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RESEARCH ARTICLE

On pre-filling probability of flood control detention facilities

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Flood control detention facilities are widely used for stormwater control in urban areas. Standard design procedures are in most cases based on the design storm approach: a single flood event at a time is considered, at the beginning of which the facility is assumed completely empty. The possibility of pre-filling from previous events is then neglected and underestimation of storage volume may occur. In this paper an analytical probabilistic approach to estimate the probability of pre-filling is presented and its effects, due to outflow rate and storage volume, are investigated. Two different strategies for the outlet control are analysed. Results are validated on a case study.

Keywords: flood control detention facilities; analytical probabilistic approach; pre-filling probability

1. Introduction

In recent decades, the significant and rapid increase of impervious surfaces in urban areas has increased the risk of both flooding, due to the overload of drainage systems during intense rainfall events, and uncontrolled polluted spills into water receivers by combined sewer overflows (CSOs). Flood detention facilities may be effective in the reduction of these risks, not only in the more traditional form of closed tanks and open ponds at the outlets of the sewer system (downstream control), but also in the distributed small basins and ponds used before the sewer inlets in sustainable urban drainage systems (SUDS) (upstream control).

Although the modelling of these facilities should be based on the analysis of the stochastic process of flood events, a simplified approach, based on the design storm, is often adopted in engineering procedures. A "critical" rainfall event of specific return period, extracted from a recorded series or defined by the combination of rainfall IDF curves and standard hyetograph patterns, is then typically used, together with a rainfall-runoff model to estimate a flood hydrograph. The storage volume needed to satisfy a maximum water release is then calculated accordingly to a specific hypothesis on the outlet characteristics.

This approach has several weaknesses, highlighted by many researchers in the last decades (e.g., Adams *et al.* 1986, Adams and Papa 2000). First of all, the return period associated to the storage volume is assumed to be the same as the hyetograph or, sometimes as the maximum rainfall intensity for the storm duration. This simplification ignores not only the effect of catchment antecedent conditions on the rainfall-runoff process and the consequent hydrograph (Wenzel 1981, Becciu and Paoletti 1997, 2000), but also the influence of the hydrograph pattern and duration on the detention process and then on storage volume. In engineering practice, these problems are often solved with some simplifying hypothesis on the catchment antecedent conditions and on the hydrograph pattern in order to have cautionary (overestimating) results.

A second deficiency is that the "critical" flood event is considered isolated from the whole stochastic process and the basin is assumed to be always empty at its beginning. Single-event design storm approaches depend, in fact, on arbitrary assumptions on the antecedent conditions and ignore the dry weather processes. The storm interevent times strongly affect runoff volume such that to estimate the flood frequency properly, the joint probabilities of both the antecedent conditions and the current rainfall event should be considered (James 1992, 1994).

Pre-filling from previous events is then neglected and underestimation of the needed storage may occur or, which is the same, the return period assumed in design is overestimated especially for low outflow rates and some management rules (Becciu and Raimondi 2012).

Some authors in the past discussed this problem but often in an incomplete way. Di Toro *et al.* (1984) observed that the total storage volume may not always be available at the beginning of a storm event since the basin may still have leftover runoff from previous events, but they didn't consider the influence of rainfall duration on pre-filling volume.

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Loganathan and Delleur (1984) only considered in the estimation of pre-filling volume, the condition in which the storage starts emptying at the beginning of the rainfall event, neglecting other strategies for runoff control (e.g., the possibility that the emptying starts at the end of the runoff event). They also proposed a method for sizing storage basins from the available storage volume at the end of the last rainfall event, neglecting the complete filling–emptying cycle.

To analyse the probability of pre-filling, the more suitable solution is the continuous simulation of the detention facility operations for a period long enough to take into account different possible combinations of storm events and dry periods. This approach, also allowing the modelling of multi-purpose facilities with complex operation modes (see e.g., Camnasio and Becciu 2011), usually gives reliable results; however, it may be difficult to apply because of the costs for data collection and processing or the unavailability of long series of data to make a reliable risk analysis. Also the relationship between the pre-filling probability and the size and operation modes of the detention facility is in this case also determined only indirectly from a posterior regression analysis among results in a number of hypothetical scenarios.

An alternative approach, able to consider the facility operation dynamics without the need of continuous simulation, is analytical probabilistic modelling. This method is based on the probabilistic analysis of functions of random variables, aimed at the analytical derivation of their distribution functions, eased by some simplifying hypotheses (Benjamin and Cornell 1970). A relevant application of this approach to hydrology dates back to the 1970s (e.g., Eagleson 1972), while significant applications to detention facilities are more recent (Guo and Adams 1998a, 1998b, 1999, Adams and Papa 2000).

