Brief paper

Plug-and-play model predictive control based on robust control invariant sets[☆]

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ABSTRACT

We consider the problem of designing decentralized controllers for large-scale linear constrained systems composed by a number of interacting subsystems. As in Riverso et al. (2013b), (i) the design of local controllers requires limited transmission of information from other subsystems and (ii) the addition/removal of a subsystem triggers the design of local controllers for child subsystems only. These properties enable Plug-and-Play (PnP) operations, and we show how to perform them while preserving global stability of the origin and constraint satisfaction. We improve several aspects of the PnP design procedure proposed in Riverso et al. (2013b) and, using recent results in the computation of Robust Control Invariant (RCI) sets, we show that all critical steps in the design of a local controller can be solved through Linear Programming (LP). Finally, an application of the proposed design procedure to a large-scale mechanical system is presented.

1. Introduction

The ever-increasing complexity and size of process plants, manufacturing systems, transportation systems and power networks has triggered a renewed interest in decentralized and distributed control schemes, that have been studied since the 1970s for unconstrained models (Lunze, 1992; Šiljak, 1991). In a nutshell, decentralized control assumes the overall plant is represented through the coupling of several subsystems for which local regulators are designed. The main advantages of this architecture are that the computation of control variables for different subsystems is parallelized and only communication between a subsystem and its local controller is required. Similar remarks also apply to distributed controllers where local controllers can also exchange information through a communication network.

In the last years, many decentralized/distributed MPC (De/DiMPC) schemes have been proposed (Scattolini, 2009), in view of the possibility of coping with constraints on system variables besides guaranteeing stability, robustness, and global optimality (Rawlings & Mayne, 2009). Available DiMPC methods span from cooperative (Stewart, Venkat, Rawlings, Wright, & Pannocchia, 2010) to non-cooperative, which require limited computational load, memory, and transmission of information (Camponogara, Jia, Krogh, & Talukdar, 2002; Farina & Scattolini, 2012; Riverso & Ferrari-Trecate, 2012; Trodden & Richards, 2010). One of the main problems of existing De/DiMPC approaches is the need of a centralized off-line design phase. In the context of large-scale systems, this can be a severe limitation because a global model of the system can be very hard or costly to obtain. Moreover, in several examples of systems of systems, units frequently enter and leave a network (Samad & Parisini, 2011) making it impractical to retune the overall controller in a centralized fashion. In these cases, a decentralized design based on local computational resources is the only viable approach.

In Riverso, Farina, and Ferrari-Trecate (2013b) we proposed a novel controller synthesis procedure based on the PnP paradigm (Stoustrup, 2009). PnP design, besides synthesis decentralization, requires limited information transmission for the synthesis of local

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controllers when subsystems are added or removed. Furthermore, the complexity of controller design and implementation, for a given subsystem, scales with the number of its parent subsystems only.

As in Riverso et al. (2013b), we propose a PnP design procedure hinging on tube MPC (Raković & Mayne, 2005) for handling coupling among subsystems, and aim at stabilizing the origin of the whole closed-loop system while guaranteeing satisfaction of constraints on local inputs and states. However, we advance the design procedure in Riverso et al. (2013b) in two main directions: (I) while in Riverso et al. (2013b) the design of local controllers requires the solution to nonlinear optimization problems, in this paper, using regulators based on RCI sets (Raković & Baric, 2010; Raković & Mayne, 2005), only the solution to Linear Programming (LP) problems is needed; (II) in Riverso et al. (2013b) stability requirements are fulfilled imposing an aggregate sufficient small-gain condition for networks, while in this paper we resort to set-based conditions that are usually less conservative. As for any decentralized synthesis procedure for general linear systems without a special structure, our method involves some degree of conservativity (Bakule & Lunze, 1988). More specifically, it requires that coupling between subsystems giving rise to loops is small enough. The potential application of our method to real-world systems is assessed through examples. In Riverso, Farina, and Ferrari-Trecate (2012, 2013a) we present an application of PnP-DeMPC to frequency control in power networks and compare results with those achievable by centralized MPC and the control scheme in Riverso et al. (2013b). In particular, our new controller outperforms the PnP controllers described in Riverso et al. (2013b). In this paper we highlight computational advantages brought about by our method by considering the control of a large array of masses connected by springs and dampers.

The paper is structured as follows. The design of decentralized controllers is introduced in Section 2. In Section 3 we discuss how to design local controllers by solving LP problems and in Section 4 we describe PnP operations. Sections 5 and 6 are devoted to a numerical example and some conclusions, respectively. Generalizations of PnP-DeMPC to distributed control architectures are given in Riverso et al. (2012). A preliminary version of this work has been presented at the 52nd IEEE Conference on Decision and Control (Riverso et al., 2013a).

Notation. We use $a:b$ for the set of integers $\{a, a+1, \dots, b\}$. The column vector with s components $v_1, \dots, v_s$ is $\mathbf{v} = (v_1, \dots, v_s)$. The function $\text{diag}(G_1, \dots, G_s)$ denotes the block-diagonal matrix composed by s block G_i, $i \in 1:s$. The symbols $\oplus$ and $\ominus$ denote the Minkowski sum and difference, respectively, i.e. $A = B \oplus C$ if $A = \{a : a = b + c, \text{ for all } b \in B \text{ and } c \in C\}$ and $A = B \ominus C$ if $a \oplus C \subseteq B$, $\forall a \in A$. Moreover, $\bigoplus_{i=1}^{s} G_i = G_1 \oplus \dots \oplus G_s$. For $\rho > 0$, $B_\rho(z) = \{x \in \mathbb{R}^n : \|x - z\| \le \rho\}$ where $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$. Given a set $\mathbb{X} \subset \mathbb{R}^n$, $\text{convh}(\mathbb{X})$ denotes its convex hull. The symbol $\mathbf{1}$ denotes a column vector of suitable dimension with all elements equal to 1.

Definition 1 (*RCI Set*)**.** Consider the discrete-time Linear Time-Invariant (LTI) system $x(t+1) = Ax(t) + Bu(t) + w(t)$, with $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $w(t) \in \mathbb{R}^n$ and subject to constraints $u(t) \in \mathbb{U} \subseteq \mathbb{R}^m$ and $w(t) \in \mathbb{W} \subset \mathbb{R}^n$. The set $\mathbb{X} \subseteq \mathbb{R}^n$ is an RCI set with respect to $w(t) \in \mathbb{W}$, if $\forall x(t) \in \mathbb{X}$ there exists $u(t) \in \mathbb{U}$ such that $x(t+1) \in \mathbb{X}$, $\forall w(t) \in \mathbb{W}$.

