POLITECNICO
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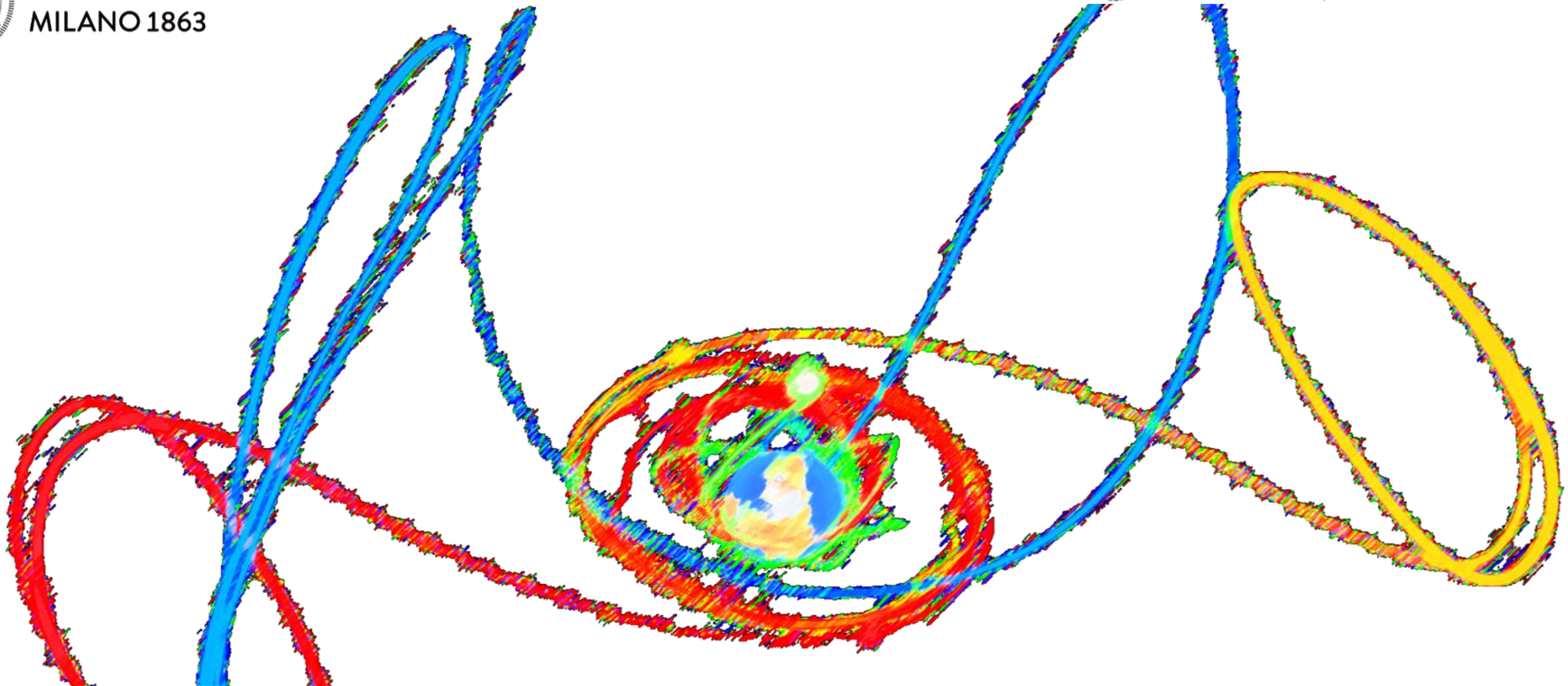


Collision avoidance manoeuvre design and application to passive deorbiting missions

Juan Luis Gonzalo, Camilla Colombo and Pierluigi Di Lizia

2018AMC$_{70}$ workshop: “between Mathematics and Astronomy”
3-5 September 2018, Pisa

Introduction

New space debris mitigation policies are currently being proposed

- Inter-Agency Space Debris Coordination Committee's 25 year guideline (Low Earth Orbit)

Drag or solar sails are cost-effective options to decrease de-orbit time

- Their large cross-sectional area may increase collision risk

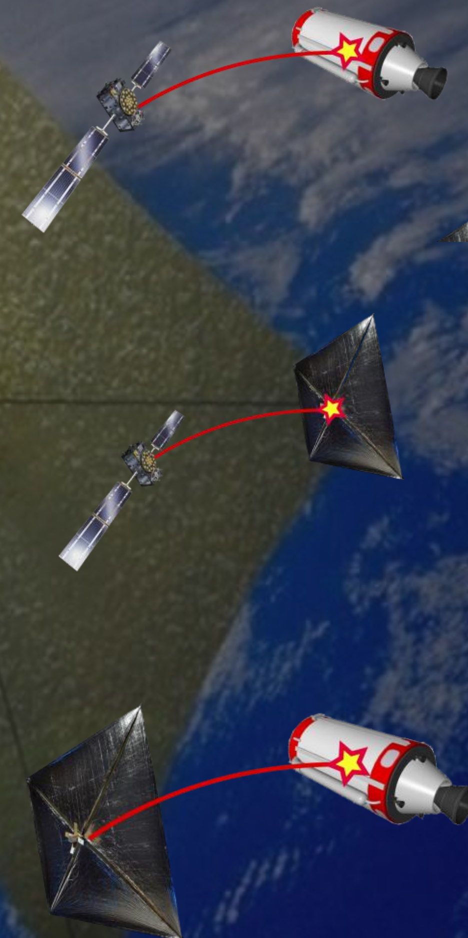
Net effect of sails and tethers on the space environment is being studied in the **ESA-funded project "Environmental aspects of passive de-orbiting devices"**

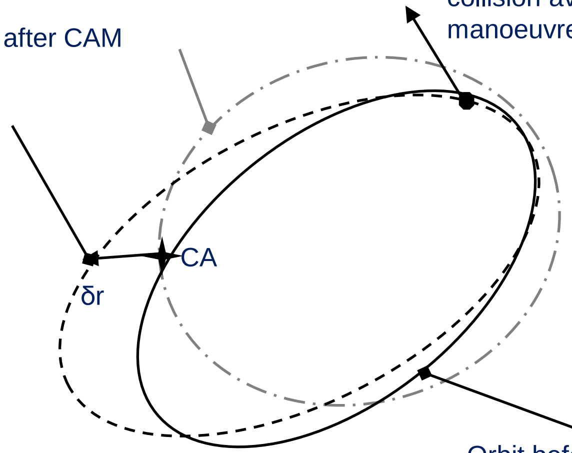
In this talk we will deal with the design of **Collision Avoidance Maneuvers (CAMs) involving sails**

- Manoeuvring either the sail or incoming object (spacecraft)
- Analytical expressions for the impulsive CAMs (maximum deviation or minimum collision probability)
- Taking into account the effect of uncertainties

Outline

- Theoretical approach
- CAM design and sensitivity analysis
  - Spacecraft against debris
  - Effect of uncertainties. Spacecraft versus sail
  - CAM by a deorbiting sail
- Conclusions





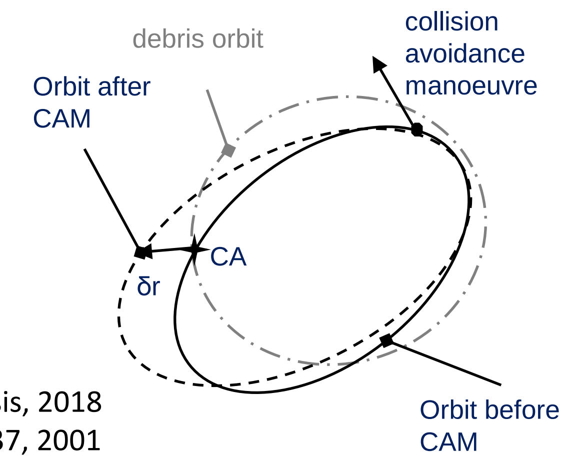
THEORETICAL APPROACH

Theoretical approach

Collision avoidance manoeuvre design

Modelling of Collision Avoidance Manoeuvre (CAM) in the b-plane

- Determine Close Approach (CA) between manoeuvrable spacecraft and debris
- CAM given at lead time Δt before the CA, modelled through Gauss planetary equations for finite differences [1]
- Analytical computation of miss distance at the CA through relative motion equations [1]
- Projection of the miss distance on the b-plane [2]
- **Maximum deviation CAM design is reduced to an eigenvector problem. [3]**



[1] M. Vasile, and C. Colombo, "Optimal impact strategies for asteroid deflection, *JGCD*, 31(4):858-872,2008

[2] M. Petit, "Optimal Deflection Of Resonant Near-Earth Objects Using The b-Plane", Master thesis, 2018

[3] B. A. Conway, "Near-optimal deflection of earth-approaching asteroids," *JGCD,* 24(5):1035-1037, 2001

Theoretical approach

Maximum deviation CAM

$$\delta\boldsymbol{\alpha}(t_{\mathrm{CAM}}) = \mathbf{G}_v(t_{\mathrm{CAM}}; \boldsymbol{\alpha})\,\delta\mathbf{v}(t_{\mathrm{CAM}})$$

Gauss planetary equations [1]

$$\delta\mathbf{r}(t_{\mathrm{CA}}) = \mathbf{A}_r(t_{\mathrm{CA}}; \boldsymbol{\alpha}, \Delta t)\,\delta\boldsymbol{\alpha}(t_{\mathrm{CAM}})$$

Linearized relative motion [2]

$$\delta\mathbf{r}(t_{\mathrm{CA}}) = \mathbf{A}_r\mathbf{G}_v\delta\boldsymbol{v}(t_{\mathrm{CAM}}) = \mathbf{T}\,\delta\mathbf{v}(t_{\mathrm{CAM}})$$