In the literature, however, the application of this approach to detention facility analysis rarely considers the possibility of pre-filling from previous events and when it happens, the associated probability and its relation with both the volume and the operation of the detention facility is not expressed directly (Becciu *et al.* 2011).

The procedure presented in this work is intended to fill this gap, deriving analytical probabilistic formulas for the estimation of pre-filling probability as a function of rainfall statistics, outlet operation rules, maximum outflow rate and storage volume.

An application to a case study in Italy is presented to test the reliability of the proposed formulas. A comparison with continuous simulation is also presented and discussed to investigate the importance of simplifying assumptions.

2. Modelling of flood control detention facilities

In the modelling of flood control detention facilities some simplifying assumptions have been used.

On-line detention facilities have been considered: they can store stormwater collected from drainage surfaces of different sizes on catchments, roof tops, etc.

To isolate independent events from a continuous record of rainfalls, a minimum interevent time, the so called interevent time definition (IETD) (USEPA 1986) has been defined. If the actual storm interevent time is smaller than the IETD, two consecutive rainfalls are joined together into a single event, otherwise they are assumed independent.

Meteorological input variables that affect most the modelling of flood control detention facilities [rainfall depth (h), duration (θ) and interevent time (d)] are considered independent and exponentially distributed:

$$f_h = \xi \cdot e^{-\xi \cdot h} \tag{1}$$

$$f_\theta = \lambda \cdot e^{-\lambda \cdot \theta} \tag{2}$$

$$f_d = \psi \cdot e^{-\psi \cdot (d - IETD)} \tag{3}$$

$\xi = 1/\mu_h$, $\lambda = 1/\mu_\theta$, $\psi = 1/(\mu_d - IETD)$

μ_x expected value of the random variable x.

In the literature, for most basins, this hypothesis has often been confirmed or considered acceptable in order to reduce the complexity of analytical derivation (Eagleson 1978, Adams *et al.* 1986, Bedient and Huber 1992).

The hydrological model is the same employed by the STORM simulation model (U.S. Army Corps of Engineers 1974). It considers an initial abstraction (IA) that fills the volume of depression storages and only if rainfall depth is higher than IA does runoff occur. Losses for infiltration are taken into account in the dimensionless runoff coefficient ϕ.

Hydrological losses are averaged over rainfall duration (Figure 1). A uniform loss equal to $(1 -)\cdot(h - IA)$, occurring after the initial depression storages have been filled, is considered.

The rainfall-runoff transformation is neglected and net rainfall intensities are considered as inflow rates in the basin. This hypothesis can be reliable for small catchments

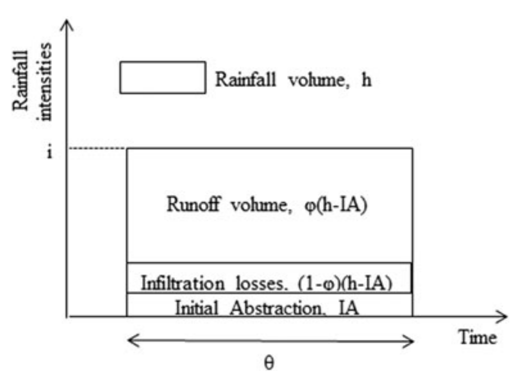

Figure 1. Hydrological losses averaged on rainfall duration.

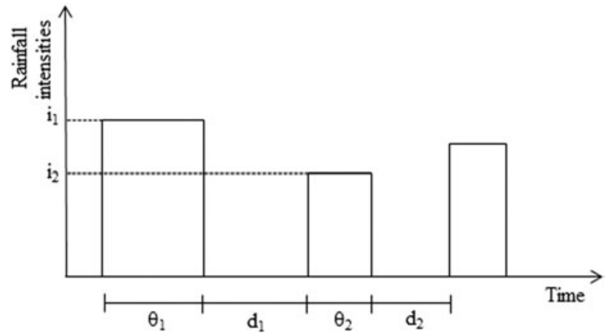

Figure 2. Scheme of storage process.

where runoff rates can be assumed approximately proportional to rainfall intensities. As a consequence, runoff duration is considered equal to rainfall duration; although in reality the duration of a runoff event is usually longer than that of a rainfall event.

Incoming hydrographs are assumed as rectangular, neglecting the temporal distribution of rainfall intensity within a storm event. This hypothesis can be considered acceptable as the design of flood control detention facilities is mainly influenced by rainfall volumes rather than by rainfall intensities. Moreover, if inflow rates are greater than constant outflow rates for the storm duration, the final stored volumes are independent from the hydrograph pattern and rectangular events may be used.

