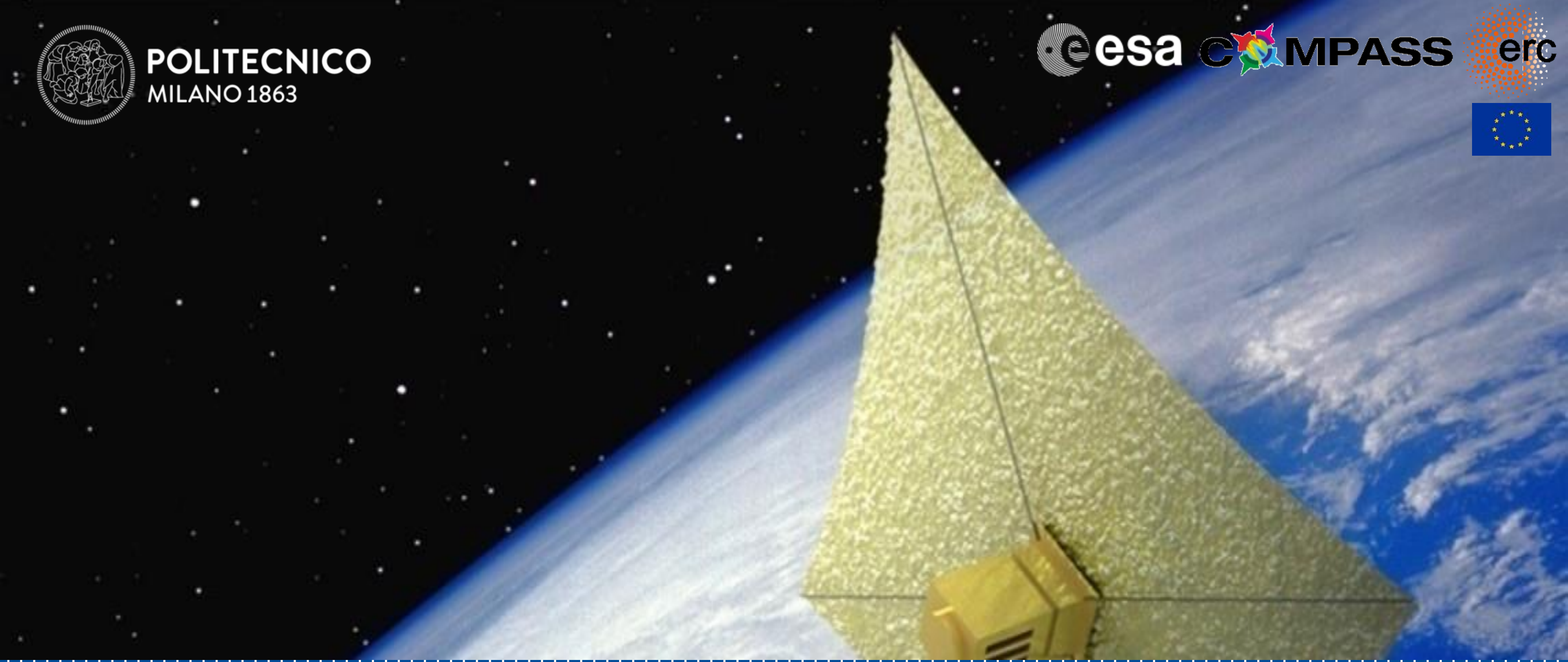
POLITECNICO
MILANO 1863

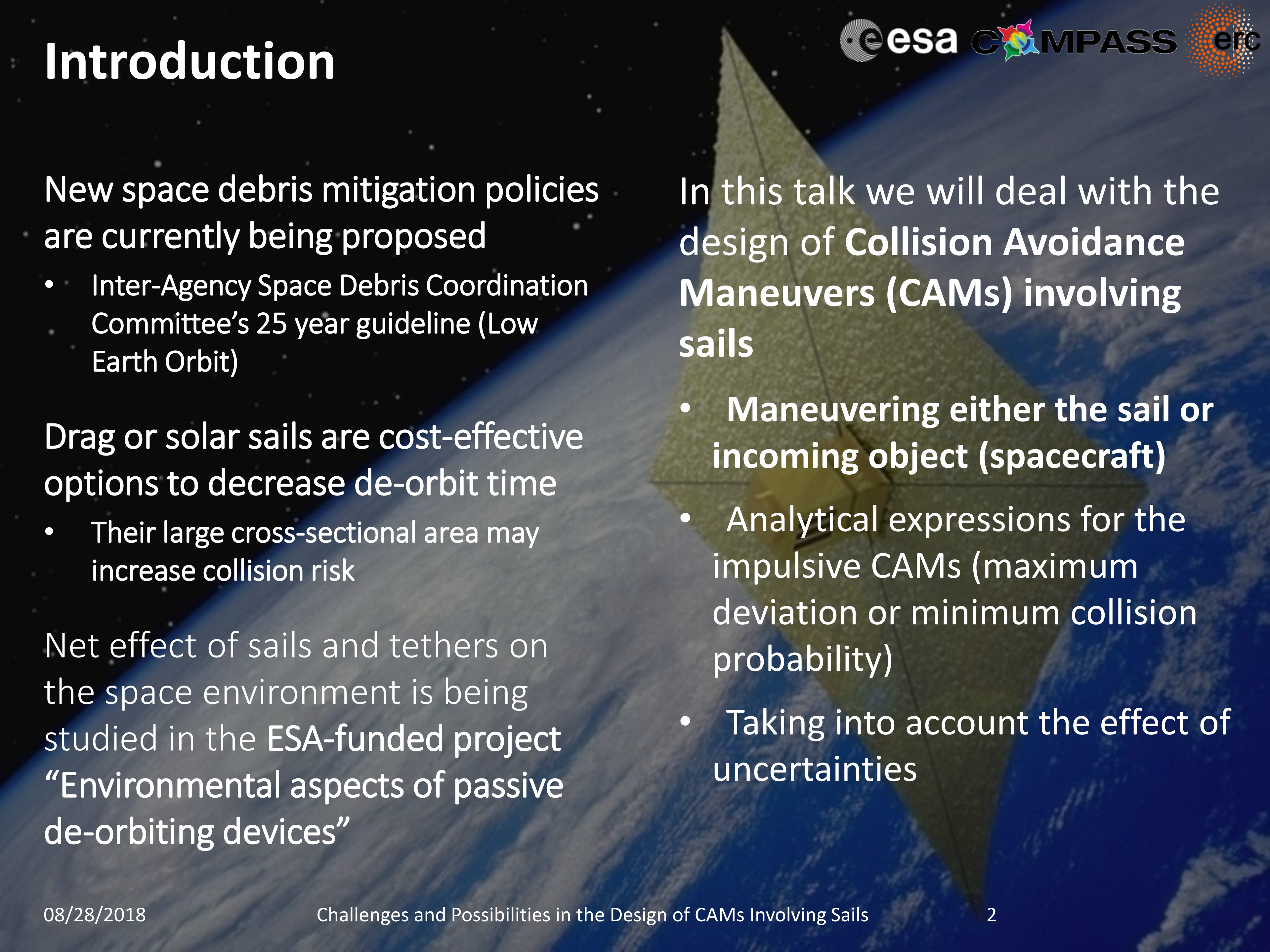


Challenges and possibilities in the design of collision avoidance maneuvers involving sails

Juan Luis Gonzalo, Camilla Colombo and Pierluigi Di Lizia

University of Arizona
28 August 2018

Introduction



New space debris mitigation policies are currently being proposed

- Inter-Agency Space Debris Coordination Committee’s 25 year guideline (Low Earth Orbit)

Drag or solar sails are cost-effective options to decrease de-orbit time

- Their large cross-sectional area may increase collision risk

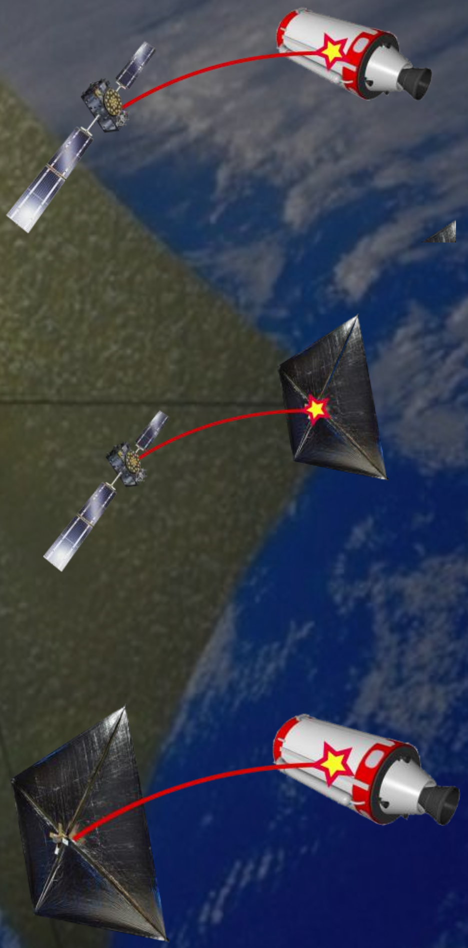
Net effect of sails and tethers on the space environment is being studied in the **ESA-funded project “Environmental aspects of passive de-orbiting devices”**

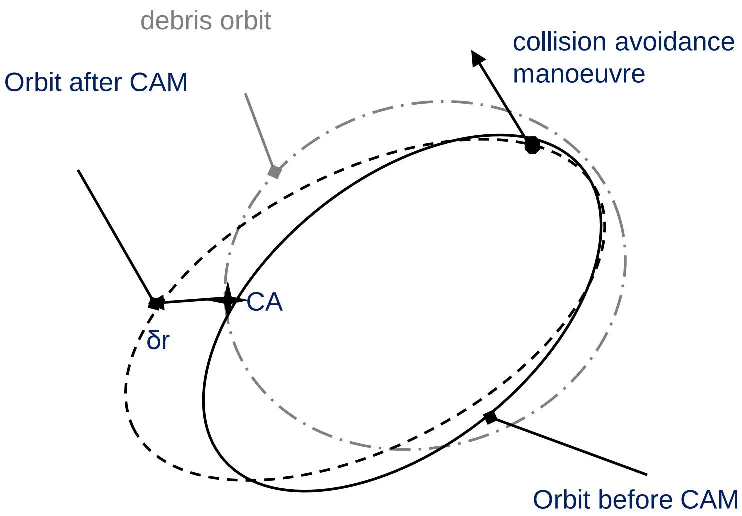
In this talk we will deal with the design of **Collision Avoidance Maneuvers (CAMs) involving sails**

- **Maneuvering either the sail or incoming object (spacecraft)**
- Analytical expressions for the impulsive CAMs (maximum deviation or minimum collision probability)
- Taking into account the effect of uncertainties

Outline

- Theoretical approach
- CAM design and sensitivity analysis
    - Spacecraft against debris
    - Effect of uncertainties. Spacecraft versus sail
    - CAM by a deorbiting sail
- Conclusions





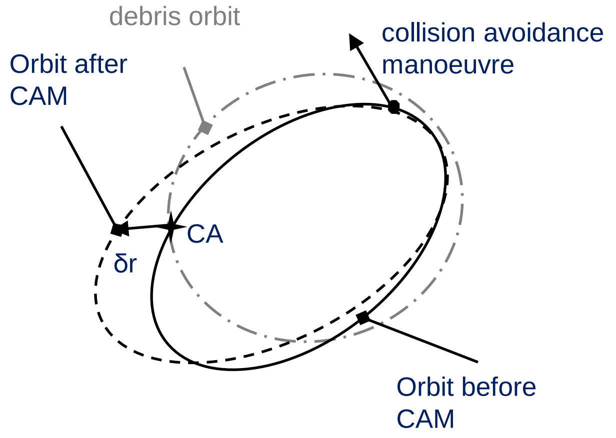
THEORETICAL APPROACH

Theoretical approach

Collision avoidance maneuver design

Modelling of Collision Avoidance Maneuver (CAM) in the b-plane

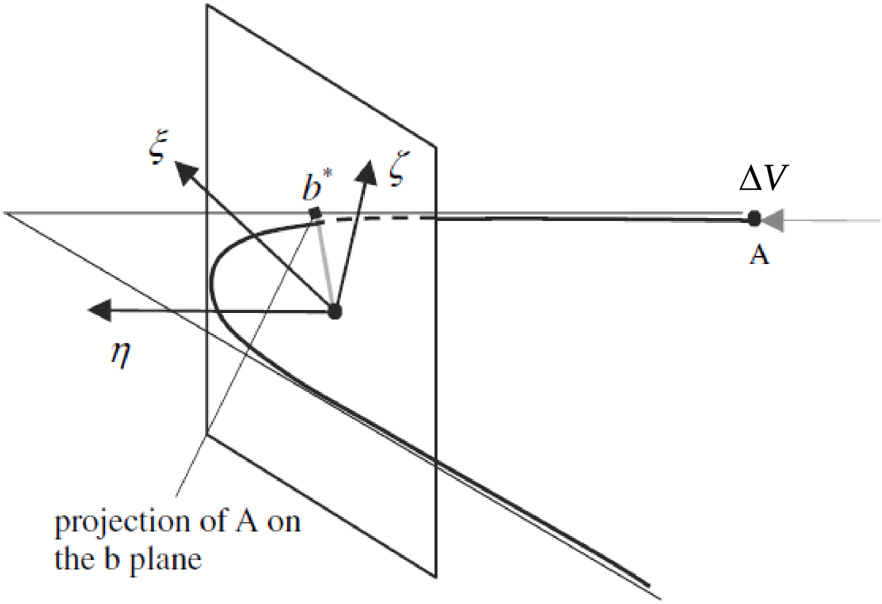
- Determine Close Approach (CA) between maneuverable spacecraft and debris
- CAM given at lead time Δt before the CA, modelled through Gauss planetary equations for finite differences
- Analytical computation of miss distance at the CA through relative motion equations
- Projection of the miss distance on the b-plane
- **Maximum deviation CAM design is reduced to an eigenvector problem.**



- Vasile and Colombo, JGCD 2008
- Conway, JSR 2005
- Petit, 2018

Theoretical approach

B-plane definition



- Intersection of the incoming asymptote and the b-plane: $\boldsymbol{b}^* =$ impact parameter
- η = 0 on the b-plane identifies the conjunction

Plane orthogonal to the s/c relative velocity at conjunction

η-axis: parallel to the relative velocity

ζ-axis: parallel to the projection on the b-plane of the debris, but in the opposite direction

ξ-axis: to complete a positively oriented reference system

➢ Öpik, 1976

Theoretical approach

Maximum deviation CAM

$$\delta\boldsymbol{\alpha}(t_{\mathrm{CAM}}) = \boxed{\mathbf{G}_v(t_{\mathrm{CAM}};\boldsymbol{\alpha})}\,\delta\mathbf{v}(t_{\mathrm{CAM}})$$

Gauss planetary equations [1]

$$\delta\mathbf{r}(t_{\mathrm{CA}}) = \boxed{\mathbf{A}_r(t_{\mathrm{CA}};\boldsymbol{\alpha},\Delta t)}\,\delta\boldsymbol{\alpha}(t_{\mathrm{CAM}})$$

Linearized relative motion [2]

$$\delta\mathbf{r}(t_{\mathrm{CA}}) = \mathbf{A}_r\mathbf{G}_v\delta\boldsymbol{v}(t_{\mathrm{CAM}}) = \mathbf{T}\,\delta\mathbf{v}(t_{\mathrm{CAM}})$$

Total displacement

$$\delta\mathbf{b}(t_{\mathrm{CA}}) = \boldsymbol{M}(t_{\mathrm{CA}})\delta\mathbf{r}(t_{\mathrm{CAM}}) = \mathbf{M}\,\mathbf{T}\,\delta\mathbf{v}(t_{\mathrm{CAM}}) = \mathbf{Z}\,\delta\mathbf{v}(t_{\mathrm{CAM}})$$

Displacement in b-plane

Optimization problem reduces to an eigenvalue/eigenvector problem [3]:

$$\max\|\delta\mathbf{r}(t_{\mathrm{CA}})\| = \mathrm{n} \quad \text{ector/value of } \mathbf{T}^{\mathrm{T}}\mathbf{T}$$

$$\mathbf{M}(t_{\mathrm{CA}}) = \begin{bmatrix} \eta_2^2+\eta_3^2 & -\eta_1\eta_2 & -\eta_1\eta_3 \\ -\eta_1\eta_2 & \eta_1^2+\eta_3^2 & -\eta_2\eta_3 \\ -\eta_1\eta_3 & -\eta_2\eta_3 & \eta_1^2+\eta_2^2 \end{bmatrix}$$

$$\max\|\delta\mathbf{b}(t_{\mathrm{CA}})\| = \mathrm{n} \quad \text{ector/value of } \mathbf{Z}^{\mathrm{T}}\mathbf{Z}$$

with $\|\delta\mathbf{v}\|$ as large as possible

[1] R. Battin, *An Introduction to the Mathematics and Methods of Astrodynamics*, 1999
[2] J. L. Junkins and H. Schaub, *Analytical mechanics of space systems*, 2009
[3] B. A. Conway, “Near-optimal deflection of earth-approaching asteroids,” *JGCD,* 24(5):1035-1037, 2001

