Image credit: Earth Observatory, NASA

Evolution of a fragment cloud in elliptical orbit using representative objects

Stefan Frey & Camilla Colombo

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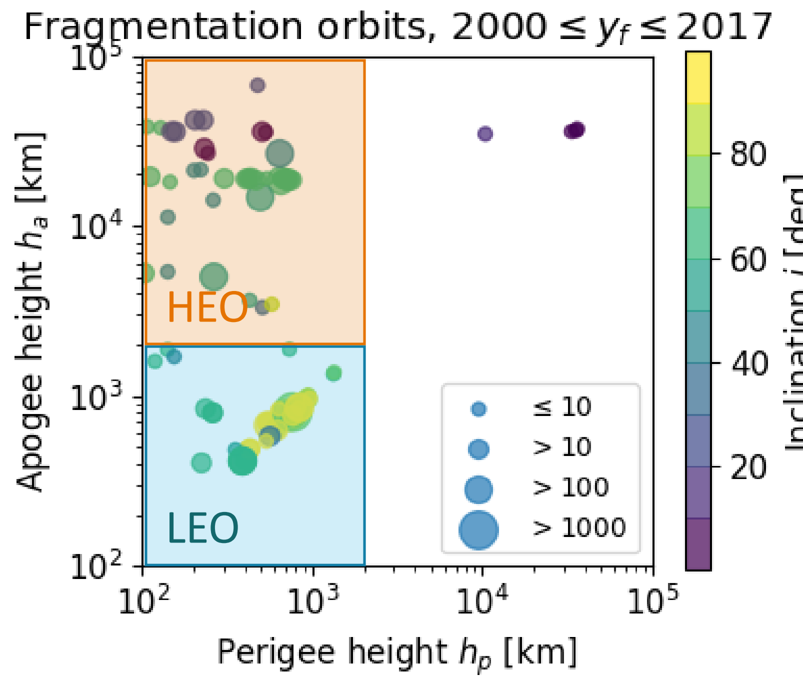
Outline

- Introduction of problem
- Fragmentation
- Gridding/Representative Objects
- Propagation
- Results
- Conclusions and Future Work

Introduction

Collision risk due to fragmentations

- 90 satellites and upper stages fragmented since 2000 alone[1]
- Ignoring collisions and deliberate explosions
- 42 out of 90 fragmented in highly eccentric orbits (HEOs)
- Subject to a multitude of perturbations
- How to predict evolution of cloud?
- How to predict **collision risk** with active missions?

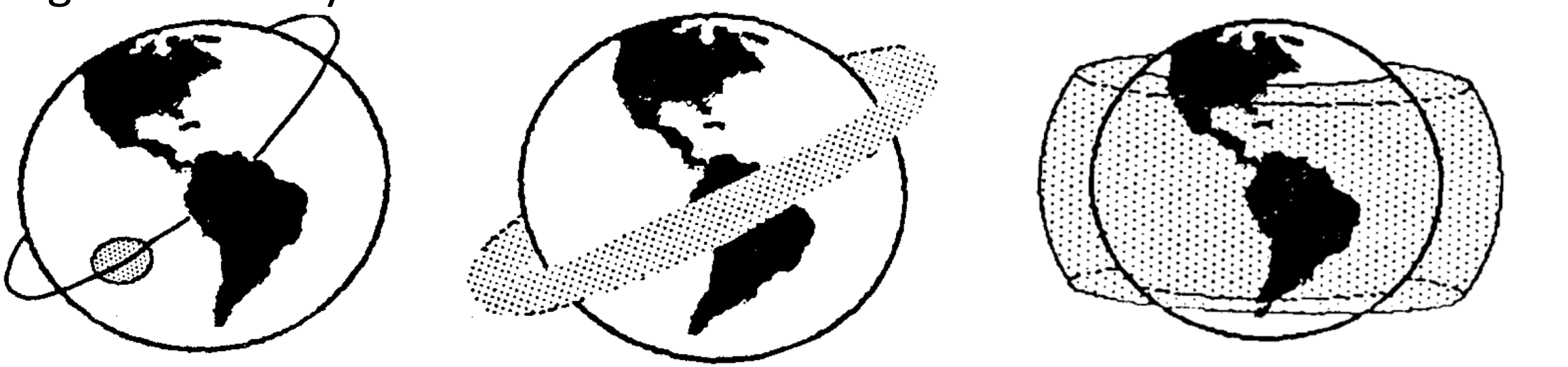


1. T. Flohrer, S. Lemmens, B. Bastida Virgili, H. Krag, H. Klinkrad, N. Sanchez, J. Oliveira, and F. Pina. DISCOS - current status and future developments. In Proceedings of the 6th European Conference on Space Debris, 2013

Time of Closure

Why is it important?