Total displacement

$$\delta\mathbf{b}(t_{\mathrm{CA}}) = \boldsymbol{M}(t_{\mathrm{CA}})\delta\mathbf{r}(t_{\mathrm{CAM}}) = \mathbf{M}\,\mathbf{T}\,\delta\mathbf{v}(t_{\mathrm{CAM}}) = \mathbf{Z}\,\delta\mathbf{v}(t_{\mathrm{CAM}})$$

Displacement in b-plane

Optimization problem reduces to an eigenvalue/eigenvector problem [3]:

$\max\|\delta\mathbf{r}(t_{\mathrm{CA}})\| = \mathrm{n}$ … ector/value of $\mathbf{T}^{\mathrm{T}}\mathbf{T}$

$\max\|\delta\mathbf{b}(t_{\mathrm{CA}})\| = \mathrm{n}$ … ector/value of $\mathbf{Z}^{\mathrm{T}}\mathbf{Z}$

$$\mathbf{M}(t_{\mathrm{CA}}) = \begin{bmatrix} \eta_2^2 + \eta_3^2 & -\eta_1\eta_2 & -\eta_1\eta_3 \\ -\eta_1\eta_2 & \eta_1^2 + \eta_3^2 & -\eta_2\eta_3 \\ -\eta_1\eta_3 & -\eta_2\eta_3 & \eta_1^2 + \eta_2^2 \end{bmatrix}$$

with $\|\delta\mathbf{v}\|$ as large as possible

[1] R. Battin, *An Introduction to the Mathematics and Methods of Astrodynamics*, 1999
[2] J. L. Junkins and H. Schaub, *Analytical mechanics of space systems*, 2009
[3] B. A. Conway, “Near-optimal deflection of earth-approaching asteroids,” *JGCD,* 24(5):1035-1037, 2001

Theoretical approach

Propagation of covariance matrix

Extending the model, the full analytic State Transition Matrix (STM) from $\delta\mathbf{s} = (\delta\mathbf{r}, \delta\mathbf{v})$ at t_{CAM} to $\delta\mathbf{s} = (\delta\mathbf{r}, \delta\mathbf{v})$ at t_{CA} is developed:

- $\mathbf{G}_r$ and $\mathbf{A}_v$ not directly available in previous references (but straightforward to derive)
- Optimizing the miss distance only required a quarter of this matrix.
- The covariance matrix can be propagated.
- Validated against Monte-Carlo simulations with nonlinear dynamics
- Drag and SRP not taken into account (i.e. not valid for sails)... for now

$$\delta\boldsymbol{\alpha}(t_{\mathrm{CAM}}) = \begin{bmatrix}\mathbf{G}_{\mathrm{r}}(t_{\mathrm{CAM}}, \boldsymbol{\alpha}) \\ \mathbf{G}_{\mathrm{v}}(t_{\mathrm{CAM}}, \boldsymbol{\alpha})\end{bmatrix} \delta\mathbf{s}(t_{\mathrm{CAM}})$$

$$\delta\mathbf{s}(t_{\mathrm{CA}}) = \begin{bmatrix}\mathbf{A}_r(t_{\mathrm{CA}}; \boldsymbol{\alpha}, \Delta t) \\ \mathbf{A}_v(t_{\mathrm{CA}}; \boldsymbol{\alpha}, \Delta t)\end{bmatrix} \delta\boldsymbol{\alpha}(t_{\mathrm{CAM}})$$

$$\delta\mathbf{s}(t_{\mathrm{CA}}) = \begin{bmatrix}\mathbf{A}_{\mathrm{r}}\mathbf{G}_{\mathrm{r}} & \mathbf{A}_{\mathrm{r}}\mathbf{G}_{\mathrm{v}} \\ \mathbf{A}_{\mathrm{v}}\mathbf{G}_{\mathrm{r}} & \mathbf{A}_{\mathrm{v}}\mathbf{G}_{\mathrm{v}}\end{bmatrix} \delta\mathbf{s}(t_{\mathrm{CAM}})$$

Theoretical approach

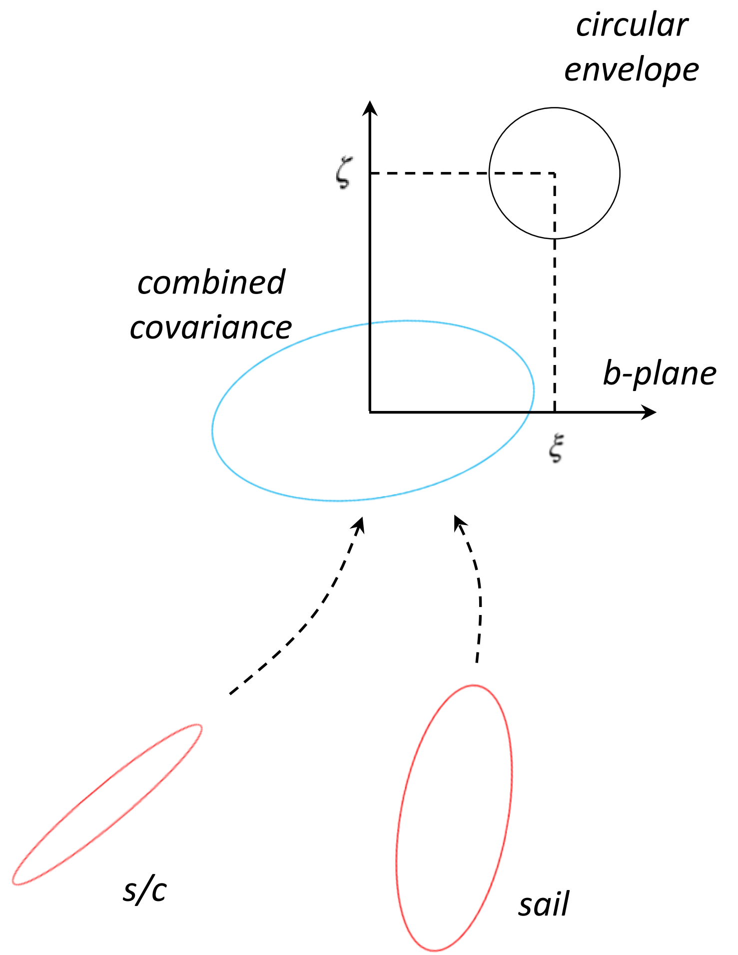
Minimum collision probability CAM design

- Given the combined covariance matrix and the circular envelope of the objects at b-plane
- CAM is designed by computing the $\delta \mathbf{v}(t_{\mathrm{CAM}})$ to minimise collision probability (Chan's approach [1])
- The optimisation problem is reduced to an eigenvalue problem that maximise

$$J_P(\Delta \mathbf{v}) = \left(\frac{\xi}{\sigma_\xi}\right)^2 + \left(\frac{\zeta}{\sigma_\zeta}\right)^2 - 2\rho_{\xi\zeta}\frac{\xi\,\zeta}{\sigma_\xi\,\sigma_\zeta}$$

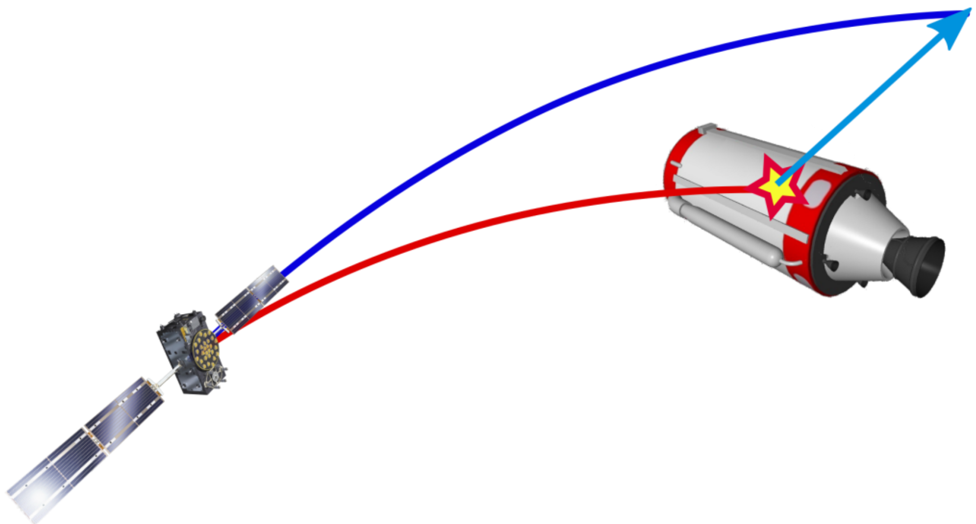
With the combined covariance at the b-plane:

$$\mathbf{C}_{\xi\zeta} = \begin{bmatrix} \sigma_\xi^2 & \rho_{\xi\zeta}\sigma_\xi\sigma_\zeta \\ \rho_{\xi\zeta}\sigma_\xi\sigma_\zeta & \sigma_\zeta^2 \end{bmatrix}$$



The previous CAM model based on **Gauss equations** and **linear relative motion** is used.

