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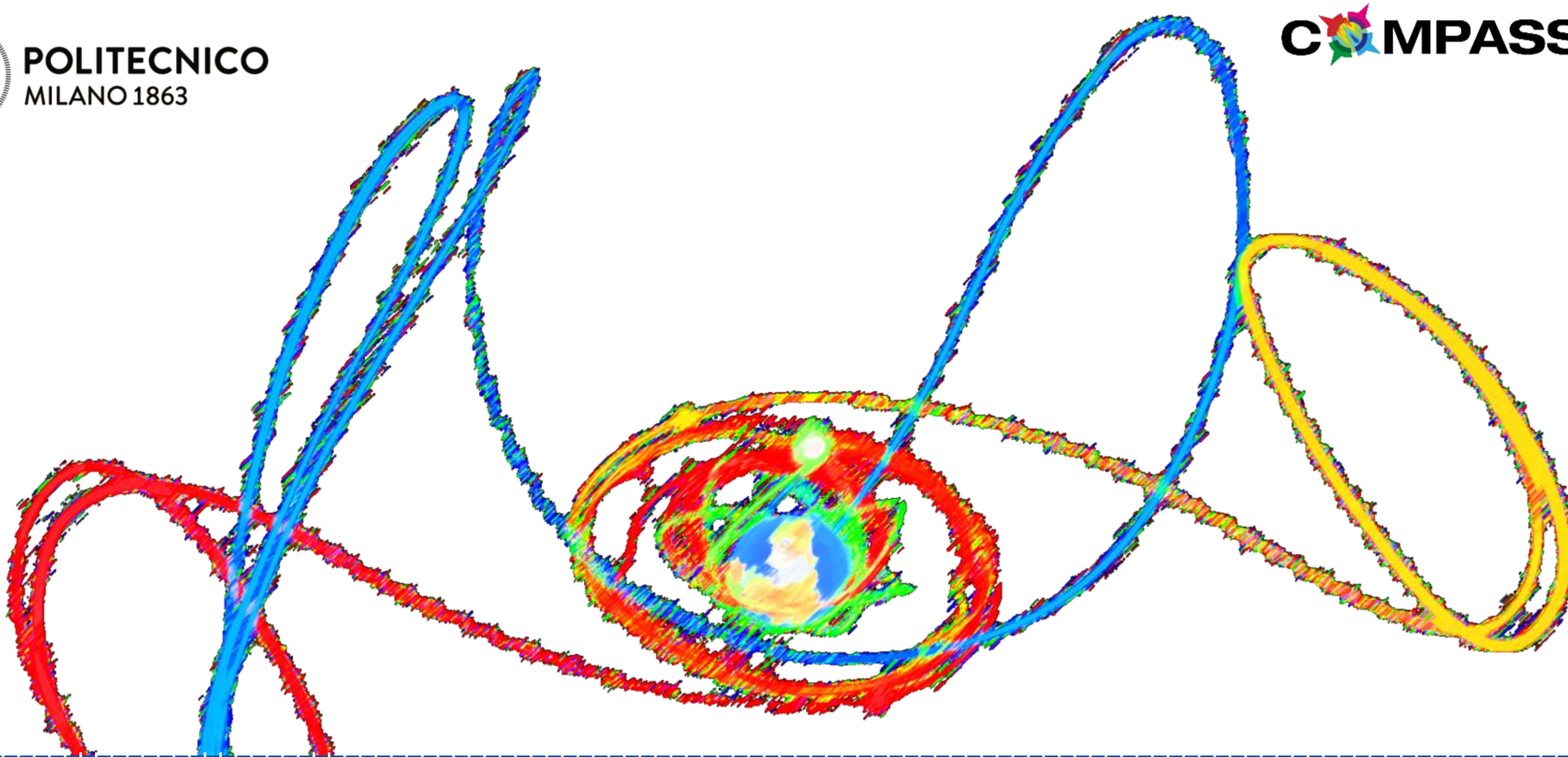

Extension of the King-Hele orbital contraction method and application to the geostationary transfer orbit re-entry prediction

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4th International Workshop on Space Debris Re-entry

ESOC, Darmstadt, 28 February 2018

INTRODUCTION

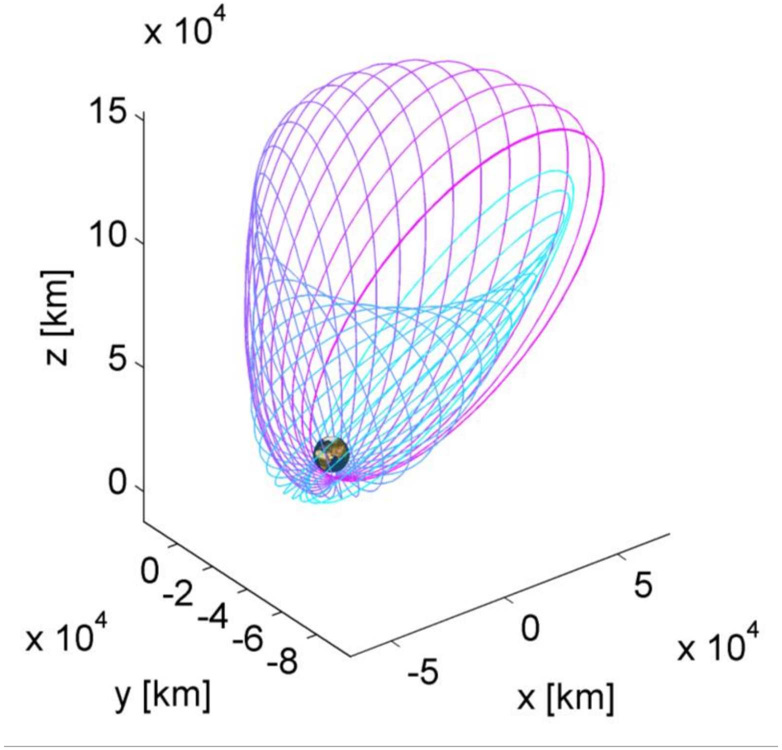
Introduction

Re-entry prediction and precise orbit propagation are a challenging task

- Complex dynamics of orbit perturbations
- Uncertainties related to spacecraft parameters and atmosphere

Semi-analytical techniques can be used:

- Reduce computational time
  - Sensitivity analysis (many initial conditions)
  - Zero-find algorithm for determination
  - Optimisation of disposal manoeuvres
  - Propagation of fragment clouds
- Give accuracy comparable with high fidelity dynamics if model is properly derived

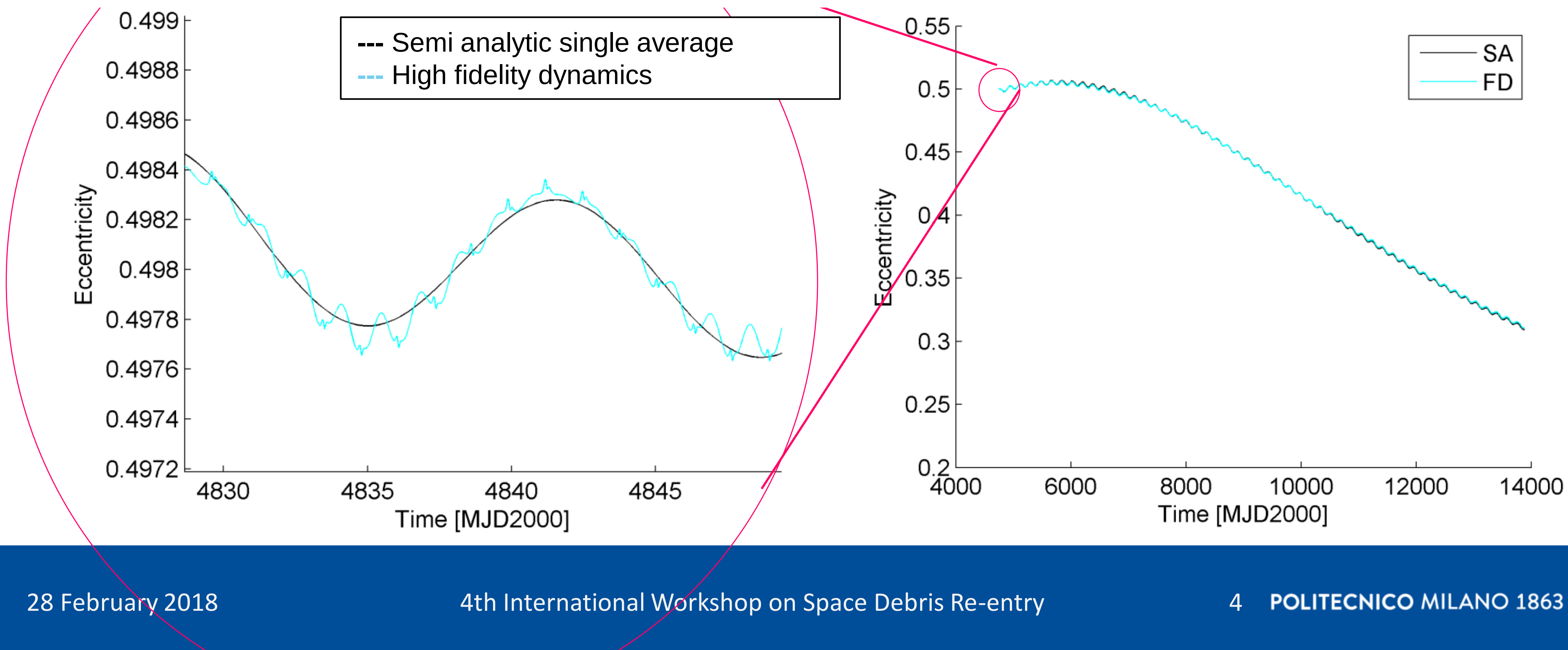


Planetary Orbital Dynamics

Why averaged dynamics

Average variation of orbital elements over one orbit revolution

- Filter high frequency oscillations
- Reduce stiffness of the problem
- Decrease computational time for long term integration



