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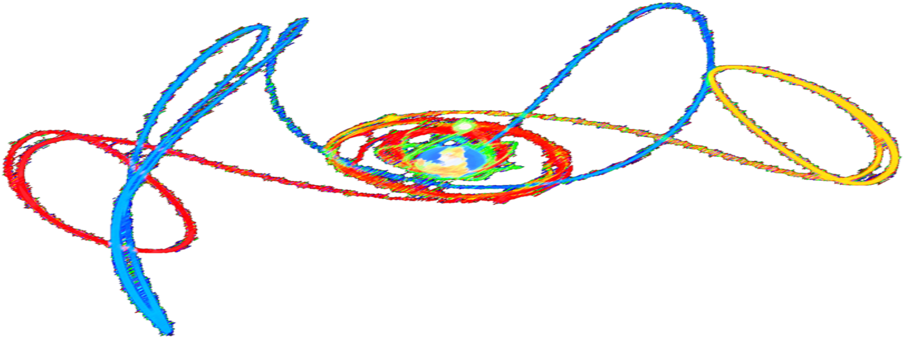

High Earth orbits' characterisation by Hamiltonian reduction on the ecliptic

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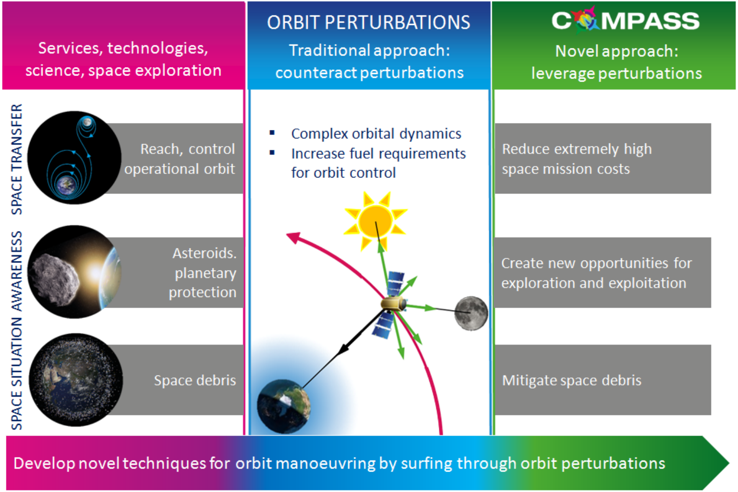
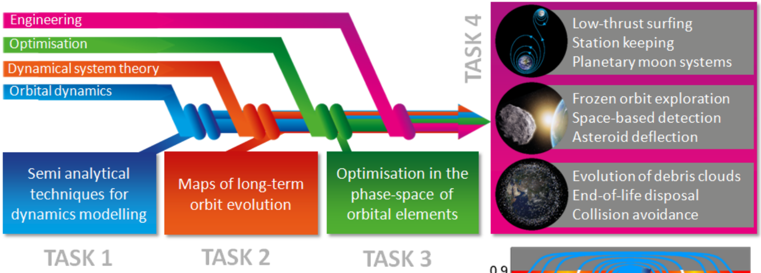
Outline

- Motivation
- Equatorial reduction
- Ecliptic reduction
- Phase-space study
- Disposal design

Motivation


Services, technologies, science, space exploration
ORBIT PERTURBATIONS
Traditional approach: counteract perturbations
COMPASS
Novel approach: leverage perturbations
SPACE TRANSFER
SPACE SITUATION AWARENESS
Reach, control operational orbit
Asteroids. planetary protection
Space debris
Complex orbital dynamics
Increase fuel requirements for orbit control
Reduce extremely high space mission costs
Create new opportunities for exploration and exploitation
Mitigate space debris
Develop novel techniques for orbit manoeuvring by surfing through orbit perturbations

Motivation



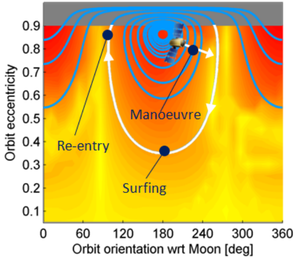
T1. Understanding of the spacecraft orbit evolution

T2. Topology of space of orbit perturbations (stability)

T3. Spacecraft surf these natural currents to the desired orbit

T4. Design of space missions

COMPASS: 5 year ERC project, 10 people-research group, interdisciplinary expertise



Model formulation

The orbit of a massless Earth's satellite in high orbit (no drag) can be modelled as a perturbed Keplerian motion

$$\mathcal{H} = H_{\rm kep} + H_{\sf zonal} + H_{\rm third\text{-}body}$$

- Keplerian part:

$$H_{\rm kep} = -\frac{\mu}{2a}$$

- Zonal Harmonics:

$$H_{\sf zonal} = -\frac{\mu}{r} \sum_{j \geq 2} \left(\frac{R_\oplus}{r} \right)^j C_{j,0} P_{j,0}(\sin\phi)$$

- Third-body attraction (Sun and Moon):

$$H_{\rm third\text{-}body} = -\frac{\mu'}{r'} \left(\frac{r'}{||\mathbf{r} - \mathbf{r}'||} - \frac{\mathbf{r} \cdot \mathbf{r}'}{r'^2} \right)$$

Equatorial reduction

Express all positions in the **equatorial frame**

- Satellite's position:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R_3(-\Omega)R_1(-i)R_3(-\theta)\begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$$

- Moon's position:

$$\begin{pmatrix} x_{☾} \\ y_{☾} \\ z_{☾} \end{pmatrix} = R_1(-\epsilon)R_3(-\Omega_{☾})R_1(-i_{☾})R_3(-\theta_{☾})\begin{pmatrix} r_{☾} \\ 0 \\ 0 \end{pmatrix}$$