[1] C. Bombardelli, and J. Hernando-Ayuso, "Optimal impulsive collision avoidance in low earth orbit", JGCD, 38(2):217-225, 2015



Spacecraft against debris

CAM DESIGN AND SENSITIVITY ANALYSIS

Spacecraft against debris

Test cases

Two tests cases from current ESA 's missions:

PROBA-2 (quasi-circular)						
ID	Epoch [UTC]	a [km]	e [-]	i [deg]	Ω [deg]	ω [deg]
36037	2018/04/20 03:18:34	7093.637	0.0014624	98.2443	303.5949	109.4990

XMM (elliptical)						
ID	Epoch [UTC]	a [km]	e [-]	i [deg]	Ω [deg]	ω [deg]
25989	2018/04/27 18:31:05	66926.137	0.8031489	70.1138	348.8689	95.9905

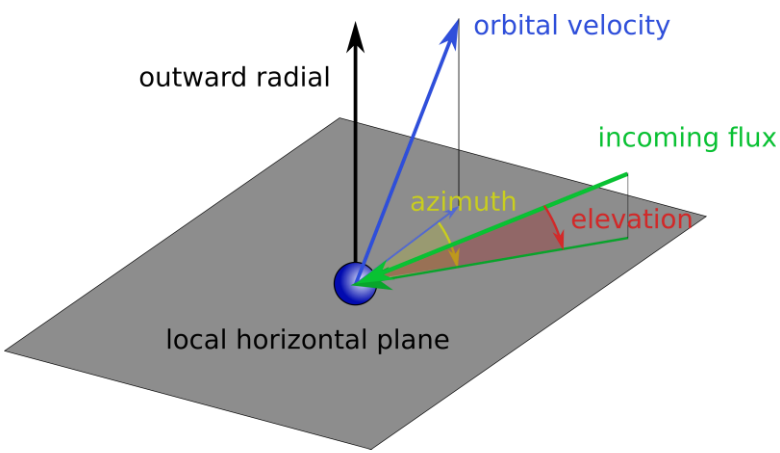
- http://www.heavens-above.com/

Images credit: ESA

Spacecraft against debris

Debris selection and conjunction geometry

- Debris orbits are constructed with **conjunction information** from ESA's MASTER-2009
  - Sources for conjunctions: launchers and mission related objects
  - Ranges for azimuth, elevation and relative velocity at the conjunction

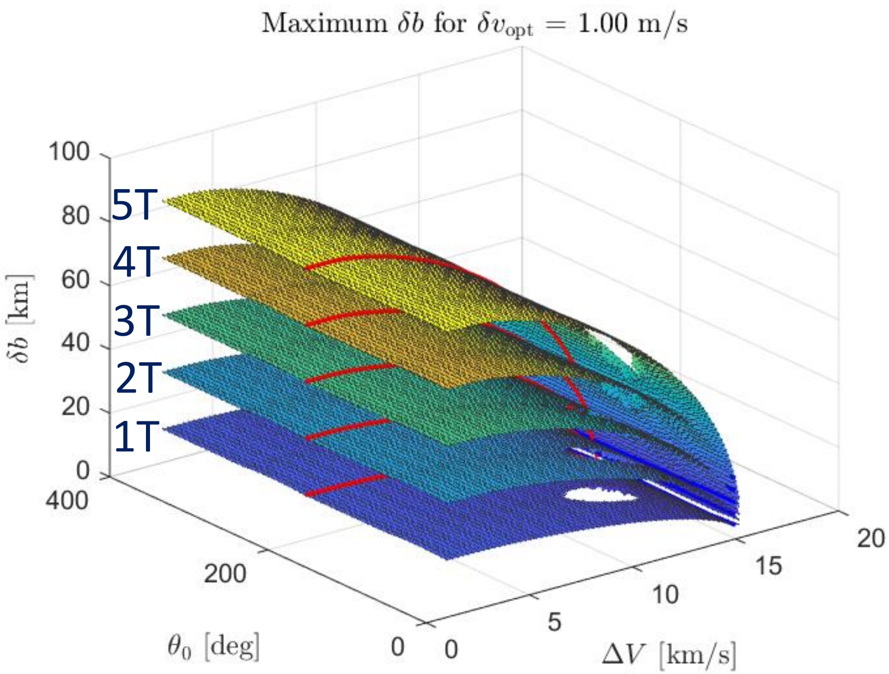


- **Four free parameters**: azimuth, elevation and magnitude of relative velocity, and true anomaly of the s/c at the conjunction
- Results are shown in terms of the **true anomaly of the s/c and the relative velocity at the conjunction**.
  - All combinations of azimuth and elevation are explored, but only the conjunction that maximises a given metric is shown

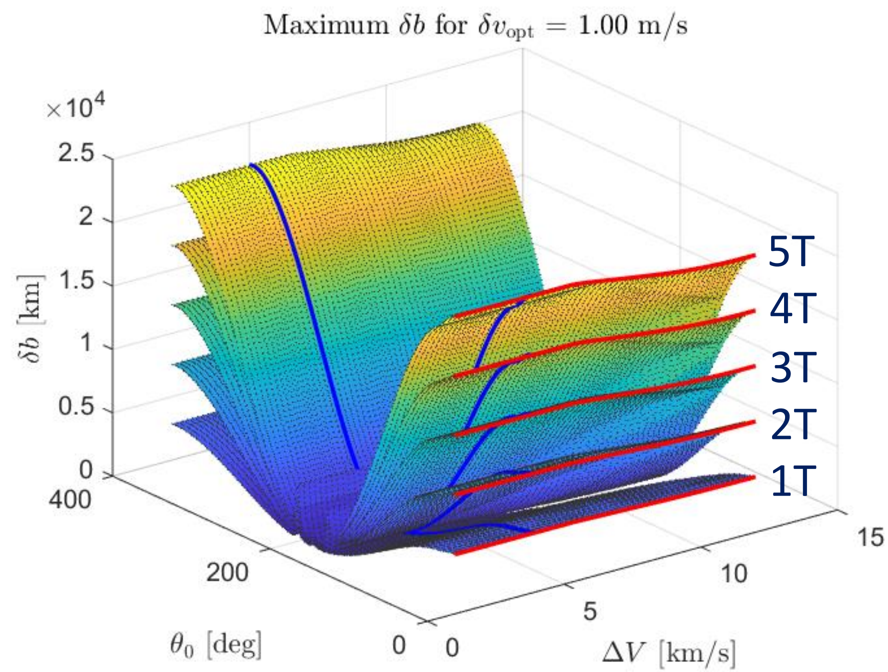
Spacecraft against debris

Sensitivity analysis: Displacement in the b-plane

Quasi-circular: PROBA-2



Elliptical: XMM

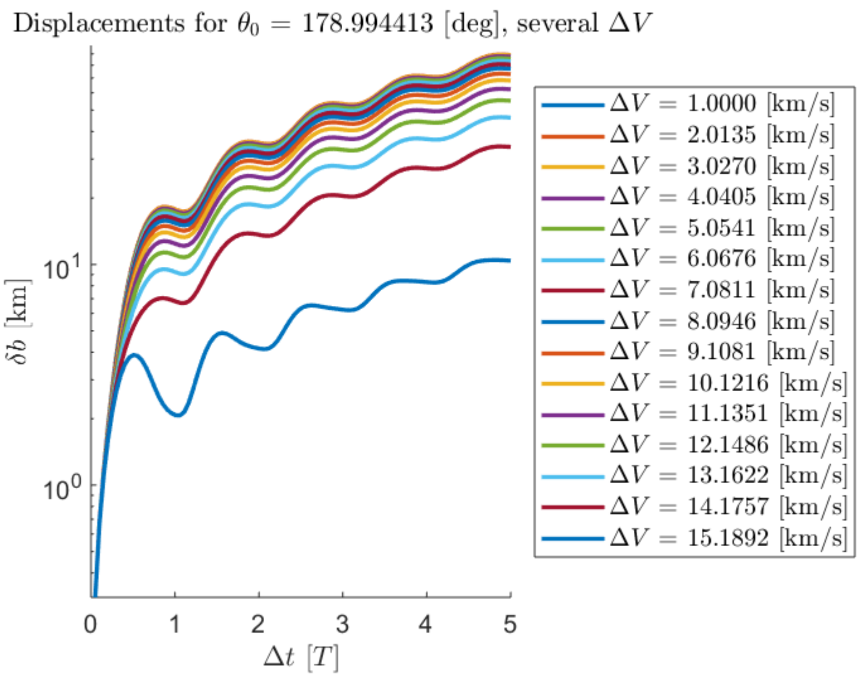


The deflection in the b-plane is strongly influenced by the geometry of the conjunction

Spacecraft against debris

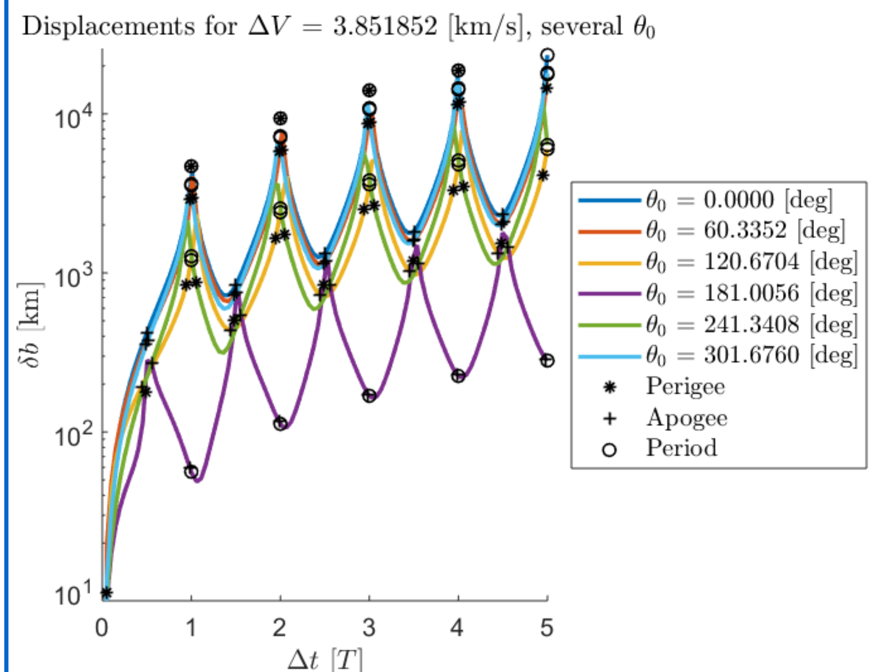
Sensitivity analysis: Time and conjunction geometry effects