Theoretical approach

Propagation of covariance matrix

Extending the model, the full analytic State Transition Matrix (STM) from $\delta\mathbf{s} = (\delta\mathbf{r}, \delta\mathbf{v})$ at t_{CAM} to $\delta\mathbf{s} = (\delta\mathbf{r}, \delta\mathbf{v})$ at t_{CA} is developed:

- $\mathbf{G}_r$ and $\mathbf{A}_v$ not directly available in previous references (but straightforward to derive)
- Optimizing the miss distance only required a quarter of this matrix.
- The covariance matrix can be propagated.
- Validated against Monte-Carlo simulations with nonlinear dynamics
- Drag and SRP not taken into account (i.e. not valid for sails)... for now

$$\left.\begin{aligned}
\delta\boldsymbol{\alpha}(t_{\mathrm{CAM}}) &= \begin{bmatrix} \mathbf{G}_{\mathrm{r}}(t_{\mathrm{CAM}}, \boldsymbol{\alpha}) \\ \mathbf{G}_{\mathrm{v}}(t_{\mathrm{CAM}}, \boldsymbol{\alpha}) \end{bmatrix} \delta\mathbf{s}(t_{\mathrm{CAM}}) \\
\delta\mathbf{s}(t_{\mathrm{CA}}) &= \begin{bmatrix} \mathbf{A}_r(t_{\mathrm{CA}}; \boldsymbol{\alpha}, \Delta t) \\ \mathbf{A}_v(t_{\mathrm{CA}}; \boldsymbol{\alpha}, \Delta t) \end{bmatrix} \delta\boldsymbol{\alpha}(t_{\mathrm{CAM}})
\end{aligned}\right\} \quad \delta\mathbf{s}(t_{\mathrm{CA}}) = \begin{bmatrix} \mathbf{A}_{\mathrm{r}}\mathbf{G}_{\mathrm{r}} & \mathbf{A}_{\mathrm{r}}\mathbf{G}_{\mathrm{v}} \\ \mathbf{A}_{\mathrm{v}}\mathbf{G}_{\mathrm{r}} & \mathbf{A}_{\mathrm{v}}\mathbf{G}_{\mathrm{v}} \end{bmatrix} \delta\mathbf{s}(t_{\mathrm{CAM}})$$

Theoretical approach

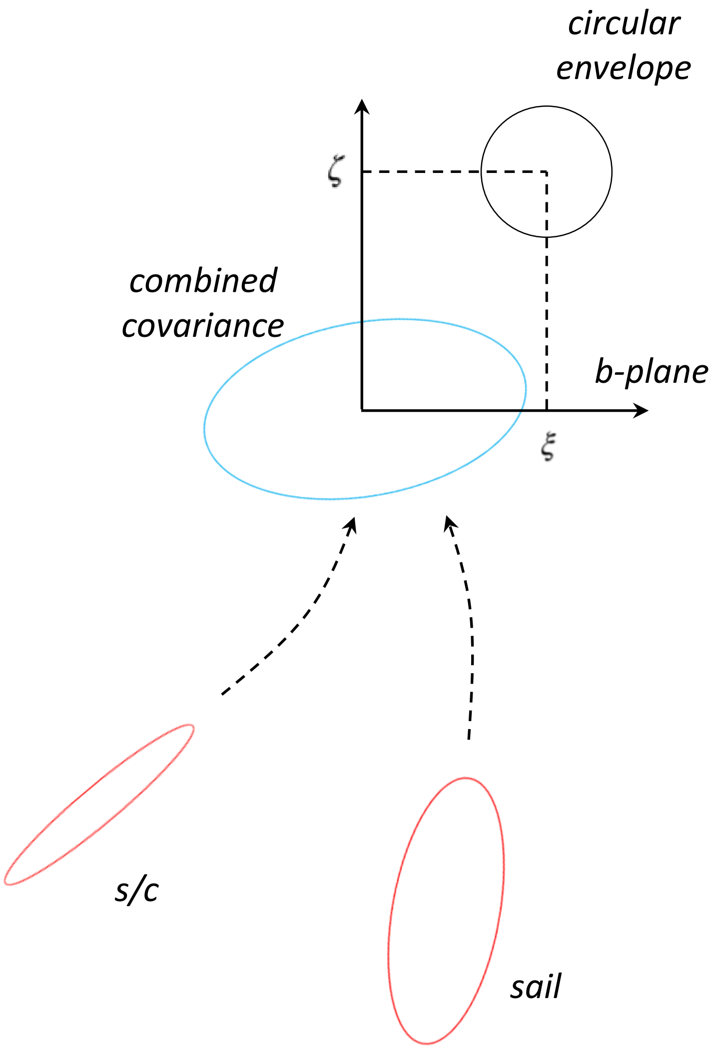
Minimum collision probability CAM design

- Given the combined covariance matrix and the circular envelope of the objects at b-plane
- CAM is designed by computing the $\delta\mathbf{v}(t_{\mathrm{CAM}})$ to minimise collision probability (Chan's approach*)
- The optimisation problem is reduced to an eigenvalue problem that maximise

$$J_P(\Delta\mathbf{v}) = \left(\frac{\xi}{\sigma_\xi}\right)^2 + \left(\frac{\zeta}{\sigma_\zeta}\right)^2 - 2\rho_{\xi\zeta}\frac{\xi\,\zeta}{\sigma_\xi\,\sigma_\zeta}$$

With the combined covariance at the b-plane:

$$\mathbf{C}_{\xi\zeta} = \begin{bmatrix} \sigma_\xi^2 & \rho_{\xi\zeta}\sigma_\xi\sigma_\zeta \\ \rho_{\xi\zeta}\sigma_\xi\sigma_\zeta & \sigma_\zeta^2 \end{bmatrix}$$



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* Bombardelli C., Hernando-Ayuso J., Optimal impulsive collision avoidance in low earth orbit, JGCD (2015)

Theoretical approach

Minimum collision probability CAM design

- Given the combined covariance matrix and the circular envelope of the objects at b-plane
- CAM is designed by computing the $\delta\mathbf{v}(t_{\text{CAM}})$ to minimise collision probability (Chan's approach[*])
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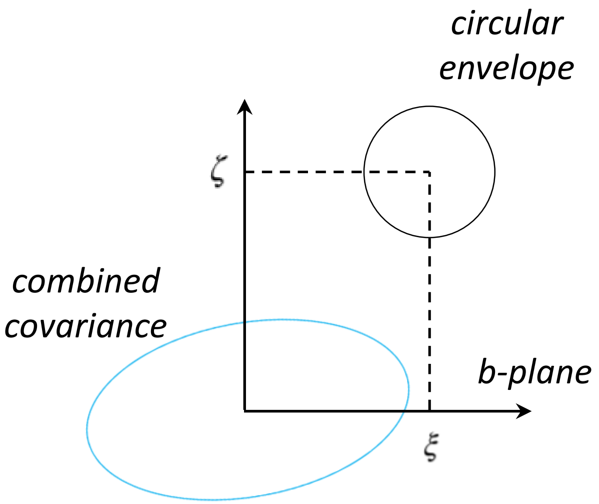
$$J_P(\Delta\mathbf{v}) = \left(\frac{\xi}{\sigma_\xi}\right)^2 + \left(\frac{\zeta}{\sigma_\zeta}\right)^2 - 2\rho_{\xi\zeta}\,\frac{\xi\,\zeta}{\sigma_\xi\,\sigma_\zeta}$$

Is the combined covariance to be **considered an input** or to be **computed by propagating the 6D covariances** of the objects?



Instead of the Dromo-based matrix used by Bombardelli and Hernando-Ayuso, the previously introduced STM based on **Gauss equations** and **linear relative motion** is applied.

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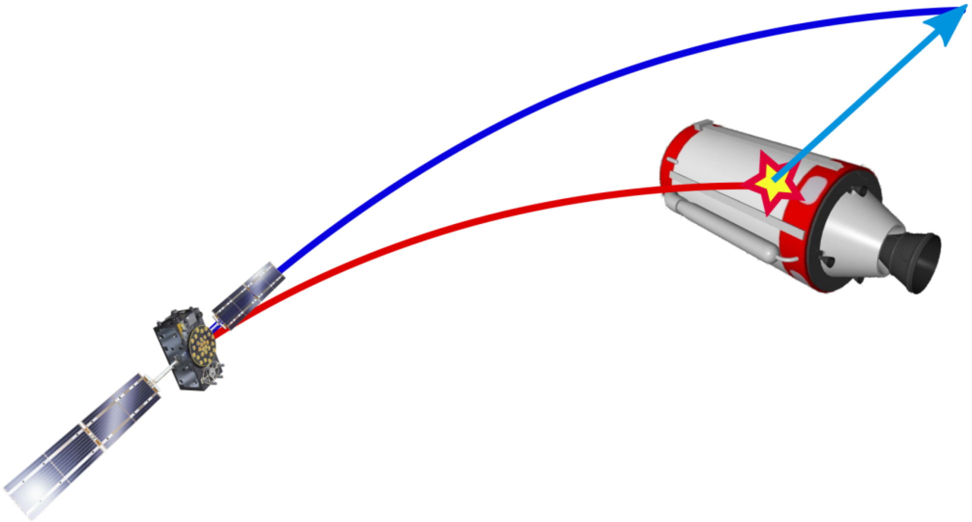
[*] Bombardelli C., Hernando-Ayuso J., Optimal impulsive collision avoidance in low earth orbit, JGCD (2015)



Spacecraft against debris

CAM DESIGN AND SENSITIVITY ANALYSIS

Spacecraft against debris

Test cases

Two tests cases from current ESA 's missions:

PROBA-2 (quasi-circular)						
ID	Epoch [UTC]	a [km]	e [-]	i [deg]	Ω [deg]	ω [deg]
36037	2018/04/20 03:18:34	7093.637	0.0014624	98.2443	303.5949	109.4990

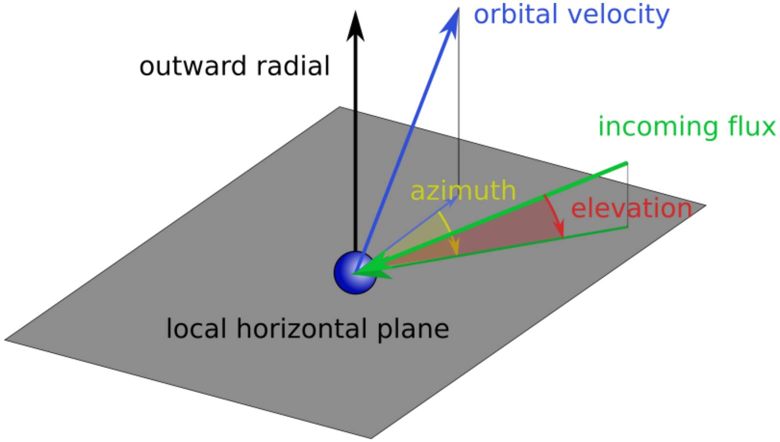
XMM (elliptical)						
ID	Epoch [UTC]	a [km]	e [-]	i [deg]	Ω [deg]	ω [deg]
25989	2018/04/27 18:31:05	66926.137	0.8031489	70.1138	348.8689	95.9905

- http://www.heavens-above.com/

Spacecraft against debris

Debris selection and conjunction geometry

- Debris orbits are constructed with conjunction information from ESA's MASTER-2009
  - Sources for conjunctions: launchers and mission related objects
  - Ranges for azimuth, elevation and relative velocity at the conjunction

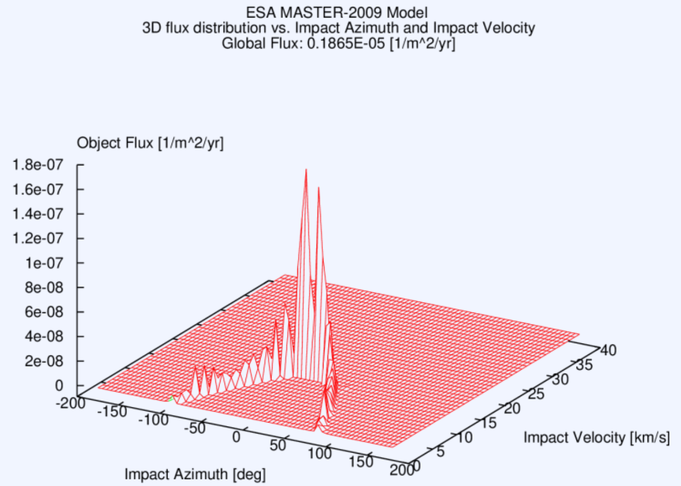


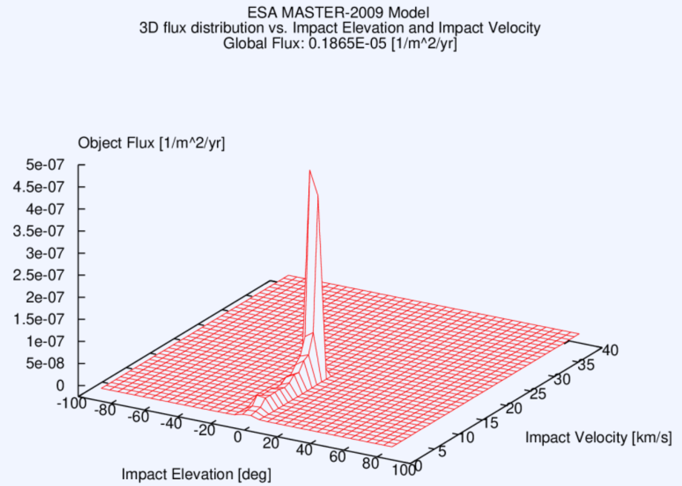
- Four free parameters: azimuth, elevation and magnitude of relative velocity, and true anomaly of the s/c at the conjunction
- Results are shown in terms of the true anomaly of the s/c and the relative velocity at the conjunction.
  - All combinations of azimuth and elevation are explored, but only the conjunction that maximises a given metric is shown

Spacecraft against debris

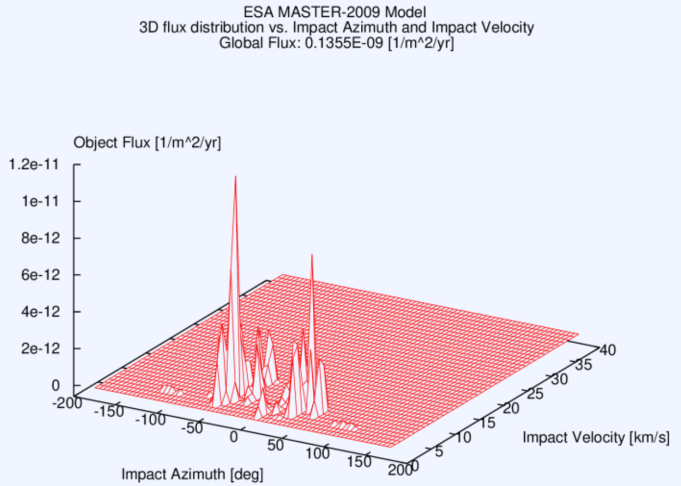
Sensitivity analysis: MASTER data

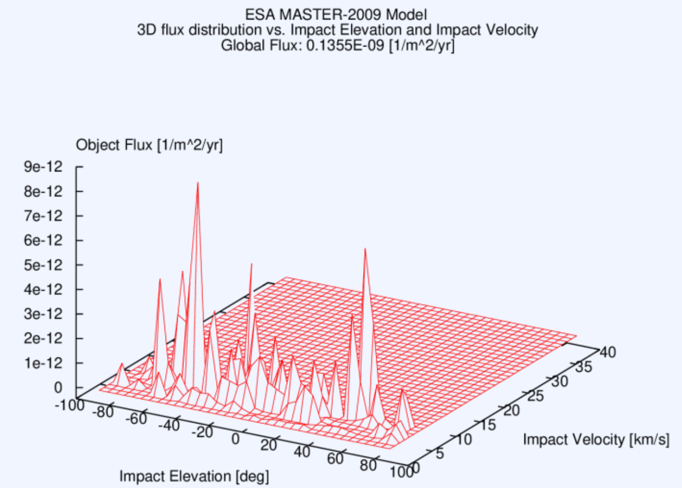
PROBA-2


ESA MASTER-2009 Model
3D flux distribution vs. Impact Azimuth and Impact Velocity
Global Flux: 0.1865E-05 [1/m^2/yr]


ESA MASTER-2009 Model
3D flux distribution vs. Impact Elevation and Impact Velocity
Global Flux: 0.1865E-05 [1/m^2/yr]

XMM


ESA MASTER-2009 Model
3D flux distribution vs. Impact Azimuth and Impact Velocity
Global Flux: 0.1355E-09 [1/m^2/yr]