Planetary Orbital Dynamics

PlanODyn suite

Space Debris Evolution, Collision risk, and Mitigation
FP7/EU Marie Curie grant 302270

End-Of-Life Disposal Concepts for Lagrange-Point, Highly Elliptical Orbit missions, **ESA GSP**

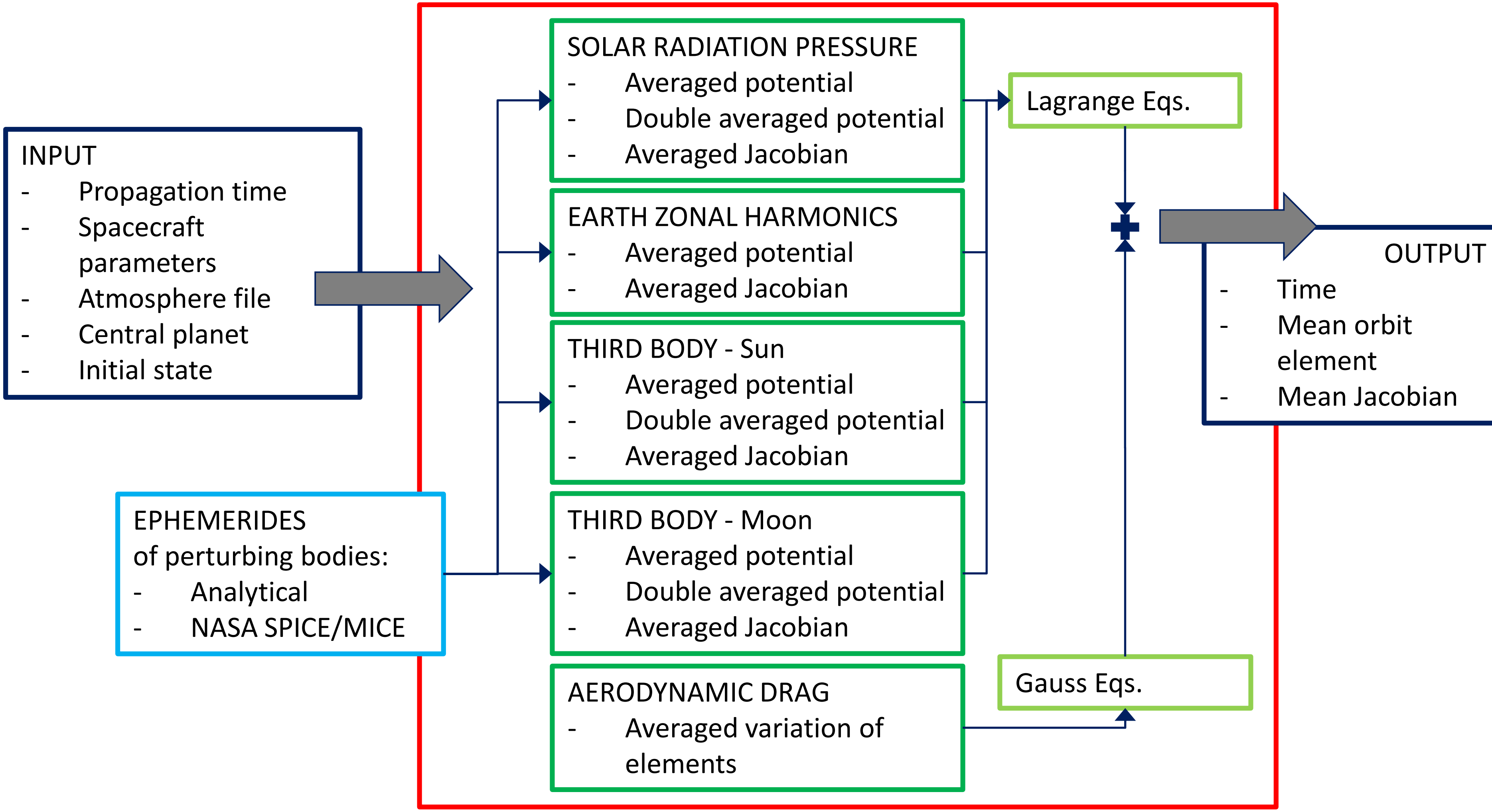
End-Of-Life Disposal Concepts Medium Earth Orbit missions, **ESA GSP**

EOL disposal in "Revolutionary Design of Spacecraft through Holistic Integration of Future Technologies"
ReDSHIFT, H2020

COMPASS, ERC "Control for orbit manoeuvring through perturbations for supplication to space systems"

Planetary Orbital Dynamics

PlanODyn: Planetary Orbital Dynamics



► *Colombo C., "Planetary Orbital Dynamics Suite for Long Term Propagation in Perturbed Environment," ICATT, ESA/ESOC, 2016.*

Planetary Orbital Dynamics

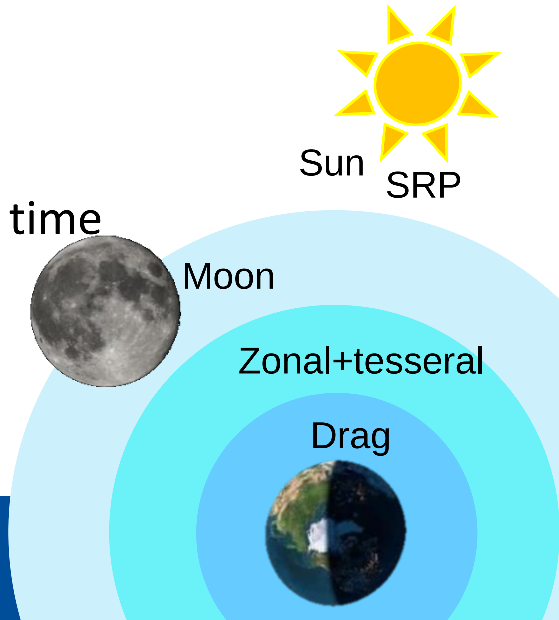
Perturbation in planet centred dynamics

- Atmospheric drag
  - Non-spherical smooth exponential model
  - J_2 short period coupling
- Earth gravity potential
  - Zonal up to order 6 with J_2^2 contribution
  - Tesseral resonant terms
- Solar radiation pressure with cannonball model
- Third body perturbation of the third body (Moon and Sun) up to order 5 in the parallax factor

Ephemerides options

- Analytical approximation based on polynomial expansion in time
- Numerical ephemerides through the NASA SPICE toolkit

Orbital elements in planet centred frame



Planetary Orbital Dynamics

Orbit propagation based on averaged dynamics

For conservative orbit perturbation effects

Disturbing potential function

$$R = R_{\mathrm{SRP}} + R_{\mathrm{zonal}} + R_{3-\mathrm{Sun}} + R_{3-\mathrm{Moon}}$$

Planetary equations in Lagrange form

$$\frac{d\boldsymbol{\alpha}}{dt} = f\left(\boldsymbol{\alpha}, \frac{\partial R}{\partial \boldsymbol{\alpha}}\right) \qquad \boldsymbol{\alpha} = \begin{bmatrix} a & e & i & \Omega & \omega & M \end{bmatrix}^T$$

Average over one orbit revolution of the spacecraft around the primary planet

$$\bar{R} = \bar{R}_{\mathrm{SRP}} + \bar{R}_{\mathrm{zonal}} + \bar{R}_{3-\mathrm{Sun}} + \bar{R}_{3-\mathrm{Moon}}$$

$$\frac{d\bar{\boldsymbol{\alpha}}}{dt} = f\left(\bar{\boldsymbol{\alpha}}, \frac{\partial \bar{R}}{\partial \bar{\boldsymbol{\alpha}}}\right)$$

Single average

Average over the revolution of the perturbing body around the primary planet

$$\bar{\bar{R}} = \bar{\bar{R}}_{\mathrm{SRP}} + \bar{R}_{\mathrm{zonal}} + \bar{\bar{R}}_{3-\mathrm{Sun}} + \bar{\bar{R}}_{3-\mathrm{Moon}}$$

$$\frac{d\bar{\bar{\boldsymbol{\alpha}}}}{dt} = f\left(\bar{\bar{\boldsymbol{\alpha}}}, \frac{\partial \bar{\bar{R}}}{\partial \bar{\bar{\boldsymbol{\alpha}}}}\right)$$