Quasi-circular: PROBA2

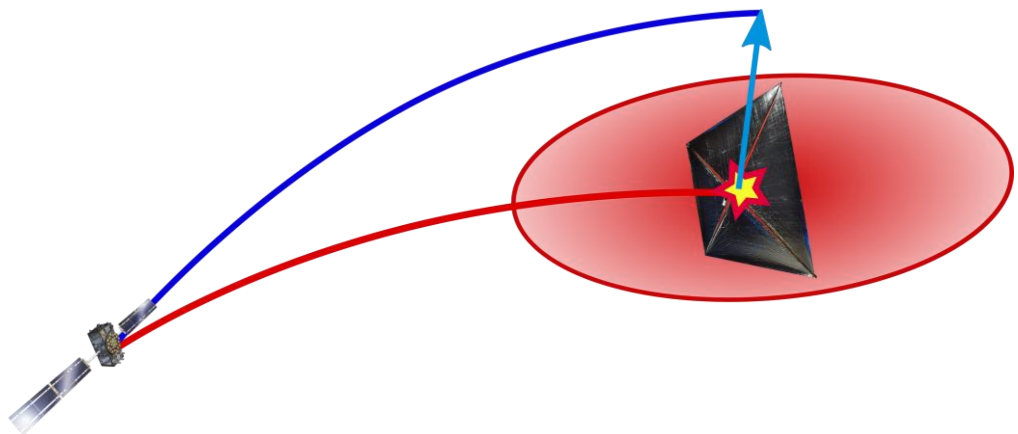


Strong influence of the conjunction geometry in the attainable deflection

Elliptical: XMM



Max/min values around perigee/apogee



Effect of uncertainties. Spacecraft versus sail

CAM DESIGN AND SENSITIVITY ANALYSIS

Effect of uncertainties

Hypothesis and method

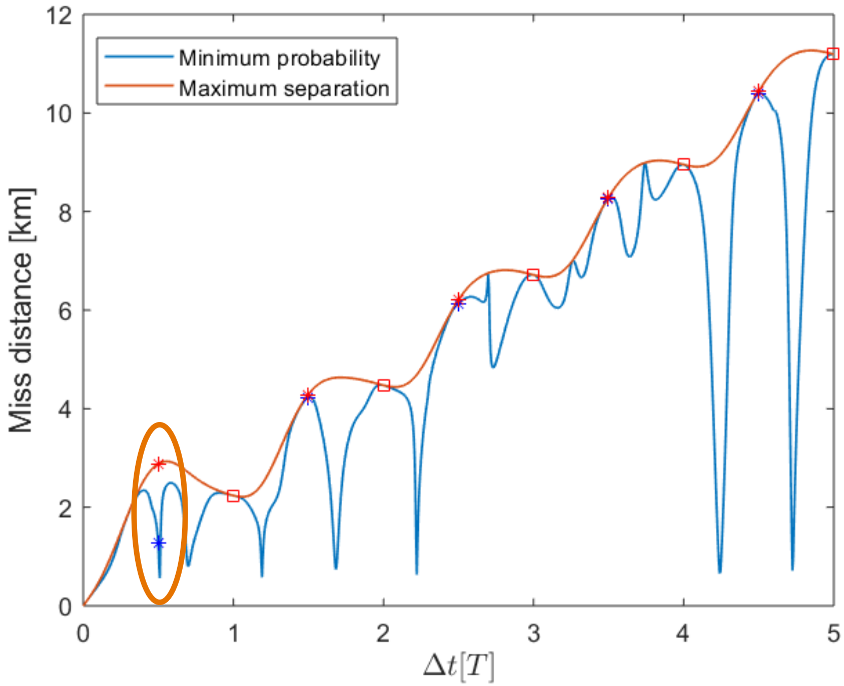
- With a longer lead time:
  - Maximum displacement for a given impulse increases
  - Uncertainties increase
- **What is the net effect on collision probability?**
- **Maximum miss distance** and **minimum collision probability** CAMs are designed and compared for the s/c versus debris case:
  - Nominal case taken from the PROBA-2 test case.
  - Realistic covariance matrix
  - Covariances known at CAM time, propagated using the analytic STM