- For the calculation of the collision risk through **spatial density**
- As soon as closure reached in variable $\beta \subset [M, \Omega, \omega]$
  - assume randomisation (drop dependence on β)
  - possibly account only for certain perturbations until next phase
- However, if time of closure is too big, cannot separate phases, as fragments decay too much



Ellipse → δP → Torus → J_2, ~~drag~~ → Band → ~~J_2~~, drag

Mc Knight, D. S. and Lorenzen, G., Collision Matrix for Low Earth Orbit Satellites, J. Spacecraft, 1989

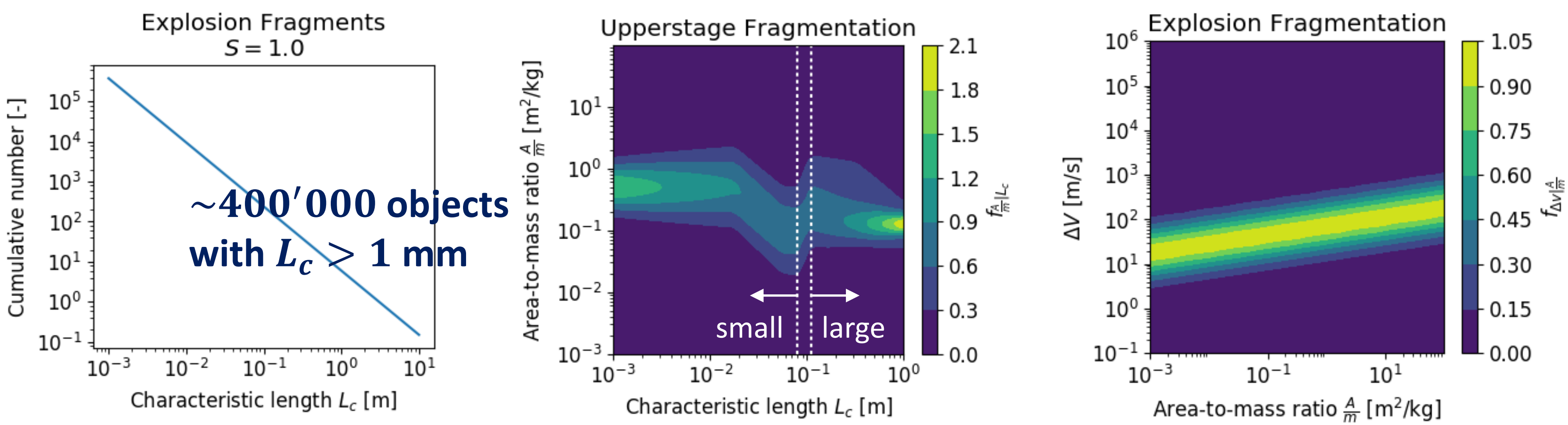
J. Ashenberg, Formulas for the phase characteristics in the problem of low earth orbital debris. Journal of Spacecraft and Rockets, 1994.

F. Letizia, C. Colombo, H. G. Lewis, Analytical Model for the Propagation of Small-Debris-Object Clouds After Fragmentations, J. GCD, 2015

Fragmentation

NASA break-up model

- NASA's break-up model[2] (empirical model):
  - Number of fragments: $N_f = f(L_c)$, with characteristic length L_c
  - Area to mass ratio $\frac{A}{m}$ PDF: $f_{\frac{A}{m}} \sim N(\mu_A, \sigma_A^2 | L_c)$
  - Impulse Δv PDF: $f_{\Delta v} \sim N(\mu_V, \sigma_V^2 | \frac{A}{m})$



2 Johnson, N.L., Krisko, P.H., Liou, J.-C., et al. NASA'S new breakup model of EVOLVE 4.0, Adv. Space Res., 28(9), 1377-1384 (2001).

Fragmentation

Common vs new approach

- Common approach in long term propagation:
  - Monte Carlo sampling: probabilistic events for fragmentation
  - Each fragmentation: draw distinct objects from prob. distribution
- Problem: each fragmentation might very differently affect a given mission in terms of collision risk

- **Instead**: describe probability of a fragment with

$$f_{L_c,\frac{A}{m},\Delta v} = f_{\Delta v|\frac{A}{m}}\, f_{\frac{A}{m}|L_c}\, f_{L_c}$$

- And propagate forward probability with representative objects
- Thus rendering Monte Carlo sampling unnecessary

Gridding

Representative objects in ${}^{A}/_{m}$ and Δv

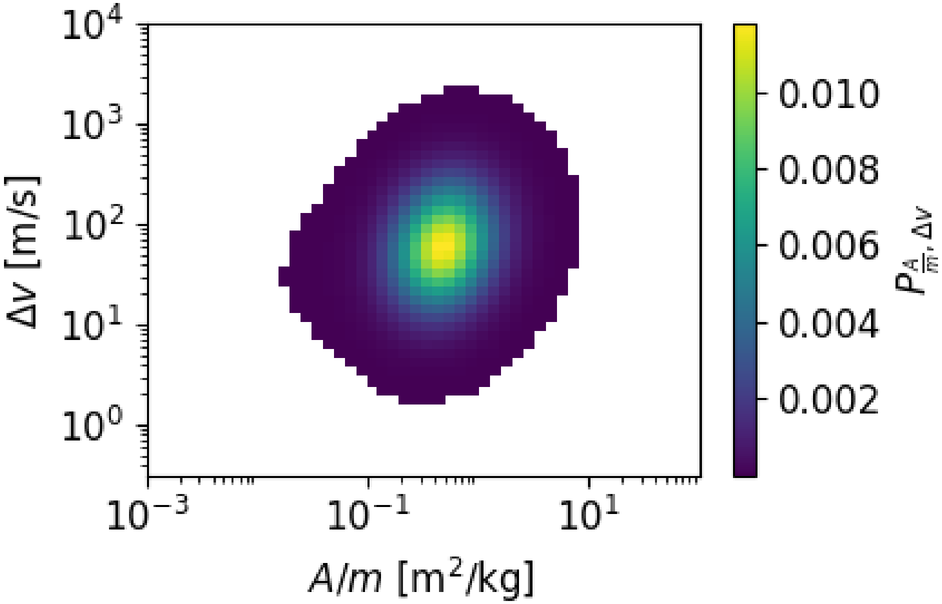
- Note that L_c is not important for propagation, probability reduces to

$$f_{\frac{A}{m},\Delta v} = f_{\Delta v|\frac{A}{m}} \int f_{\frac{A}{m}|L_c} f_{L_c}\, dL_c$$