2. Decentralized MPC for linear systems

We consider the discrete-time LTI system

$$\mathbf{x}^+ = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{u} \in \mathbb{R}^m$ are the state and the input, respectively, at time t and $\mathbf{x}^+$ stands for $\mathbf{x}$ at time $t+1$. The notation $\mathbf{x}(t)$, $\mathbf{u}(t)$ is used only if necessary. The state $\mathbf{x} = (x_{[1]}, \dots, x_{[M]})$ is partitioned into the M vectors $x_{[i]} \in \mathbb{R}^{n_i}$, where $i \in \mathcal{M} = 1:M$ and $n = \sum_{i \in \mathcal{M}} n_i$. Similarly, $\mathbf{u} = (u_{[1]}, \dots, u_{[M]})$ where $u_{[i]} \in \mathbb{R}^{m_i}$, $i \in \mathcal{M}$ and $m = \sum_{i \in \mathcal{M}} m_i$. Let the ith subsystem be given by

$$\Sigma_{[i]} : x^+_{[i]} = A_{ii} x_{[i]} + B_i u_{[i]} + w_{[i]} \tag{2}$$

$$w_{[i]} = \sum_{j \in \mathcal{N}_i} A_{ij} x_{[j]} \tag{3}$$

where $A_{ij} \in \mathbb{R}^{n_i \times n_j}$, $i, j \in \mathcal{M}$, $B_i \in \mathbb{R}^{n_i \times m_i}$ and $\mathcal{N}_i = \{j \in \mathcal{M} : A_{ij} \neq 0, i \neq j\}$ is the set of parents to subsystem i. Subsystems Σ_i are state coupled and input decoupled. Moreover, under the following assumption, they are equivalent to (1).

Assumption 1. Matrix $\mathbf{A}$ is composed by blocks A_{ij}, $i, j \in \mathcal{M}$ and $\mathbf{B} = \text{diag}(B_1, \dots, B_M)$.

We equip subsystems $\Sigma_{[i]}$, $i \in \mathcal{M}$ with the constraints

$$x_{[i]} \in \mathbb{X}_i, \qquad u_{[i]} \in \mathbb{U}_i. \tag{4}$$

Moreover, we define the sets $\mathbb{X} = \prod_{i \in \mathcal{M}} \mathbb{X}_i$, $\mathbb{U} = \prod_{i \in \mathcal{M}} \mathbb{U}_i$ and add to system (1) the constraints

$$\mathbf{x} \in \mathbb{X}, \qquad \mathbf{u} \in \mathbb{U}. \tag{5}$$

We consider the following assumptions.

Assumption 2. The matrix pairs (A_{ii}, B_i) $\forall i \in \mathcal{M}$ are controllable.

Assumption 3. Constraints $\mathbb{X}_i$ and $\mathbb{U}_i$, $i \in \mathcal{M}$ are compact and convex polytopes containing the origin in their nonempty interior.

For the design of suitable decentralized regulators, local controllers are designed following the tube MPC scheme in Raković and Mayne (2005) (see also Rawlings & Mayne, 2009). To this purpose, we treat $w_{[i]} \in \mathbb{W}_i = \bigoplus_{j \in \mathcal{N}_i} A_{ij} \mathbb{X}_j$ as a disturbance and define the nominal (unperturbed) system $\hat{\Sigma}_{[i]}$ as

$$\hat{\Sigma}_{[i]} : \hat{x}^+_{[i]} = A_{ii} \hat{x}_{[i]} + B_i v_{[i]} \tag{6}$$

where $v_{[i]} \in \mathbb{R}^{m_i}$ is the input. We want to confine $x_{[i]}$ in a tube of section $\mathbb{Z}_i$ centered in $\hat{x}_{[i]}$, i.e. to obtain that

$$x_{[i]}(0) \in \hat{x}_{[i]}(0) \oplus \mathbb{Z}_i \Rightarrow x_{[i]}(t) \in \hat{x}_{[i]}(t) \oplus \mathbb{Z}_i, \quad \forall t \ge 0. \tag{7}$$

This can be achieved (Raković & Mayne, 2005) if (a) $\mathbb{Z}_i$ is a nonempty RCI set for the constrained subsystem (2) with respect to the disturbance w_i; (b) for $\bar{x} = \hat{x}$ the local controller

$$\mathcal{C}_{[i]} : u_{[i]} = v_{[i]} + \bar{\kappa}_i (x_{[i]} - \bar{x}_{[i]}) \tag{8}$$

is used, where $\bar{\kappa}_i : \mathbb{Z}_i \to \mathbb{U}_i$ is any feedback control law[2] guaranteeing $x_{[i]} \in \mathbb{Z}_i \Rightarrow x^+_{[i]} \in \mathbb{Z}_i$, $\forall w \in \mathbb{W}$.

Following Raković and Mayne (2005), in (8) we set

$$v_{[i]}(t) = v_{[i]}(0|t), \qquad \bar{x}_{[i]}(t) = \hat{x}_{[i]}(0|t) \tag{9}$$

where $v_{[i]}(0|t)$ and $\hat{x}_{[i]}(0|t)$ are optimal values of the variables $v_{[i]}(0)$ and $\hat{x}_{[i]}(0)$, respectively, appearing in the MPC-i problem $\mathbb{P}_i^N(x_{[i]}(t))$

$$\min_{\substack{v_{[i]}(0:N_i-1) \\ \hat{x}_{[i]}(0)}} \sum_{k=0}^{N_i-1} \ell_i(\hat{x}_{[i]}(k), v_{[i]}(k)) + V_{f_i}(\hat{x}_{[i]}(N_i)) \tag{10a}$$

$$x_{[i]}(t) - \hat{x}_{[i]}(0) \in \mathbb{Z}_i \tag{10b}$$

[2] Definition 1 guarantees the existence of a function $\bar{\kappa}_i$.

$$\hat{x}_{[i]}(k+1) = A_{ii}\hat{x}_{[i]}(k) + B_i v_{[i]}(k), \quad k \in 0 : N_i - 1 \tag{10c}$$

$$\hat{x}_{[i]}(k) \in \hat{\mathbb{X}}_i, \quad k \in 0 : N_i - 1 \tag{10d}$$

$$v_{[i]}(k) \in \mathbb{V}_i, \quad k \in 0 : N_i - 1 \tag{10e}$$

$$\hat{x}_{[i]}(N_i) \in \hat{\mathbb{X}}_{f_i}. \tag{10f}$$

In (10), $N_i \in \mathbb{N}$ is the control horizon, $\ell_i : \mathbb{R}^{n_i \times m_i} \to \mathbb{R}_+$ is the stage cost, $V_{f_i} : \mathbb{R}^{n_i} \to \mathbb{R}_+$ is the final cost and $\hat{\mathbb{X}}_{f_i}$ is the terminal set. Moreover, from (10c), the nominal system $\hat{\Sigma}_i$, equipped with suitable constraints $\hat{x}_{[i]} \in \hat{\mathbb{X}}_i$ and $v_{[i]} \in \mathbb{V}_i$, is used for obtaining the state predictions over the control horizon. We highlight that in (10b) the initial state of the nominal system is an optimization variable: comments on this feature are provided in Raković and Mayne (2005).

As shown in Raković and Mayne (2005), constraints (4) can be fulfilled using (8)–(10) if there exist sets $\hat{\mathbb{X}}_i$ and $\mathbb{V}_i$, $i \in \mathcal{M}$ verifying

$$\hat{\mathbb{X}}_i \oplus \mathbb{Z}_i \subseteq \mathbb{X}_i, \qquad \mathbb{V}_i \oplus \mathbb{U}_{z_i} \subseteq \mathbb{U}_i \tag{11}$$

where $\mathbb{U}_{z_i} = \bar{\kappa}_i(\mathbb{Z}_i)$. The existence of such sets is guaranteed by the following assumption.