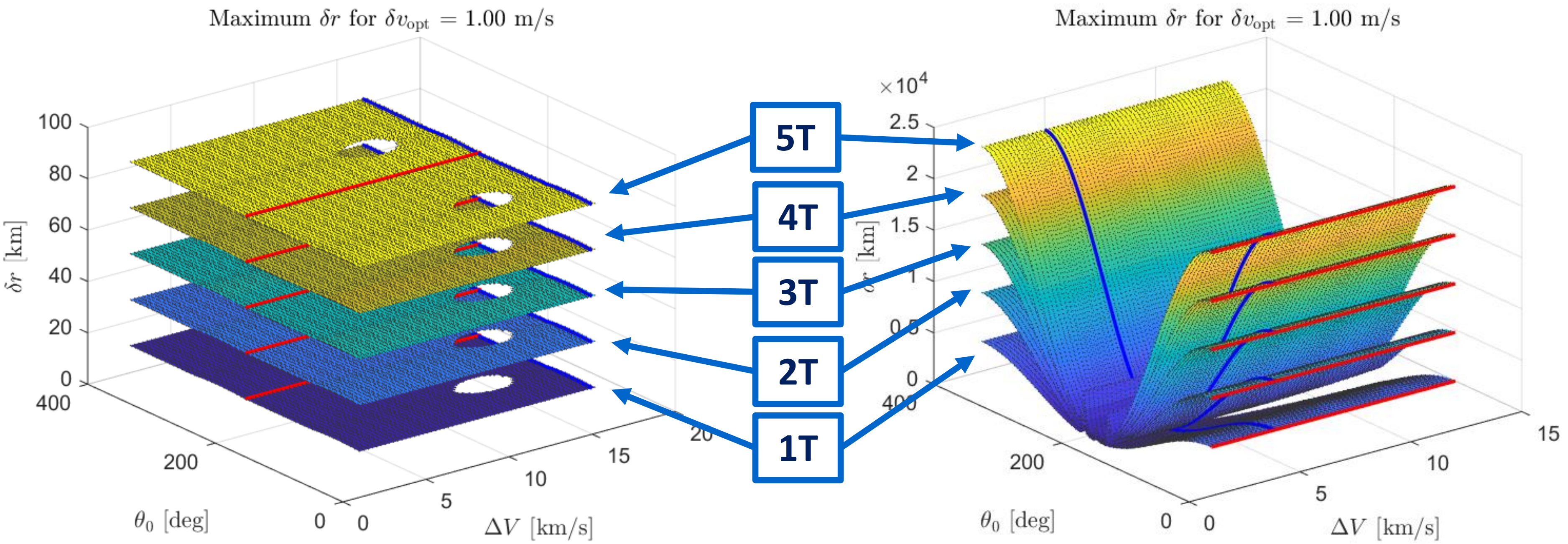

ESA MASTER-2009 Model
3D flux distribution vs. Impact Elevation and Impact Velocity
Global Flux: 0.1355E-09 [1/m^2/yr]

Spacecraft against debris

Sensitivity analysis: Maximum displacement

Quasi-circular: PROBA2

Elliptical: XMM

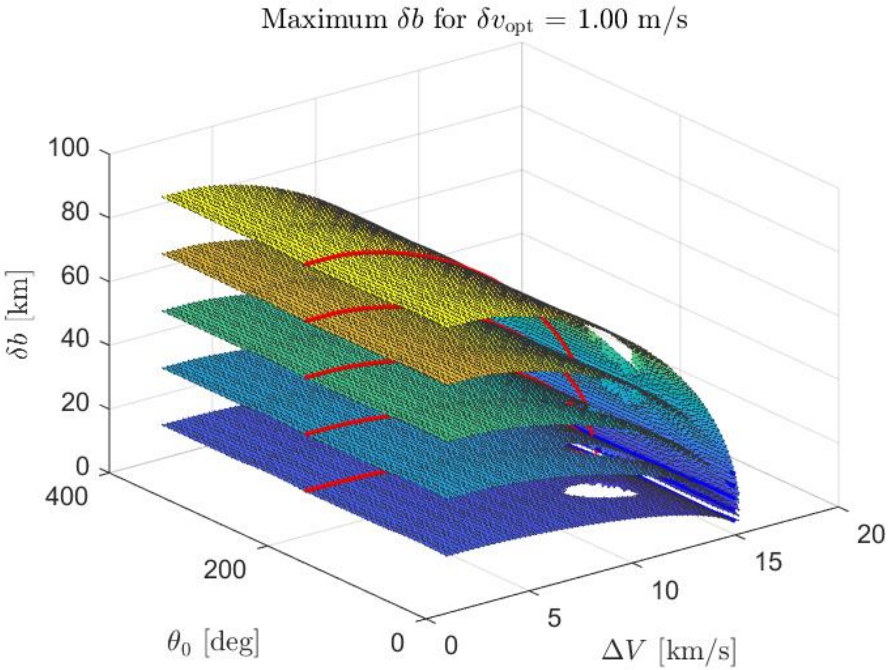


- Results nearly independent on conjunction geometry (ΔV, azimuth, elevation)
- For elliptical case, dependence on the s/c true anomaly at CA

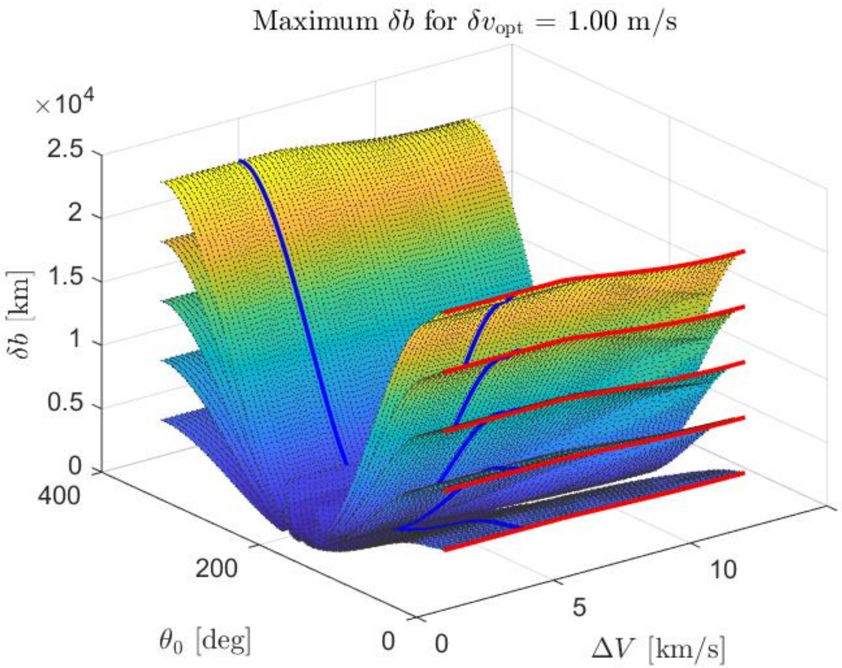
Spacecraft against debris

Sensitivity analysis: Displacement in the b-plane

Quasi-circular: PROBA2



Elliptical: XMM

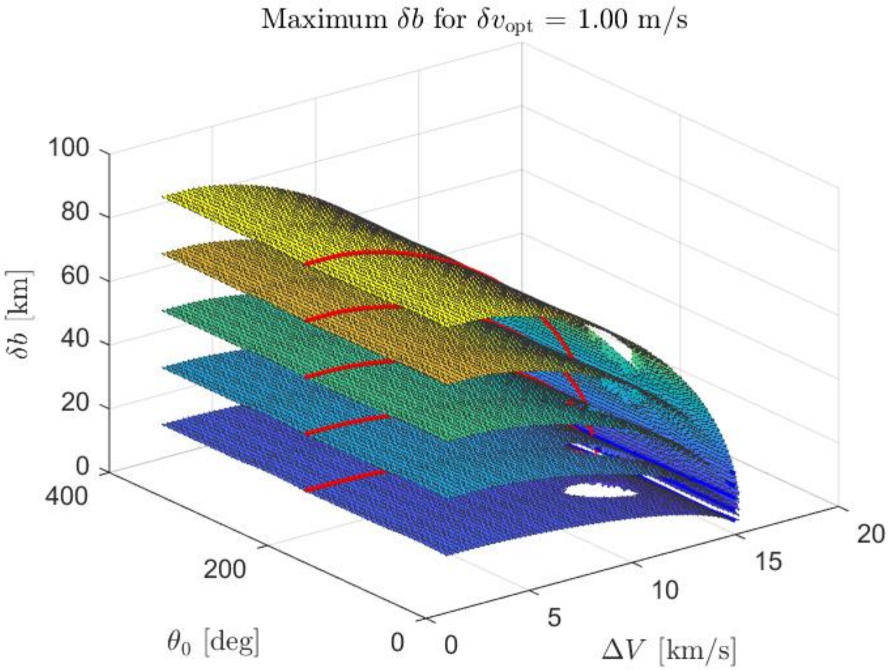


The deflection in the b-plane is strongly influenced by the geometry of the conjunction

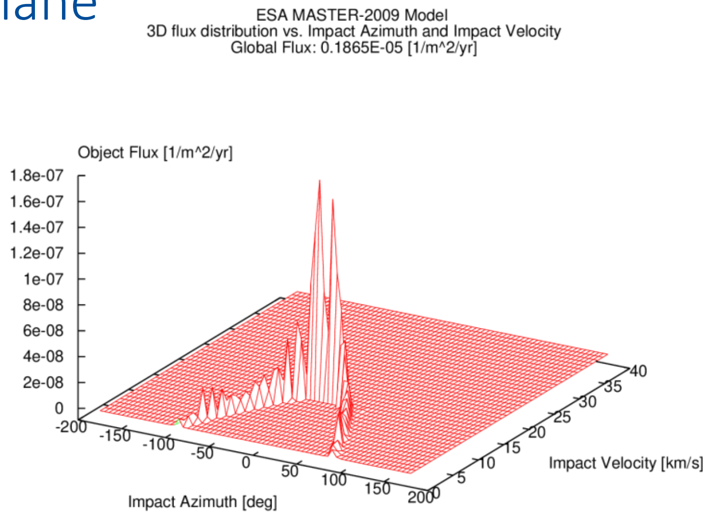
Spacecraft against debris

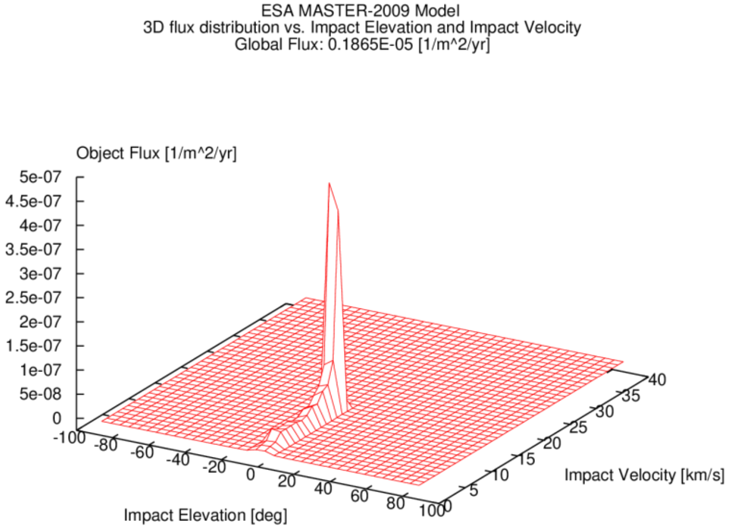
Sensitivity analysis: Displacement in the b-plane

Quasi-circular: PROBA2



As ΔV increases, conjunction becomes a head-on collision

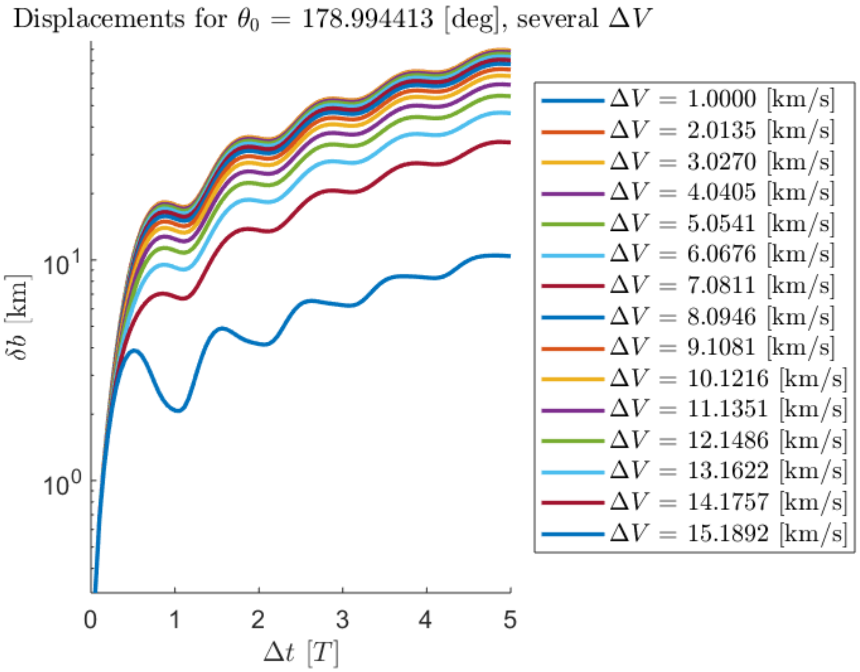




Spacecraft against debris

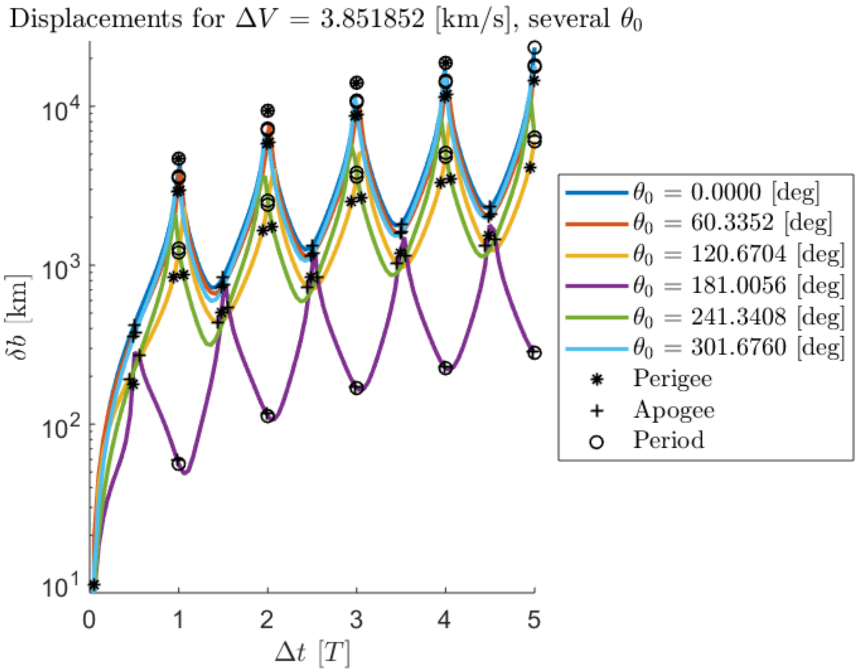
Sensitivity analysis: Time and conjunction geometry effects