- Representative fragments: $\frac{A}{m} \in d\left(\frac{A}{m}\right)_i, \Delta v \in d(\Delta v)_j$, with probability

$$P_{i,j} = \iint f_{\frac{A}{m},\Delta v}\, d(\Delta v)_j d\left(\frac{A}{m}\right)_i$$

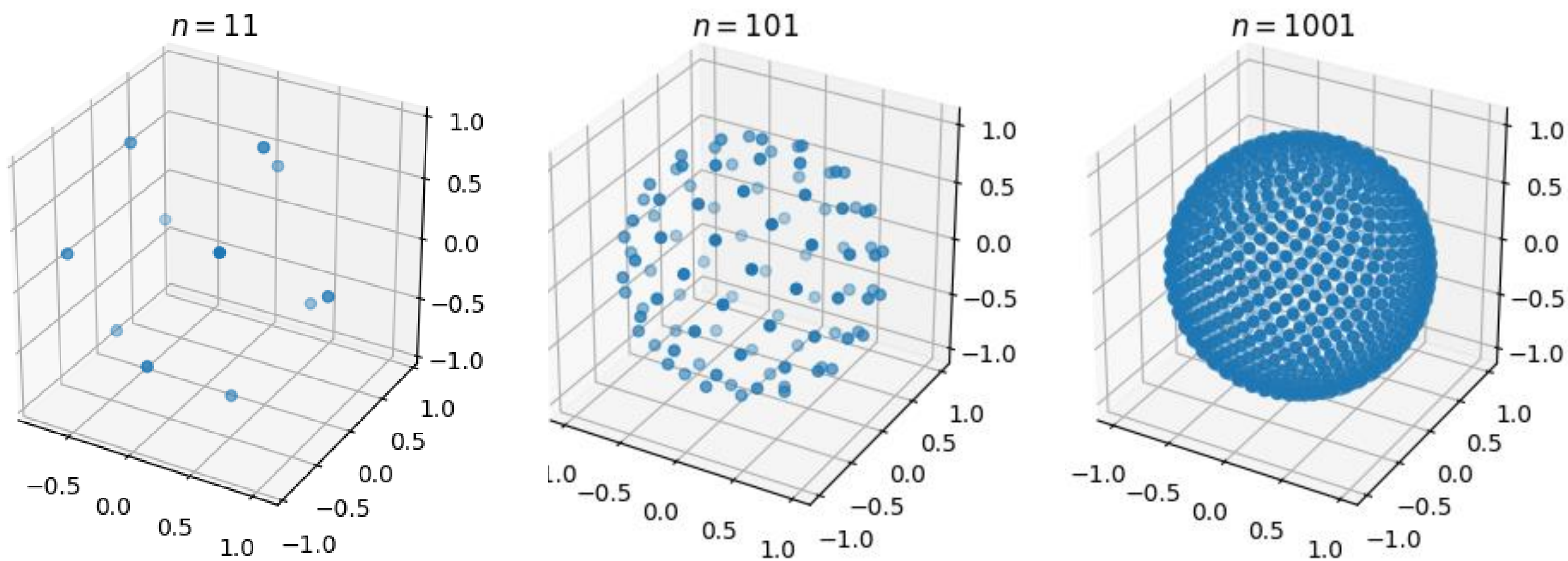
- Using log-spaced grid
- Only retain 99.9% of all fragments



Gridding

Isotropic direction

- Use Fibonacci Lattice: n_K (odd) quasi equally distributed points on sphere
- Each direction (θ_i, φ_i) with equal probability $P_k = \frac{1}{n_K}$