Assumption 4. There exist $\rho_{i,1} > 0$, $\rho_{i,2} > 0$ such that $\mathbb{Z}_i \oplus B_{\rho_{i,1}}(0) \subseteq \mathbb{X}_i$ and $\mathbb{U}_{z_i} \oplus B_{\rho_{i,2}}(0) \subseteq \mathbb{U}_i$, where $B_{\rho_{i,1}}(0) \subset \mathbb{R}^{n_i}$ and $B_{\rho_{i,2}}(0) \subset \mathbb{R}^{m_i}$.

Assumption 4 implies that the coupling of subsystems connected in a cyclic fashion must be sufficiently small. As an example, for two subsystems Σ_1 and Σ_2 where each one is the parent of the other one, Assumption 4 implies that $\mathbb{Z}_1 \subseteq \mathbb{X}_1$ and $\mathbb{Z}_2 \subseteq \mathbb{X}_2$. Since, by construction, $\mathbb{Z}_i \supseteq \mathbb{W}_i$, one has $A_{21}\mathbb{X}_1 \subseteq \mathbb{X}_2$ and $A_{12}\mathbb{X}_2 \subseteq \mathbb{X}_1$ that implies $A_{12}A_{21}\mathbb{X}_1 \subseteq \mathbb{X}_1$. These conditions are similar to the ones arising in the small gain theorem for networks (Dashkovskiy, Rüffer, & Wirth, 2007).

In order to stabilize the origin of the closed-loop system, we introduce a customary assumption in MPC (Rawlings & Mayne, 2009).

Assumption 5. For all $i \in \mathcal{M}$, there exist an auxiliary control law $\kappa_i^{\mathrm{aux}}(\hat{x}_{[i]})$ and a $\mathcal{K}_\infty$ function $\mathcal{B}_i$ such that:

(i) $\ell_i(\hat{x}_{[i]}, v_{[i]}) \geq \mathcal{B}_i(\|(\hat{x}_{[i]}, v_{[i]})\|)$, for all $\hat{x}_{[i]} \in \mathbb{R}^{n_i}$, $v_{[i]} \in \mathbb{R}^{m_i}$ and $\ell_i(0, 0) = 0$;
(ii) $\hat{\mathbb{X}}_{f_i} \subseteq \hat{\mathbb{X}}_i$ is an invariant set for $\hat{x}^+_{[i]} = A_{ii}\hat{x}_{[i]} + B_i\kappa_i^{\mathrm{aux}}(\hat{x}_{[i]})$;
(iii) $\forall \hat{x}_{[i]} \in \hat{\mathbb{X}}_{f_i}$, $\kappa_i^{\mathrm{aux}}(\hat{x}_{[i]}) \in \mathbb{V}_i$;
(iv) $\forall \hat{x}_{[i]} \in \hat{\mathbb{X}}_{f_i}$, $V_{f_i}(\hat{x}^+_{[i]}) - V_{f_i}(\hat{x}_{[i]}) \leq -\ell_i(\hat{x}_{[i]}, \kappa_i^{\mathrm{aux}}(\hat{x}_{[i]}))$.

We highlight that there are several methods, discussed e.g. in Rawlings and Mayne (2009), for computing $\ell_i(\cdot)$, $V_{f_i}(\cdot)$ and $\mathbb{X}_{f_i}$ verifying Assumption 5.

In summary, the controller $\mathcal{C}_{[i]}$ is given by (8)–(10) and depends upon quantities of system $\Sigma_{[i]}$ only. Therefore the collective controller for (1) is decentralized. The main problem that still has to be solved in the design of local controllers is the following one.

Problem $\mathcal{P}$. Compute nonempty RCIs $\mathbb{Z}_i$, $i \in \mathcal{M}_i$ for (2), if they exist, verifying Assumption 4. □

In the next section we show how to solve Problem $\mathcal{P}$, under Assumptions 2 and 3 through a distributed and computationally efficient algorithm based on LP. We also show how sets $\hat{\mathbb{X}}_i$ and $\mathbb{V}_i$ verifying (11) and functions $\bar{\kappa}_i$ in (8) can be readily computed.

3. Decentralized synthesis of DeMPC

From Assumption 3 we define the sets $\mathbb{X}_i$ and $\mathbb{U}_i$ as

$$\mathbb{X}_i = \{x_{[i]} \in \mathbb{R}^{n_i} : c^T_{x_{i,r}} x_{[i]} \leq 1, \ \forall r \in 1 : g_i\} \tag{12}$$

$$\mathbb{U}_i = \{u_{[i]} \in \mathbb{R}^{m_i} : c^T_{u_{i,r}} u_{[i]} \leq 1, \ \forall r \in 1 : l_i\} \tag{13}$$

where $c_{x_{i,r}} \in \mathbb{R}^{n_i}$ and $c_{u_{i,r}} \in \mathbb{R}^{m_i}$. Using the procedure proposed in Section VI of Raković and Baric (2010), we compute an RCI set $\mathbb{Z}_i \subset \mathbb{X}_i$ using an appropriate parametrization, i.e., we define the set of variables θ_i as

$$\theta_i = \{\bar{z}^{(s,f)}_{[i]} \in \mathbb{R}^{n_i} \quad \forall s \in \mathcal{A}^5_i, \ \forall f \in \mathcal{A}^1_i; \tag{14a}$$

$$\bar{u}^{(s,f)}_{[i]} \in \mathbb{R}^{m_i} \quad \forall s \in \mathcal{A}^3_i, \ \forall f \in \mathcal{A}^1_i; \tag{14b}$$

$$\rho_i^{(f_1,f_2)} \in \mathbb{R} \quad \forall f_1 \in \mathcal{A}^1_i, \ \forall f_2 \in \mathcal{A}^1_i; \tag{14c}$$

$$\psi_i^{(\tau,s)} \in \mathbb{R} \quad \forall r \in \mathcal{A}^2_i, \ \forall s \in \mathcal{A}^3_i; \tag{14d}$$

$$\gamma_i^{(\tau,s)} \in \mathbb{R} \quad \forall r \in \mathcal{A}^4_i, \ \forall s \in \mathcal{A}^3_i; \tag{14e}$$

$$\alpha_i \in \mathbb{R}\} \tag{14f}$$

with $\mathcal{A}^1_i = 1 : q_i$, $\mathcal{A}^2_i = 1 : \tau^u_i$, $\mathcal{A}^3_i = 0 : k_i - 1$, $\mathcal{A}^4_i = 1 : \tau^x_i$ and $\mathcal{A}^5_i = 0 : k_i$, where k_i, $q_i \in \mathbb{N}$ are parameters of the procedure that can be chosen by the user as well as the set

$$\bar{\mathbb{Z}}^0_i = \mathrm{convh}(\{\bar{z}^{(0,f)}_{[i]} \in \mathbb{R}^{n_i}, \ \forall f \in \mathcal{A}^1_i\}) \tag{15}$$