Quasi-circular: PROBA2


Displacements for $\theta_0 = 178.994413$ [deg], several ΔV

Strong influence of the conjunction geometry in the attainable deflection

Elliptical: XMM

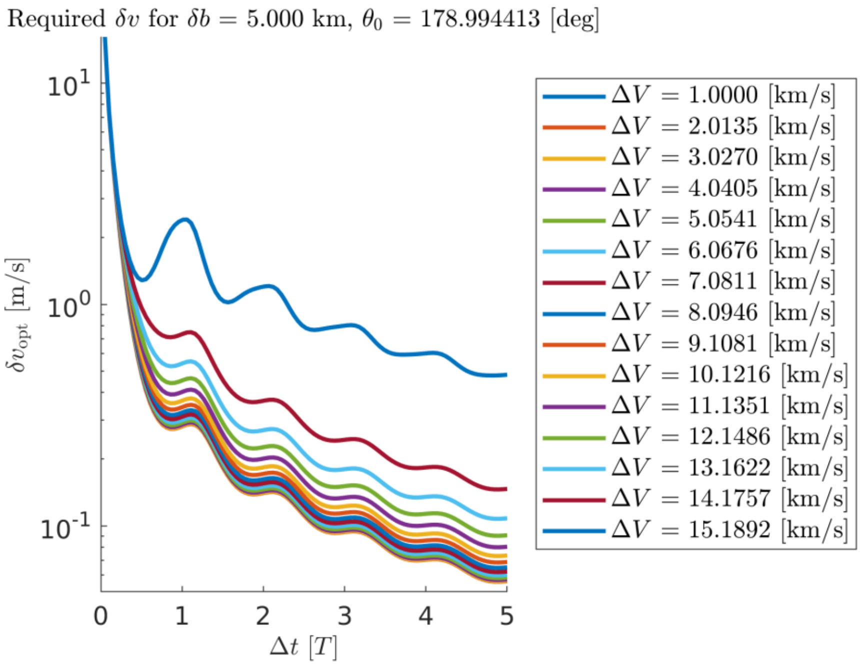

Displacements for $\Delta V = 3.851852$ [km/s], several θ_0

Max/min values around perigee/apogee

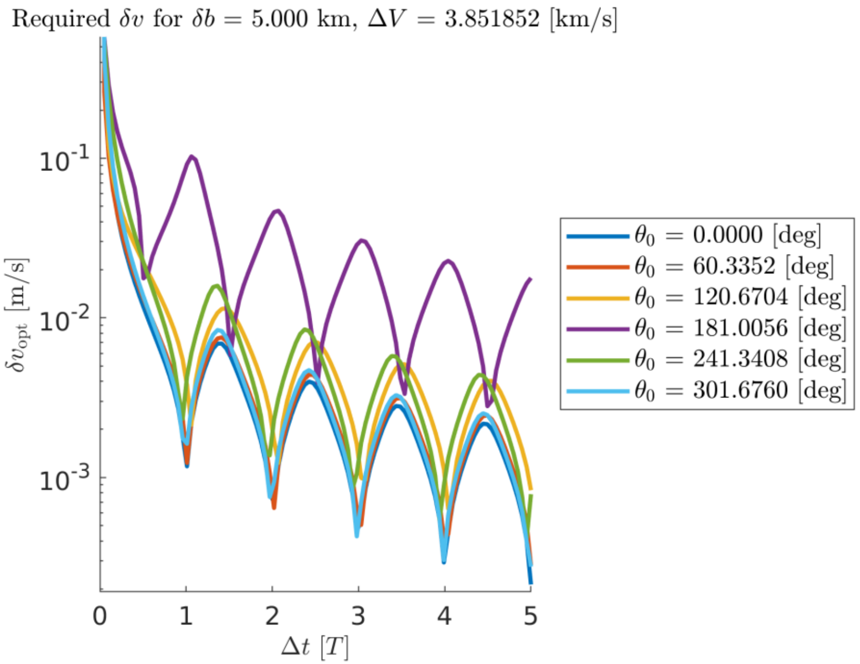
Spacecraft against debris

Sensitivity analysis: Delta-v requirements for a 5 km displacement

Quasi-circular: PROBA2



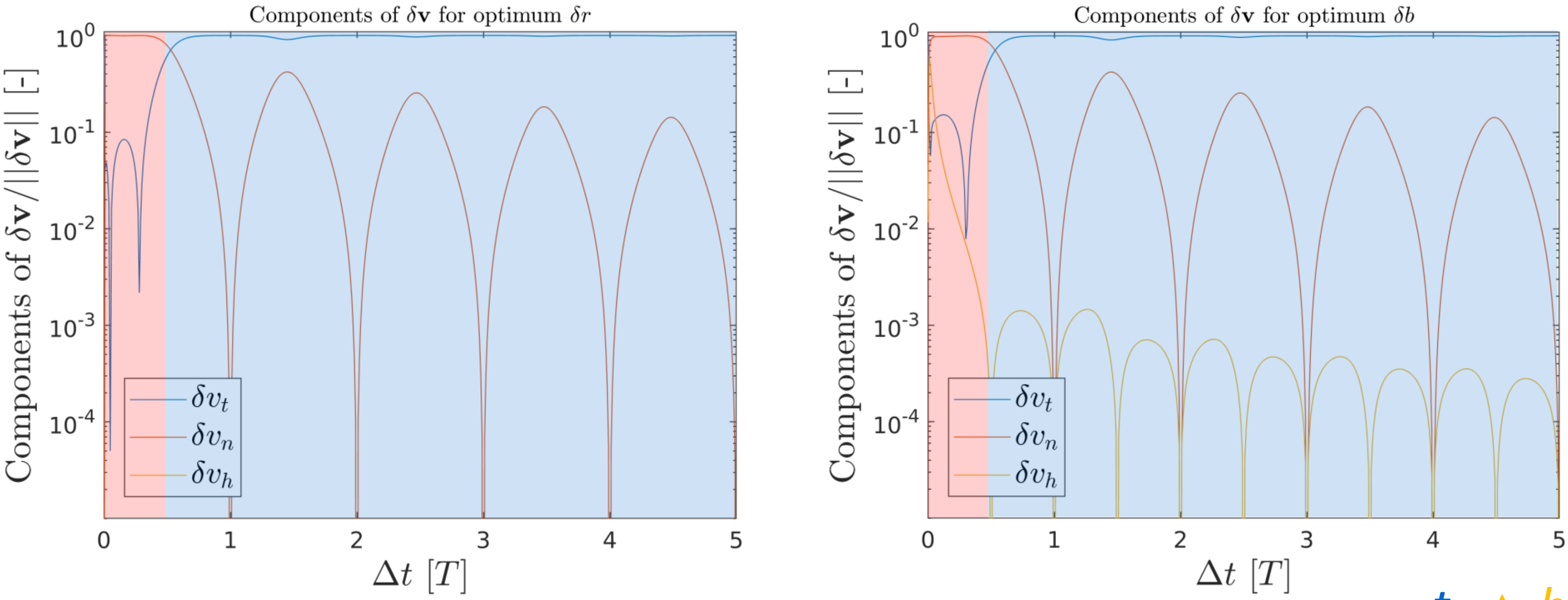
Elliptical: XMM



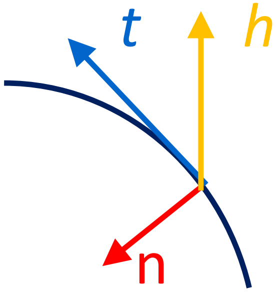
Spacecraft against debris

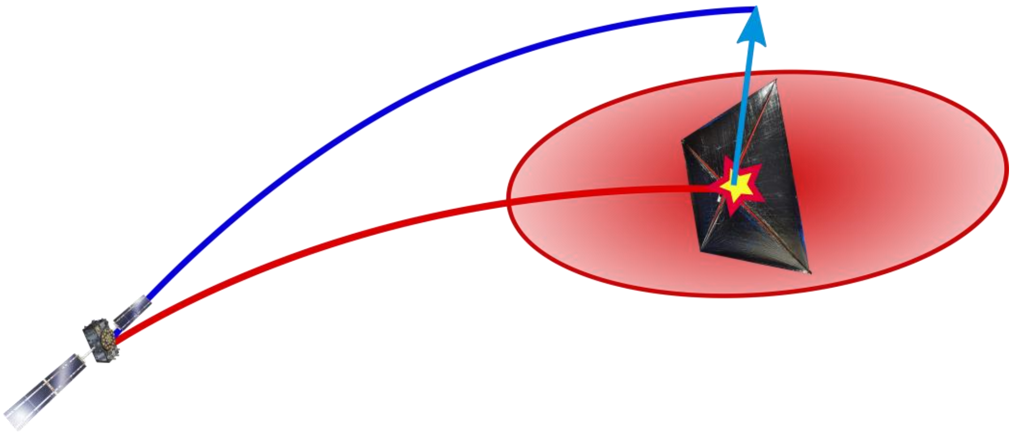
Optimal delta-v orientation: eccentric orbit

XMM mission (e=0.8)



- The normal direction is dominant during the first half period
- The tangential direction is dominant for longer times
- The out-of-plane direction is negligible (but noticeably larger for the b-plane displacement)





Effect of uncertainties. Spacecraft versus sail

CAM DESIGN AND SENSITIVITY ANALYSIS

Effect of uncertainties

Hypothesis and method

- With a longer lead time:
  - Maximum displacement for a given impulse increases
  - Uncertainties increase
- **What is the net effect on collision probability?**
- **Maximum miss distance** and **minimum collision probability** CAMs are designed and compared for the s/c versus debris case:
  - Nominal case taken from the PROBA-2 test case.
  - Realistic reference covariance matrix
  - Covariances known at CAM time, propagated using the analytic STM

Effect of uncertainties

Test case: Input data

- Keplerian elements at conjunction:

	a [km]	e [-]	i [deg]	Ω [deg]	ω [deg]	θ_0 [deg]
PROBA-2	7093.637	0.0014624	98.2443	303.5949	109.4990	179.4986
Debris	7782.193	0.08716212	88.6896	142.7269	248.1679	1.2233

- Distance at conjunction: **0 (direct impact)**
- Sample covariance matrix taken from NORAD ID 33874 (IRIDIUM 33 DEB)

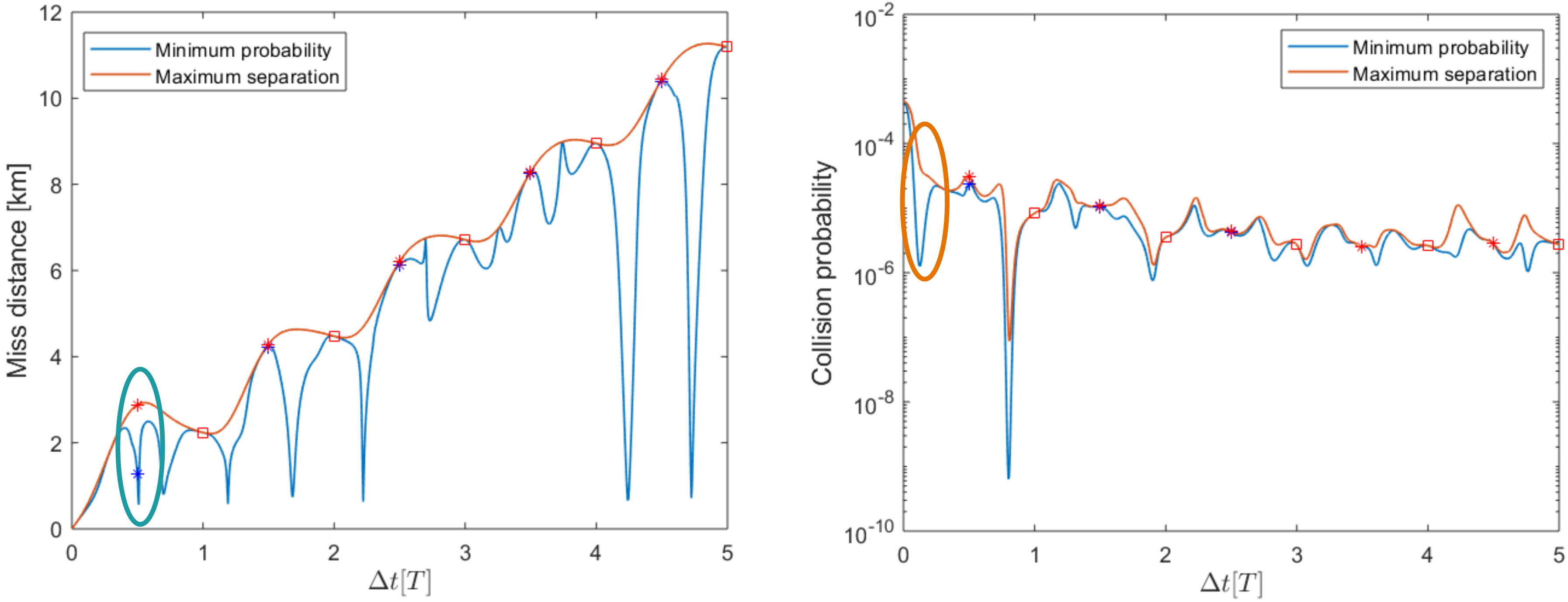
x [km]	y [km]	z [km]	vx [km/s]	vy [km/s]	vz [km/s]
1.1554603167E-02	-2.3144336551E-03	-1.1731962249E-03	+4.5252954759E-07	-5.6795909134E-07	-1.0945466309E-05
-2.3144336551E-03	+1.9146944058E-02	+1.4167201661E-02	-1.2286501559E-05	-2.5535535854E-06	-3.3049394326E-06
-1.1731962249E-03	+1.4167201661E-02	+3.0870283719E-01	-2.8750137394E-04	-8.6187778997E-05	-1.2493173453E-06
+4.5252954759E-07	-1.2286501559E-05	-2.8750137394E-04	+2.8850679902E-07	+7.9940432768E-08	+1.1511416229E-09
-5.6795909134E-07	-2.5535535854E-06	-8.6187778997E-05	+7.9940432768E-08	+4.5996583428E-08	+1.4570092897E-09
-1.0945466309E-05	-3.3049394326E-06	-1.2493173453E-06	+1.1511416229E-09	+1.4570092897E-09	+1.2022009401E-08

Effect of uncertainties

Test case: maximum miss distance and minimum collision prob. CAMs

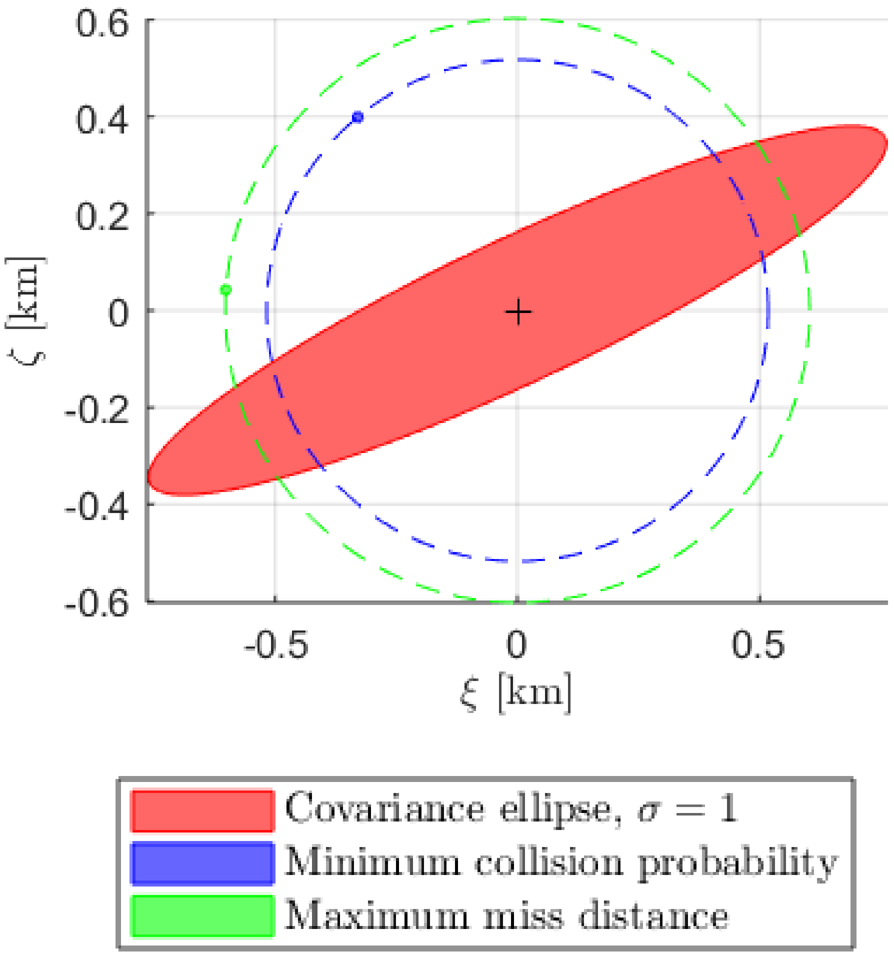
- Greatest qualitative differences are observed during the first period
  - Different distance at perigee (solutions at later perigees are very close)
  - Highest difference in probabilities

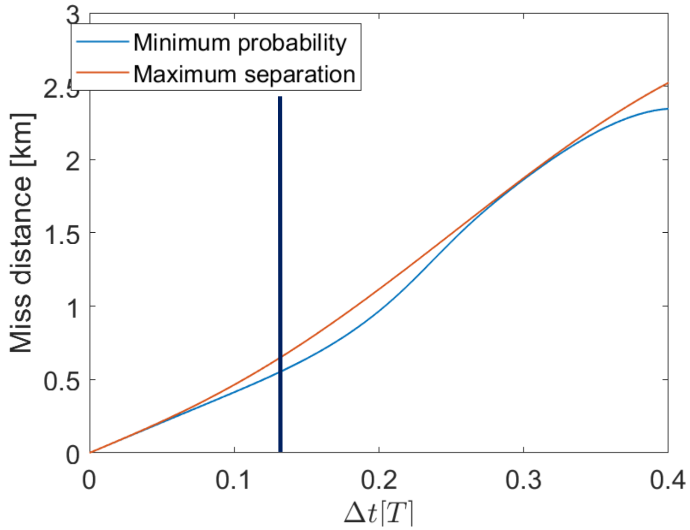
$$\Delta v = 0.7\ \mathrm{m/s},\ \ r_A = 10\ m$$