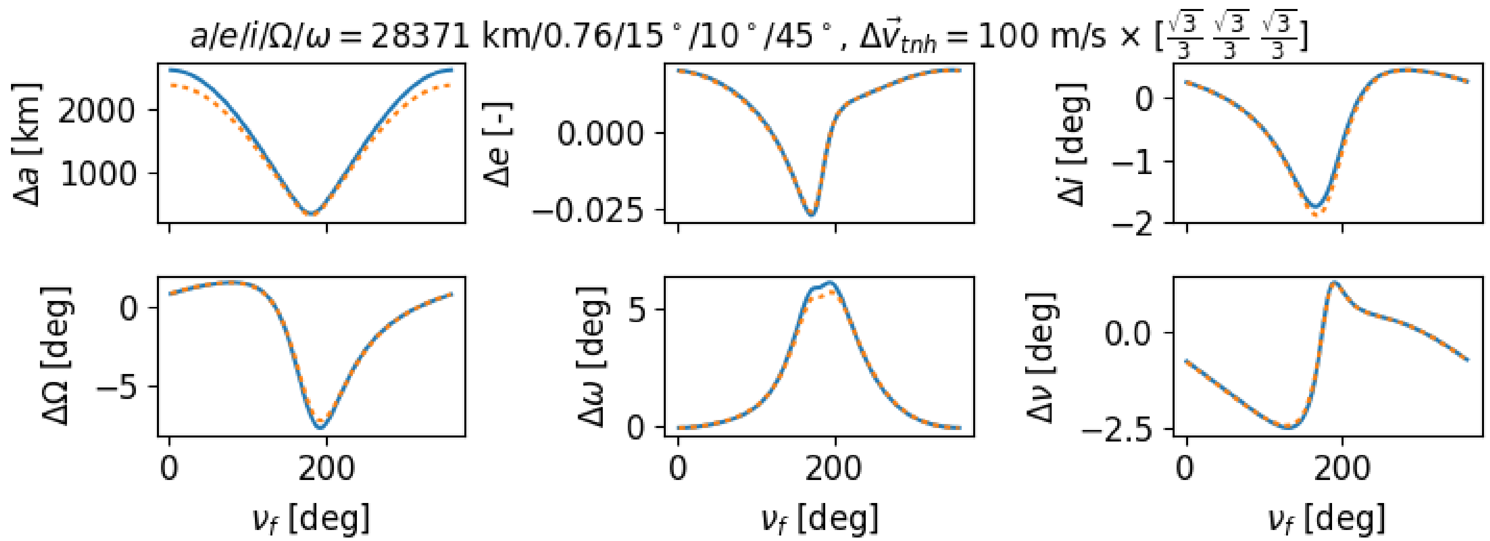


Gridding

Conversion

1. Transformation of initial conditions: $k_f = (a, e, i, \Omega, \omega, f) \rightarrow \vec{x}_f, \vec{v}_f$
2. Addition of perturbation $\Delta\vec{v} = \Delta\vec{v}(\Delta v, \theta, \varphi)$ for each grid point
3. Back-transformation $\vec{x}_f, \vec{v}_f + \Delta\vec{v} \rightarrow k_p$

In work: direct transformation of $k_p = k_p(k_f, \Delta\vec{v})$ using 2nd order Gauss planetary equations



Propagation

Perturbations

- J_2: important for time of closure in Ω, ω, as $\dot{\Omega}, \dot{\omega} \sim J_2$
- Drag: important for time of re-entry
  - New atmospheric contraction model proposed (next slide)
- Still to be taken into account:
  - Solar radiation pressure (important)
  - Third body perturbations (important)

Propagation

Orbital Decay

- Pre-requisites
  - Fast (i.e. semi-analytical) → King-Hele[3]
  - Accurate, especially for HEOs → Superimposed King-Hele
- Atmosphere model: **Smooth exponential atmosphere**, increasing speed

$$\rho_S(h) = \sum_{p}^{P} \rho_{0,p} \exp\left(-\frac{h}{H_p}\right)$$

- Contraction model: **Superimposed King-Hele**, increasing accuracy

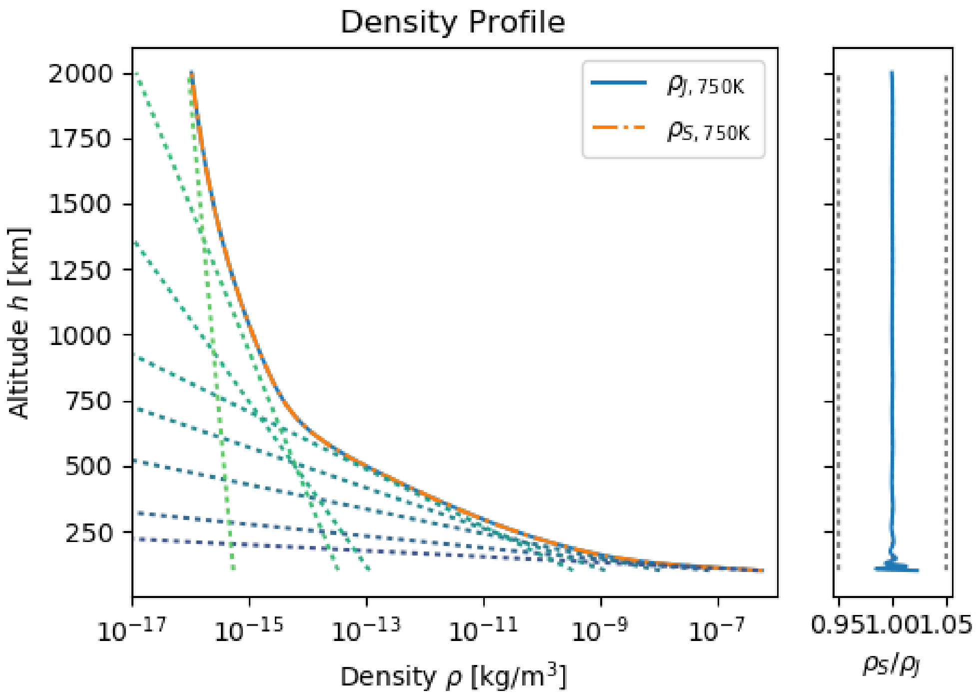
$$\Delta a = \sum_{p} \Delta a_p \qquad \Delta e = \sum_{p} \Delta e_p$$