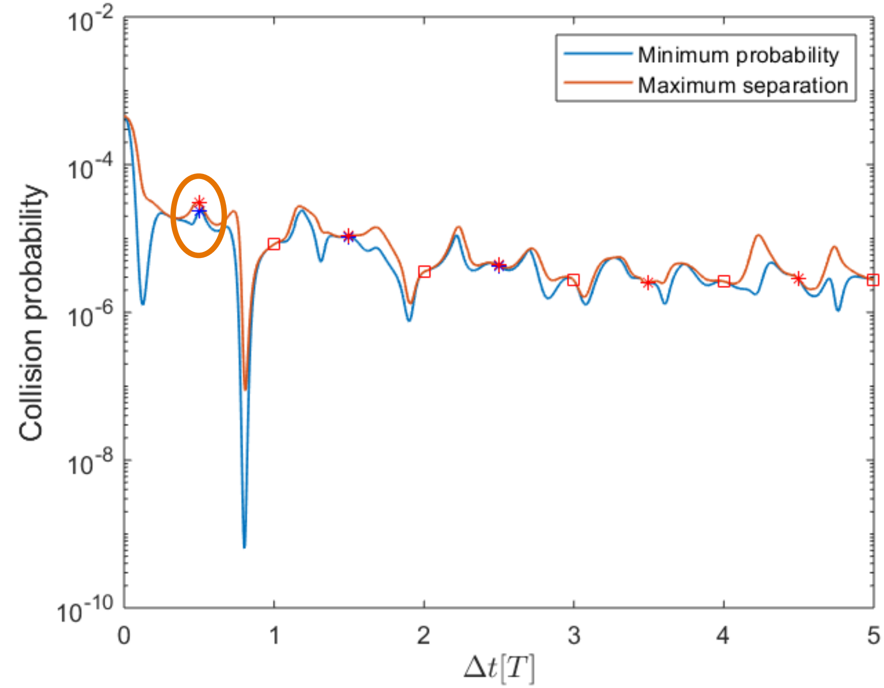
Effect of uncertainties

Test case: maximum miss distance and minimum collision prob. CAMs

- Greatest qualitative differences are observed during the first period

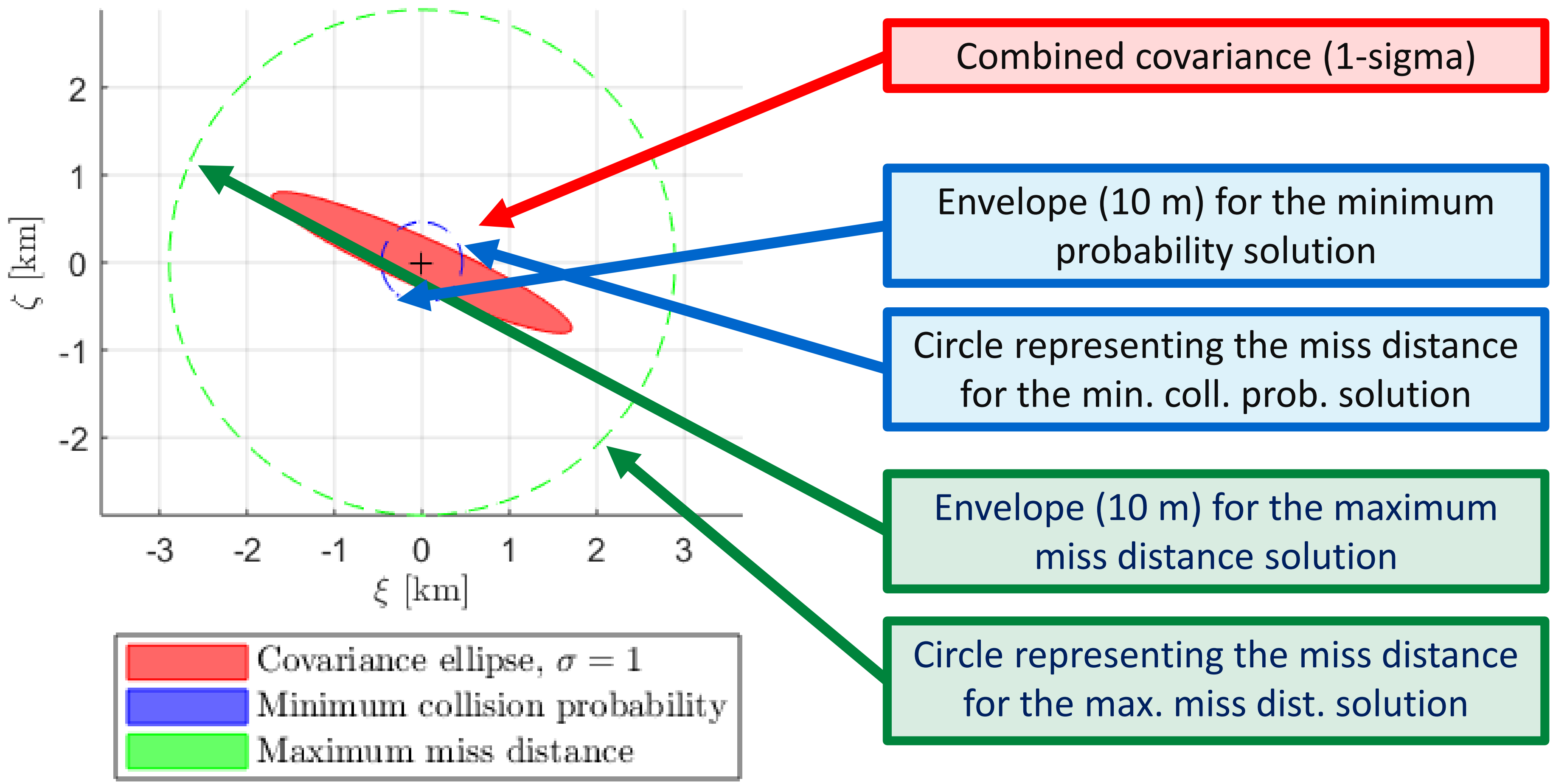
$\Delta v = 0.7$ m/s, $r_A = 10\ m$





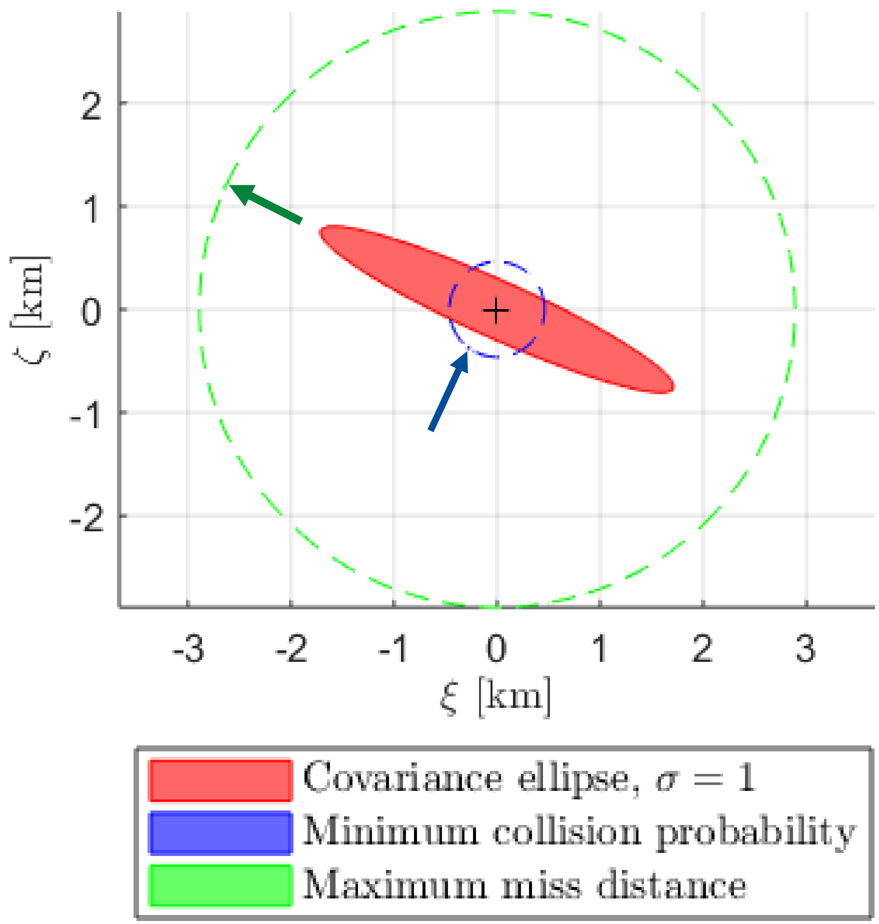
Effect of uncertainties

Test case: Highest difference in miss distance, comparable probability



Effect of uncertainties

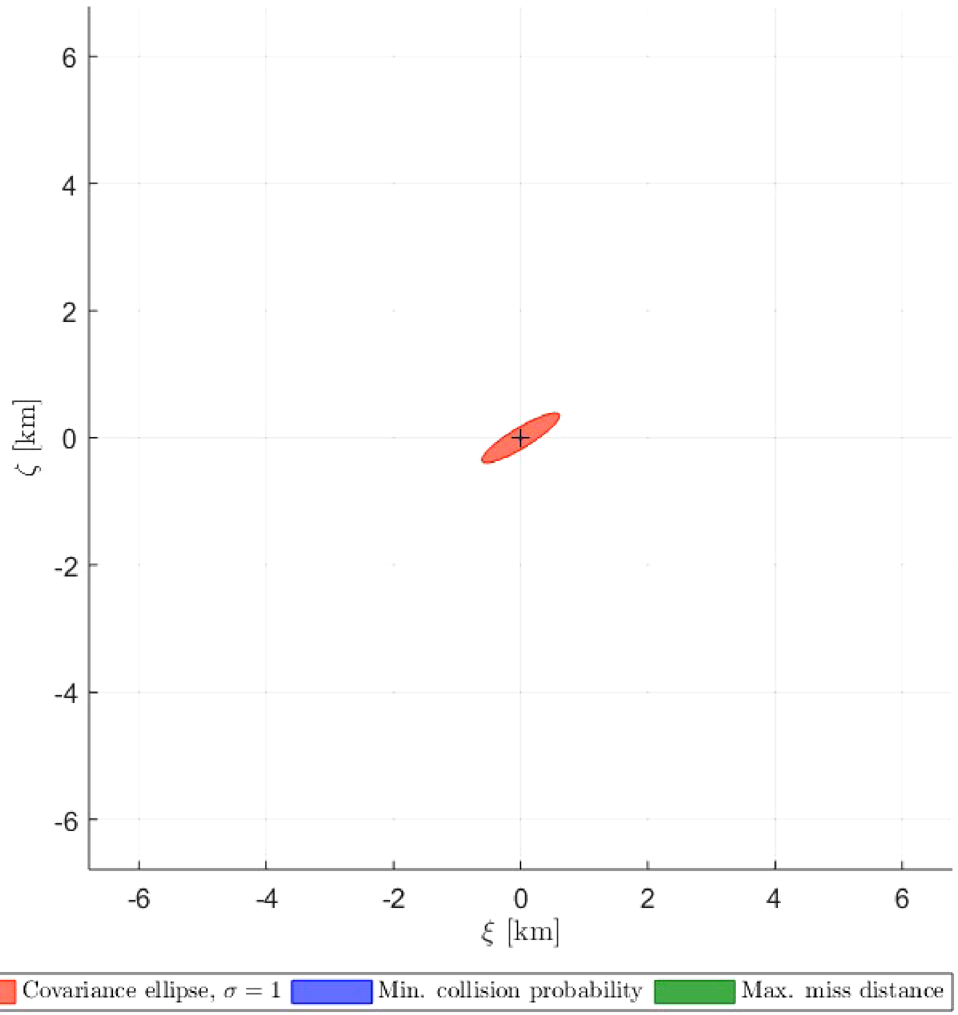
Test case: Highest difference in miss distance, comparable probability



A very similar collision probability is achieved with very different miss distances, due to the orientation with respect to the axis of the covariance ellipse