The outflow rate from the basin is also assumed to be constant. This is not easy to accomplish, since most of outlets have a linear logarithmic relationship between headwater depth and discharge. Also when the basin is emptied by a pump, the efficiency of the system can vary with the submergence of the pump, but in this case the outflow rate may be assumed constant.

As discussed by Raimondi (2012), the estimation of the pre-filling probability assuming just water carryover from one previous event may be acceptable only when a sufficiently long IETD and high outflow rates are considered. For flood control detention facilities with low outflow rates (e.g., infiltration basins) or when strict limitations on discharges in the downstream drainage system are imposed, this hypothesis can lead to underestimation of the pre-filling probability. To take into account all different conditions of discharge, two previous events have been considered

Two storage management rules are analysed, according to the more frequent strategies of discharge control:

Management rule A. The flood control detention facility is emptied with a constant outflow rate (q), starting as soon as it begins to fill. Considering rectangular events with inflow rates greater than outflow rates, this means soon after the beginning of each event (Figure 3). This is typical with on-line flood control detention facilities.

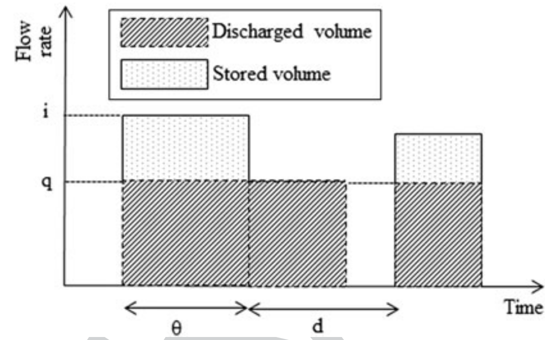

Figure 3. Management rule A.

Management rule B. The constant outflow (q) starts after the end of each event. It continues until the basin is empty or the next event begins (Figure 4). Management rule B can be used in real time control (RTC) applications, when it is necessary to temporarily retain a certain volume to reduce the risk of downstream system overload, or for water quality control purposes.

3. Pre-filling probability

With both management rules, it is possible to have a non-zero probability of a pre-filling volume greater than zero only if the maximum emptying time of storage volume (w_0) is greater than the minimum IETD. If volumes and flow rates are expressed per unit of effective catchment area ($\varphi \cdot S$ where φ is runoff coefficient and S is the catchment area), as it is in all of the following formulas, this condition leads to the equivalent inequalities:

$$\frac{w_0 \cdot (1-\alpha)}{q} > IETD \tag{4}$$

$$q < q_M = \frac{w_0}{IETD} \tag{5}$$

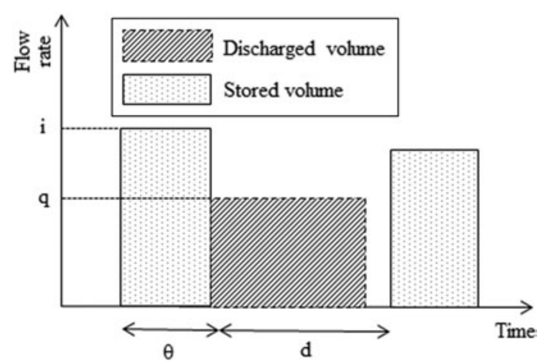

Figure 4. Management rule B.

$$w \geq w^* = q \cdot IETD \tag{6}$$

where q_M is the flow rate to empty the volume w_0 in time $IETD$, w^* is the volume emptied in time $IETD$ with a flow rate equal to q and α is the percentage of storage volume (w_0) pre-filled ($0 \leq \alpha \leq 1$).

Expressions for the estimation of the pre-filling probability are then derived considering the above two described management rules.

3.1 Management rule A

If a couple of rainfall events with the same depth, duration and interevent time are considered (with reference to Figure 2, setting $h = h_1 = h_2$, $\theta = \theta_1 = \theta_2$, $d = d_1 = d_2$), the pre-filling volume at the beginning of the second rainfall event ($w_{pr,1}$) can be expressed as:

$$w_{pr,1} = \begin{cases} h - IA - q \cdot \theta - q \cdot d & Case\ I \\ w_0 - q \cdot d & Case\ II \\ 0 & Otherwise \end{cases} \tag{7}$$

Case I : *if* $w_{sp,1} = h - IA - q \cdot \theta - w_0 \leq 0 \cup h - IA - q \cdot \theta - q \cdot d > 0$
Case II : *if* $w_{sp,1} = h - IA - q \cdot \theta - w_0 > 0 \cup w_0 - q \cdot d > 0$

where $w_{sp,1}$ is the spilled volume at the end of the first rainfall event.

