanical Systems and Signal Processing* doi:10.1016/j.ymssp.2010.03.004
- Limongelli M.P**.** (2014). "Seismic health monitoring of an instrumented multistorey building using the Interpolation Method". *Earthquake Engng. Struct. Dyn*. 43, pp 1581-1602. doi: 10.1002/eqe.2411
- Limongelli M.P. (2019). The surface interpolation method for damage localization in plates". *Mechanical Systems and Signal Processing* 118 (2019) 171–194. <https://doi.org/10.1016/j.ymssp.2018.08.032>
- Lu Q., Ren G. Zhao Y. (2002). Multiple damage location with flexibility curvature and relative frequency change for beam structures. *Journal of sound and vibration*. 253(5), 1101-1114
- Moaveni B, He X, Conte JP, Restrepo JI.(2010). Damage Identification Study of a Seven-Story Full-scale Building Slice Tested on the UCSD-NEES Shake Table. *Structural Safety* 2010; **32**(5):347-356.
- Moaveni B, He X, Conte JP, Restrepo JI Panagiotou M. (2011). System Identification Study of a Seven-Story Full-scale Building Slice Tested on the UCSD-NEES Shake Table. *Journal of Structural Engineering* 2011; ASCE, 137(6). NEES Network for Earthquake Engineering Simulation Project [\(http://nees.ucsd.edu/7story.html\)](http://nees.ucsd.edu/7story.html).
- Necati Catbas F., Brown D.L., Aktan A.E., (2006). Use of modal flexibility for damage detection and condition assessment: case studies and demonstrations on large structures, *Journal of Structural Engineering ASCE* 132. 1699–1712.
- Page D.L., Sun Y., Koschan A.F., Paik J., Abidi M.A. (2002). Normal Vector Voting: Crease Detection and Curvature Estimation on Large, Noisy Meshes. *Graphical Models*, Volume 64, Issues 3-4, 2002, Pages 199-229, ISSN 1524-0703, <https://doi.org/10.1006/gmod.2002.0574>
- Pai P.F., Jin S., (2000). Locating structural damage by detecting boundary effects*, Journal of Sound and Vibration* 231(4) (2000) 1079–1110.
- Panagiotou M, Restrepo J, Conte J. Shake-Table Test of a Full-Scale 7-Story Building Slice. (2011). Phase I: Rectangular Wall. *Journal of Structural Engineering*; 10.1061/(ASCE)ST.1943-541X.0000332, 691-704.
- Pandey, A. K., Biswas, M., and Samman, M. M. (1991) "Damage Detection from Changes in Curvature Mode Shapes". *Journal of Sound and Vibration*, 145(2) 321–332.
- Parloo, E.; Guillaume, P.; Van Overmeire, M. (2003). Damage assessment using mode shape sensitivities. *Mechanical Systems and Signal Processing*, 17.3: 499-518.
- Ratcliffe C.P. (1997) "Damage Detection Using A Modified Laplacian Operator On Mode Shape Data".. *Journal of Sound and Vibration* 204(3) 505 517. Damage location using vibration mode shapes. *International Modal Analysis Conference*, 933–939.
- Salawu, O.S. and Williams, C. (1994). Damage location using vibration mode shapes. *International Modal Analysis Conference*, 933–939.
- Sampaio, R. P. C.; Maia, N. M. M.; Silva, J. M. M. (1999). Damage detection using the frequency response function curvature method. *Journal of sound and vibration*, 226.5: 1029- 1042.
- Shi, Z. Y.; Law, S. S.; Zhang, L. M. (1998). Structural damage localization from modal strain energy change. *Journal of Sound and Vibration*, 218.5: 825-84.
- Stubbs N., Kim JT, Topole K. (1992). "An efficient and robust algorithm for damage localization in offshore structures", 10th. *ASCE Structures Conference*, American Society of Civil Engineers, New York, NY, pp 543–546.
- Stubbs N., Kim JT (1995). "Model-uncertainty impact and damage-detection accuracy in plate girders", *Journal of Structural Engineering*, [Vol. 121, Issue 10](https://ascelibrary.org/toc/jsendh/121/10)  [https://doi.org/10.1061/\(ASCE\)0733-9445\(1995\)121:10\(1409\)](https://ascelibrary.org/toc/jsendh/121/10)
- Surace C., Saxena R., Gherlone M., Darwich H., (2014). Damage localisation in plate likestructures using the two-dimensional polynomial annihilation edge detection method, *Journal of Sound and Vibration* 333(2014) 5412-5426
- USGS. Earthquake hazard program. UCLA Factor Seismic Array. (Available from: [http://fac](http://factor.gps.caltech.edu/node/61)[tor.gps.caltech.edu/node/61.\)](http://factor.gps.caltech.edu/node/61). Last visited October 2013.
- Yan, A.M., Golinval J.C. (2006). Null subspace-based damage detection of structures using vibration measurements. *Mechanical Systems and Signal Processing*, 2006, 20.3: 611-626.
- Wahab M.A.; De Roeck, G.. (1999). Damage detection in bridges using modal curvatures: application to a real damage scenario. *Journal of sound and vibration*, 226.2: 217-235
- Zhang Z., Aktan A.E. (1998) Application of Modal Flexibility and Its Derivatives in Structural Identification, *Journal of Research in Nondestructive Evaluation*, 10:1, 43-61. <https://doi.org/10.1080/09349849809409622>
- Zhang Y., Lie S.T., Xiang Z., (2013). Damage detection method based on operating deflection shape curvature extracted from dynamic response of a passing vehicle, *Mechanical Systems and Signal Processing* 35 (2013) 238–254.