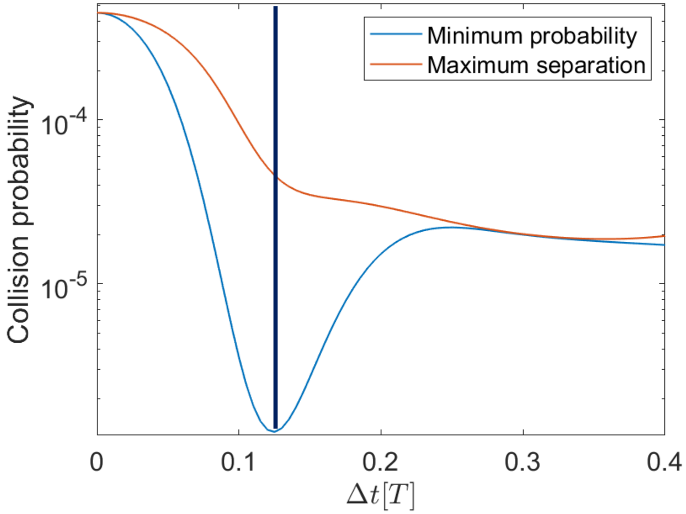


Effect of uncertainties

Test case: First minimum in probability

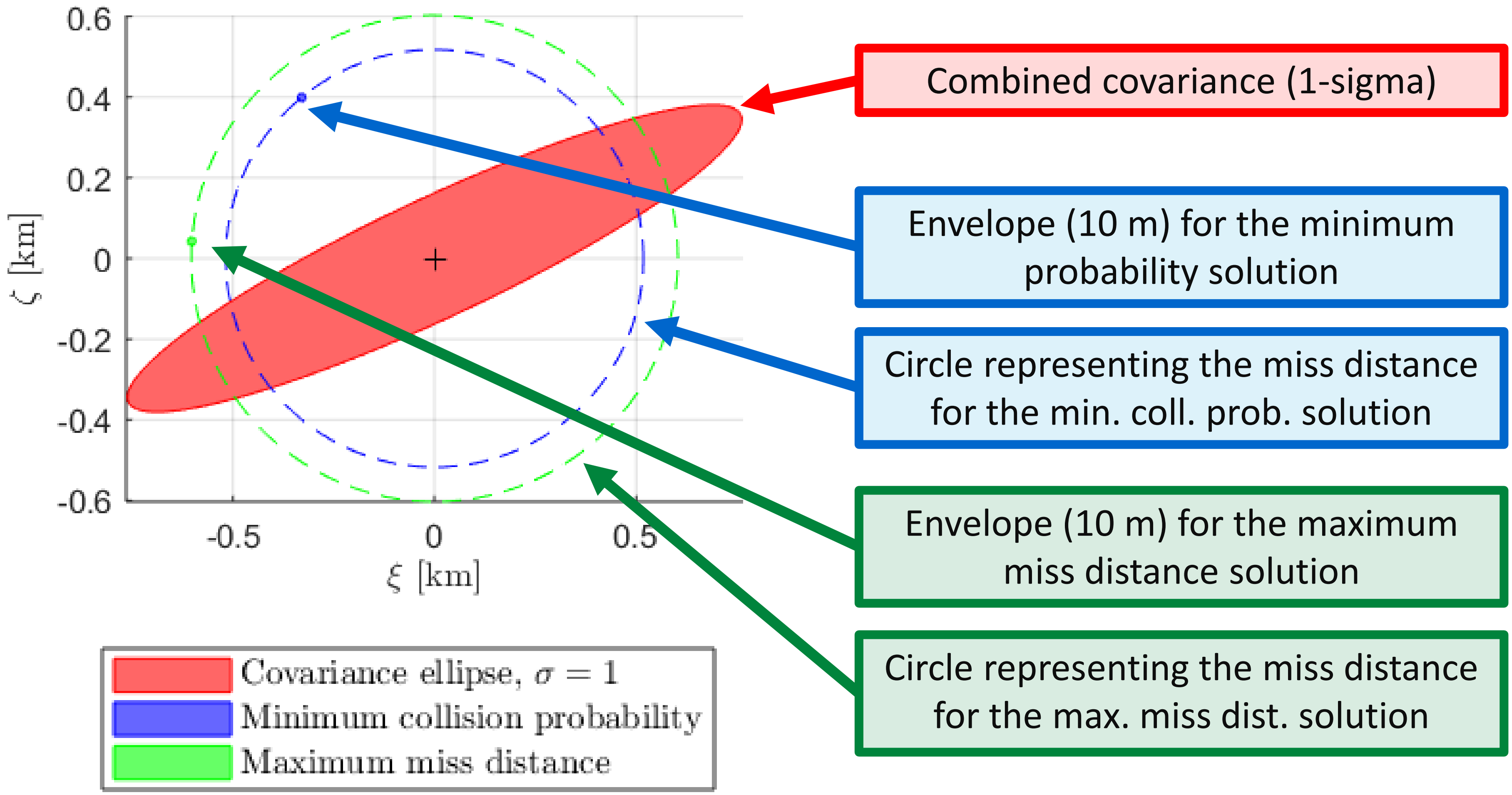






Effect of uncertainties

Test case: First minimum in probability



Combined covariance (1-sigma)

Envelope (10 m) for the minimum probability solution

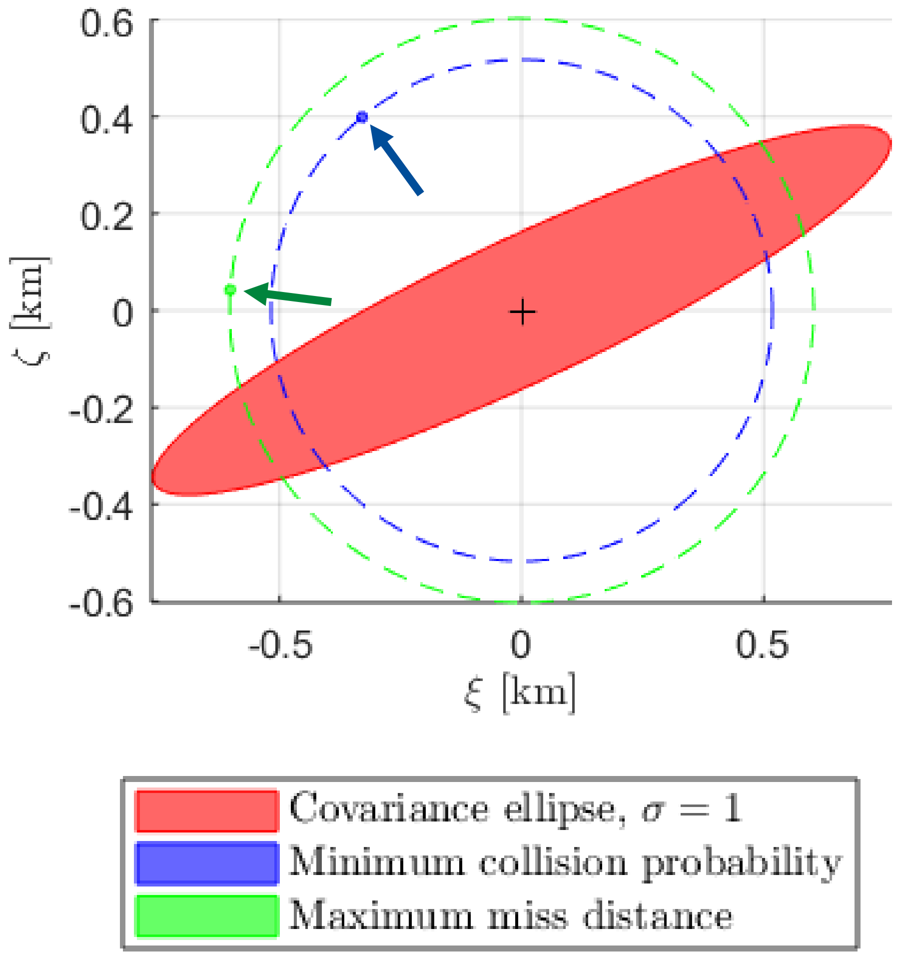
Circle representing the miss distance for the min. coll. prob. solution

Envelope (10 m) for the maximum miss distance solution

Circle representing the miss distance for the max. miss dist. solution

Effect of uncertainties

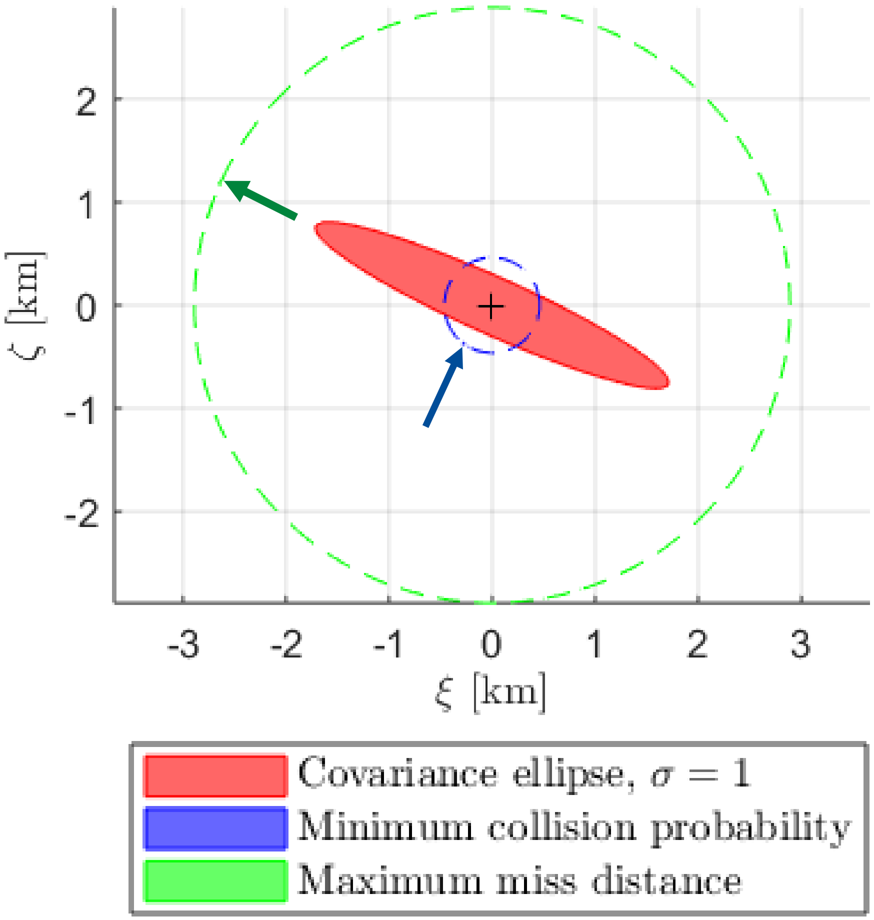
Test case: First minimum in probability

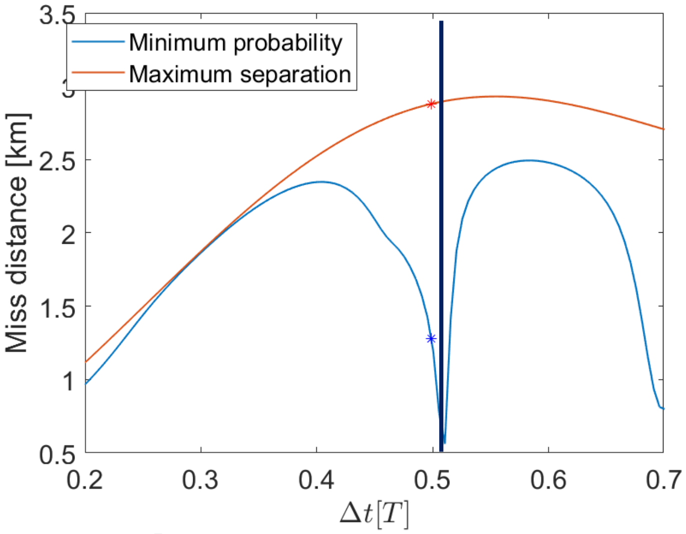


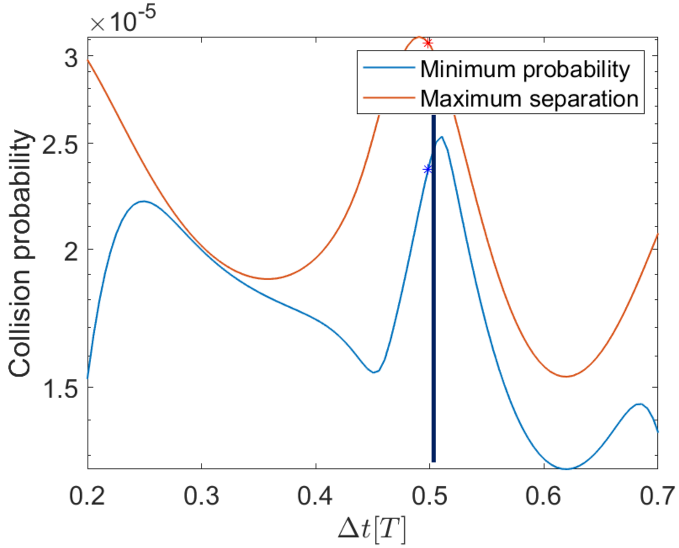
Minimum collision probability CAM sacrifices miss distance in order to align itself with the semi-minor axis of the covariance ellipse

Effect of uncertainties

Test case: Highest difference in miss distance, comparable probability

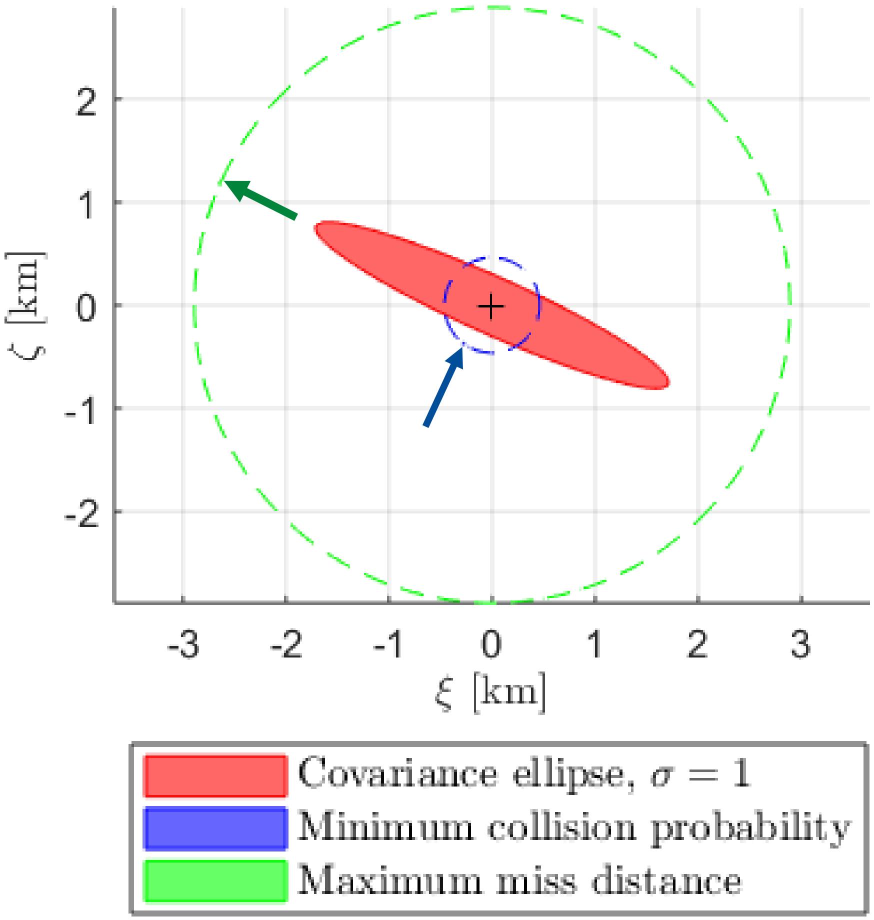






Effect of uncertainties

Test case: Highest difference in miss distance, comparable probability

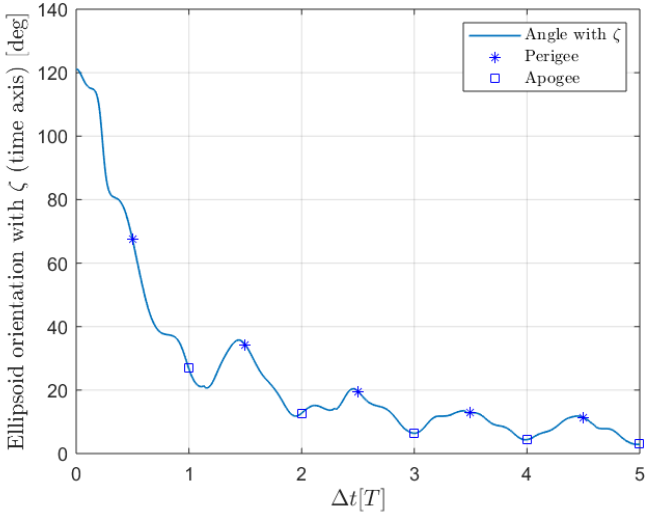


A very similar collision probability is achieved with very different miss distances, due to the orientation with respect to the axis of the covariance ellipse