3 D. King-Hele. Theory of satellite orbits in an atmosphere. London Butterworths, 1964.

Propagation

Atmosphere Model

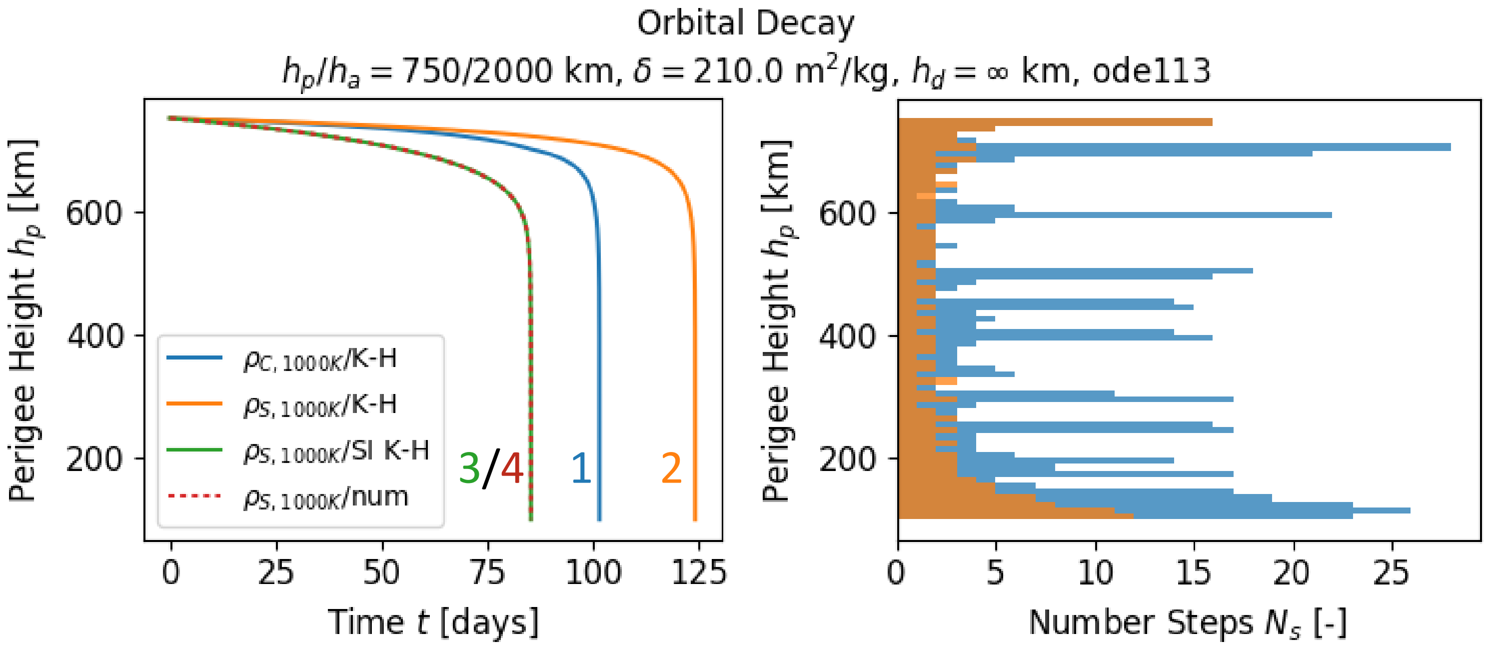
- Smooth model fit to Jacchia-77[4]
- Error below 0.1% $\forall\, h > 218$ km and below 1% $\forall\, h > 118$ km
- Advantage: **fixed scale heights $\boldsymbol{H}$**, easily extendable to time-varying



4 L. G. Jacchia. Thermospheric temperature, density, and composition: new models. SAO Special Report, 375, 1977.

Propagation

Comparison Orbit Contraction



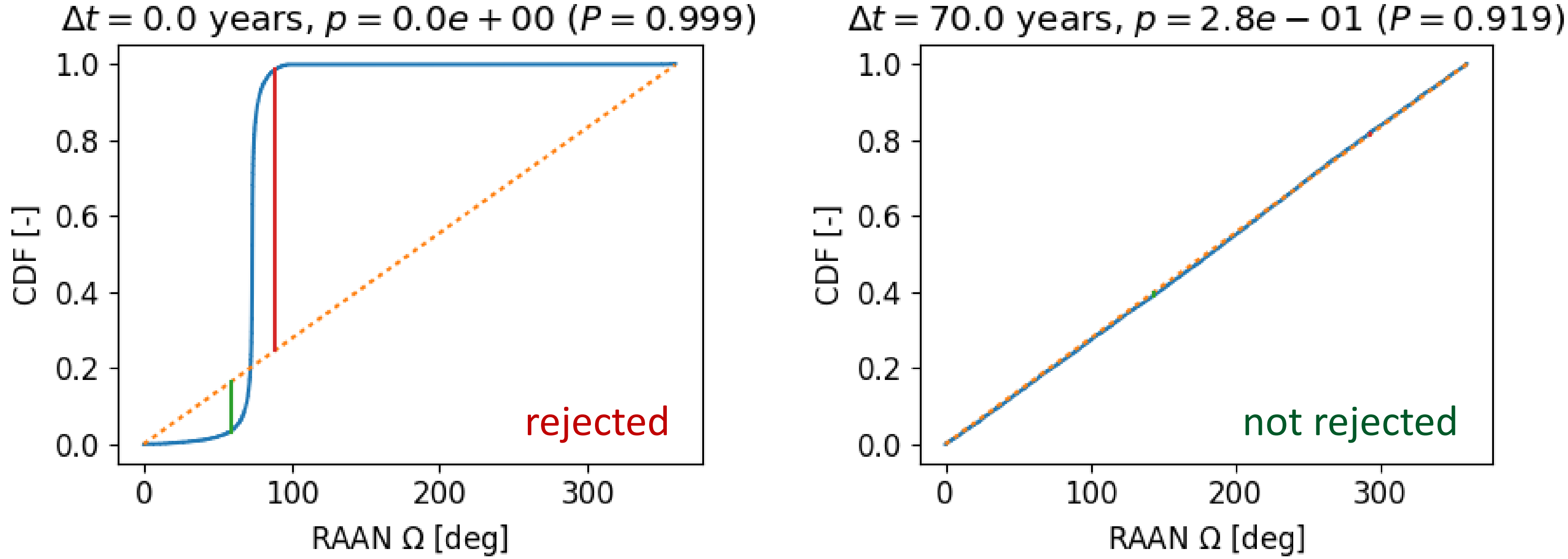
MATLAB's ode113: variable-step, variable-order Adams-Bashforth-Moulton. tol: 1e-9