Effect of uncertainties

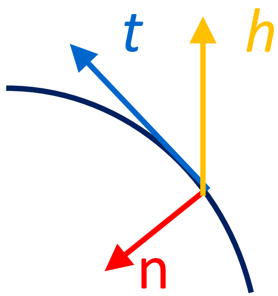
Test case: Time evolution

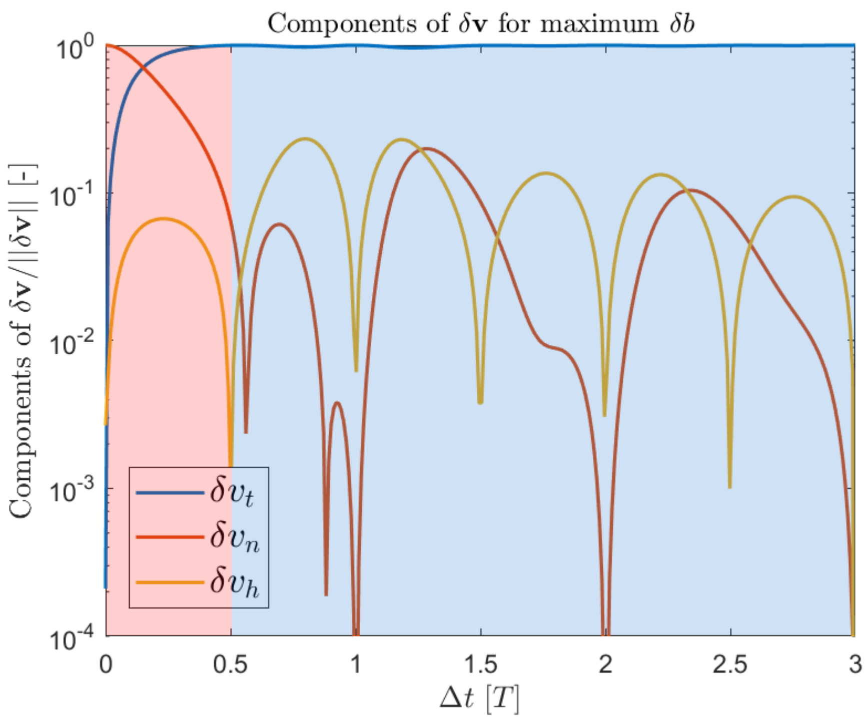


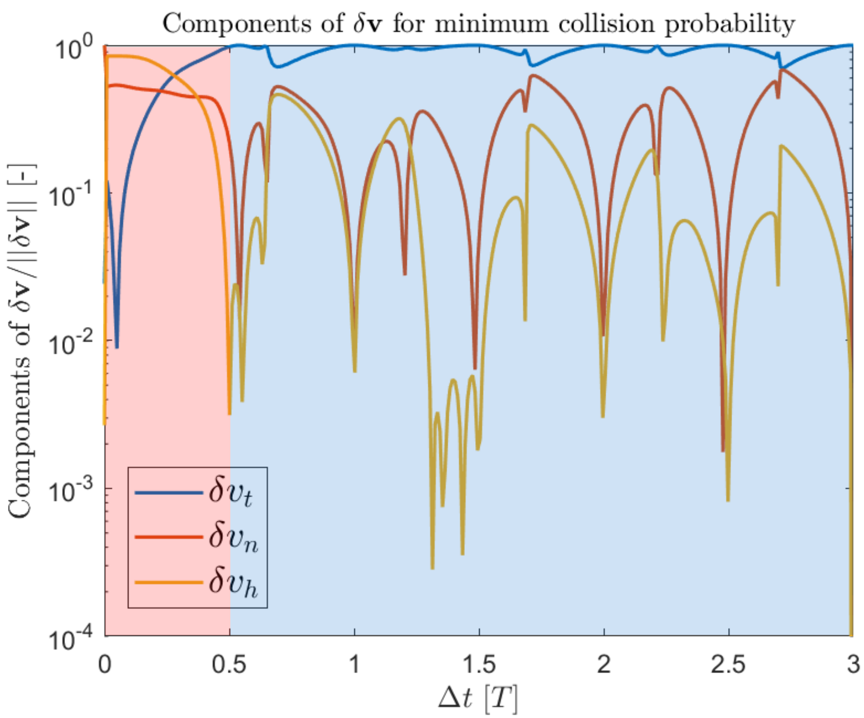
Effect of uncertainties

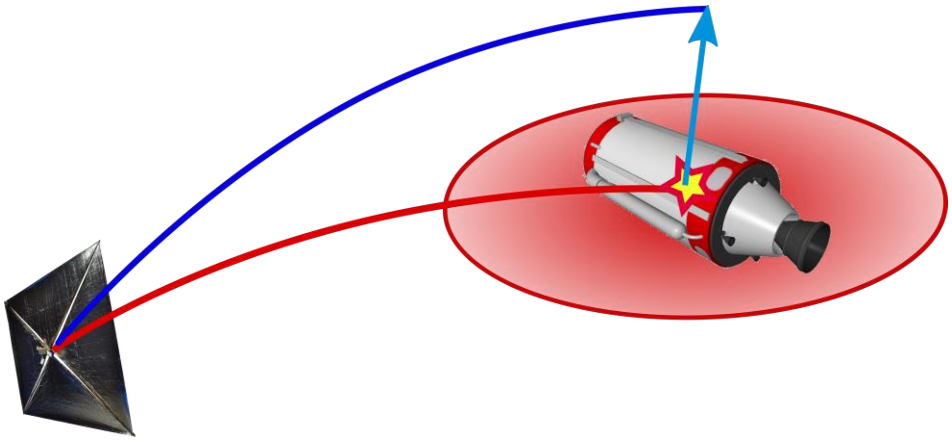
Components of optimum δ**v**

Both for maximum miss distance and minimum collision probability, the impulsive manoeuvre aligns with the transversal direction for $\Delta t > 5T$









CAM by a de-orbiting sail

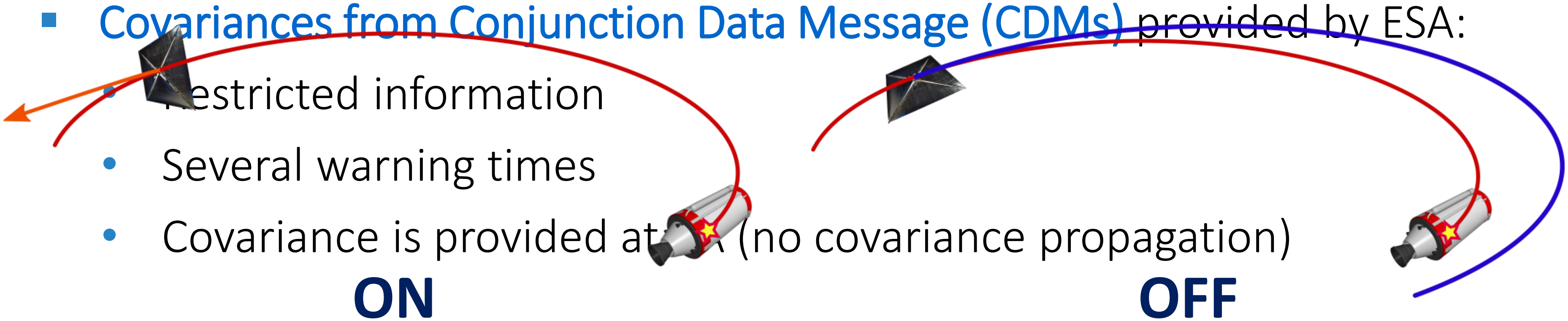
CAM DESIGN AND SENSITIVITY ANALYSIS

CAM by a de-orbiting sail

Method and hypotheses

- Limited control capability:
  - Sail **ON** (perpendicular to the main force)/**OFF** (at feather)
  - **For drag sail, tangential thrust**
  - Effect on CAM is like a **phasing manoeuvre**
- *A/m* represents the 'control authority', i.e., is the parameter for our tests.

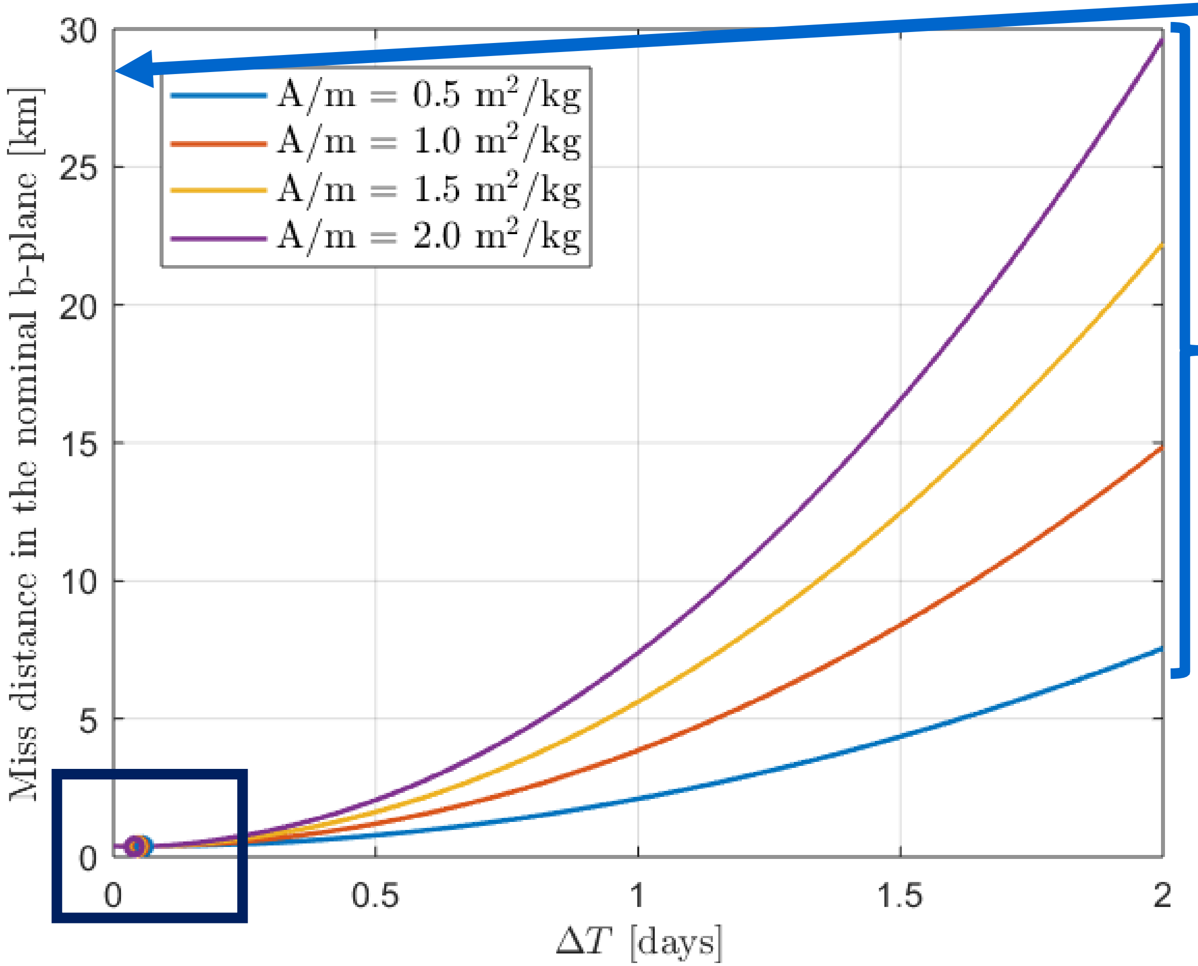
- Covariances from Conjunction Data Message (CDMs) provided by ESA:
  - Restricted information
  - Several warning times
  - Covariance is provided at TCA (no covariance propagation)