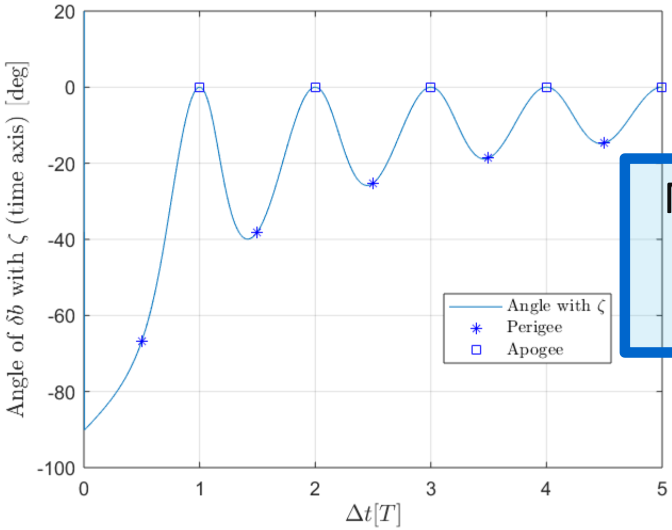
Effect of uncertainties

Test case: alignment with time axis

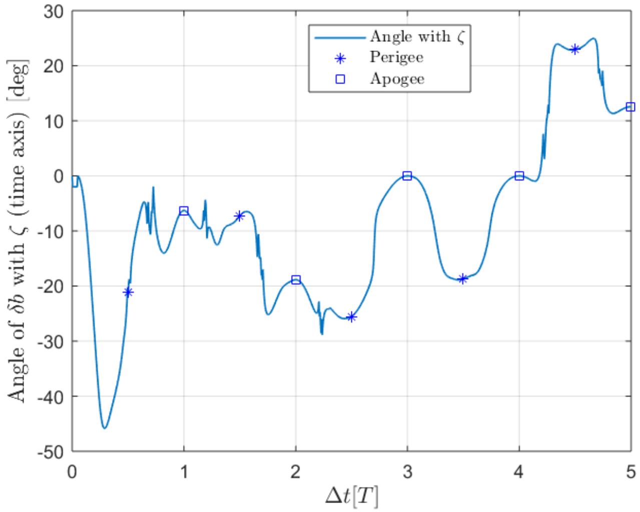
Covariance Ellipse (b-plane)

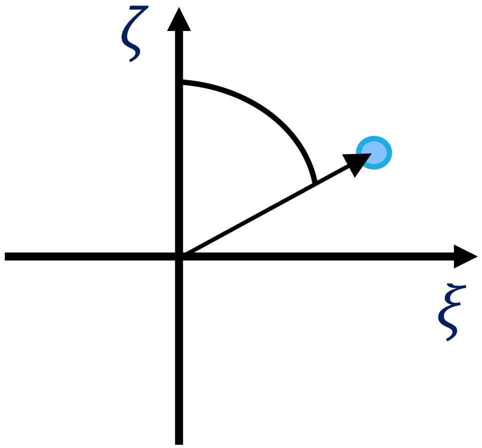


Maximum Deviation CAM



Minimum Coll. Prob. CAM

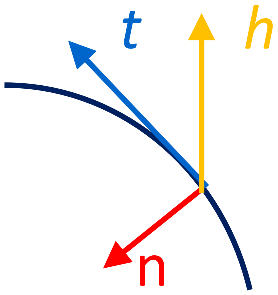


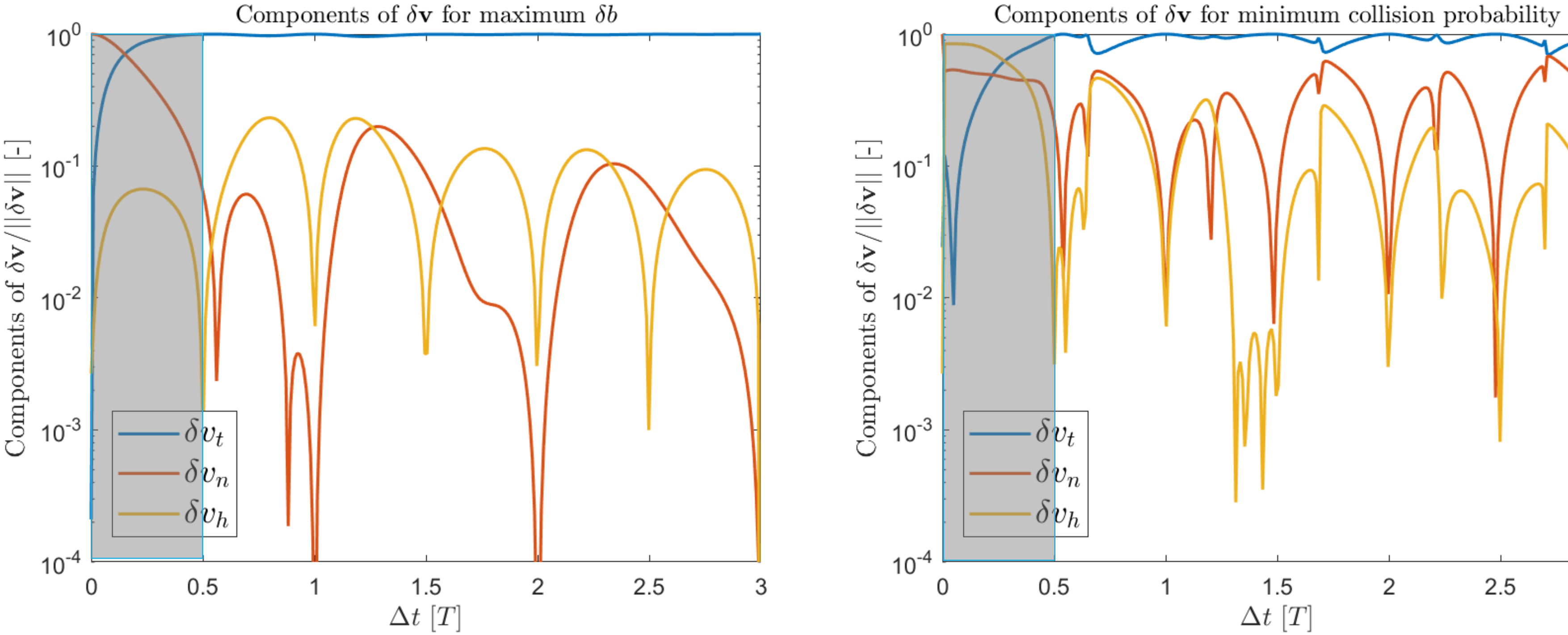


Effect of uncertainties

Components of optimum δ**v**

Both for maximum miss distance and minimum collision probability, the impulsive manoeuvre tends to align with the transversal direction for lead times greater than half a period

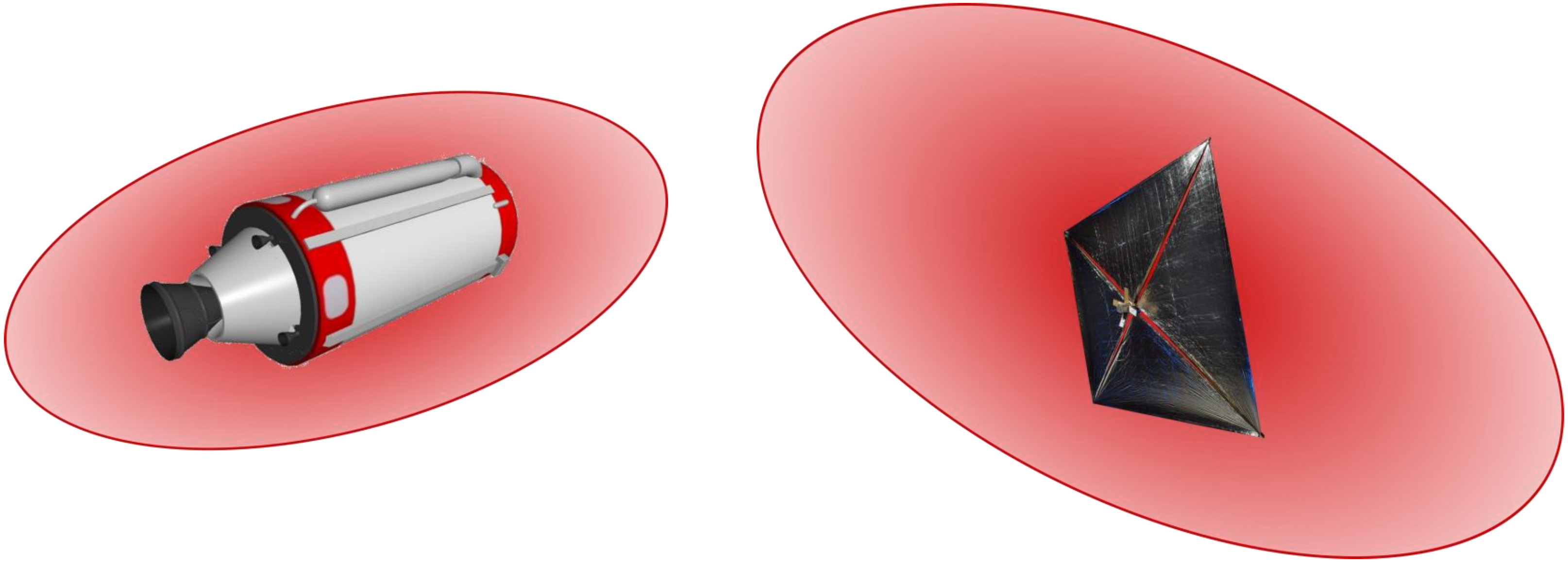




Effect of uncertainties

Effect of drag and SRP on uncertainties

- If the debris is a sail, the effects of **drag and SRP** cannot be neglected:
  - Previous STM is no longer valid
  - Orbit propagation using averaged dynamics with PlanODyn
  - Covariance propagation using Monte Carlo methods



Effect of uncertainties

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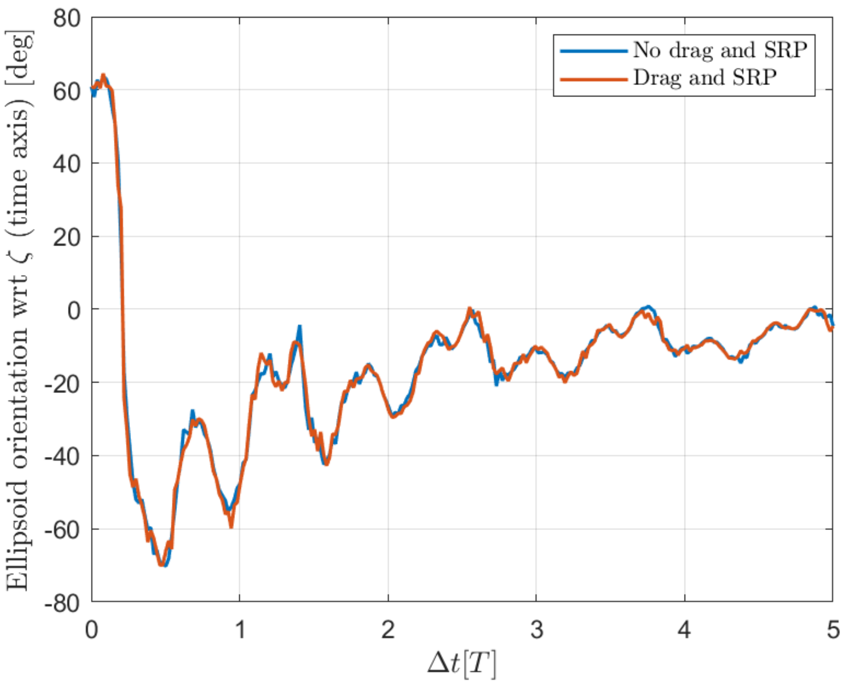
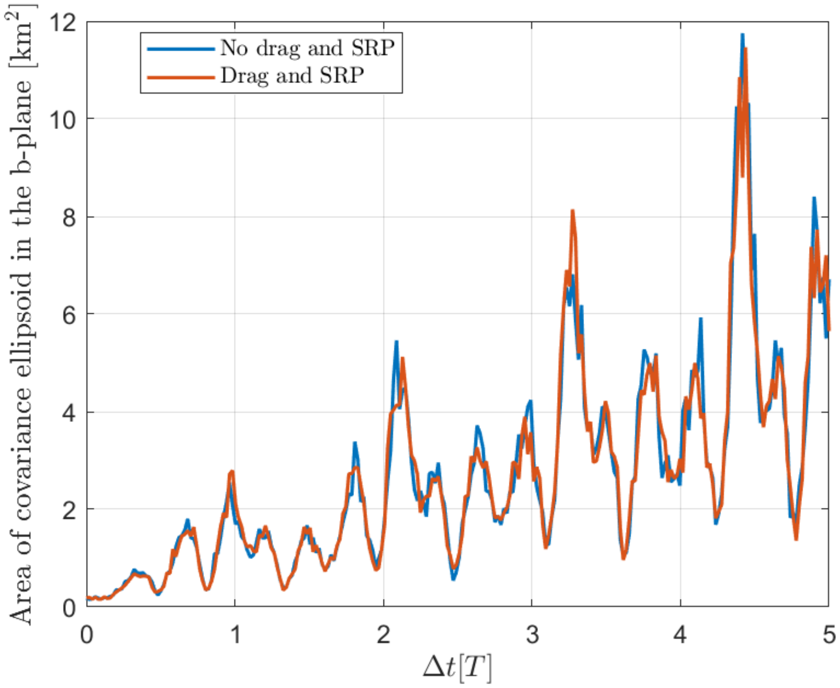
- CAM design for s/c versus sail presents the same qualitative characteristics already analysed for s/c versus debris. The next slides focus on:
  - Effect of drag and SRP on covariance evolution
  - Influence of the envelope size on the required δv

Effect of uncertainties

Effect of drag and SRP: covariance ellipse in b-plane

PROBA-2 vs debris (MASTER) test case + $\boldsymbol{A/m = 2\, m^2/s,\ c_D = 2.1,\ c_R = 1.8}$



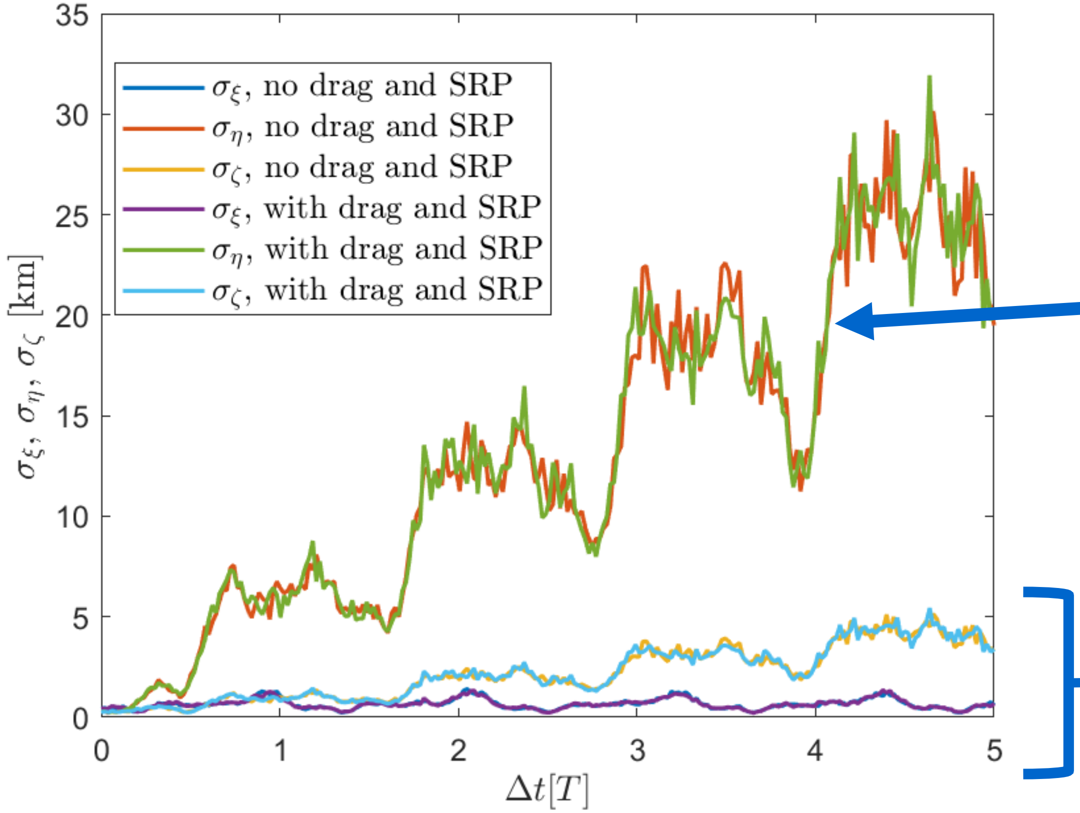


For short lead times (typical of impulsive CAMs), the effect of drag and SRP on the covariance ellipse orientation and size is very small

Effect of uncertainties

Effect of drag and SRP: covariance elements

PROBA-2 vs debris (MASTER) test case + $A/m = 2\ m^2/s,\ c_D = 2.1,\ c_R = 1.8$



The covariance experiments its largest variation along the η axis (normal to the b-plane).

Inside the b-plane, the covariance tends to grow along the time axis ζ.