where $\bar{z}^{(0,1)}_{[i]} = 0$. Let us define the sets

$$\bar{\mathbb{Z}}^s_i = \mathrm{convh}(\{\bar{z}^{(s,f)}_{[i]} \in \mathbb{R}^{n_i}, \ \forall f \in \mathcal{A}^1_i\}), \quad \forall s \in \mathcal{A}^5_i, \tag{16}$$

$$\bar{\mathbb{U}}^s_{z_i} = \mathrm{convh}(\{\bar{u}^{(s,f)}_{[i]} \in \mathbb{R}^{m_i}, \ \forall f \in \mathcal{A}^1_i\}), \quad \forall s \in \mathcal{A}^3_i, \tag{17}$$

where $\bar{z}^{(s,1)}_{[i]} = 0$, $\bar{u}^{(p,1)}_{[i]} = 0$, and consider the following set of affine constraints on the decision variable θ_i

$$\Theta_i = \{\theta_i :$$

$$\alpha_i < 1, \qquad -\alpha_i \leq 0 \tag{18a}$$

$$z_{[i]}{}^{(k_i,f_1)} = \sum_{f_2=1}^{q_i} \rho_i^{(f_1,f_2)} z_{[i]}{}^{(0,f_2)}, \quad \forall f_1 \in \mathcal{A}^1_i; \tag{18b}$$

$$-\alpha_i + \sum_{f_2=1}^{q_i} \rho_i^{(f_1,f_2)} \leq 0, \quad \forall f_1 \in \mathcal{A}^1_i; \tag{18c}$$

$$-\rho_i^{(f_1,f_2)} \leq 0, \quad \forall f_1 \in \mathcal{A}^1_i, \ \forall f_2 \in \mathcal{A}^1_i; \tag{18d}$$

$$\sum_{s=0}^{k_i-1} \psi_i^{(\tau,s)} + \alpha_i < 1, \quad \forall \tau \in \mathcal{A}^2_i; \tag{18e}$$

$$c^T_{u_{i,\tau}} \bar{u}^{(s,f)}_{[i]} \leq \psi_i^{(\tau,s)}, \quad \forall \tau \in \mathcal{A}^2_i, \ \forall s \in \mathcal{A}^3_i, \ \forall f \in \mathcal{A}^1_i; \tag{18f}$$

$$\sum_{s=0}^{k_i-1} \gamma_i^{(\tau,s)} + \alpha_i < 1, \quad \forall \tau \in \mathcal{A}^4_i; \tag{18g}$$

$$c^T_{x_{i,\tau}} \bar{z}^{(s,f)}_{[i]} \leq \gamma_i^{(\tau,s)}, \quad \forall \tau \in \mathcal{A}^4_i, \ \forall s \in \mathcal{A}^3_i, \ \forall f \in \mathcal{A}^1_i; \tag{18h}$$

$$\bar{z}^{(s+1,f)}_{[i]} = A_{ii}\bar{z}^{(s,f)}_{[i]} + B_i\bar{u}^{(s,f)}_{[i]}, \quad \forall s \in \mathcal{A}^3_i, \ \forall f \in \mathcal{A}^1_i\}. \tag{18i}$$

We introduce the following assumption on the choice of sets $\bar{\mathbb{Z}}^0_i$.

Assumption 6. The set $\bar{\mathbb{Z}}^0_i$ is such that there is $\omega_i > 0$ verifying $\mathbb{W}_i \oplus B_{\omega_i}(0) \subseteq \bar{\mathbb{Z}}^0_i$.

Note that, since $0 \in \mathbb{W}_i$, Assumption 6 can be fulfilled only if $0 \in \bar{\mathbb{Z}}_i^0$, that is guaranteed by the use of $\bar{z}_{[i]}^{(0,1)} = 0$ in (15). The relation between elements of Θ_i and the RCI sets $\mathbb{Z}_i$ is established in the next proposition.

Proposition 7. *Let Assumptions 2 and 6 hold and sets $\mathbb{X}_i$ and $\mathbb{U}_i$ be defined as in (12) and (13) respectively. Let $k_i > 0$. If there exists $\theta_i \in \Theta_i$, then*

$$\mathbb{Z}_i = (1-\alpha_i)^{-1} \bigoplus_{s=0}^{k_i-1} \bar{\mathbb{Z}}_i^s \tag{19}$$

is an RCI set and the corresponding set $\mathbb{U}_{z_i}$ is given by

$$\mathbb{U}_{z_i} = (1-\alpha_i)^{-1} \bigoplus_{p=0}^{k_i-1} \bar{\mathbb{U}}_{z_i}^p. \tag{20}$$

Proof. The proof directly follows from Section 6-a and Theorem 4.3 of Raković and Baric (2010). □

Remark 1. The feasibility problem (18) is an LP problem, since the constraints in Θ_i are affine.[3] In Raković and Baric (2010) the authors propose to compute $\theta \in \Theta_i$ while minimizing different cost functions. In our context we minimize α_i that, in view of (19), corresponds to the minimization of the size of $\mathbb{Z}_i$. This leads to "bigger" constraint sets $\hat{\mathbb{X}}_i$ and $\mathbb{V}_i$ in (10) (see (21) below). Also note that the inclusion of $\bar{z}_{[i]}^{(s,1)} = 0$ in the definition of $\bar{\mathbb{Z}}_i^s$, $s \in 0 : k_i$, ensures that all sets $\bar{\mathbb{Z}}_i^s$ contain the origin and hence, under Assumption 6, $\mathbb{Z}_i$ contains the origin in its nonempty interior. If $\mathbb{W}_i$ is full dimensional, we can set $\bar{\mathbb{Z}}_i^0 = (1+\epsilon)\mathbb{W}_i$, with $\epsilon > 0$ sufficiently small.

We highlight that the set of constraints Θ_i depends only upon local fixed parameters $\{A_{ii}, B_i, \mathbb{X}_i, \mathbb{U}_i\}$, fixed parameters $\{A_{ij}, \mathbb{X}_j\}_{j\in\mathcal{N}_i}$ of parents of $\hat{\Sigma}_{[i]}$ (because from Assumption 6 the set $\bar{\mathbb{Z}}_i^0$ must be chosen in a way such that $\bar{\mathbb{Z}}_i^0 \supseteq \mathbb{W}_i = \bigoplus_{j\in\mathcal{N}_i} A_{ij}\mathbb{X}_j$) and local tunable parameters θ_i (the decision variables (14)). Moreover, Θ_i does not depend on tunable parameters of parents. This implies that the computation of sets $\mathbb{Z}_i$ and $\mathbb{U}_{z_i}$ in (19) and (20) does not influence the choice of $\mathbb{Z}_j$ and $\mathbb{U}_{z_j}$, $j \neq i$. Therefore Problem $\mathcal{P}$ is decomposed in the following independent LP problems for $i \in \mathcal{M}$.

Problem $\mathcal{P}_i$. Solve the feasibility LP problem $\theta_i \in \Theta_i$.