Combining conditions from Equation (7) together and using PDFs of rainfall depth (Equation (1)), duration (Equation (2)) and interevent time (Equation (3)), the probability of pre-filling considering only a previous rainfall event results is:

$$P_A(w_{pr,1} > \alpha \cdot w_0) = \int_{\theta_A}^{\theta_B} f_\theta \cdot d\theta \int_{h_A}^{h_B} f_h \cdot dh \int_{d_A}^{d_B} f_d \cdot dd + \int_{\theta_C}^{\theta_D} f_\theta \cdot d\theta \int_{h_C}^{h_D} f_h \cdot dh \int_{d_C}^{d_D} f_d \cdot dd$$

where:
$\theta_A = \theta_C = 0$
$\theta_B = \theta_D = \infty$
$d_A = d_C = IETD$
$d_B = d_D = w_0 \cdot (1 - \alpha)/q$
$h_A = IA + q \cdot \theta + q \cdot d + \alpha \cdot w_0$
$h_B = h_C = IA + q \cdot \theta + w_0$
$h_d = \infty$

$$P_A(w_{pr,1} > \alpha \cdot w_0) = e^{-\xi \cdot IA} \cdot \left(\frac{1 - \beta}{1 + q^*}\right) \cdot \left[e^{-\xi \cdot q \cdot IETD - \xi \cdot \alpha \cdot w_0} - e^{\psi \cdot IETD - \frac{\psi}{q} \cdot w_0 \cdot (1 - \alpha) - \xi \cdot w_0}\right] \tag{8}$$

where: $\beta = \frac{q \cdot \xi}{q \cdot \xi + \psi}$ and $q^* = \frac{q \cdot \xi}{\lambda}$.

With reference to Figure 2, the pre-filling volume at the beginning of the third rainfall event ($w_{pr,2}$) can be expressed as:

$$w_{pr,2} = \begin{cases} 2 \cdot \left(h - IA - q \cdot \theta - q \cdot d\right) & Case\ I \\ w_0 - q \cdot d & Case\ II \\ 0 & Otherwise \end{cases} \tag{9}$$

Case I : *if* $w_{sp,1} = h - IA - q \cdot \theta - w_0 \leq 0 \cup w_{pr,1} = h - IA - q \cdot \theta - q \cdot d > 0 \cup w_{sp,2} = 2 \cdot (h - IA - q \cdot \theta) - q \cdot d - w_0 \leq 0 \cup 2 \cdot (h - IA - q \cdot \theta - q \cdot d) > 0$
Case II : *if* $w_{sp,1} = h - IA - q \cdot \theta - w_0 \leq 0 \cup w_{pr,1} = h - IA - q \cdot \theta - q \cdot d > 0 \cup w_{sp,2} = 2 \cdot (h - IA - q \cdot \theta) - q \cdot d - w_0 > 0 \cup w_0 - q \cdot d > 0$
or $w_{sp,1} = h - IA - q \cdot \theta - w_0 > 0 \cup w_{pr,1} = w_0 - q \cdot d > 0 \cup w_{sp,2} = w_0 - q \cdot d + h - IA - q \cdot \theta - w_0 > 0 \cup w_0 - q \cdot d > 0$

where $w_{sp,2}$ is the spilled volume at the end of the second rainfall event.

The respective probability of pre-filling results are as follows:

$$P_A(w_{pr,2} > \alpha \cdot w_0) = \int_{\theta_A}^{\theta_B} f_\theta \cdot d\theta \int_{h_A}^{h_B} f_h \cdot dh \int_{d_A}^{d_B} f_d \cdot dd + \int_{\theta_C}^{\theta_D} f_\theta \cdot d\theta \int_{h_C}^{h_D} f_h \cdot dh \int_{d_C}^{d_D} f_d \cdot dd + \int_{\theta_E}^{\theta_F} f_\theta \cdot d\theta \cdot \int_{h_E}^{h_F} f_h \cdot dh \cdot \int_{d_E}^{d_F} f_d \cdot dd$$

where:
$\theta_A = \theta_C = \theta_E = 0$
$\theta_B = \theta_D = \theta_F = \infty$
$d_A = d_C = d_E = IETD$
$d_B = d_D = d_F = w_0 \cdot (1 - \alpha)/q$
$h_A = IA + q \cdot \theta + q \cdot d + (\alpha \cdot w_0)/2$
$h_B = h_C = IA + q \cdot \theta + (w_0 + q \cdot d)/2$
$h_D = h_E = IA + q \cdot \theta + w_0$
$h_F = \infty$

$$P_A(w_{pr,2} > \alpha \cdot w_0) = e^{-\xi \cdot IA} \cdot \left(\frac{1 - \beta}{1 + q^*}\right) \cdot \left[e^{-\xi \cdot q \cdot IETD - \xi \cdot \alpha \cdot w_0} - e^{\psi \cdot IETD - \frac{\psi}{q} \cdot w_0 \cdot (1 - \alpha) - \xi \cdot w_0}\right] \tag{10}$$