- Orbit propagation using averaged dynamics with **PlanODyn**

CAM by a de-orbiting sail

Test case: 2 days warning

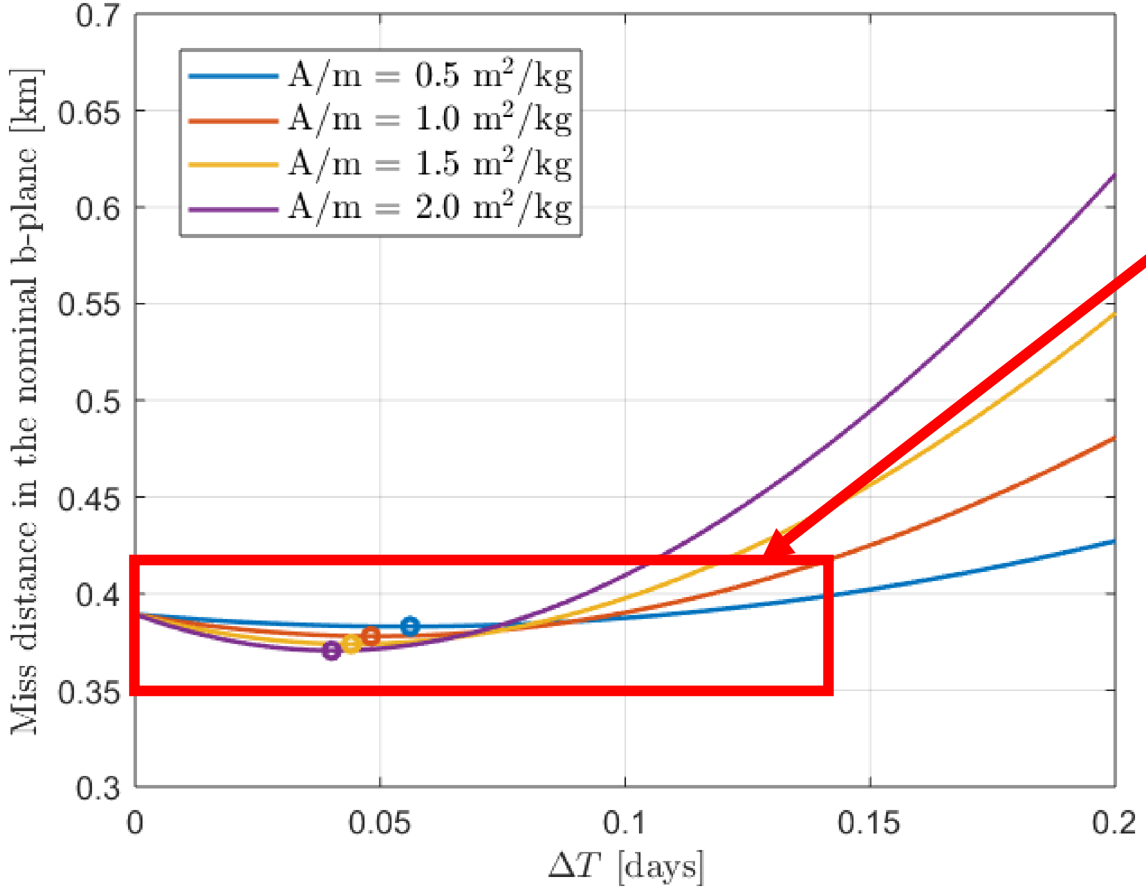


Miss distance can be increased greatly with enough lead time

Area-to-mass is the 'control authority'. A proportional increase is miss distance is observed.

CAM by a de-orbiting sail

Test case: 2 days warning

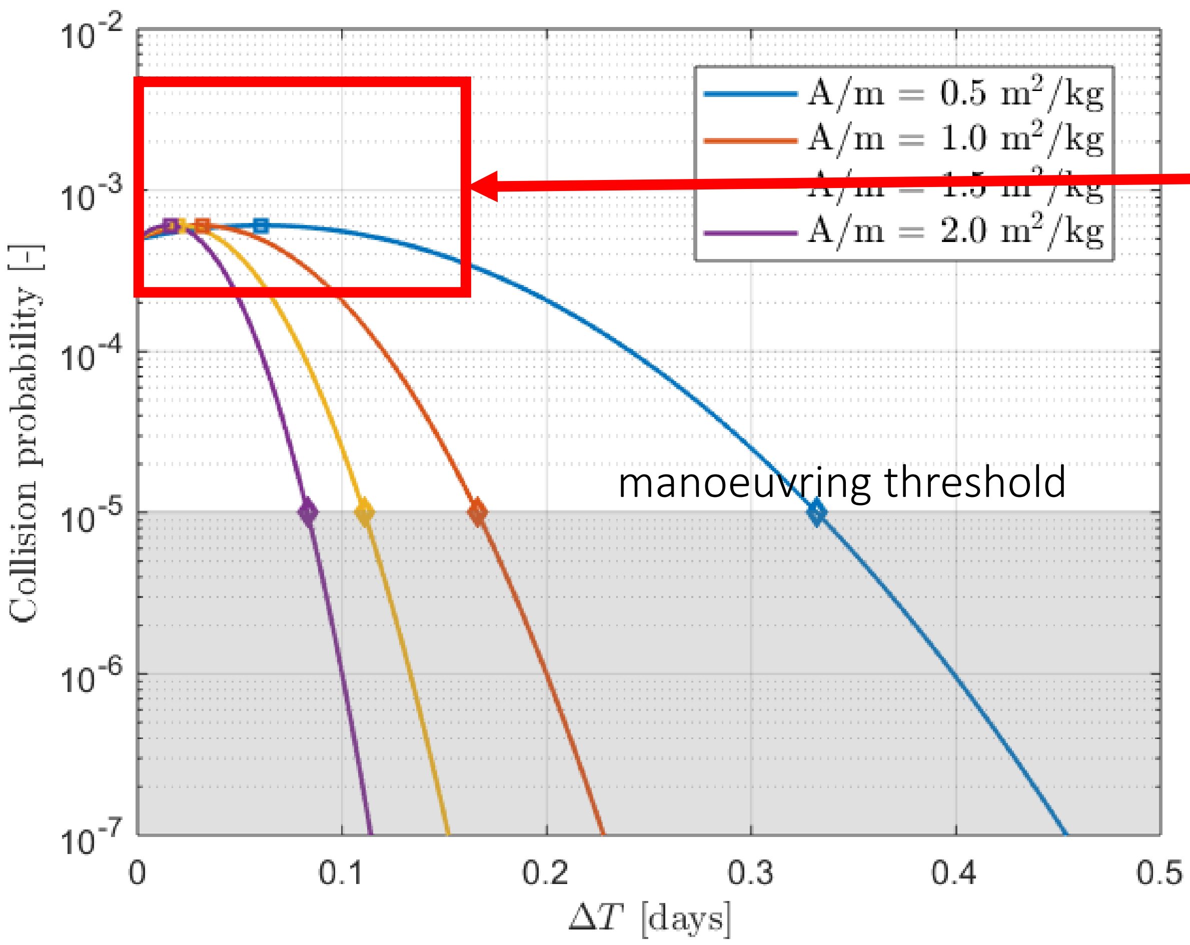


Control can actually reduce miss distance for small lead times.

Is collision probability increased?

CAM by a de-orbiting sail

Test case: 2 days warning



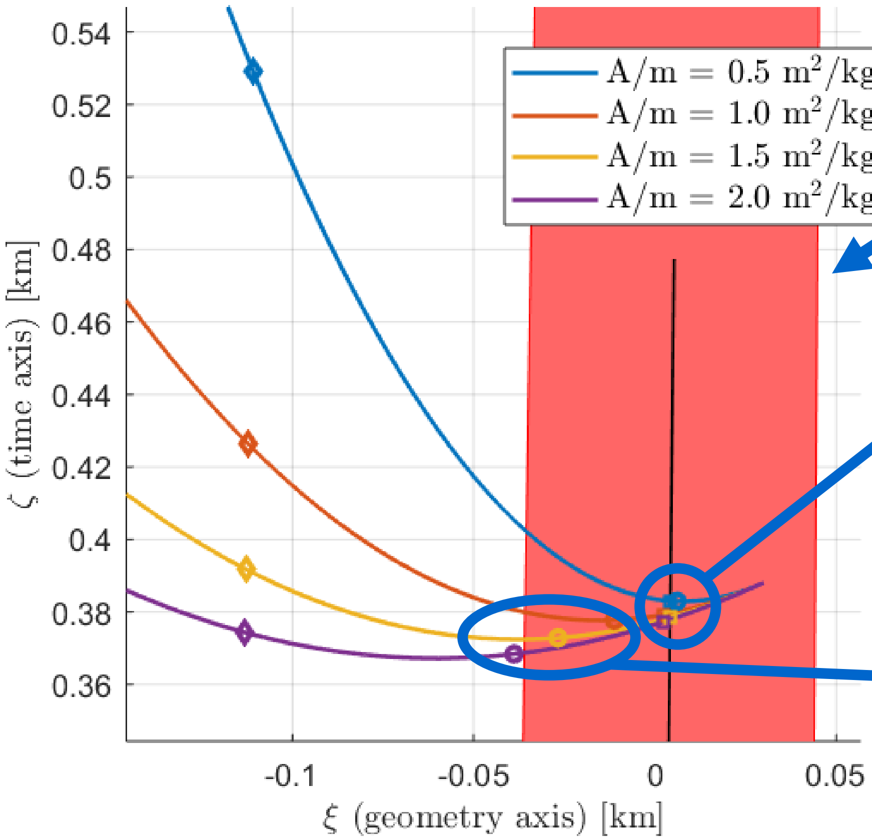
There is an appreciable initial increase of the collision probability

Answers can be found in the b-plane

CAM by a de-orbiting sail

Test case: 2 days warning

Short lead time behaviour in the b-plane justifies the differences in the collision probability