Double average

Dynamical model

Third body potential

- Series expansion of third body potential around $\delta = a/r' = 0$
- Expressed as function of orientation of orbit eccentricity vector and semi-latus rectum vector with respect to third body

$$R_{3B}(r,r') = \frac{\mu'}{r'} \sum_{k=2}^{\infty} \delta^k F_k(A,B,e,E)$$

μ' gravitational coefficient third body

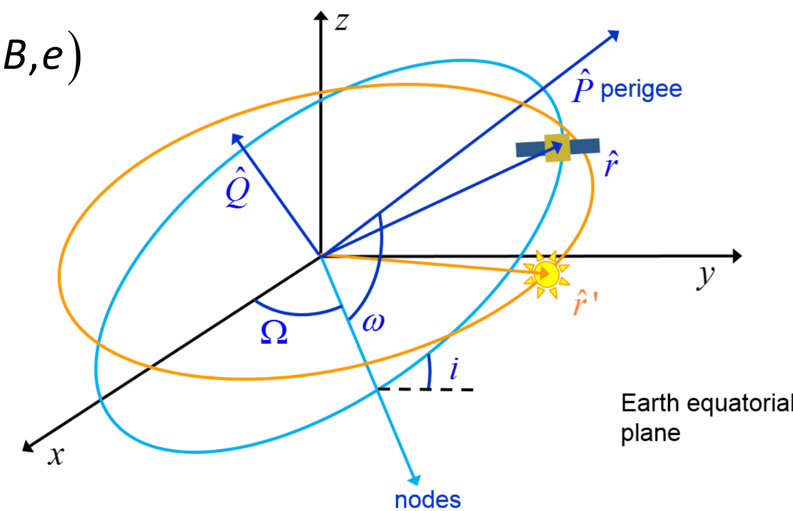
$\mathbf{r}'$ position vector of third body

E eccentric anomaly

- Average over one orbit revolution

$$\bar{R}_{3B}(r,r') = \frac{\mu'}{r'} \sum_{k=2}^{\infty} \delta^k \bar{F}_k(A,B,e)$$

- Calculate partial derivatives for Lagrange equations

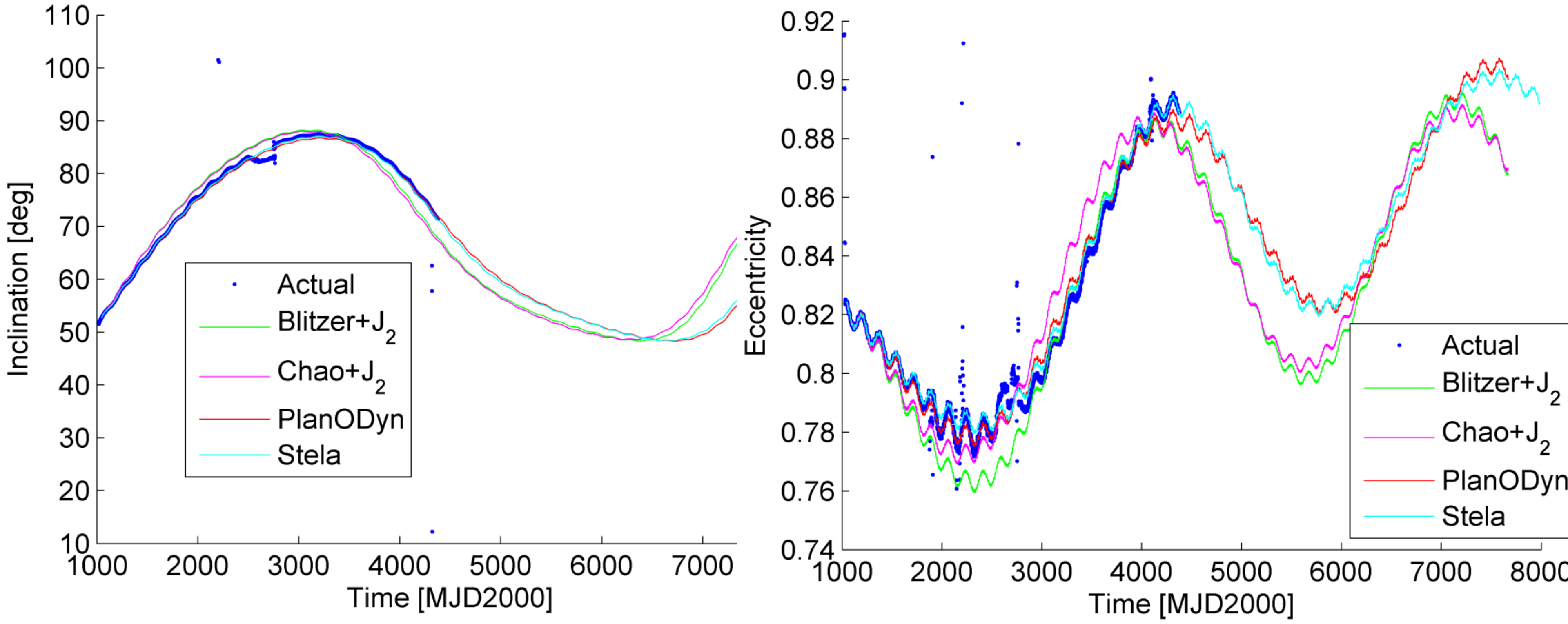


► *Kaufman and Dasenbrock, NASA report, 1979*

Dynamical model

Order of the luni-solar potential expansion

For HEO third-body perturbing potential of the Moon at least up to the fourth order of the power expansion



- *Blitzer L., Handbook of Orbital Perturbations, Astronautics, 1970*
- *Chao-Chun G. C., Applied Orbit Perturbation and Maintenance, 2005*

EXTENSION OF KING-HELE ORBITAL CONTRACTION METHOD

Orbital Contraction

Averaging

- Average out fast moving variable (f, E or M), assuming the other mean elements to be fixed

$$\bar{\dot{x}} = \frac{\Delta x}{P} = \frac{1}{P}\int_0^{2\pi} \frac{dx}{dE} dE \qquad x \in [a, e]$$

- The change $\frac{dx}{dE}$ is a function the Keplerian elements, $\boldsymbol{k}$, the density, ρ, at altitude, h, and the effective area-to-mass ratio, $\delta = c_D \frac{A}{m}$

$$\frac{dx}{dE} = f\left(\boldsymbol{k}, \rho\left(h(\boldsymbol{k})\right), \delta\right) \qquad \boldsymbol{k}^{\mathrm{T}} = (\mathrm{a}, \mathrm{e}, \mathrm{i}, \Omega, \omega, E)$$

$$h = h_m + \Delta h_\varepsilon + \Delta h_{J_2}$$

- Δh_{J_2} → Short periodic variation
- Δh_ε → Altitude above ellipsoid variation
- h_m → Mean altitude

Orbital Contraction

Averaging method

- The integrals can be approximated quickly numerically or analytically
  - E.g. *Gauss-Legendre* (GL) quadrature
    - \+ Flexible: can work with any drag model
    - \+ Valid for any eccentricity, i.e. series expansion avoided
    - − Multiple density evaluations (default $\mathrm{N} = 33$)
  - E.g. *King-Hele* (KH) method
    - − Requires exponentially decaying atmosphere model (next slide)
    - − Series expansion in eccentricity (solved for low and high eccentricities by KH)
    - \+ Only one density evaluation
    - \+ Analytical estimation of the Jacobian available
- Both are implemented in *PlanODyn*, with the (Superimposed) King-Hele method as default

➢ *Liu, J. J. F., Alford, R. L., An Introduction to Gauss-Legendre Quadrature, Northrop Services, Inc., 1973.*

➢ *King-Hele, D., Theory of Satellite Orbits in an Atmosphere, London Butterworths, 1964*

Orbital Contraction

Superimposed Atmosphere (ρ_S) and Superimposed King-Hele (SI-KH)

- KH requires atmosphere to decay exponentially
- Fit superimposed partial exponential atmospheres to any desired model

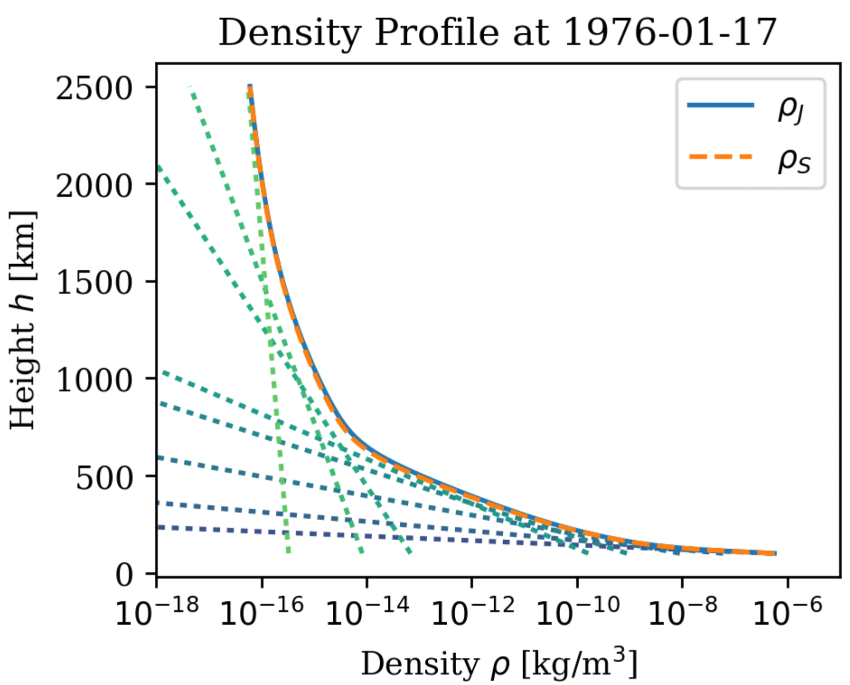
$$\rho_S(h) = \sum_p \rho_{0,p} \exp - \frac{h}{H_p}$$