For the hypothesis that $h = h_1 = h_2$, $\theta = \theta_1 = \theta_2$, $d = d_1 = d_2$, Equation (10) represents both the probability $P_A(w_{pr,2} > \alpha \cdot w_0)$ and the conditional probability $P_A(w_{pr,2} > \alpha \cdot w_0 | w_{pr,1} > \alpha \cdot w_0)$. The total pre-filling probability $P_A(w_{pr} > \alpha \cdot w_0)$ for management rule A results as follows:

$$\begin{aligned} P_A(w_{pr} > \alpha \cdot w_0) &= P_A(w_{pr,1} > \alpha \cdot w_0) \cup P_A(w_{pr,2} > \alpha \cdot w_0) = P_A(w_{pr,1} > \alpha \cdot w_0) + P_A(w_{pr,2} > \alpha \cdot w_0) - P_A(w_{pr,2} > \alpha \cdot w_0 | w_{pr,1} > \alpha \cdot w_0) \cdot P(w_{pr,1} > \alpha \cdot w_0) = 2 \cdot e^{-\xi \cdot IA} \cdot \left(\frac{1-\beta}{1+q^*}\right) \\ &\cdot \left[e^{-\xi \cdot q \cdot IETD - \xi \cdot \alpha \cdot w_0} - e^{\psi \cdot IETD - \frac{\psi}{q} \cdot w_0 \cdot (1-\alpha) - \xi \cdot w_0}\right] \\ &- \left\{e^{-\xi \cdot IA} \cdot \left(\frac{1-\beta}{1+q^*}\right) \cdot \left[e^{-\xi \cdot q \cdot IETD - \xi \cdot \alpha \cdot w_0} - e^{\psi \cdot IETD - \frac{\psi}{q} \cdot w_0 \cdot (1-\alpha) - \xi \cdot w_0}\right]\right\}^2 \end{aligned} \quad (11)$$

3.2 Management rule B

For this strategy of control of discharges, the detention facility starts emptying only at the end of the event; as a consequence, the pre-filling volume is independent of rainfall duration. Considering a couple of rainfall events with the same depth, duration and interevent time ($h = h_1 = h_2$, $d = d_1 = d_2$ with reference to Figure 2), pre-filling volume at the beginning of the second rainfall event ($w_{pr,1}$) can be expressed as:

$$w_{pr,1} = \begin{cases} h - IA - q \cdot d & \textit{Case I} \\ w_0 - q \cdot d & \textit{Case II} \\ 0 & \textit{Otherwise} \end{cases} \quad (12)$$

Case I : *if* $w_{sp,1} = h - IA - w_0 \le 0 \cup h - IA - q \cdot d > 0$
Case II : *if* $w_{sp,1} = h - IA - w_0 > 0 \cup w_0 - q \cdot d > 0$

Combining conditions from Equation (12) together and using the PDFs of rainfall depth (Equation (1)) and interevent time (Equation (3)), the probability of pre-filling results are as follows:

$$P_B(w_{pr,1} > \alpha \cdot w_0) = \int_{h_A}^{h_B} f_h \cdot dh \int_{d_A}^{d_B} f_d \cdot dd + \int_{h_C}^{h_D} f_h \cdot dh \int_{d_C}^{d_C} f_d \cdot dd$$

where:
$d_A = d_C = IETD$
$d_B = d_D = w_0 \cdot (1-\alpha)/q$
$h_A = IA + q \cdot d + \alpha \cdot w_0$
$h_B = h_C = IA + w_0$
$h_d = \infty$

$$P_B(w_{pr,1} > \alpha \cdot w_0) = e^{-\xi \cdot IA} \cdot (1-\beta) \cdot \left[e^{-\xi \cdot q \cdot IETD - \xi \cdot \alpha \cdot w_0} - e^{\psi \cdot IETD - \frac{\psi}{q} \cdot w_0 \cdot (1-\alpha) - \xi \cdot w_0}\right] \quad (13)$$

The pre-filling volume at the beginning of the third rainfall events ($w_{pr,2}$) can be expressed as:

$$w_{pr,2} = \begin{cases} 2 \cdot (h - IA - q \cdot d) & \textit{Case I} \\ w_0 - q \cdot d & \textit{Case II} \\ 0 & \textit{Otherwise} \end{cases} \quad (14)$$