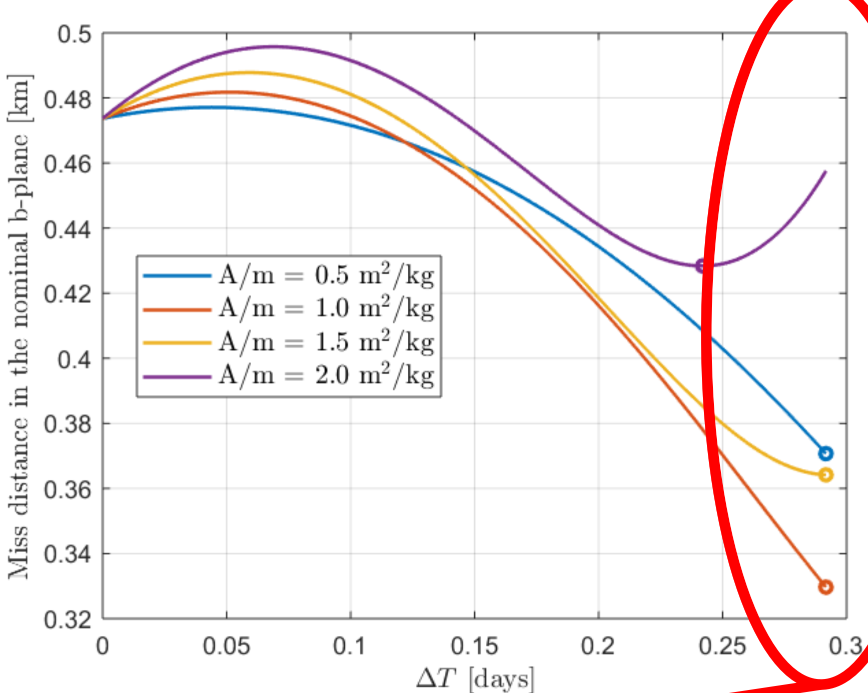
1-σ covariance ellipsoid

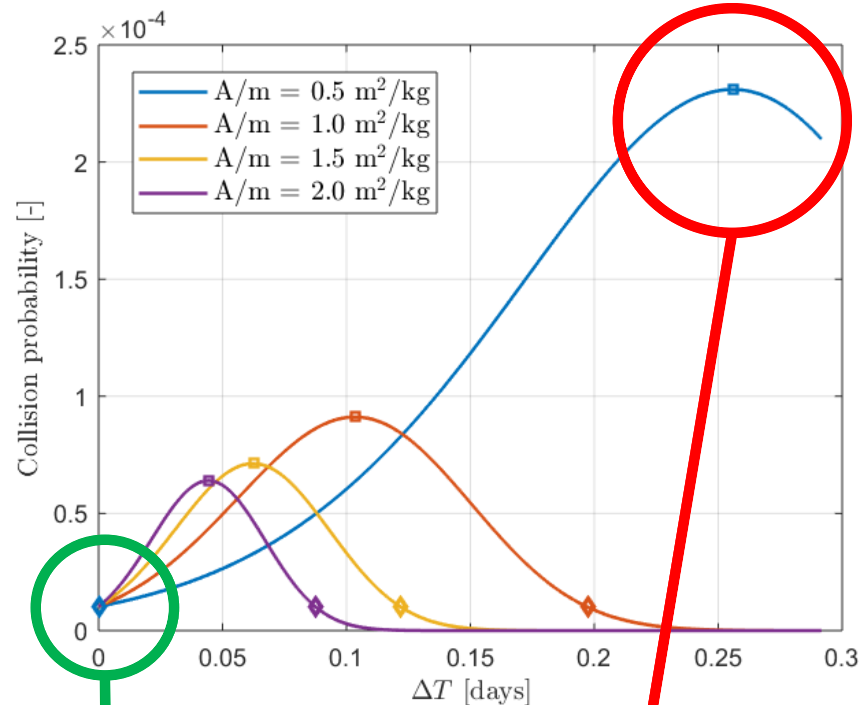
Evolution in the b-plane crosses the semimajor axis of the covariance ellipse, leading to the maximum in collision probability

Minimum miss distance is not only associated to MOID, because distance in the time axis never crosses zero

CAM by a de-orbiting sail

Test case: 7 hours warning





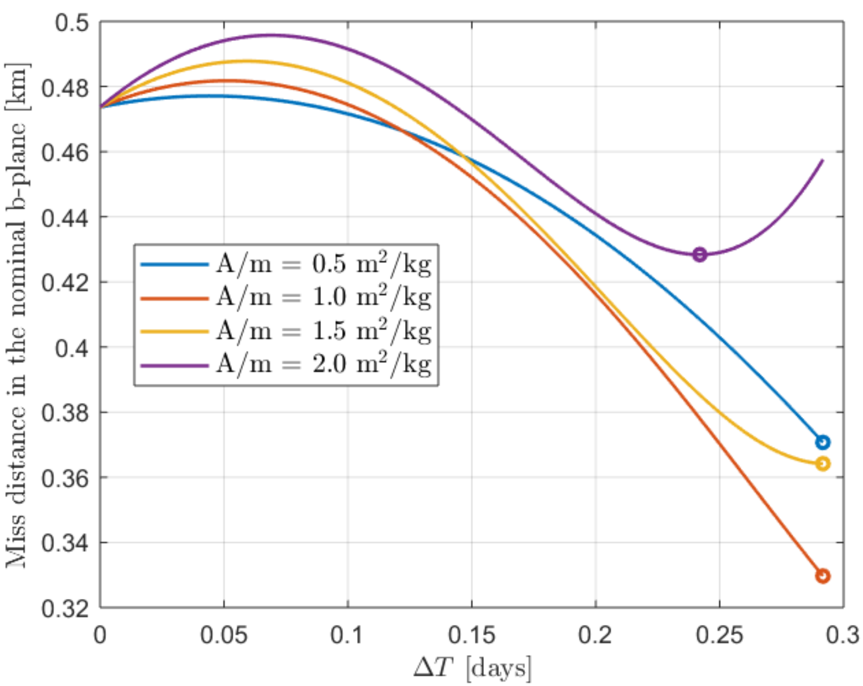
If the decision on the CAM is delayed too much, it will not be possible to increase miss distance

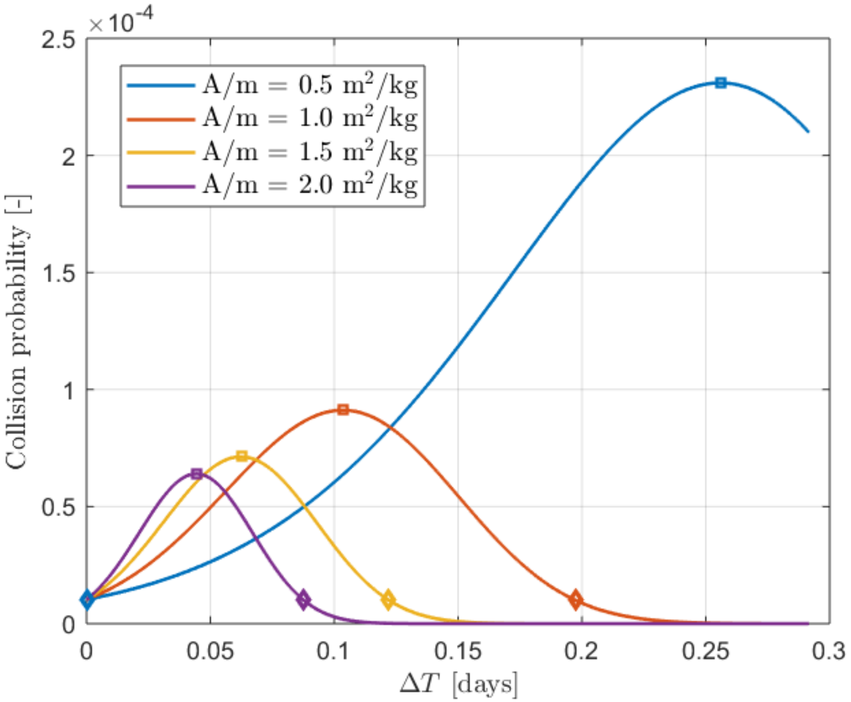
CAM is not actually needed

The smaller sail cannot reach the 10^{-5} collision probability threshold

CAM by a de-orbiting sail

Test case: 7 hours warning





Waiting strategy:

- With the updated CDM the manoeuvre may not be needed
- The effectiveness of the eventual CAM is reduced

CONCLUSIONS

Conclusions

Impulsive CAMs (s/c versus debris or sail)

- Analytical method for maximum deviation (in b-plane) and minimum collision probability impulsive CAMs
  - Extensive sensitivity analysis for the effects of conjunction geometry and true anomaly of the s/c at CA
  - STM for analytic propagation of covariance (without sail)

- As lead time increases, both covariance ellipse and maximum miss distance CAMs in the b-plane tend to align with ζ (time axis)
  - This limits the decrease in collision probability.
- Minimum collision probability CAM moves along ξ (geometrical axis) for some configurations.
- $\delta \boldsymbol{v}$ for both CAMs is mostly transversal for lead times $> 0.5\ T$

Conclusions

Manoeuvring sail

- **Effective CAMs for a deorbiting sail** can be designed through a simple **ON/OFF control law**
  - **A minimum Δt is needed** (depending on encounter geometry and A/m)
  - May **require more anticipation** from satellite operators than impulsive CAMs (**more unneeded manoeuvres?**)

The research performed for this paper has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme within the project COMPASS (grant agreement No 679086).

The simulations have been performed within the study contract "Environmental aspects of passive de-orbiting devices" funded by the European Space Agency (Space Debris Office) (contract number 4000119560/17/F/MOS).

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