- Then simply superimposed orbital contractions from KH

$$\Delta a = \sum_p \Delta a_p \qquad \Delta e = \sum_p \Delta e_p$$

- Can include temporal changes

E.g. fit to Jacchia-77, ρ_J

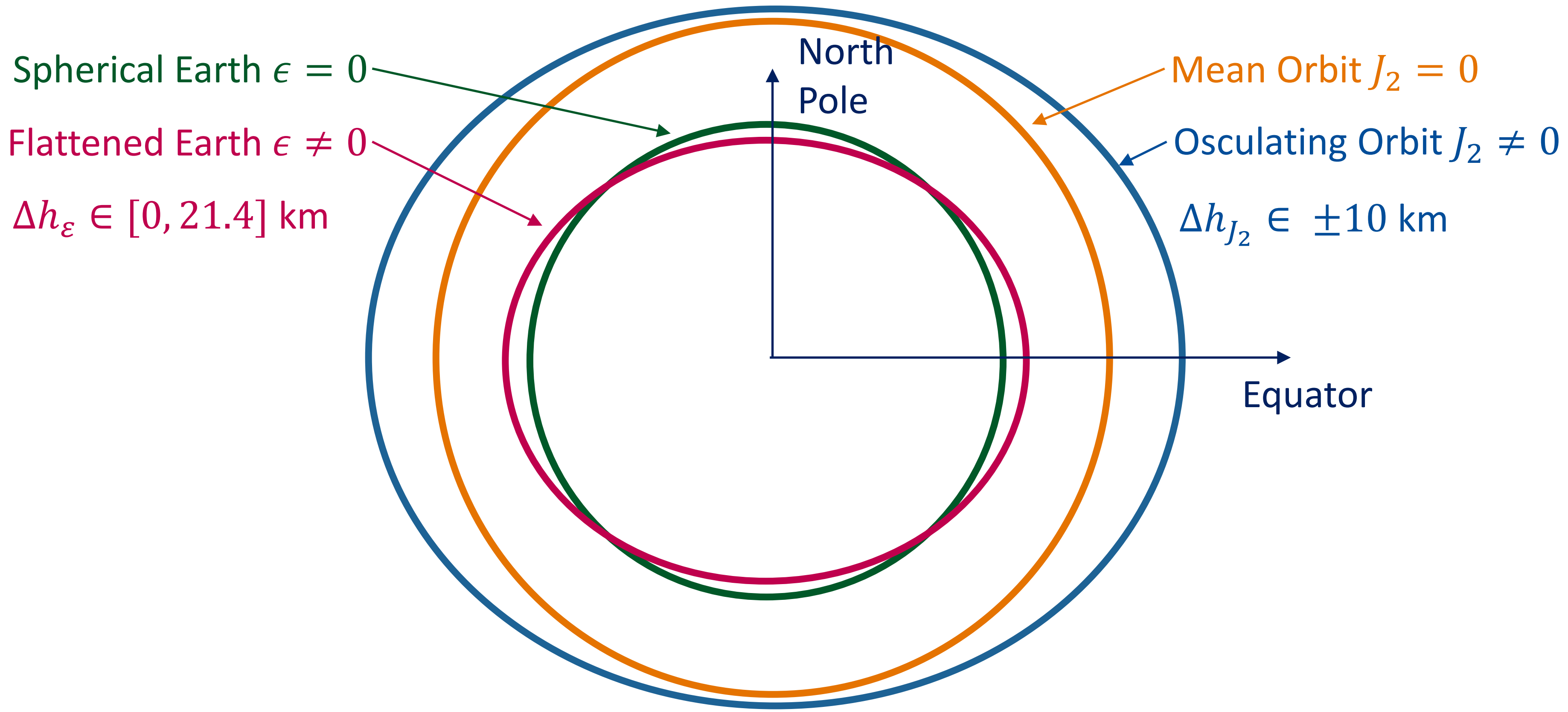


➢ *Jacchia, L. G., Thermospheric temperature, density, and composition: new models. SAO Special Report, 1977.*

Orbital Contraction

Non-Spherical Earth and Atmosphere

- Non-Spherical Atmosphere and coupling of Earth flattening and Drag



Orbital Contraction

Non-Spherical Earth and Atmosphere

- Mean Height

$$h_m = a(1 - e\ \cos E) - R_\oplus$$

- Height above non-spherical Earth surface ($\varepsilon \neq 0$)

$$\Delta h_\varepsilon \approx \varepsilon R_\oplus \sin^2 i \sin^2(\omega + f)$$

- Short periodic variation due to flattening ($J_2 \neq 0$)

$$\Delta h_{J_2} = \frac{J_2\, {R_\oplus}^2}{4a(1-e^2)}\left[\sin^2 i \cos(2(\omega + f)) + (3\sin^2 i - 2)\left\{1 + \frac{e\cos f}{1+\sqrt{1-e^2}} + \frac{2\sqrt{1-e^2}}{1+e\cos f}\right\}\right]$$

- During averaging, assume changes divided by scale height to be small

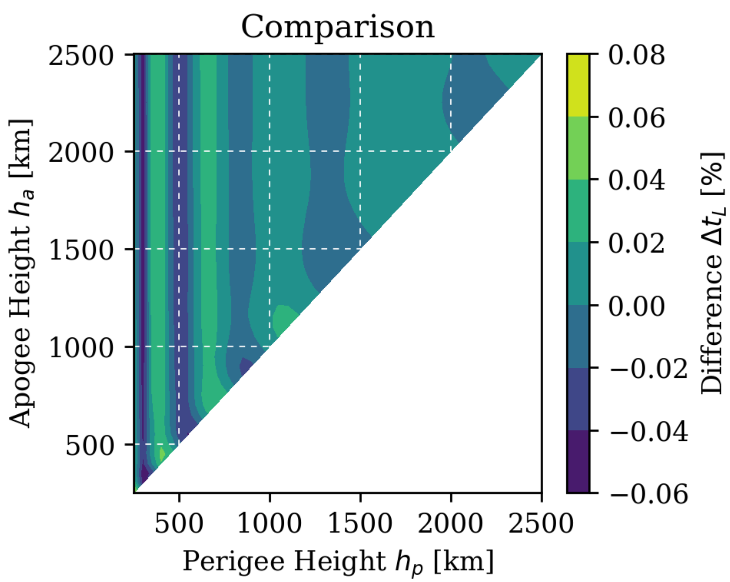
$$\exp\left(\frac{h_m + \Delta h}{H}\right) = \exp\left(\frac{h_m}{H}\right)\exp\left(\frac{\Delta h}{H}\right) \qquad \exp(x) \approx \left[1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \cdots\right]$$

➢ *Liu, J.J.F, Alford, R.L., Semi analytic Theory for a Close-Earth Artificial Satellite. Journal of Guidance and Control, 1980.*

Orbital Contraction

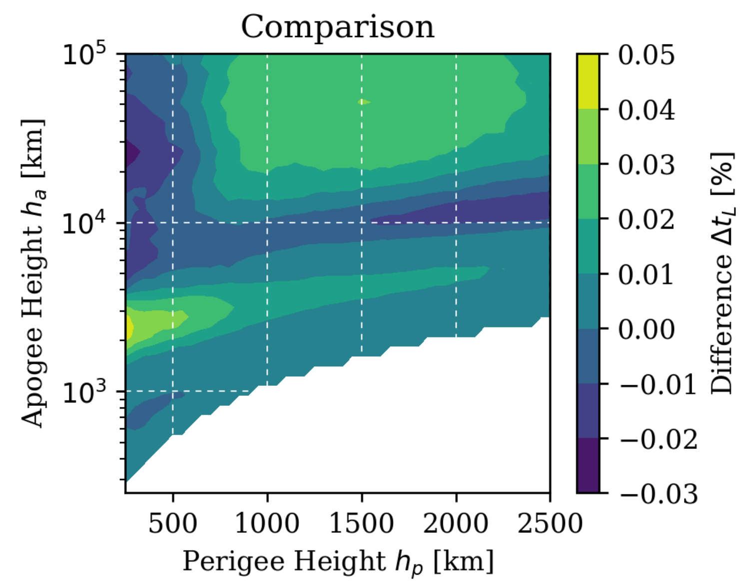
Validation: Spherical Earth, T_∞ fixed

- Comparing ρ_J with ρ_S
- Using GL quadrature
- Area-to-mass ratio $^A/_m = 1$ m²/kg