	Atmosphere Model	Contraction Model	Δt [days]	N_f [-]
1	Non-smooth	Classical King-Hele	101.5	1239
2	Smooth	Classical King-Hele	124.1	430
3	Smooth	Superimposed King-Hele	**85.282**	**426**
4	Smooth	Numerical propagation	85.282	97819

Time of Closure

Definition

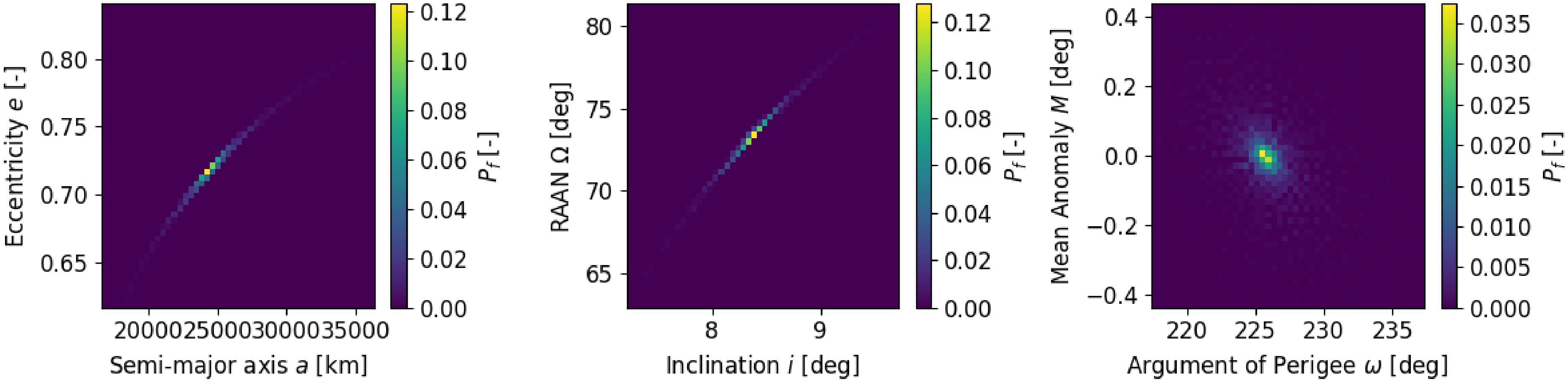
- Kuiper's test: Ideal for distribution on a circle
- Closure when failing to reject $\boldsymbol{H_0}$: **uniform distribution**
- Fail to reject H_0 if p-value $p > \alpha = 0.05$ (pre-defined)
- p: probability of getting a more *extreme* result, given H_0 is true



Results

After fragmentation

- Simulated Fragmentation of Upper Stage (based on real event in 2001)
- GTO: 515 x 35700 km, inclination $i = 8°$
- Fragmentation at perigee, $f = 0°$