If Problem $\mathcal{P}_i$ is solved, then we can compute sets $\hat{\mathbb{X}}_i$ and $\mathbb{V}_i$ in (10d) and (10e) as

$$\hat{\mathbb{X}}_i = \mathbb{X}_i \ominus \mathbb{Z}_i, \qquad \mathbb{V}_i = \mathbb{U}_i \ominus \mathbb{U}_{z_i}. \tag{21}$$

The overall procedure for the decentralized synthesis of local controllers $\mathcal{C}_{[i]}$, $i \in \mathcal{M}$ is given in Algorithm 1, the properties of which are summarized in the next proposition.

Proposition 8. *Under Assumptions 2 and 3 if, for all $i \in \mathcal{M}$, controllers $\mathcal{C}_{[i]}$ are designed according to Algorithm 1, then also Assumptions 4–6 are verified.*

Proof. Assumptions 5 and 6 are enforced in Steps (iii-iii) and (i) of Algorithm 1, respectively. As for Assumption 4, because of the inequality in (18e), constraints (18e)–(18f) guarantee the existence of $\rho_{i,2} > 0$ such that $\mathbb{U}_{z_i} \oplus B_{\rho_{i,2}}(0) \subseteq \mathbb{U}_i$. Similarly, because of the inequality in (18g), one has that (18g) and (18h) imply the existence of $\rho_{i,1} > 0$ such that $\mathbb{Z}_i \oplus B_{\rho_{i,1}}(0) \subseteq \mathbb{X}_i$. □

If in Step (ii) of Algorithm 1 the LP problem is infeasible, we can restart it with a different k_i, although there is no guarantee that the

[3] As customary in optimization, strict inequalities (18e) and (18g) can be replaced by nonstrict ones using small positive tolerances.

Algorithm 1 Design of controller $\mathcal{C}_{[i]}$ for system $\Sigma_{[i]}$

Input: A_{ii}, B_i, $\mathbb{X}_i$, $\mathbb{U}_i$, $\mathcal{N}_i$, $\{A_{ij}\}_{j\in\mathcal{N}_i}$, $\{\mathbb{X}_j\}_{j\in\mathcal{N}_i}$, $k_i > 0$
Output: controller $\mathcal{C}_{[i]}$ given by (8), (9) and (10)

(i) Compute the set $\mathbb{W}_i = \bigoplus_{j\in\mathcal{N}_i} A_{ij}\mathbb{X}_j$ and choose $\bar{\mathbb{Z}}_i^0$ such that $\mathbb{X}_i \supseteq \bar{\mathbb{Z}}_i^0 \supseteq \mathbb{W}_i \oplus B_{\omega_i}(0)$ for a sufficiently small $\omega_i > 0$. If $\bar{\mathbb{Z}}_i^0$ does not exist **stop** (the controller $\mathcal{C}_{[i]}$ cannot be designed)
(ii) Solve the feasibility LP problem $\theta_i \in \Theta_i$. If it is infeasible **stop** (the controller $\mathcal{C}_{[i]}$ cannot be designed).
(iii) Design controller MPC-i by
 (iii-i) Computing $\mathbb{Z}_i$ as in (19) and $\mathbb{U}_{z_i}$ as in (20).
 (iii-ii) Computing $\hat{\mathbb{X}}_i$ and $\mathbb{V}_i$ as in (21).
 (iii-iii) Choosing $\ell_i(\cdot)$, $V_{f_i}(\cdot)$ and $\mathbb{X}_{f_i}$ verifying Assumption 5.
(iv) Choose the function $\bar{\kappa}_i$ in (8).

LP problem is feasible for some values of k_i (Raković & Baric, 2010). Steps (iii)-(i) and (iii)-(ii) of Algorithm 1, which provide constraints in (10), are the most computationally expensive because they involve Minkowski sums and differences of polytopic sets. In the next sections we show how to avoid burdensome computations exploiting results from Raković and Baric (2010) and how to compute a suitable function $\bar{\kappa}_i$ through LP.

3.1. Implicit representation of sets $\mathbb{Z}_i$ and $\mathbb{U}_{z_i}$

In this section we show how to rewrite the constraints (10b) by exploiting the implicit representation of set $\mathbb{Z}_i$ proposed in Section VI.B of Raković and Baric (2010). Recalling (19), we have that $\tilde{z}_{[i]}^s \in \bar{\mathbb{Z}}_i^s$ if, for all $f \in 1 : q_i$, $\exists \beta_i^{(s,f)} \geq 0$ such that

$$\sum_{f=1}^{q_i} \beta_i^{(s,f)} = 1, \qquad \tilde{z}_{[i]}^s = \sum_{f=1}^{q_i} \beta_i^{(s,f)} \bar{z}_{[i]}^{(s,f)}.$$

Hence, $x_{[i]}(t) - \hat{x}_{[i]}(0|t) \in \mathbb{Z}_i$ if and only if, for all $f \in 1 : q_i$ and $s \in 0 : k_i - 1$, there exist $\beta_i^{(s,f)} \in \mathbb{R}$ such that

$$\begin{aligned} &\beta_i^{(s,f)} \geq 0, \quad \sum_{f=1}^{q_i} \beta_i^{(s,f)} = 1 \\ &x_{[i]}(t) - \hat{x}_{[i]}(0|t) = (1-\alpha_i)^{-1} \sum_{s=0}^{k_i-1} \sum_{f=1}^{q_i} \beta_i^{(s,f)} \bar{z}_{[i]}^{(s,f)}. \end{aligned} \tag{22}$$

In other words we add to the optimization problem (10) the variables $\beta_i^{(s,f)}$ and replace (10b) with constraints (22).

With similar arguments, we can also provide an implicit representation of sets $\mathbb{U}_{z_i}$. In particular, we have that $u_{z[i]} \in \mathbb{U}_{z_i}$ if and only if $\forall f \in 1 : q_i$, $\forall s \in 0 : k_i - 1$ there exist $\phi_i^{(s,f)} \in \mathbb{R}$ such that

$$\phi_i^{(s,f)} \geq 0 \tag{23a}$$

$$\sum_{f=1}^{q_i} \phi_i^{(s,f)} = 1 \tag{23b}$$

$$u_{z[i]} = (1-\alpha_i)^{-1} \sum_{s=0}^{k_i-1} \sum_{f=1}^{q_i} \phi_i^{(s,f)} \bar{u}_{[i]}^{(s,f)}. \tag{23c}$$

3.2. Computation of sets $\hat{\mathbb{X}}_i$ and $\mathbb{V}_i$

In this section we show how to compute sets $\hat{\mathbb{X}}_i$ and $\mathbb{V}_i$ in (21) using the implicit representation of $\mathbb{Z}_i$ and $\mathbb{U}_{z_i}$.