- Speed increase: 6.4x

- Comparing SI-KH with full numerical integration
- Using ρ_S
- Lifetime of ~1 year

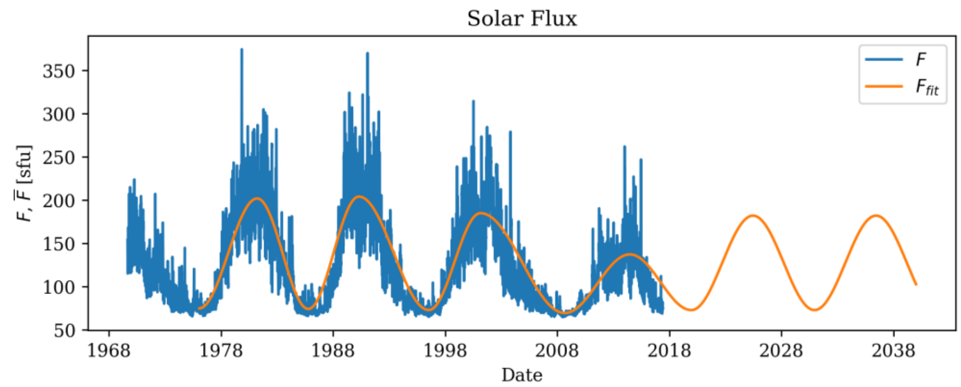


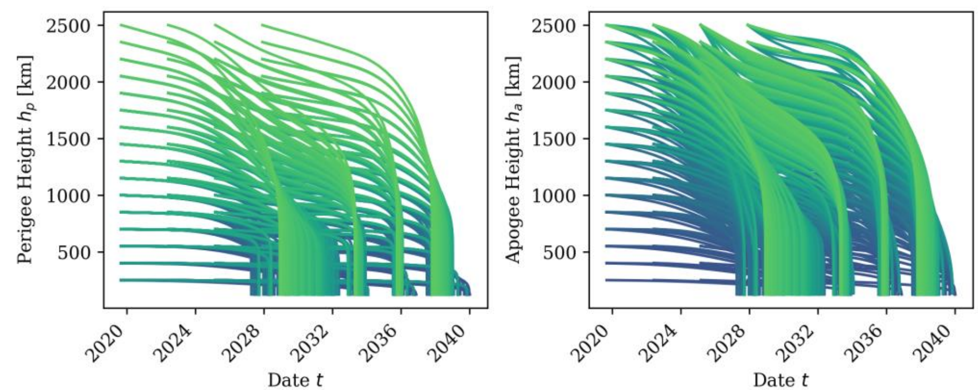
- Speed increase: 560x

Orbital Contraction

Validation: Spherical Earth, T_∞-dependence

- 544 initial conditions:
  - $h_p = 250 - 2500$ km
  - $h_a = 250 - 2500$ km
  - $t_0 = 0$, ¼, ½ and ¾ through predicted future solar cycle 2019-2030
- $^A/_m$ s. t. re-enters ~11 years
- ρ_S /SI-KH vs ρ_J/ GL





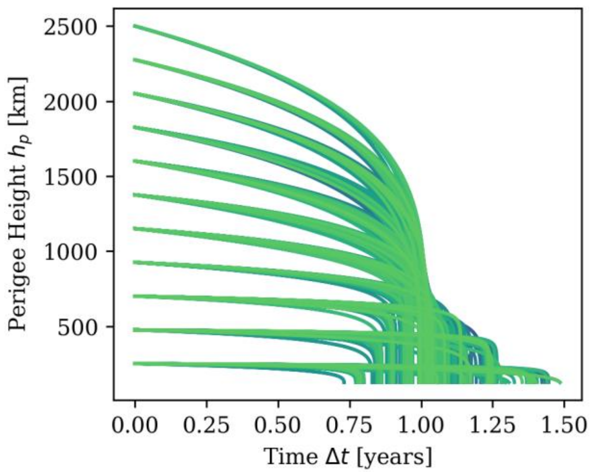
- Accuracy: $\frac{t_L(\rho_S/SI{-}KH)}{t_L(\rho_J/GL)}$

min	$q_{5\%}$	$q_{50\%}$	$q_{95\%}$	max	x CPU
0.9957	0.9987	0.9999	1.0005	1.0012	6.2

Orbital Contraction

Validation: J_2 and ε, T_∞ fixed

- 1092 initial conditions:
  - $h_p = 250 - 2500$ km
  - $h_a = 250 - 2500$ km
  - $i = 1, 45, 63.4, 90°$
  - $\omega = 0, 45, 90°$
- A/m s. t. re-enters ~1 year
- Using ρ_S
- SI-KH(ε, J_2) vs Full numerical



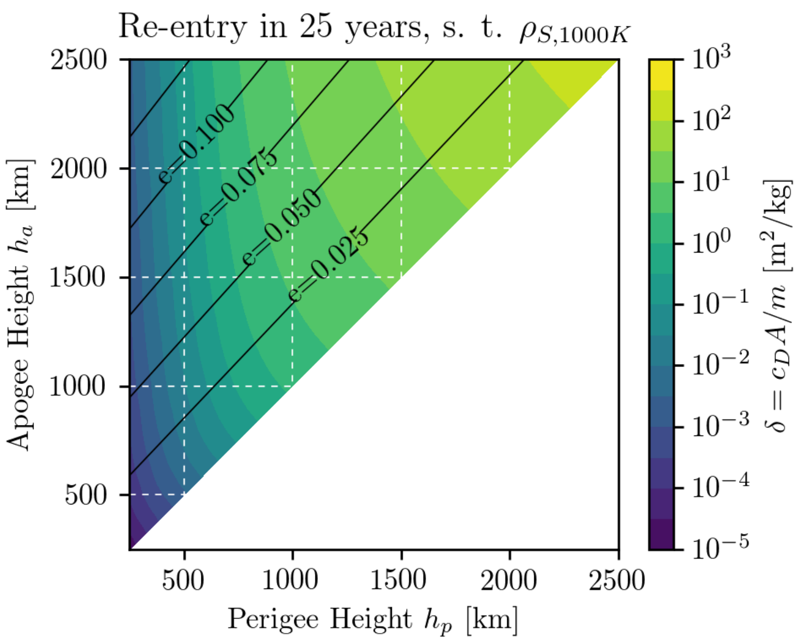
$$\frac{t_L(SI - KH)}{t_L(Full\ Num.)}$$

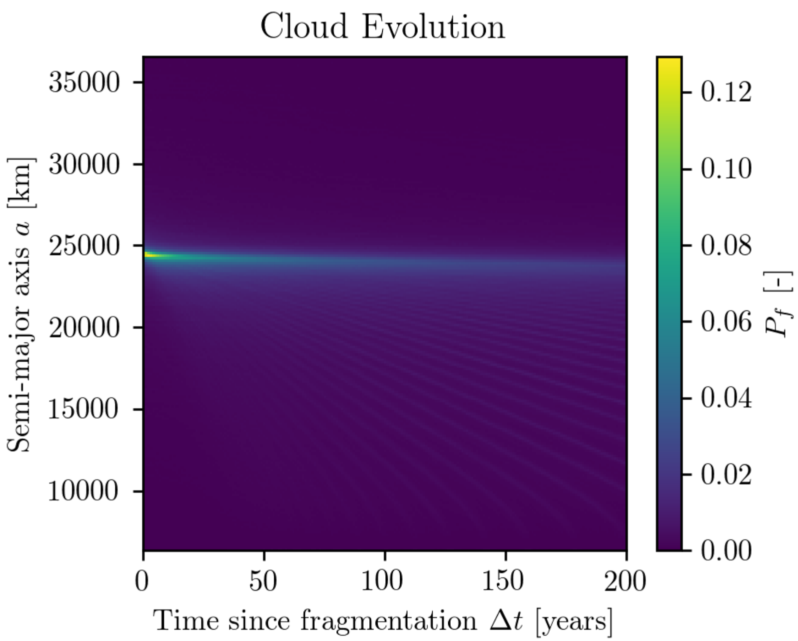
	min	$q_{5\%}$	$q_{50\%}$	$q_{95\%}$	max	x CPU
ε, J_2	0.739	0.858	1.035	1.380	1.531	323
ε, J_2	0.739	0.852	0.998	1.086	1.355	295
ε, J_2	0.996	0.999	1.031	1.256	1.414	320
ε, J_2	0.979	0.999	1.001	1.008	1.032	331

Applications

Drag induced re-entry: two examples

- Maps of effective area-to-mass ratio required for re-entry in x years (optimisation)
- Evolution of clouds of fragments (collision or explosion) or entire space debris population





➢ *Frey, S., Colombo, C., Lemmens, S., Krag H., Evolution of Fragmentation Cloud in Highly Eccentric Orbit using Representative, Proceedings of the 68th IAC, 2017*

GEOSTATIONARY TRANSFER ORBIT RE-ENTRY PREDICTION

TLE based re-entry prediction

Background

ESA study (DINAMICA, Uni of Southampton, CNRS)

Technology for improving re-entry prediction of European upper stages through dedicated observations, ESA-GSP study ITT 8155, 2015