Case I : *if* $w_{sp,1} = h - IA - w_0 \le 0 \cup w_{pr,1} = h - IA - q \cdot d > 0 \cup w_{sp,2} = 2 \cdot (h - IA) - q \cdot d - w_0 \le 0 \cup 2 \cdot (h - IA - q \cdot d) > 0$
Case II : *if* $w_{sp,1} = h - IA - w_0 \le 0 \cup w_{pr,1} = h - IA - q \cdot d > 0 \cup w_{sp,2} = 2 \cdot (h - IA) - q \cdot d - w_0 > 0 \cup w_0 - q \cdot d > 0$
or $w_{sp,1} = h - IA - w_0 > 0 \cup w_{pr,1} = w_0 - q \cdot d > 0 \cup w_{sp,2} = w_0 - q \cdot d + h - IA - w_0 > 0 \cup w_0 - q \cdot d > 0$

And the respective probability of pre-filling results are as follows:

$$P_B(w_{pr,1} > \alpha \cdot w_0) = \int_{h_A}^{h_B} f_h \cdot dh \int_{d_A}^{d_B} f_d \cdot dd + \int_{h_C}^{h_D} f_h \cdot dh \int_{d_C}^{d_C} f_d \cdot dd + \int_{h_E}^{h_F} f_h \cdot dh \cdot \int_{d_E}^{d_F} f_d \cdot dd$$

where:
$d_A = d_C = d_E = IETD$
$d_B = d_D = d_F = w_0 \cdot (1-\alpha)/q$
$h_A = IA + q \cdot d + (\alpha \cdot w_0)/2$
$h_B = h_C = IA + (w_0 + q \cdot d)/2$
$h_D = h_E = IA + w_0$
$h_F = \infty$

$$P_B(w_{pr,2} > \alpha \cdot w_0) = e^{-\xi \cdot IA} \cdot (1-\beta) \cdot \left[e^{-\xi \cdot q \cdot IETD - \xi \cdot \alpha \cdot w_0} - e^{\psi \cdot IETD - \frac{\psi}{q} \cdot w_0 \cdot (1-\alpha) - \xi \cdot w_0}\right] \quad (15)$$

For the hypothesis that $h = h_1 = h_2$, $\theta = \theta_1 = \theta_2$, $d = d_1 = d_2$, Equation (15) represents both the probability $P_B(w_{pr,2} > \alpha \cdot w_0)$ and the conditional probability $P_B(w_{pr,2} > \alpha \cdot w_0 | w_{pr,1} > \alpha \cdot w_0)$. The total pre-filling probability $P_B(w_{pr} > \alpha{\cdot}w_0)$ in this case results as follows:

$$\begin{aligned} P_B(w_{pr} > \alpha{\cdot}w_0) &= P_B(w_{pr,1} > \alpha{\cdot}w_0) \cup P_B(w_{pr,2} \\ &> \alpha{\cdot}w_0) = P_B(w_{pr,1} > \alpha{\cdot}w_0) + P_B(w_{pr,2} \\ &> \alpha{\cdot}w_0) - P_B(w_{pr,2} > \alpha{\cdot}w_0 | w_{pr,1} \\ &> \alpha{\cdot}w_0){\cdot}P(w_{pr,1} > \alpha{\cdot}w_0) = 2{\cdot}e^{-\xi{\cdot}IA}{\cdot}(1-\beta) \\ &\cdot\left[e^{-\xi{\cdot}q{\cdot}IETD-\xi{\cdot}\alpha{\cdot}w_0} - e^{\psi{\cdot}IETD-\frac{\psi}{q}{\cdot}w_0{\cdot}(1-\alpha)-\xi{\cdot}w_0}\right] \\ &- \Big\{e^{-\xi{\cdot}IA}{\cdot}(1-\beta) \\ &\cdot\left[e^{-\xi{\cdot}q{\cdot}IETD-\xi{\cdot}\alpha{\cdot}w_0} - e^{\psi{\cdot}IETD-\frac{\psi}{q}{\cdot}w_0{\cdot}(1-\alpha)-\xi{\cdot}w_0}\right]\Big\}^2 \end{aligned} \tag{16}$$

4. Effects due to outflow rate and storage volume

For both management rules, the pre-filling probability depends also on both a minimum IETD and IA.

Resulting formulas for the two different strategies of control of discharges are similar, except for the independence from rainfall duration with management rule B. Comparing Equations (11) and (16), results in:

$$P_A(w_{pr} > \alpha{\cdot}w_0) = \frac{P_B(w_{pr} > \alpha{\cdot}w_0)}{1+q^*} \tag{17}$$

As expected, pre-filling probability is lower with rule A than with rule B.