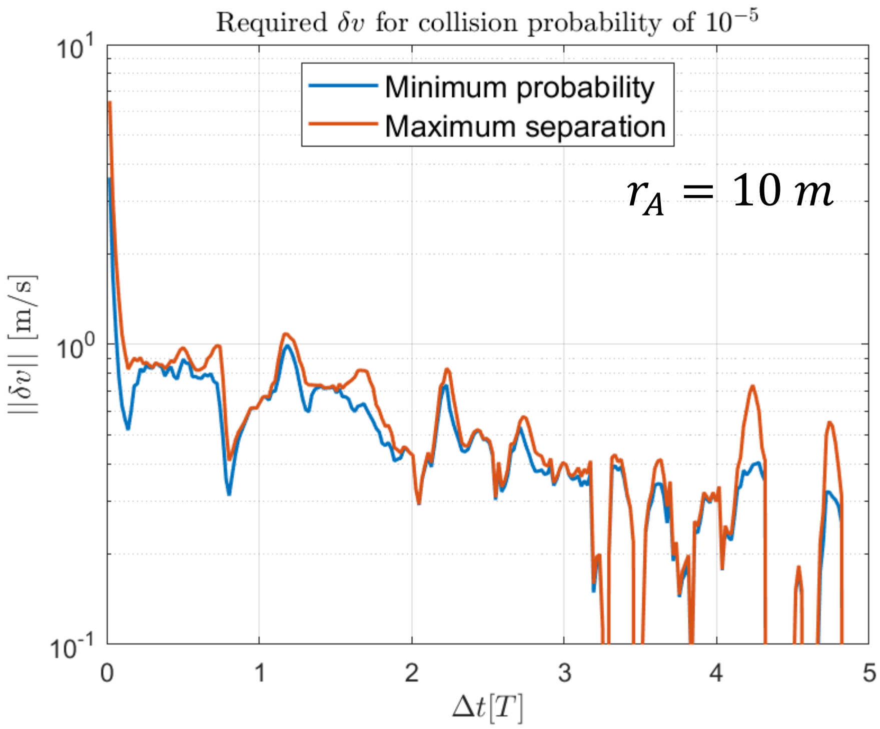
No significant differences due to SRP and drag for short lead times

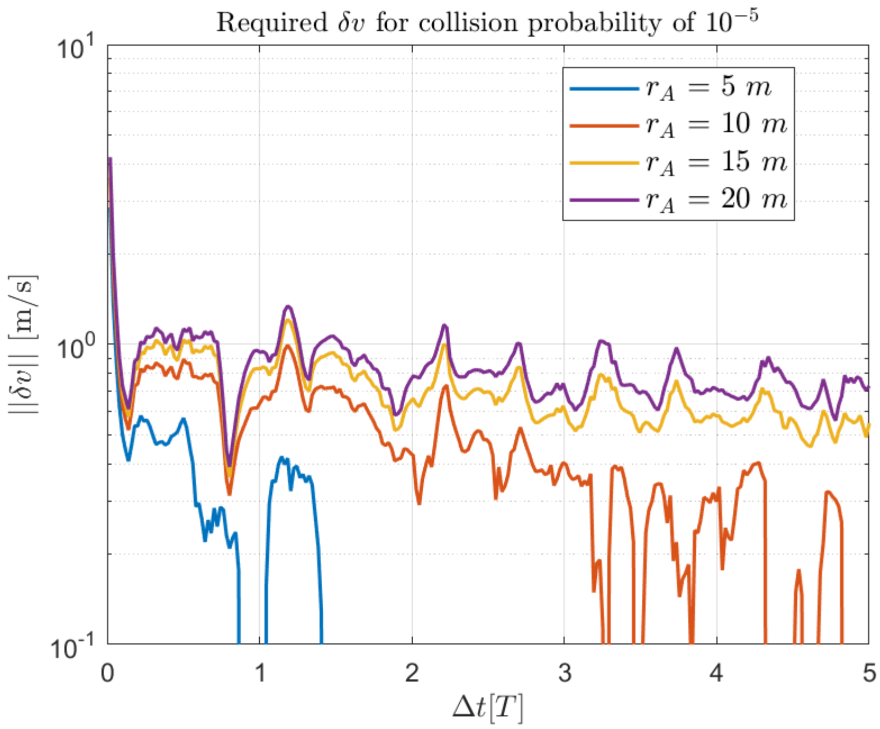
Effect of uncertainties

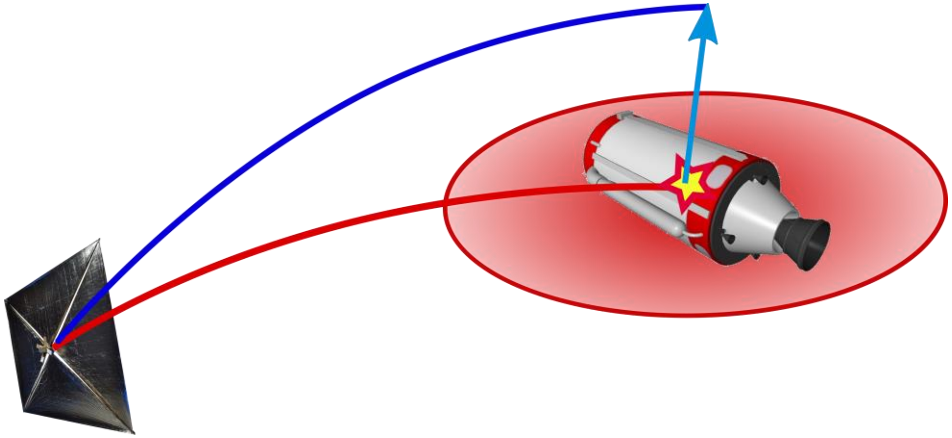
Requirements for the impulsive CAM

PROBA-2 vs debris (MASTER) test case + $\boldsymbol{A/m = 2\ m^2/s,\ c_D = 2.1,\ c_R = 1.8}$

Required δv to reach a collision probability of 10^{-5}







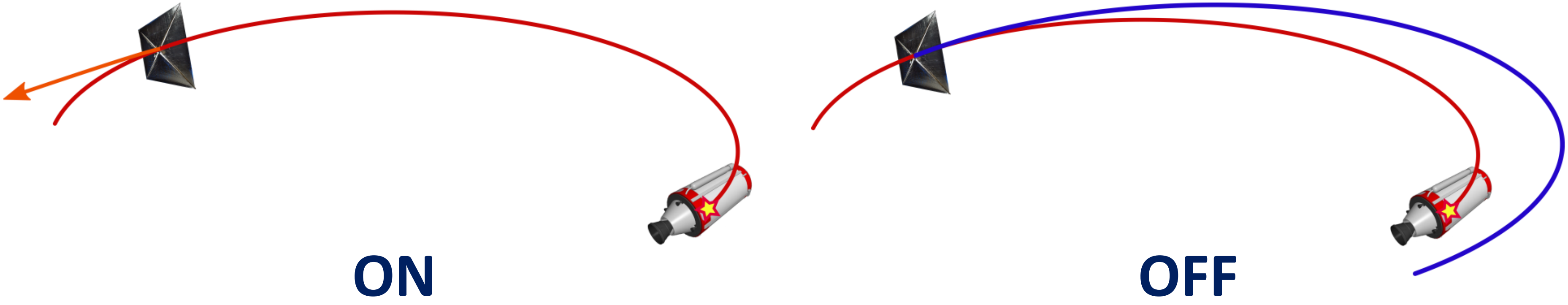
CAM by a de-orbtiting sail

CAM DESIGN AND SENSITIVITY ANALYSIS

CAM by a de-orbtiting sail

Method and hypotheses

- Limited control capability:
  - Sail ON (perpendicular to the main force)/OFF (at feather)
  - **For drag sail, tangential thrust**
  - Effect on CAM is like a **phasing maneuver**
- *A*/*m* represents the 'control authority', i.e., is the parameter for our tests.



CAM by a de-orbtiting sail

Method and hypotheses

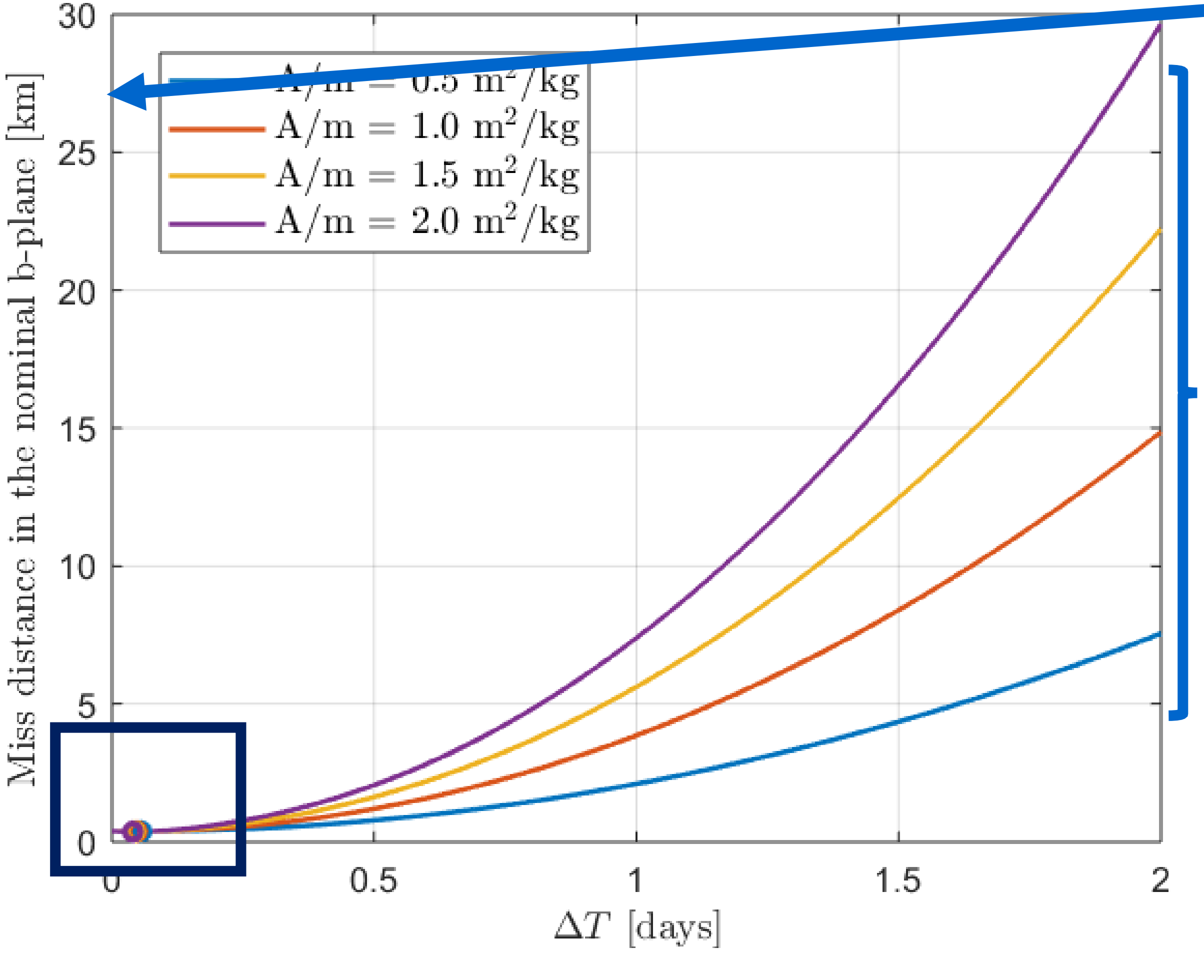
- Limited control capability:
  - Sail ON (perpendicular to the main force)/OFF (at feather)
  - **For drag sail, tangential thrust**
  - Effect on CAM is like a **phasing maneuver**
- *A/m* represents the 'control authority', i.e., is the parameter for our tests.

- Covariances from Conjunction Data Message (CDMs) provided by ESA:
  - Restricted!
  - Several warning times
  - Covariance is provided at CA (no covariance propagation)

- Orbit propagation with PlanODyn

CAM by a de-orbtiting sail

Test case: 2 days warning

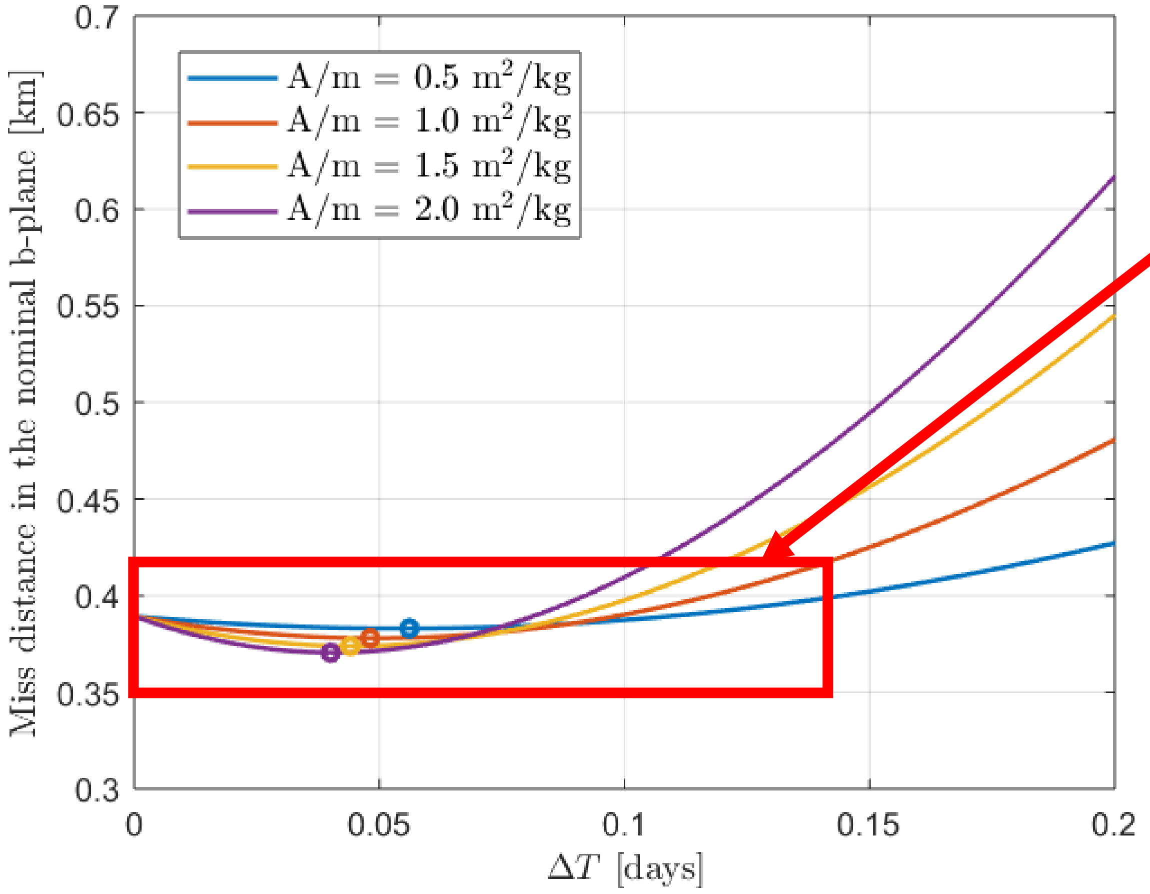


Miss distance can be increased greatly with enough lead time

Area-to-mass is the 'control authority'. A proportional increase is miss distance is observed.

CAM by a de-orbtiting sail

Test case: 2 days warning

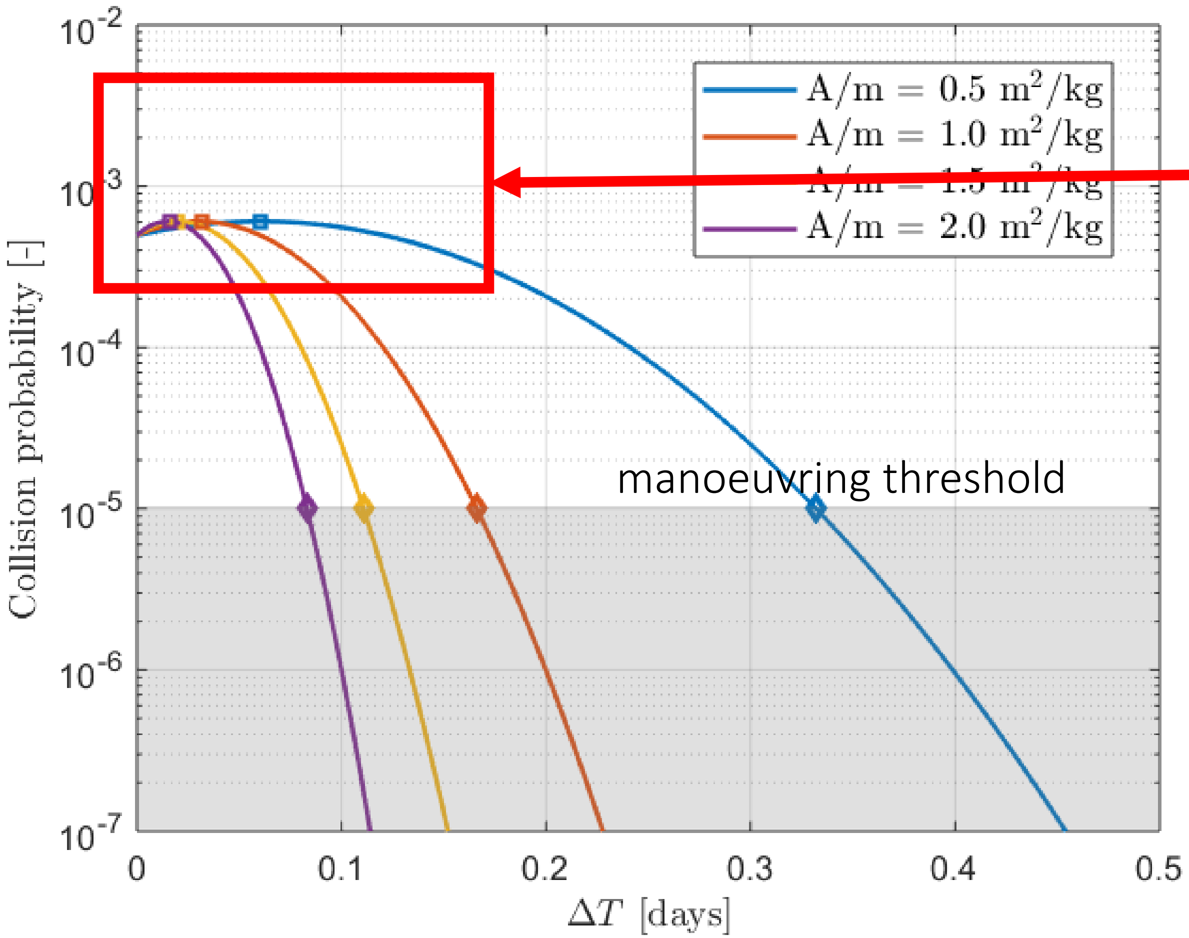


Control can actually reduce miss distance for small lead times.