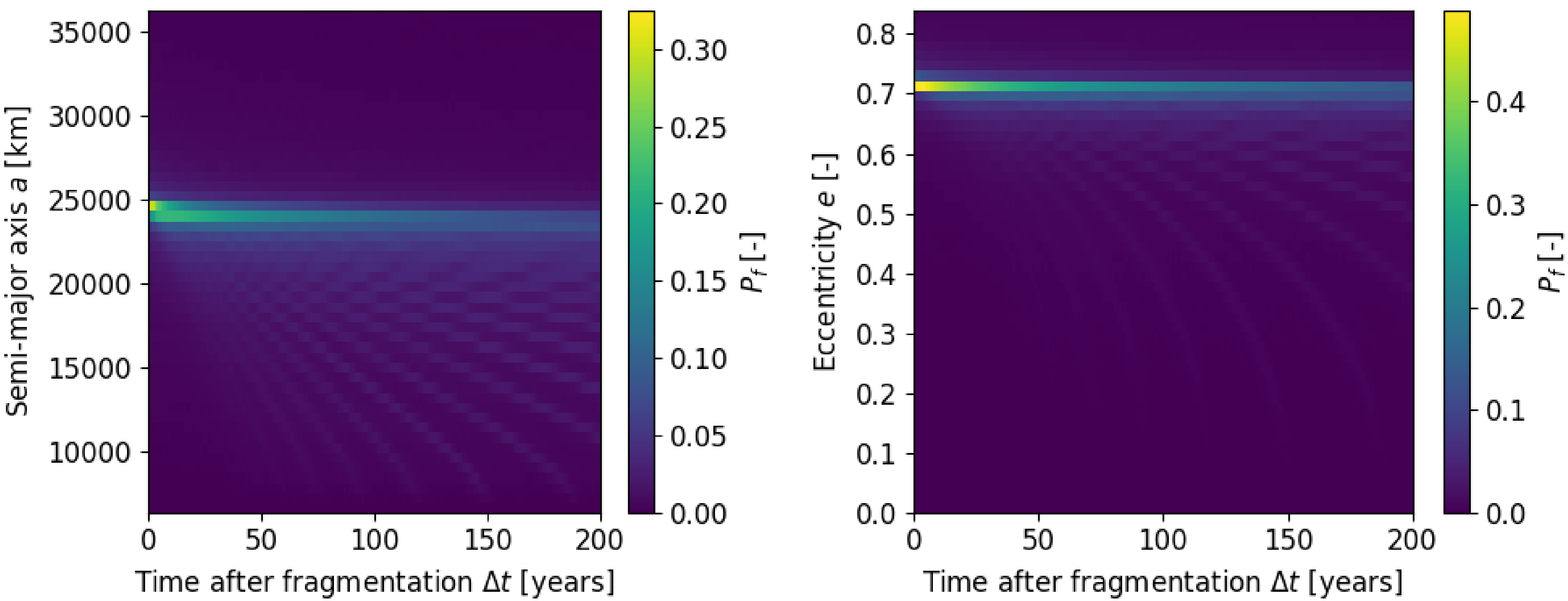


History of on-orbit satellite fragmentations, 14th Edition. NASA's Orbital Debris Program Office, 2008

Results

Evolution

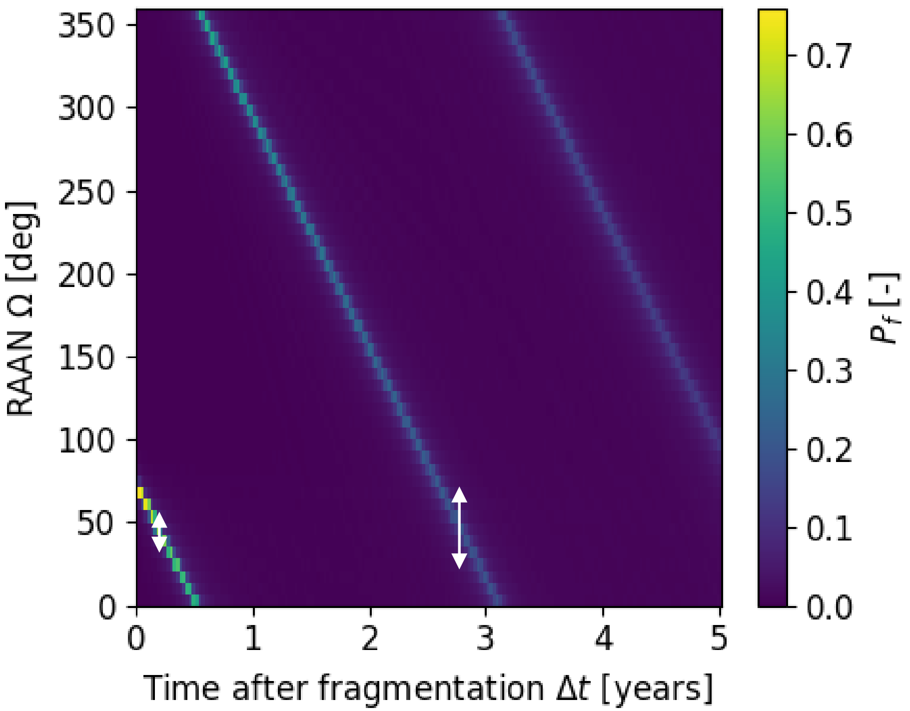
- Evolution based on fragmentation of GTO example mentioned earlier, for 200 years