As q tends to zero, P_A and P_B tend to the same limit value:

$$\begin{aligned} lim_{q=0}P_A(w_{pr} > \alpha{\cdot}w_0) &= lim_{q=0}P_B(w_{pr} > \alpha{\cdot}w_0) \\ &= P_0 = e^{-\xi{\cdot}IA} \end{aligned} \tag{18}$$

while as q tends to q_M, both P_A and P_B tend to zero (Figure 5).

Considering the effects of storage volume (w_0), from Equation (6) it follows that pre-filling probability is, for both management rules, greater than zero for values greater than $w_0 = w^* = q{\cdot}IETD$. As w_0 tends to infinity, pre-filling probabilities tend to the constant values $P_{lim,A}$ and $P_{lim,B}$ (Figure 6):

$$lim_{w_0\to\infty}P_A = P_{lim,A} = \left(\frac{1-\beta}{1+q^*}\right){\cdot}e^{-\xi{\cdot}IA+(\xi{\cdot}q+\psi){\cdot}IETD} \tag{19}$$

$$lim_{w_0\to\infty}P_B = P_{lim,B} = (1-\beta){\cdot}e^{-\xi{\cdot}IA+(\xi{\cdot}q+\psi){\cdot}IETD} \tag{20}$$

5. Case study

Formulas for the estimation of pre-filling probability have been applied using a series of rainfall events recorded in the period 1991–2005 at the rain gauge of Monviso in the city of Milano, Italy. An IETD = 10 hours has been selected, identifying $N = 1647$ independent rainfall events (Raimondi 2012) and an IA has been assumed. The mean, standard deviation and coefficient of variation of rainfall depth, duration and interevent time of the recorded series of events are reported in Table 1.

As can be deduced by the coefficient of variation, only the hypothesis of an exponential distribution of rainfall duration seems correct. Bacchi *et al.* (2008), tested that for most Italian basins the Weibull probability distribution function, better fits the probability distribution of the meteorological input variables (rainfall depth h, rainfall duration θ and interevent time d). However, its use would involve a considerable complication in the integration of the expressions for the estimation of pre-filling probability. Becciu and Raimondi (2012) assumed a double-exponential distribution that better fitted the frequency distribution of the observed data for rainfall depth and interevent time, maintaining the exponential distribution for rainfall duration. The double-exponential distribution may be easily integrated but derived expressions are more complex. Their application highlighted that the use of the double-exponential distribution little improves the accuracy of results only for increasing outflow rates. The bias due to the use of the exponential distribution is negligible when compared to its simplicity of integration.

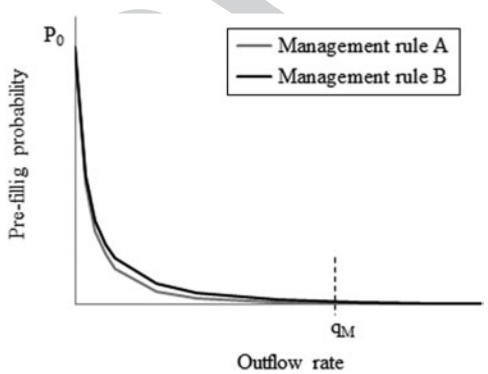

Figure 5. Pre-filling probability vs. outflow rate.

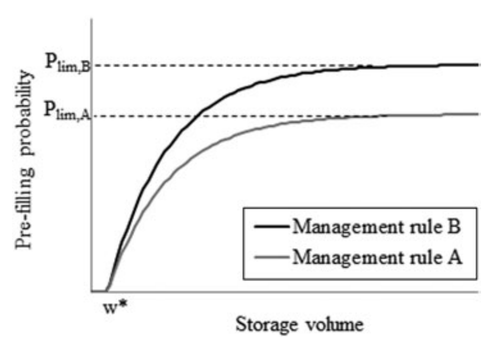

Figure 6. Pre-filling probability vs. storage volume.

Table 1. Mean, standard deviation and coefficient of variation of rainfall variables.

μ_h [mm] = 18.49	σ_h [mm] = 21.33	V_h [−] = 1.15
μ_θ [hours] = 14.37	σ_θ [hours] = 14.81	V_θ [−] = 1.03
μ_d [hours] = 172.81	σ_d [hours] = 223.89	V_d [−] = 1.30

To test the simplifying hypothesis of the independence of rainfall variables, correlation coefficients are calculated (Table 2). While rainfall depth and interevent time, as well as rainfall duration and interevent time, are only weakly correlated, rainfall depth and duration could not be assumed as independent. Also in this case, the assumption of independence in the proposed approach will cause a bias in the results.