Using (19) we can rewrite $\hat{\mathbb{X}}_i = \mathbb{X}_i \ominus (1-\alpha_i)^{-1} \bigoplus_{s=0}^{k_i-1} \bar{\mathbb{Z}}_i^s$. Recalling that sets $\bar{\mathbb{Z}}_i^s$, $s \in 0 : k_i - 1$ are defined as the convex hull of points $\bar{z}_{[i]}^{(s,f)}$, $f \in 1 : q_i$, we can compute the set $\hat{\mathbb{X}}_i$ using Algorithm 2. In particular, the operation in Step (ii)-(ii) of Algorithm 2 amounts to LP problems. In a similar way we can compute the set $\mathbb{V}_i$ using the implicit representation of $\mathbb{U}_{z_i}$ given in (23). Indeed, it suffices to use Algorithm 2 replacing $\mathbb{X}_i$ with $\mathbb{U}_i$, defined in (13), and points $\bar{z}_{[i]}^{(s,f)}$ with points $\bar{u}_{[i]}^{(s,f)}$, $\forall s \in 0 : k_i - 1, \forall f \in 1 : q_i$.

Algorithm 2

Input: set $\mathbb{X}_i$ defined as in (12), points $\bar{z}_{[i]}^{(s,f)}$, $\forall s \in 0 : k_i - 1$, $\forall f \in 1 : q_i$ and scalar α_i.
Output: set $\hat{\mathbb{X}}_i$.

(i) $\bar{C}_i = (c_{x_{i,1}}^T, \ldots, c_{x_{i,g_i}}^T) \in \mathbb{R}^{g_i \times n_i}$ and $\bar{D}_i = \mathbf{1}$
(ii) **For each** $s \in 0 : k_i - 1$
  (ii-i) **For each** $f \in 1 : q_i$
  $\bar{C}_i = (\bar{C}_i, \bar{C}_i)$ and $\bar{D}_i = (\bar{D}_i, \bar{D}_i - (1-\alpha_i)^{-1}\bar{C}_i \bar{z}_{[i]}^{(s,f)})$
  (ii-ii) Remove redundant constraints from $\bar{C}_i \hat{x}_{[i]} \leq \bar{D}_i$ so obtaining the inequalities $\bar{C}_i \hat{x}_{[i]} \leq \bar{D}_i$
(iii) Set $\hat{\mathbb{X}}_i = \{\hat{x}_{[i]} : \bar{C}_i \hat{x}_{[i]} \leq \bar{D}_i\}$ where $\bar{C}_i \in \mathbb{R}^{\hat{g}_i \times n_i}$ and $\bar{D}_i \in \mathbb{R}^{\hat{g}_i}$

3.3. Evaluation of the control law $\bar{\kappa}_i(\cdot)$

Recalling (8), since $\bar{\kappa}_i(\cdot)$ depends on $\hat{x}_{[i]}$, one has to solve the MPC-i problem (10) first and then evaluate $\bar{\kappa}_i(z_{[i]})$. The control law $\bar{\kappa}(z_{[i]}) \in \mathbb{U}_{z_i}$ guarantees that if $x_{[i]}(t) - \hat{x}_{[i]}(0|t) \in \mathbb{Z}_i$ (i.e. MPC-i problem (10) is feasible), then there is $\lambda_i \in [0,1]$ such that $x_{[i]}(t+1) - \hat{x}_{[i]}(1|t) \in \lambda_i \mathbb{Z}_i$. In order to minimize the contractivity parameter λ_i and take advantage of the representation (22) of the set $\mathbb{Z}_i$, we exploit the results of Blanchini (1991) and Raković and Baric (2010) and propose to compute $\bar{\kappa}_i(z_{[i]})$ solving the following LP problem:

$$\bar{\mathbb{P}}_i(z_{[i]}) : \quad \min_{\mu, \beta_i^{(s,f)}} \mu \tag{24a}$$

$$\beta_i^{(s,f)} \geq 0, \quad \forall f \in 1 : q_i, \ \forall s \in 0 : k_i - 1 \tag{24b}$$

$$\sum_{f=1}^{q_i} \beta_i^{(s,f)} = \mu, \quad \forall s \in 0 : k_i - 1 \tag{24c}$$

$$\mu \geq 0 \tag{24d}$$

$$z_{[i]} = (1-\alpha_i)^{-1} \sum_{s=0}^{k_i-1} \sum_{f=1}^{q_i} \beta_i^{(s,f)} \bar{z}_{[i]}^{(s,f)} \tag{24e}$$

and setting

$$\begin{aligned} \bar{\kappa}_i(z_{[i]}) &= (1-\alpha_i)^{-1} \sum_{s=0}^{k_i-1} \bar{\kappa}_i^s(z_{[i]}) \\ \bar{\kappa}_i^s(z_{[i]}) &= \sum_{f=1}^{q_i} \bar{\beta}_i^{(s,f)} \bar{u}_{[i]}^{(s,f)} \end{aligned} \tag{25}$$

where $\bar{\beta}_i^{(s,f)}$ are the optimizers to (24) and $\bar{u}_{[i]}^{(s,f)}$ are defined in (14b) (see also (17)).

Remark 2. In Riverso et al. (2013b) we used the control law (8) with the linear function $\bar{\kappa}_i(x_{[i]} - \hat{x}_{[i]}) = K_i(x_{[i]} - \hat{x}_{[i]})$, $K_i \in \mathbb{R}^{m_i \times n_i}$. This choice has the disadvantage of requiring the computation of matrices K_i, $i \in \mathcal{M}$ through the solution to nonlinear optimization problems, fulfilling a global stability assumption. Differently, as shown in Section 3.4, we guarantee stability for the closed-loop collective system through the computation of suitable RCI sets $\mathbb{Z}_i$ and functions $\bar{\kappa}_i$ solving LP problems only.

3.4. Analysis of the closed-loop system

Defining the collective variables $\hat{\mathbf{x}} = (\hat{x}_{[1]}, \ldots, \hat{x}_{[M]}) \in \mathbb{R}^n$, $\mathbf{v} = (v_{[1]}, \ldots, v_{[M]}) \in \mathbb{R}^m$ and the function $\bar{\kappa}(x) = (\bar{\kappa}_1(x_{[1]}), \ldots, \bar{\kappa}_M(x_{[M]})) : \mathbb{R}^n \to \mathbb{R}^m$, from (2) and (8) one obtains the collective model

$$\mathbf{x}^+ = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{v} + \mathbf{B}\bar{\kappa}(\mathbf{x} - \hat{\mathbf{x}}). \tag{26}$$

The next theorem, proved in Appendix A in Riverso et al. (2012), summarizes the key properties of the closed-loop system (26).

Theorem 9. *Let Assumptions 2 and 3 hold. Assume controllers $\mathcal{C}_{[i]}$ in (8) are computed using Algorithm 1 and let the function $\bar{\kappa}_i$ be given by (25). Then, the origin of (26) is asymptotically stable, $\mathbb{X}^N = \prod_{i \in \mathcal{M}} \mathbb{X}_i^N$, with*

$$\mathbb{X}_i^N = \{s_{[i]} \in \mathbb{X}_i : (10) \text{ is feasible for } x_{[i]}(t) = s_{[i]}\},$$

is a region of attraction and $\mathbf{x}(0) \in \mathbb{X}^N$ guarantees constraints (5) are fulfilled at all time instants.