Is collision probability increased?

CAM by a de-orbtiting sail

Test case: 2 days warning



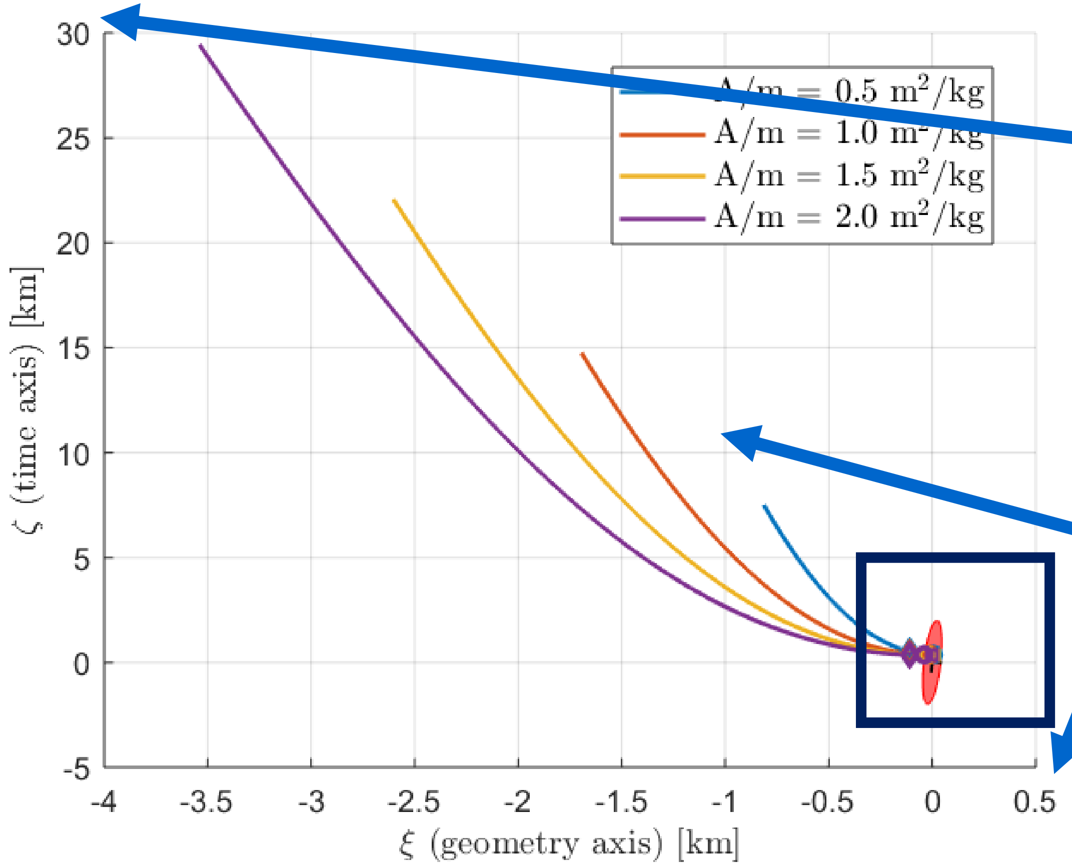
There is an appreciable initial increase of the collision probability

Answers can be found in the b-plane

CAM by a de-orbtiting sail

Test case: 2 days warning

Studying the dynamics in the b-plane helps justify these behaviours



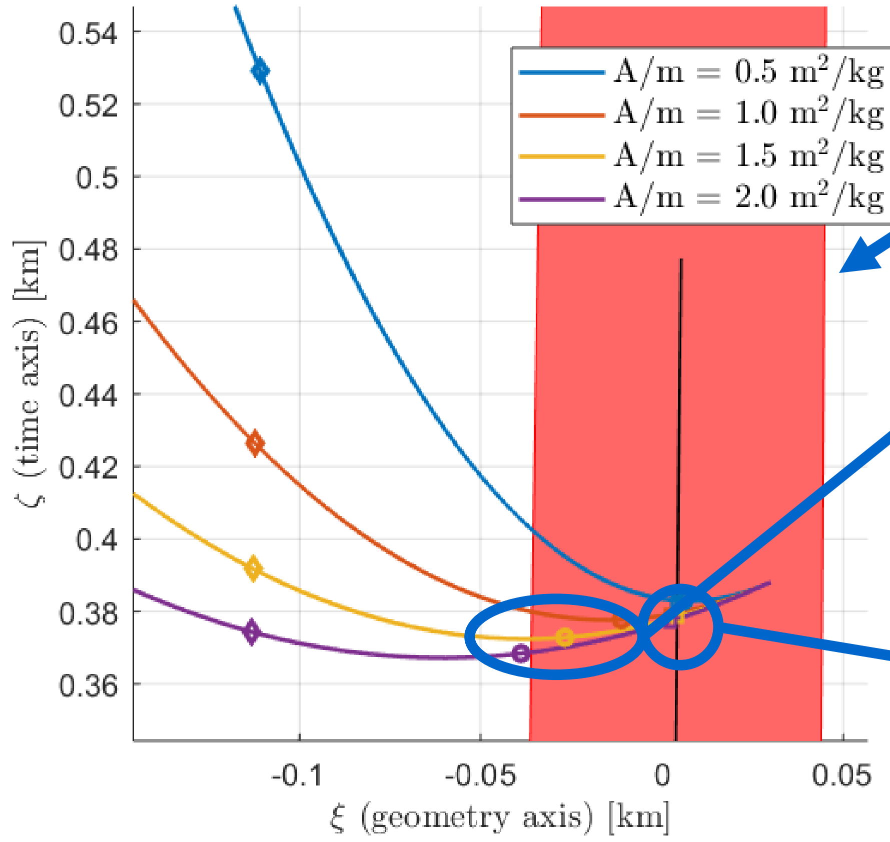
Displacement is mostly associated to time delay, not orbit geometry change.

No periodic behavior (unlike the s/c versus sail case)

CAM by a de-orbtiting sail

Test case: 2 days warning

Short lead time behaviour justifies the differences in the collision probability



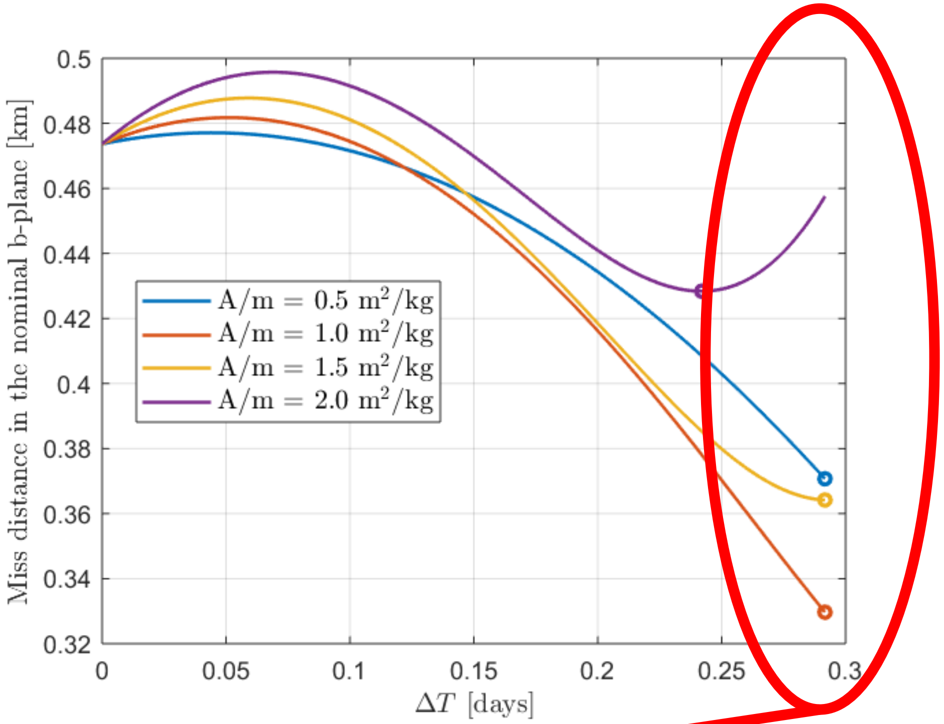
1-σ covariance ellipsoid

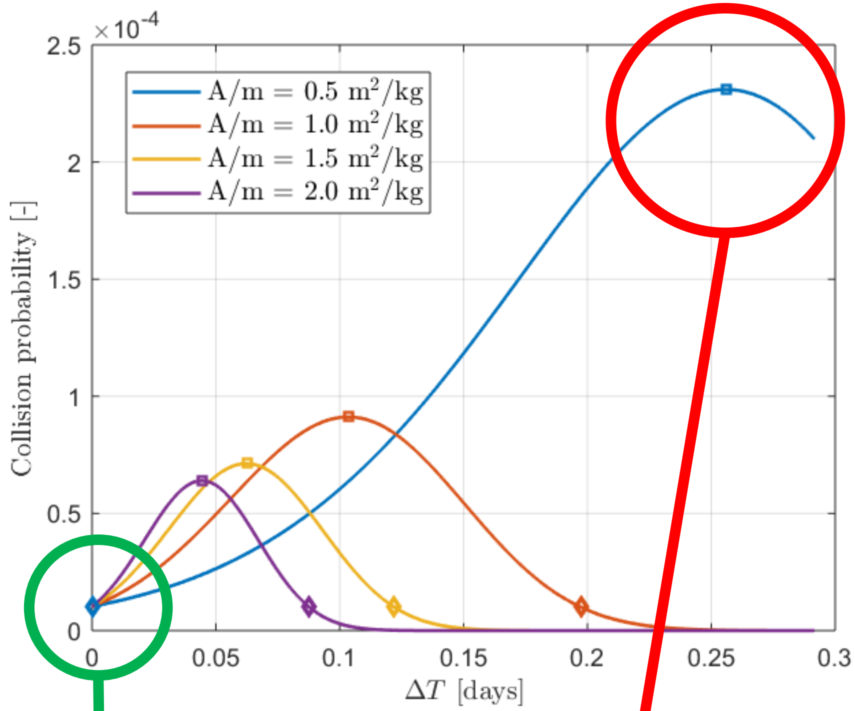
Minimum miss distance is not only associated to MOID, because distance in the time axis never crosses zero

Evolution in the b-plane crosses the semimajor axis of the covariance ellipse, leading to the maximum in collision probability

CAM by a de-orbtiting sail

Test case: 7 hours warning





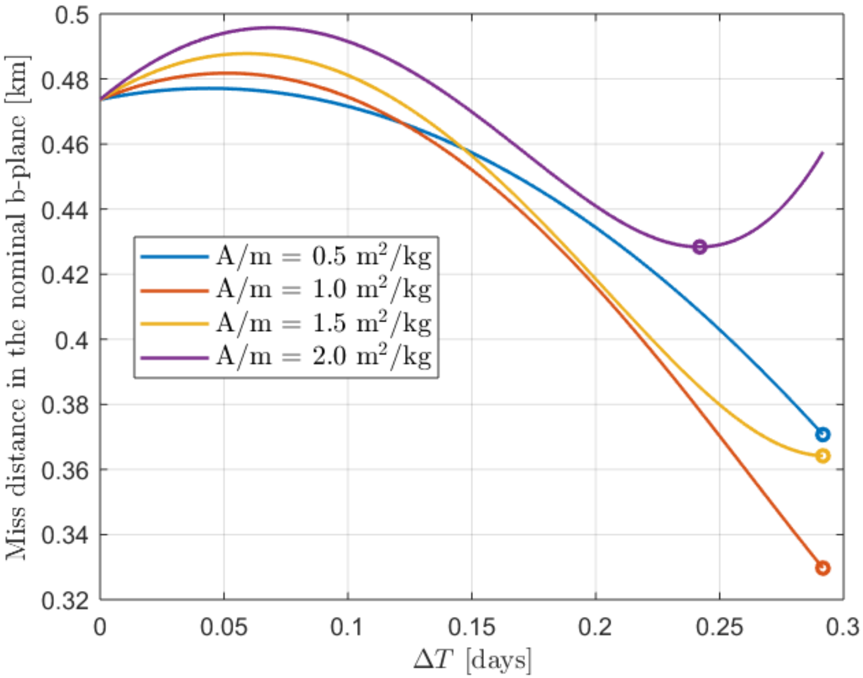
If the decision on the CAM is delayed too much, it will not be possible to increase miss distance

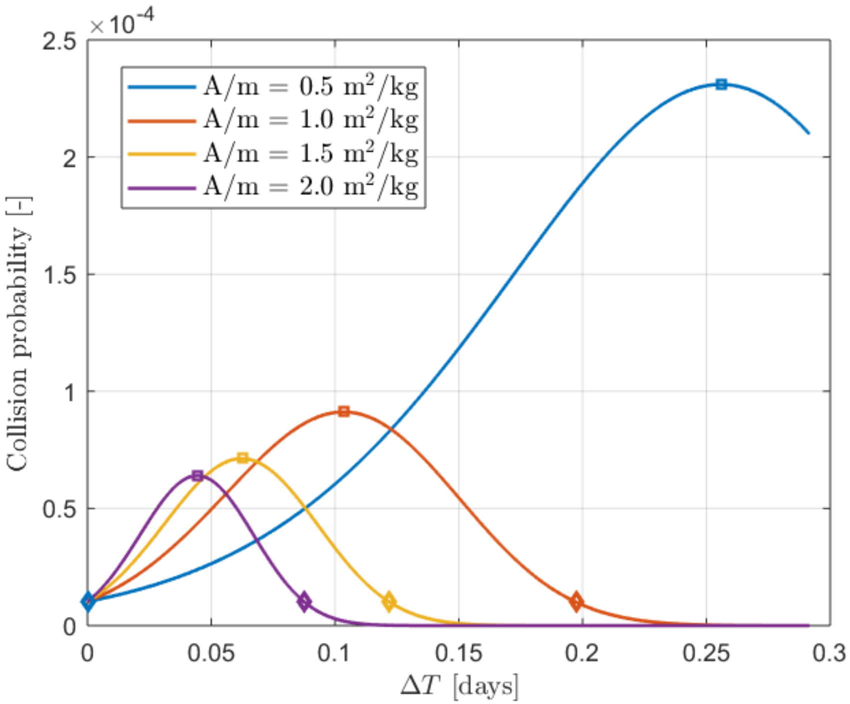
CAM is not actually needed

The smaller sail cannot reach the 10^{-5} collision probability threshold

CAM by a de-orbtiting sail

Test case: 7 hours warning





Waiting strategy:

- With the updated CDM the maneuver may not be needed
- The effectiveness of the eventual CAM is reduced

CONCLUSIONS

Conclusions

- Analytical method for maximum deviation (in b-plane) and minimum collision probability impulsive CAMs
  - Extensive sensitivity analysis for the effects of conjunction geometry and true anomaly of the s/c at CA
  - STM for analytic propagation of covariance (without sail)

- As lead time increases, both covariance ellipse and maximum miss distance CAM in the b-plane tend to align with ζ (time axis)
  - This limits the decrease in collision probability.
- Minimum collision probability CAM moves along ξ (geometrical axis) for some configurations.
- $\boldsymbol{\delta v}$ for both CAMs is mostly transversal for lead times $> 0.5\ T$

Conclusions

- Drag and SRP do not introduce significant changes in the covariance matrix for short lead times
  - 3 days lead times [computed but not shown] yield similar results.
  - Uncertainties in A/m have not been considered
- δv requirements computed for several envelope sizes and lead times
  - Technologically feasible values for the impulsive CAM

- Effective CAMs for a deorbiting sail can be designed through a simple ON/OFF control law
  - A minimum $\boldsymbol{\Delta t}$ is needed (depending on encounter geometry and A/m)
  - May require more anticipation from satellite operators than impulsive CAMs (**more unneeded maneuvers?**)

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Challenges and possibilities in the design of collision avoidance maneuvers involving sails

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28 August 2018