- TLE-based parameter estimation
  - Develop BC estimation method
  - Develop BC and SRPC estimation method
- TLE based state estimation
  - OD state estimation method

TLE based re-entry prediction

Background

- Ballistic coefficient
  - Estimate depends on:
    - Initial state (perigee height)
    - Force model: Atmosphere model (density)
      Others forces (coupling)
  - B* parameter:

```
1 25496U 98057B   98284.49064516  .00114495 -27097-6  39493-2 0  9997
2 25496 024.9619 183.2533 7300304 180.6546 177.3823 02.29699499    34
```

- Fitting parameter in TLE
- Ballistic coefficient from B*: $BC = \frac{2B^{*}}{\rho_0 R_{\text{Earth}}} = 12.741621\, B^{*}$

TLE based re-entry prediction

BC estimation method

- BC estimation is based on comparing the change in semi-major axis from the TLE data to the change in semi-major axis computed from accurate orbit propagation between two epochs
  1. Compute the change in semi-major axis between two TLE epochs from the mean motion, n

     $$\Delta a_{TLE} = a_{TLE_2} - a_{TLE_1}$$

  2. Compute the change in semi-major axis between two TLE propagating the object trajectory

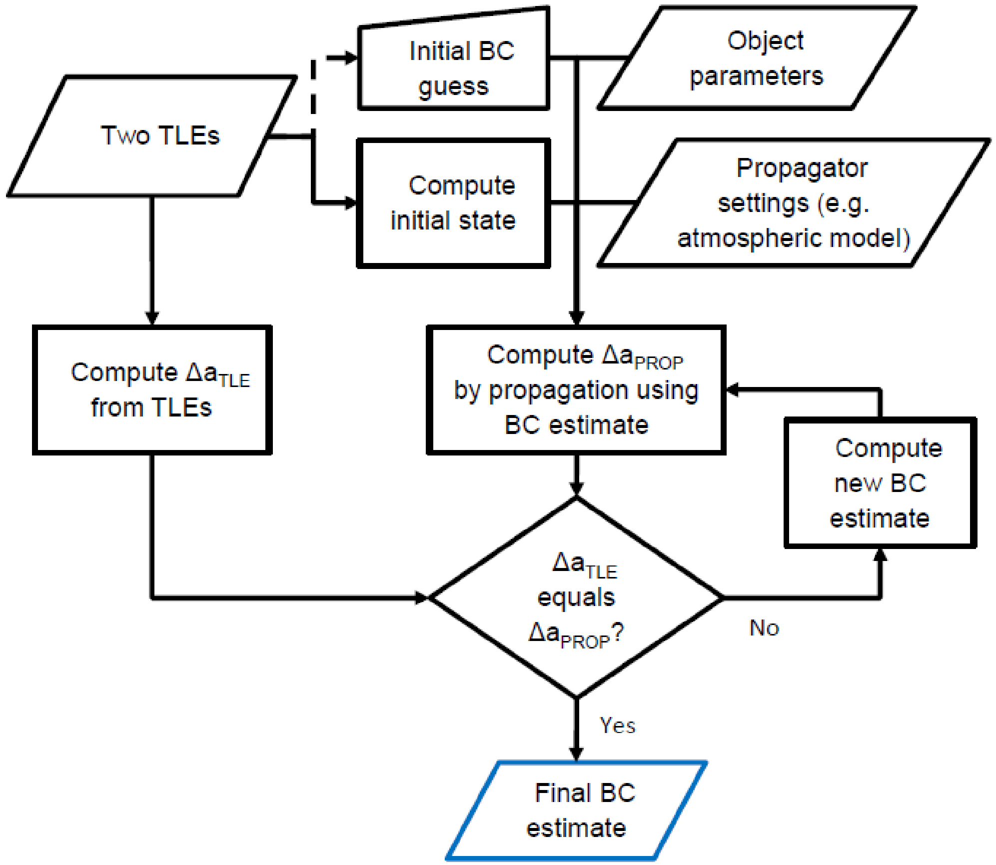
     $$\Delta a_{PROP} = \int_{TLE_1}^{TLE_2} \left(\frac{da}{dt}_{drag} \right) dt = f\left(BC_{guess}\right)$$

  3. Compute BC iteratively such that $\Delta a_{PROP} = \Delta a_{TLE}$
     - Δa_{PROP} is computed using the **average** semi-major axis because Δa_{TLE} is the change in **mean** semi-major axis
     - Δa_{PROP} can be computed by **backward** propagation to avoid re-entry during estimation

➢ *Saunders et al, 2012*

TLE based re-entry prediction

BC estimation method



- *D. J. Gondelach, R. Armellin, and A. A. Lidtke, Ballistic Coefficient Estimation for Re-entry Prediction of Rocket Bodies in Eccentric Orbits Based on TLE Data, Mathematical Problems in Engineering*

TLE based re-entry prediction

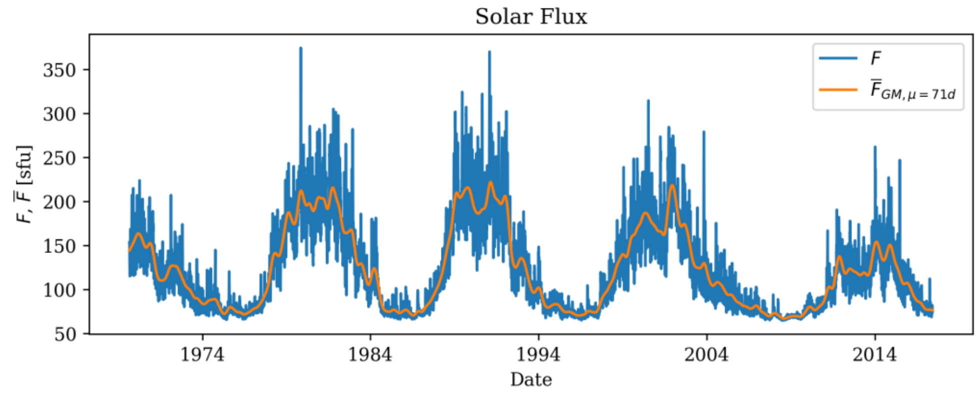
Propagation method: AIDA dynamics

- Geopotential acceleration:
  - EGM2008gravity model up to degree and order 10
- Atmosphere drag:
  - NRLMSISE-00 atmospheric model with updated weather files
  - Rotating atmosphere
- Solar radiation pressure:
  - Earth and/or Moon shadow
  - Cylindrical or biconical shadow
- Moon and Sun perturbations:
  - Moon and Sun ephemeris from NASA's SPICE kernels
  - SPICE toolkit used to time and reference frame transformations

TLE based re-entry prediction

Propagation method: PlanODyn dynamics

- Geopotential acceleration:
  - Zonal harmonics up to order 6
- Atmosphere drag:
  - T_∞-dependent smooth exponential atmosphere model, fit to Jacchia-77
  - Solar flux using Gaussian mean with standard deviation of 3 solar rotations
  - No atmospheric rotation
- Solar radiation pressure:
  - Cannonball model
  - No shadow considered
- Moon and Sun perturbations:
  - Moon and Sun ephemeris from NASA's SPICE kernels
  - Expansion of third body Legendre potential in a/a_3 up to order 5



TLE based re-entry prediction

Results

30 days and 180 days re-entry predictions of 83 and 92 objects to obtain a better understanding