The complete proof of Theorem 9 has been omitted for strict space constraints and it can be found in Riverso et al. (2012). Next, we just provide a sketch of it. By using standard arguments in MPC theory, from Assumption 5 and constraints (10b)–(10f) one can show recursive feasibility (i.e. that $x_{[i]}(t) \in \mathbb{X}_i^N$, $\forall i \in \mathcal{M}$ implies $x_{[i]}(t+1) \in \mathbb{X}_i^N$) and that $\hat{x}_{[i]}(0|t) \to 0$ and $v_{[i]}(0|t) \to 0$ as $t \to \infty$. Proving the state converges to the origin is much more challenging and requires two separate steps relying on set-theoretical and geometric arguments. In the first step we prove that, if $\mathbf{x}(0) \in \mathbb{X}^N$, there is $\tilde{T} > 0$ such that $\mathbf{x}(\tilde{T}) \in \mathbb{Z} = \prod_{i \in \mathcal{M}} \mathbb{Z}_i$. In the second step we show that, if $\mathbf{x}(\tilde{T}) \in \mathbb{Z}$, then $\mathbf{x}(t) \to 0$ as $t \to +\infty$. The main difference from standard tube MPC, that assumes persistent disturbances, is that the "disturbance" (i.e. coupling terms) affecting each subsystem is influenced by the controllers of parent subsystems. Accounting for these interactions is fundamental for proving closed-loop stability of the origin, which does not hold when disturbances are persistent.

4. PnP operations

In this section we discuss the synthesis of new controllers and the redesign of existing ones when subsystems are added to or removed from system (2). The goal is to preserve stability of the origin and constraint satisfaction for the new closed-loop system. Note that plugging in and unplugging of subsystems are here considered as off-line operations, i.e. they do not induce switching dynamics. As a starting point, we consider a plant composed by subsystems $\Sigma_{[i]}$, $i \in \mathcal{M}$ equipped with local controllers $\mathcal{C}_{[i]}$, $i \in \mathcal{M}$ produced by Algorithm 1. We also define $\mathcal{S}_k = \{i \in \mathcal{M} : k \in \mathcal{N}_i\}$ as the set of children to $\Sigma_{[k]}$.

4.1. Plugging in operation

Assume subsystem $\Sigma_{[M+1]}$, characterized by parameters $A_{M+1\,M+1}$, B_{M+1}, $\mathbb{X}_{M+1}$, $\mathbb{U}_{M+1}$, $\mathcal{N}_{M+1}$ and $\{A_{ij}\}_{j \in \mathcal{N}_{M+1}}$, is plugged in. For building the controller $\mathcal{C}_{[M+1]}$ we execute Algorithm 1 that needs information only from subsystems $\Sigma_{[j]}$, $j \in \mathcal{N}_{M+1}$. If there is no solution to the feasibility LP problem in Step (ii) of Algorithm 1, we declare that $\Sigma_{[M+1]}$ cannot be plugged in. Since each subsystem $\Sigma_{[j]}, j \in \mathcal{S}_{M+1}$ has the new parent $\Sigma_{[M+1]}$, the set $\mathbb{W}_j$ gets

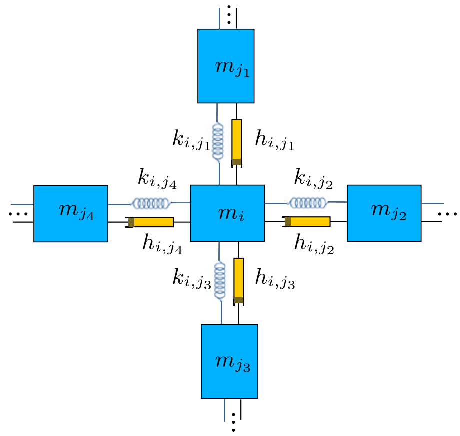


Fig. 1. Array of masses: details of interconnections.

bigger. Therefore the set $\bar{\mathbb{Z}}_j^0$ must be recomputed and the controller $\mathcal{C}_{[j]}$ must be redesigned. Again, if Algorithm 1 stops in Step (ii), we declare that $\Sigma_{[M+1]}$ cannot be plugged in.

Note that redesign of controllers $\mathcal{C}_{[i]}, i \notin \{M+1\} \bigcup \mathcal{S}_{M+1}$ is not required in order to guarantee convergence to zero of the origin and constraint satisfaction for the new closed-loop system.

4.2. Unplugging operation

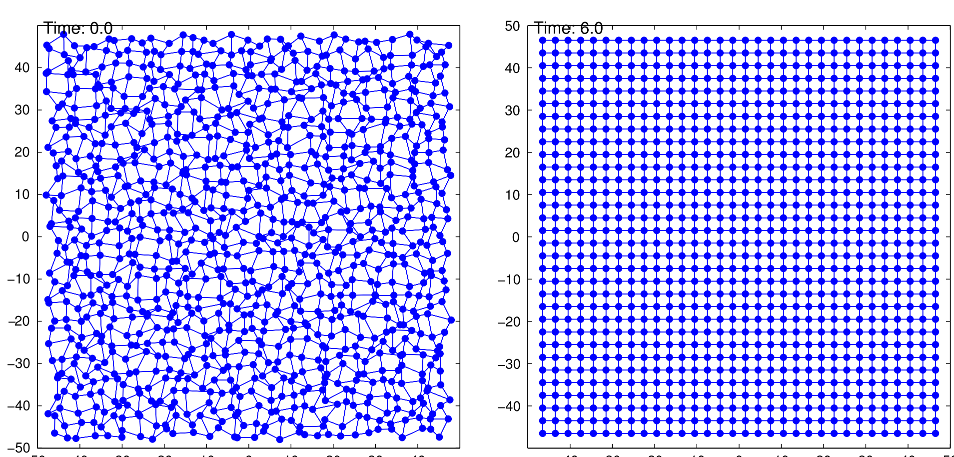
Assume that subsystem $\Sigma_{[k]}$, $k \in \mathcal{M}$ gets unplugged. Since for each $i \in \mathcal{S}_k$ the set $\mathcal{N}_i$ gets smaller, also $\mathbb{W}_i$ gets smaller and the set $\bar{\mathbb{Z}}_i^0$ already computed still verifies the inclusions in Step (i) of Algorithm 1. This means that, for each $i \in \mathcal{S}_k$, the previously computed θ_i in Step (ii) of Algorithm 1 still verifies $\theta_i \in \Theta_i$ and hence the controller $\mathcal{C}_{[i]}$ does not have to be redesigned. Also controllers $\mathcal{C}_{[j]}$, $j \notin \{k\} \bigcup \mathcal{S}_k$ do not have to be redesigned because sets $\mathcal{N}_j$ do not change. However, we highlight that since systems $\Sigma_{[i]}$, $i \in \mathcal{S}_k$ have one parent less, the redesign of controllers $\mathcal{C}_{[i]}$ through Algorithm 1 could improve the performance.