To verify the accuracy of proposed formulas, also considering the simplifications adopted by the probability scheme, frequencies of pre-filling have been calculated assuming the recorded series of rainfall events as a series of rectangular floods incoming to a flood control detention facility. Storage volumes (w_0) ranging between 50 and 600 m^3/ha_{imp} and outflow rates (q) of 1 and 3 $l/(s{\cdot}ha_{imp})$ have been considered. The coefficient α has been set to zero. Figure 7 and Figure 8 show results obtained for both management rule A and B.

As can be seen, the pre-filling probabilities estimated with analytical Equations (12) and (19) fit well to frequencies calculated by continuous simulation of the recorded series of rainfall. For comparison, results from the application of the method considering only a previous rainfall event are reported in the figures. They highlight the importance of taking into account at least two previous rainfalls especially for low discharge rates.

It is worth noting that the hypothesis of independence of rainfall variables doesn't influence results from management rule B, since the storage process depends only on rainfall depth and interevent time, which are weakly correlated. In this case, the proposed formulas are better fit to observed frequencies (Figure 8). However, as can be observed for management rule A (Figure 7), the bias of considering the simplifying hypothesis of independence of rainfall variables is negligible. In particular, for rule A and $q = 3\,l/(s{\cdot}ha_{imp})$, it would be sufficient to consider only one previous rainfall event. This because the probability of pre-filling for management strategy A is lower than for rule B.

Table 2. Correlation indexes among rainfall variables.

$\rho_{\theta,h}$	0.62
$\rho_{d,h}$	0.11
$\rho_{\theta,d}$	0.11

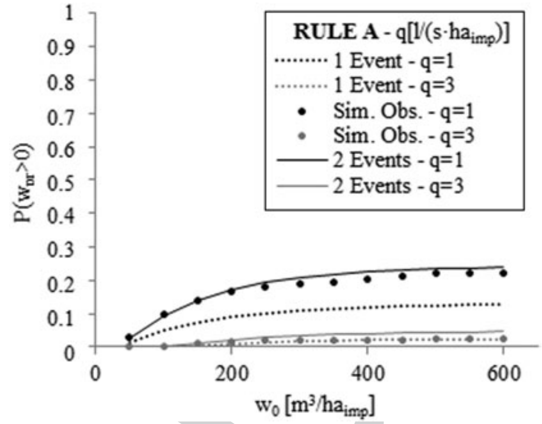

Figure 7. Pre-filling CDFs. Management rule A.

6. Conclusions

Flood control detention facilities are effective tools for runoff control. Their capacity must be carefully estimated to avoid uncontrolled spills into receivers. In some cases pre-filling of storage volume from previous storms can occur and the capacity of the facility can be underestimated.

An analytical probabilistic method has been proposed, useful to perform a simple and direct estimation of the probability of pre-filling for two difference strategies of control of discharges. Derived expressions depend on the stochastic process of the rainfall, on the storage volume and on the outflow rate. A minimum IETD and hydrological losses at the beginning of the event (IA) have also been considered.

Proposed formulas for the estimation of pre-filling probability have been tested, comparing results with those from continuous simulation in a case study. Results have been very satisfactory, showing very good agreements from the two approaches, showing also that the effects of simplifying hypothesis are negligible in most cases. Application to the case study also shows that the pre-filling

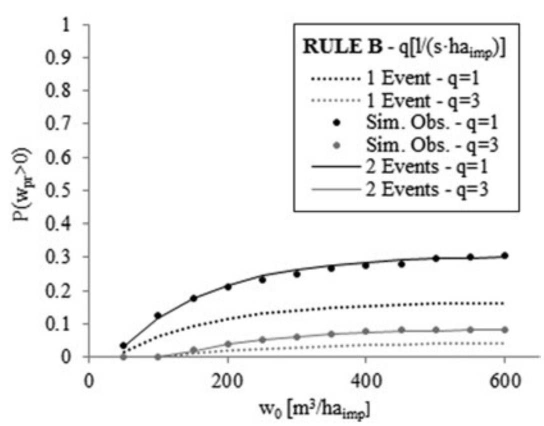

Figure 8. Pre-filling CDFs. Management rule B.

volumes may be significant and can't be neglected in the case of strict discharge limits in the downstream water system or of infiltration basins with low permeability soils.

The simplicity and the accuracy of the approach make it suitable for many engineering applications. Both the design and verification of stormwater storage facilities can benefit from a more accurate estimation of pre-filling probabilities. Proposed formulas can provide this estimation when continuous simulation of series of recorded storm events is not possible or reliable according to the amount and quality of rainfall records.

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