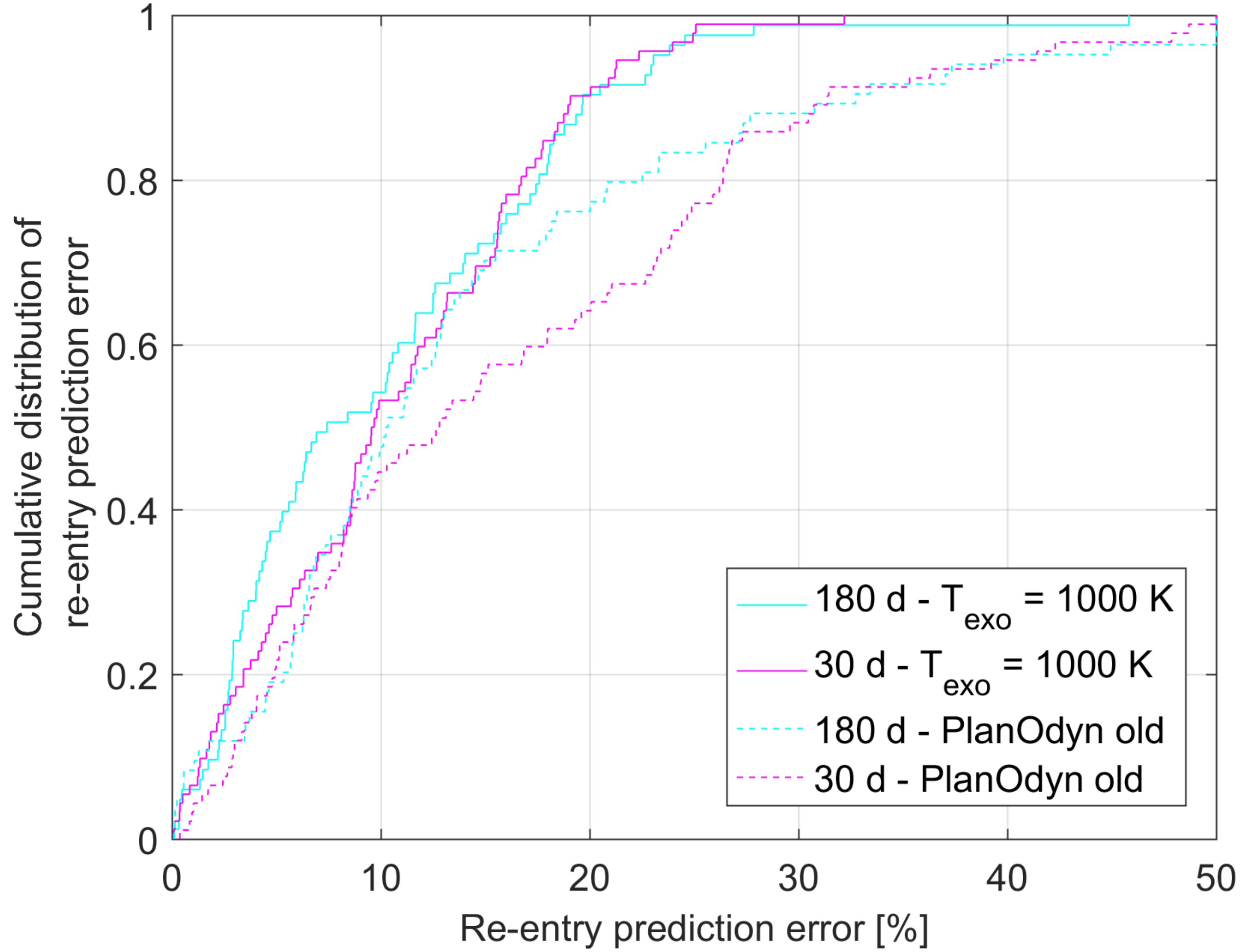
- Re-entry prediction accuracy
- Effect of dynamics

Error computation

$$error[\%] = \frac{t_{predicted} - t_{actual}}{t_{actual} - t_{lastUsedTLE}} \cdot 100$$