5. Simulation example

We consider a large-scale system composed by 1024 masses coupled as in Fig. 2(b) through springs and dampers arranged as in Fig. 1. Each mass $i \in \mathcal{M} = 1 : 1024$, is a continuous-time subsystem with state $x_{[i]} = (x_{[i,1]}, x_{[i,2]}, x_{[i,3]}, x_{[i,4]})$ and input $u_{[i]} = (u_{[i,1]}, u_{[i,2]})$, where $x_{[i,1]}$ and $x_{[i,3]}$ are the displacements with respect to a given equilibrium position (equilibria lie on a regular grid as in Fig. 2(b)), $x_{[i,2]}$ and $x_{[i,4]}$ are the horizontal and vertical velocities and $100u_{[i,1]}$ (respectively $100u_{[i,2]}$) is the force applied to mass i in the horizontal (respectively, vertical) direction. The values of m_i have been extracted randomly in the interval [5, 10] while spring constants and damping coefficients are identical and equal to 0.5. Subsystems are equipped with the state constraints $\|x_{[i,j]}\|_\infty \leq 1.5$, $j = 1, 3$, $\|x_{[i,l]}\|_\infty \leq 0.8$, $i \in \mathcal{M}$, $l = 2, 4$ and the input constraints $\|u_{[i]}\|_\infty \leq \Gamma_i$, where Γ_i have been generated randomly in the interval [1, 1.5]. We obtain subsystem $\Sigma_{[i]}$ by discretizing continuous-time models with 0.2 s sampling time, using zero-order hold discretization for the local dynamics and treating $x_{[j]}$, $j \in \mathcal{N}_i$ as exogenous signals. We synthesized controllers $\mathcal{C}_{[i]}$, $i \in \mathcal{M}$ using Algorithm 1 and plugging-in a new mass at each iteration. In the worst case the time required to solve Step (ii) of Algorithm 1 is 0.2598 s (best case 0.0140 s). Note also that the use of a centralized MPC is prohibitive since the overall system has $\mathbf{x} \in \mathbb{R}^{4096}$, $\mathbf{u} \in \mathbb{R}^{2048}$ and therefore 8192 + 4096 scalar affine constraints. Modeling, discretization and design of controllers have been performed in MatLab using the *PnPMPC-toolbox* that offers facilities for handling the interconnections of constrained subsystems (Riverso, Battocchio, & Ferrari-Trecate, 2012). In Figs. 2 and 3 we show a simulation where, at time $t = 0$, the masses are still and placed as in Fig. 2(a). At all time steps t, the control action $u_{[i]}(t)$ computed by the controller $\mathcal{C}_{[i]}$, for all $i \in \mathcal{M}$, is kept constant during the sampling interval and applied to the continuous-time system. In the worst case, the computation of the control law (8) requires 0.1047 s on a processor Intel Core i7-2600 3.4 GHz, Ram 8 GB 1.33 GHz running MatLab *r*2011*b*. Convergence is obtained for all masses to their equilibrium position while fulfilling input and state constraints. In Riverso et al. (2012) we show that the coupling is relevant since, by neglecting it in the design of local MPC controllers, recursive feasibility can be compromised. For this large-scale system, we also have considered the use of PnP-DeMPC controllers proposed in Riverso et al. (2013b), but since the design of local controllers requires the solution to nonlinear optimization problems we did not obtain conclusive results after several hours of computation.

6. Conclusions

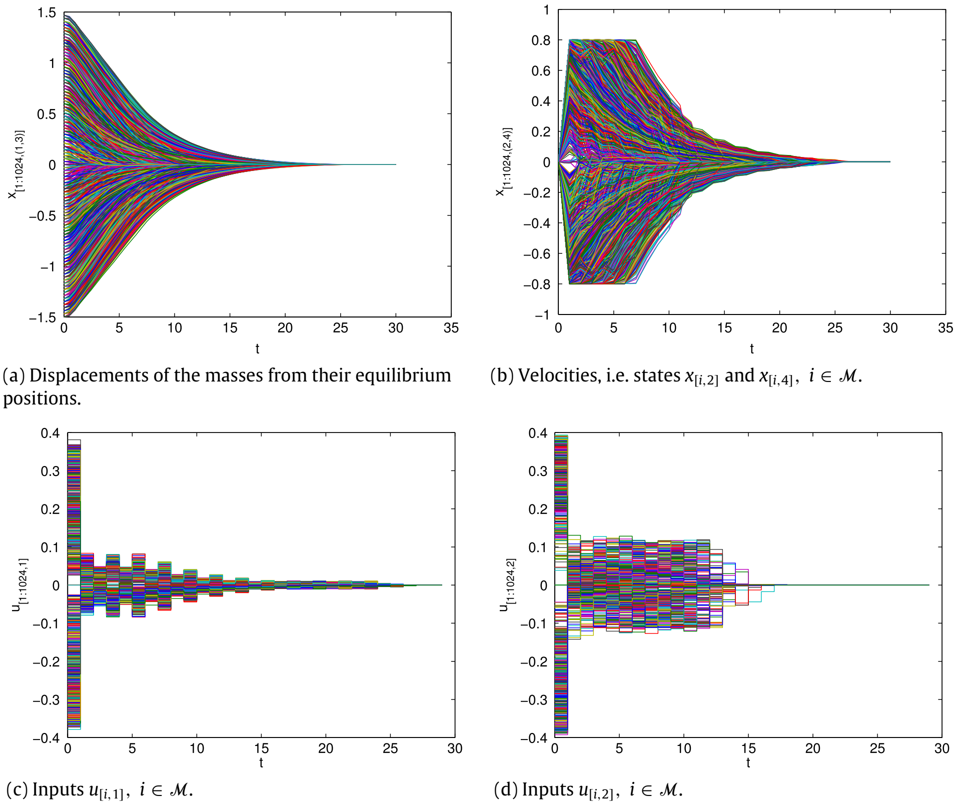
In this paper we proposed a De/DiMPC architecture using the notion of tube MPC based on RCI sets. Our control scheme guarantees closed-loop asymptotic stability and constraints satisfaction at each time instant. The design procedure enables



(a) Position of the masses at initial time.

(b) Position of the masses at time 6 s.

Fig. 2. Position of the 1024 masses on the plane.



(a) Displacements of the masses from their equilibrium positions.

(b) Velocities, i.e. states $x_{[i,2]}$ and $x_{[i,4]}$, $i \in \mathcal{M}$.

(c) Inputs $u_{[i,1]}$, $i \in \mathcal{M}$.

(d) Inputs $u_{[i,2]}$, $i \in \mathcal{M}$.

Fig. 3. State and input trajectories of the 1024 masses with initial position as in Fig. 2(a).

PnP operations, and differently from Riverso et al. (2013b) local controllers are computed solving LP problems only. In order to achieve decentralization of controller design and online operations, two sources of conservativity are introduced: coupling terms are treated as disturbances and there must be an RCI set that is the Cartesian product of local RCI sets. This implies that PnPMPC can be applied only if coupling between subsystems is small enough. To overcome these limitations, in the future we will investigate the use of alternative robust control approaches (see e.g. Blanchini, 1990) as well as the introduction of a coupling attenuation layer based on distributed control architectures. Future works also include the design of output-feedback PnP schemes combining the state-feedback PnP controller proposed in this paper and the state estimator presented in Riverso, Farina, Scattolini, and Ferrari-Trecate (2013).

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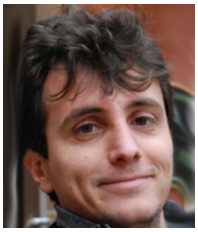

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