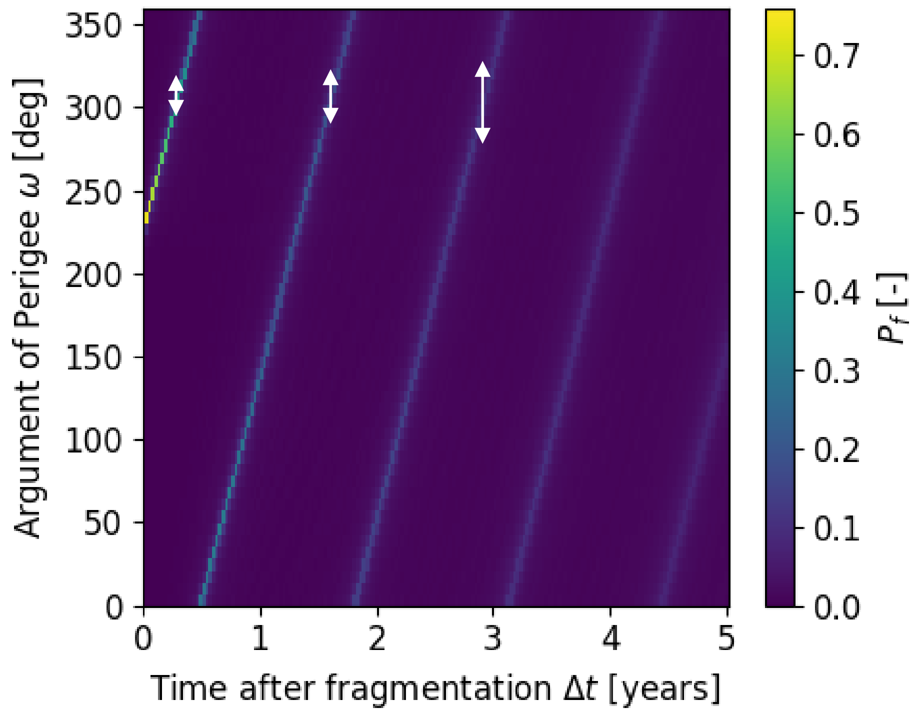


Results

Time of Closure

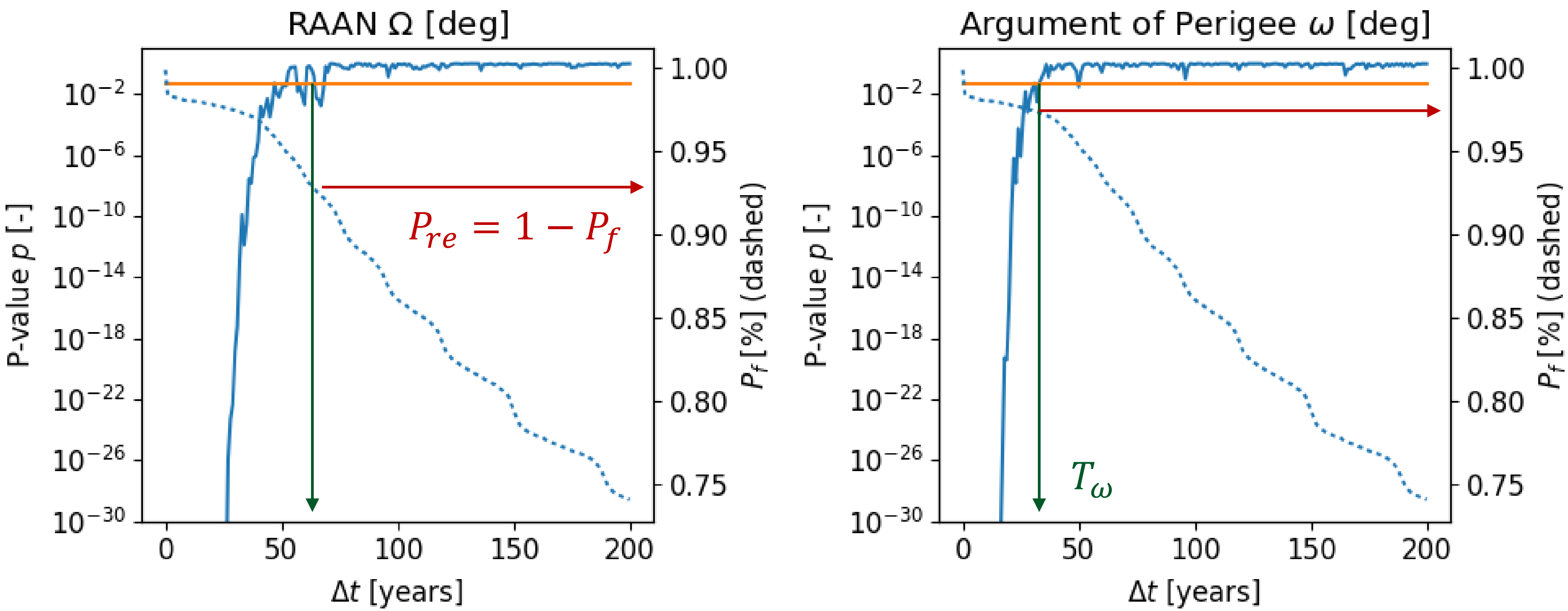
- Evolution of Ω, ω in the first 5 years
- Spreading due to J_2





Results

Time of Closure



- Still, most of fragments present until formation of band, but significant spreading in a took place

Conclusion

- Studied case: 515 x 35700 km, $i = 8°$, fragmentation at perigee
- Closure times:
  - Ω: 50 – 70 years
  - ω: 30 – 50 years
  - M: << 1 year
- Probability of re-entry (very conservative $T_\infty = 750$ K was assumed):
  - 10% after 80 years
  - 20% after 150 years
  - 25% after 200 years
- While most of the collision risk will come after closure, the 50 – 70 years from before cannot be neglected and need to be treated separately

Future Work

- Remove sensitivity on grid points, e.g. differential algebra
- Study different inclinations and perigees
- Include solar radiation pressure and third body perturbations
- Switch to density based propagation instead of representative objects
- Calculate collision risk

- Questions?

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Contact
Stefan Frey, PhD Candidate
Aerospace Science and Technology Department
Politecnico di Milano
stefan.frey@polimi.it

Image credit: Earth Observatory, NASA