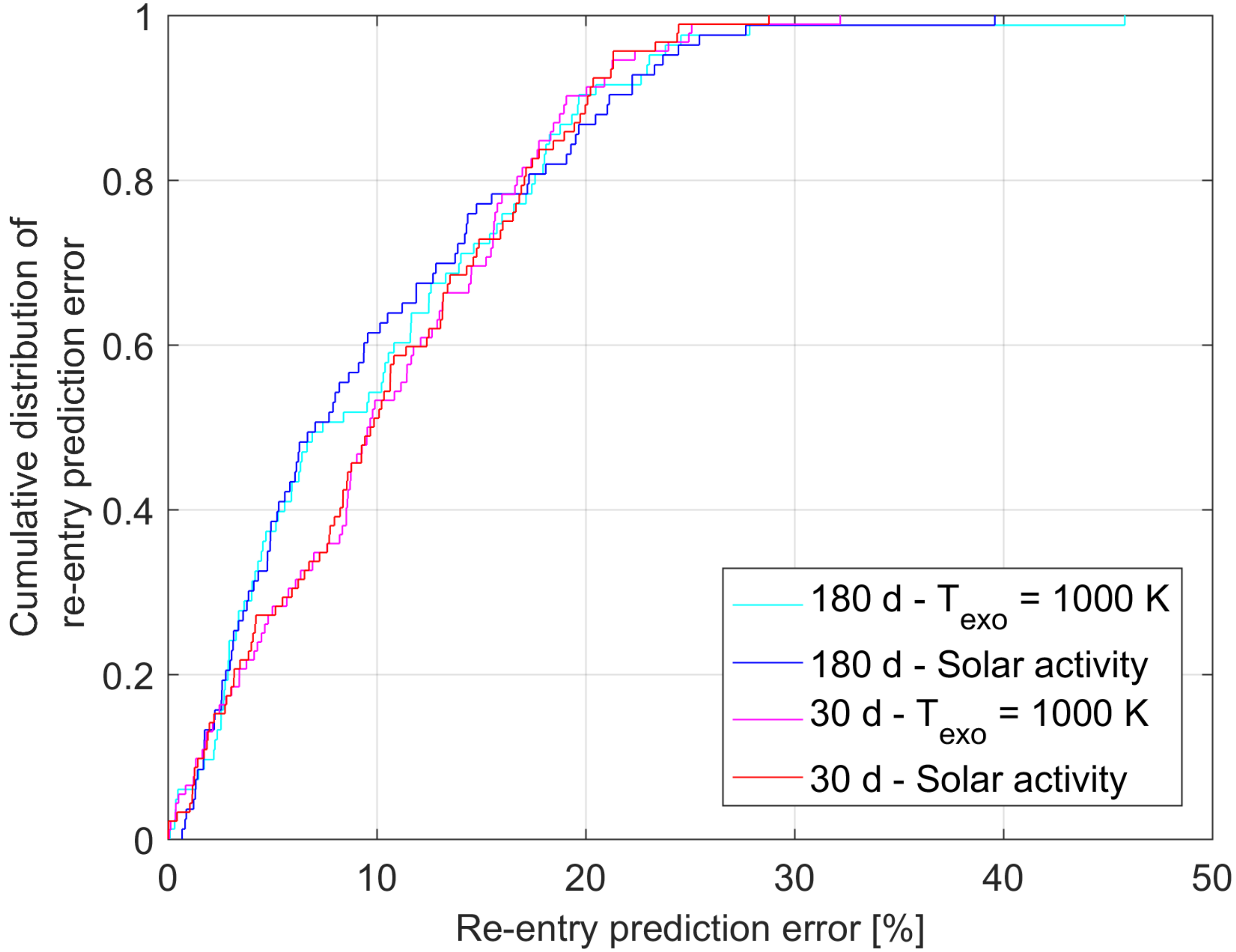
TLE based re-entry prediction

Improvement in the PlanODyn suite



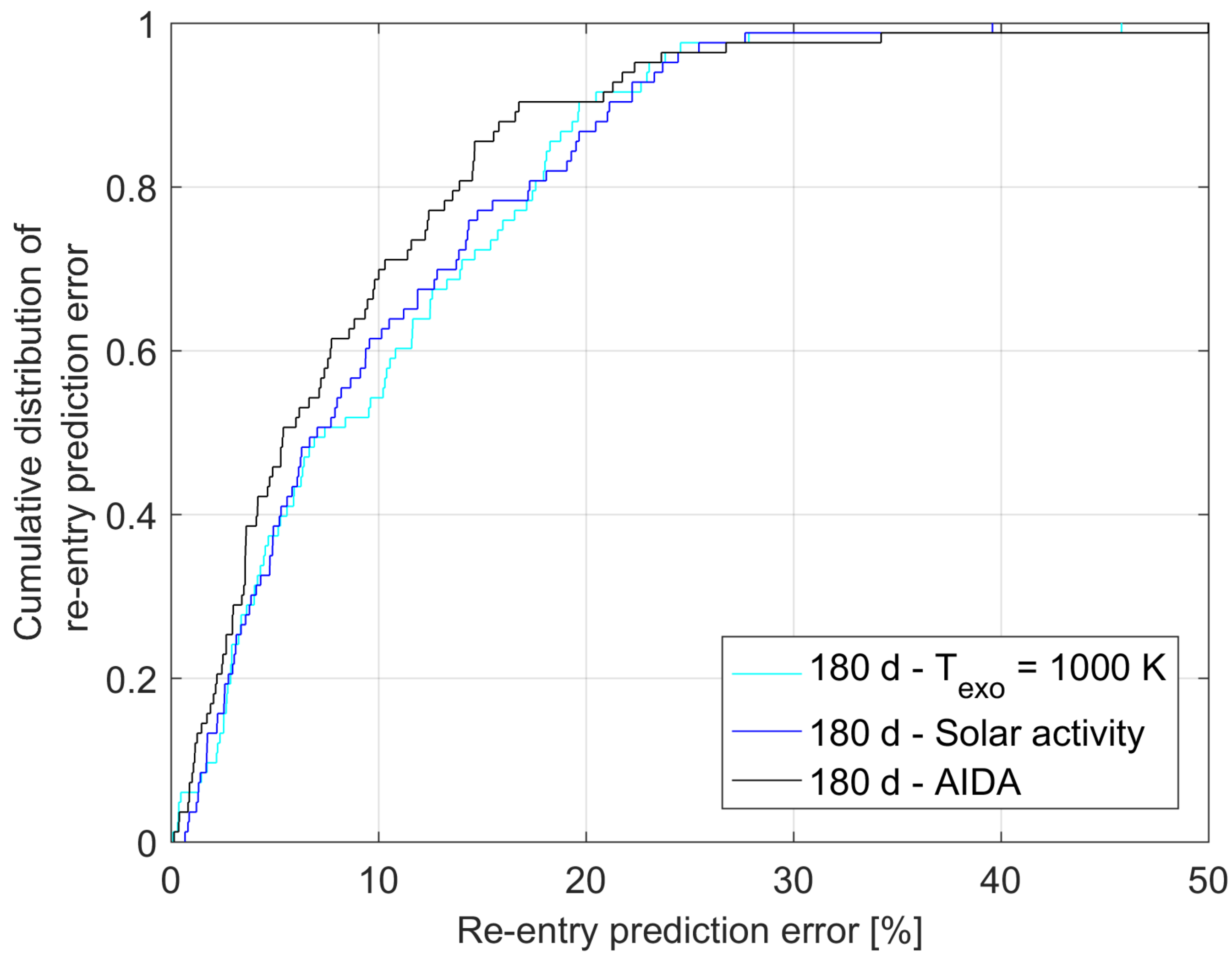
TLE based re-entry prediction

Effect of solar activity



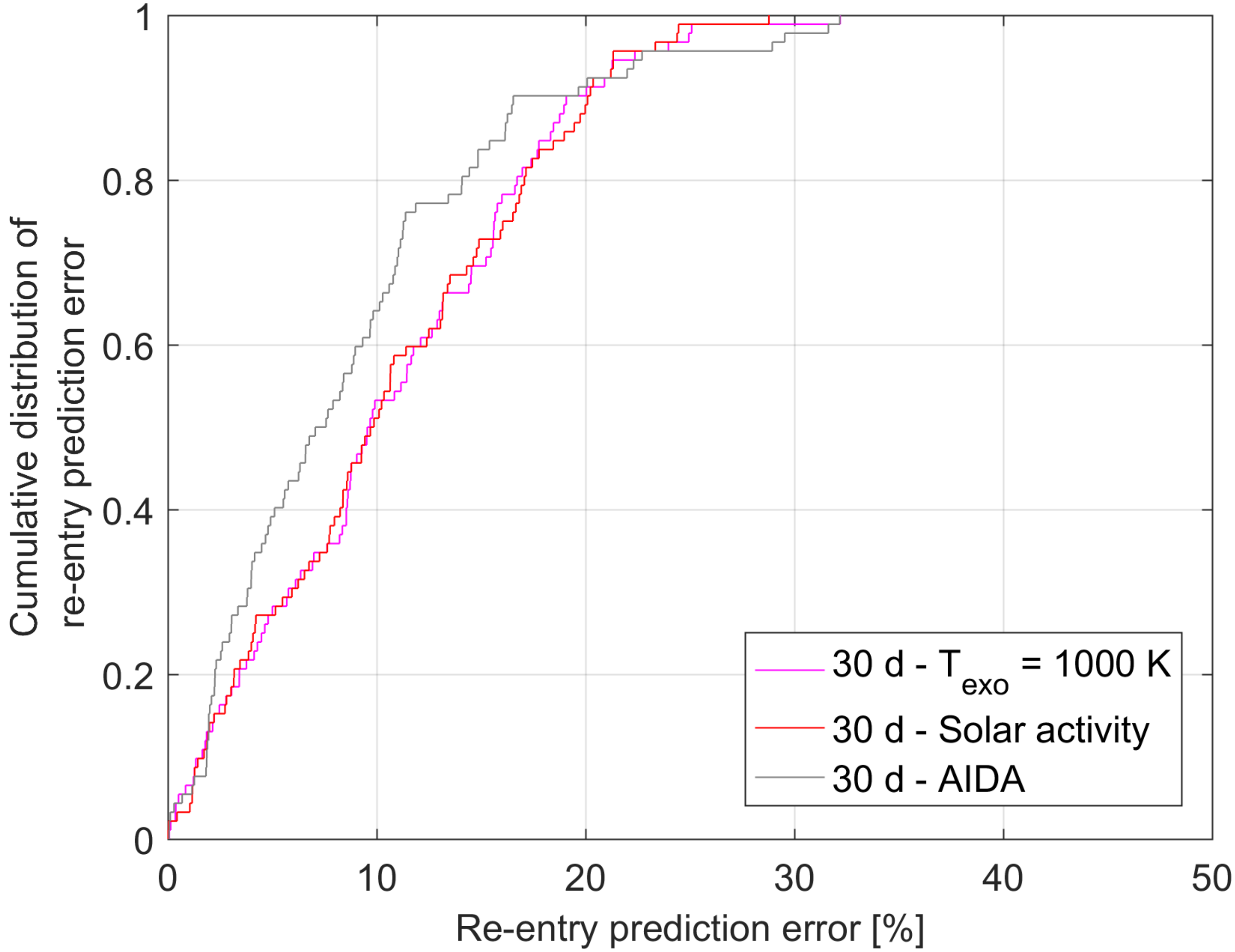
TLE based re-entry prediction

Semi-analytical (PlanODyn) versus high fidelity (AIDA)



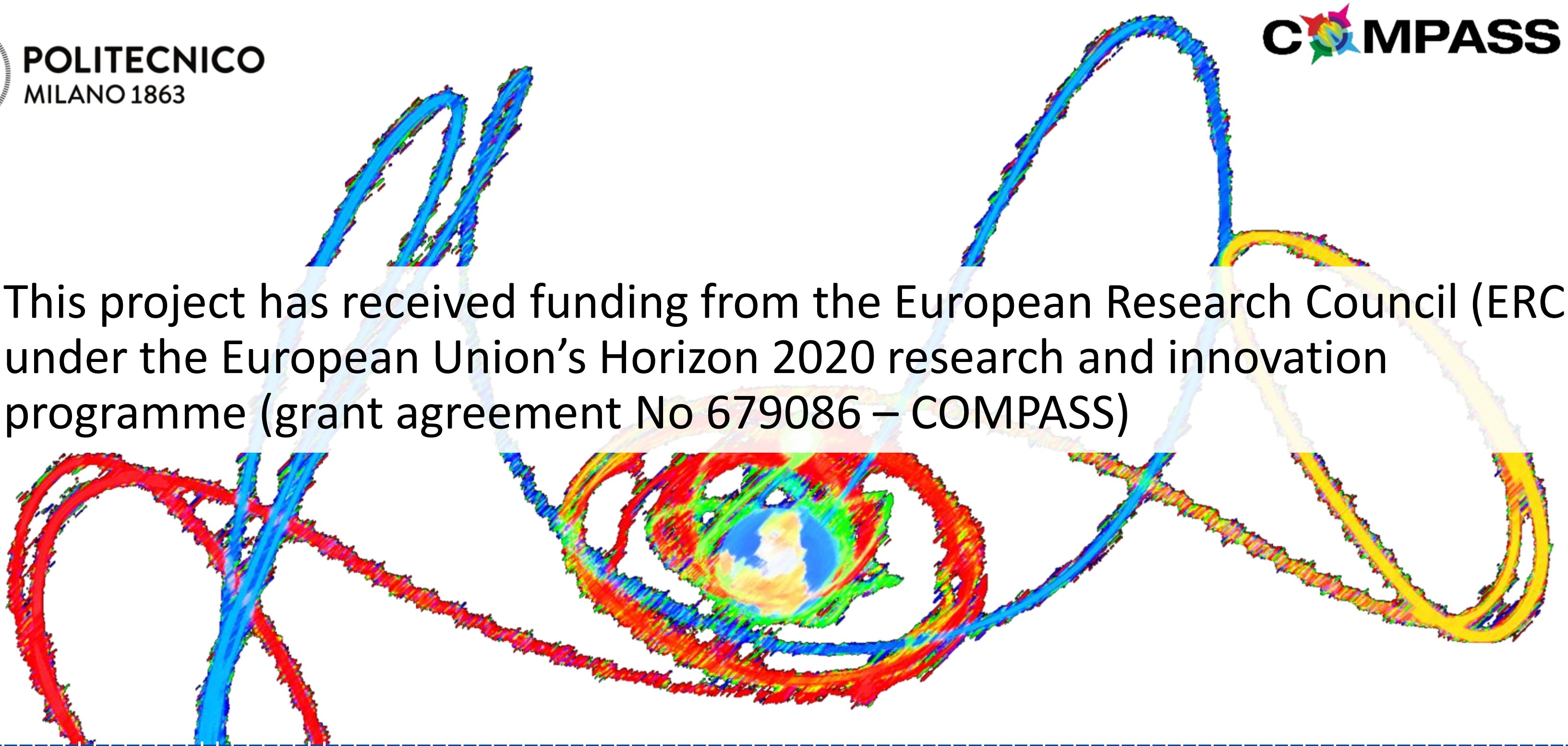
TLE based re-entry prediction

Semi-analytical (PlanODyn) versus high fidelity (AIDA)



Conclusions

- Semi-analytical methods shows accuracy against numerical propagation
  - Especially for conservative forces
  - Also for drag induced forces up until shortly before re-entry
- Future work for improving re-entry prediction
  - Inclusion of tesseral terms
  - Inclusion of equator precession
  - Rotation of the atmosphere
  - Verify long-term re-entry prediction
- Possible applications
  - Disposal trajectory design
  - Re-entry modelling and orbit determination
  - Sensitivity analysis to spacecraft parameters and model uncertainties



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Extension of the King-Hele orbital contraction method and application to the geostationary transfer orbit re-entry prediction

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