- Sun's position:

$$\begin{pmatrix} x_{\odot} \\ y_{\odot} \\ z_{\odot} \end{pmatrix} = R_1(-\epsilon)R_3(-\theta_{\odot})\begin{pmatrix} r_{\odot} \\ 0 \\ 0 \end{pmatrix}$$

Equatorial reduction

Reduction of the J_2 part of the Hamiltonian:

$$H_{J_2} = \frac{\mu}{r}\left(\frac{R_\oplus}{r}\right)^2 J_2 P_2(\sin\phi)$$

where

$$\sin\phi = \frac{z}{r}$$

We average over the satellite's mean anomaly to get:

$$\bar{H}_{J_2} = \bar{H}_{J_2}(a, e, i, -, -, -; \mu, J_2, R_\oplus) = \frac{J_2 R_\oplus \mu (3\sin^2 i - 2)}{4a^3\eta^3}$$

with $\eta = \sqrt{1-e^2}$.

Equatorial reduction

Reduction of the Sun's perturbing effect

$$H_{\odot} = -\frac{n_{\odot} a_{\odot}^{3}}{r_{\odot}} \left(\frac{r}{r_{\odot}}\right)^{2} P_{2}(cos\psi_{\odot})$$

where

$$\cos(\psi_{\odot}) = \frac{xx_{\odot} + yy_{\odot} + zz_{\odot}}{rr_{\odot}}$$

$$H_{\odot} = H_{\odot}(a, e, i, \Omega, \omega, M, \theta_{\odot}; n_{\odot}, a_{\odot}, \epsilon)$$

We average in closed form over the satellite's mean anomaly

$$\bar{H}_{\odot} = \bar{H}_{\odot}(a, e, i, \Omega, \omega, -, \theta_{\odot}; n_{\odot}, a_{\odot}, \epsilon)$$

Equatorial reduction

Reduction of the Moon's perturbing effect

$$H_{☾} = -\beta \frac{n_{☾} a_{☾}^3}{r_{☾}} \left(\frac{r}{r_{☾}} \right)^2 P_2(cos\psi_{☾})$$

where

$$\cos(\psi_{☾}) = \frac{xx_{☾} + yy_{☾} + zz_{☾}}{rr_{☾}}$$

$$H_{☾} = H_{☾}(a, e, i, \Omega, \omega, M, \Omega_{☾}, \theta_{☾}; \beta, n_{☾}, a_{☾}, i_{☾}, \epsilon)$$

We average in closed form over the satellite's mean anomaly

$$\bar{H}_{☾} = \bar{H}_{☾}(a, e, i, \Omega, \omega, -, \Omega_{☾}, \theta_{☾}; \beta, n_{☾}, a_{☾}, i_{☾}, \epsilon)$$

Equatorial reduction

We average one more time again in closed form, over the Moon's mean anomaly

$$\bar{\bar{H}}_{☾} = \bar{\bar{H}}_{☾}\left(a, e, i, \Omega, \omega, -, \Omega_{☾}, -; \beta, n_{☾}, a_{☾}, i_{☾}, \epsilon, \eta_{☾}\right)$$

The full system is

$$\bar{\bar{H}} = \bar{H} + \bar{H}_{\odot} + \bar{\bar{H}}_{☾}$$

and has **2.5** degrees of freedom

$$\bar{\bar{H}} = \bar{\bar{H}}(a, e, i, \Omega, \omega, -, \Omega_{☾}, \theta_{\odot}; \mu, J_2, R_{\oplus}, \epsilon, n_{\odot}, a_{\odot}, n_{☾}, a_{☾}, \eta_{☾})$$

If we try to further reduce the system by an **elimination of the satellite's node**, time-dependent terms associated with $\Omega_{☾}$ and $\theta_{\odot}$ still remain.

Ecliptic reduction

Express all positions in the **ecliptic frame**

- Satellite's position:

$$\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} = R_3(-\Omega)R_1(-i)R_3(-\theta)\begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix}$$

- Moon's position:

$$\begin{pmatrix} \xi_{☾} \\ \eta_{☾} \\ \zeta_{☾} \end{pmatrix} = R_3(-\Omega_{☾})R_1(-i_{☾})R_3(-\theta_{☾})\begin{pmatrix} r_{☾} \\ 0 \\ 0 \end{pmatrix}$$

- Sun's position:

$$\begin{pmatrix} \xi_{\odot} \\ \eta_{\odot} \\ \zeta_{\odot} \end{pmatrix} = R_3(-\theta_{\odot})\begin{pmatrix} r_{\odot} \\ 0 \\ 0 \end{pmatrix}$$

Ecliptic reduction

Reduction of the J_2 part of the Hamiltonian:

$$H_{J_2} = \frac{\mu}{r}\left(\frac{R_\oplus}{r}\right)^2 J_2 P_2(\sin\phi)$$

The relation between equatorial and ecliptic coordinates is simply

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R_1(-\epsilon)\begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix}$$

and

$$\sin\phi = \frac{z}{r} = \frac{\zeta\cos(\epsilon) + \eta\sin(\epsilon)}{r}$$

We average in closed form over the satellite's mean anomaly

$$\bar{H}_{J_2} = \bar{H}_{J_2}(a, e, i, \Omega, -, -; \mu, J_2, R_\oplus, \epsilon)$$

Ecliptic reduction

Reduction of the Sun's perturbing effect

$$H_\odot = -\frac{n_\odot a_\odot^3}{r_\odot} \left(\frac{r}{r_\odot} \right)^2 P_2(cos \psi_\odot)$$

where

$$\cos(\psi_\odot) = \frac{\xi \xi_\odot + \eta \eta_\odot + \zeta \zeta_\odot}{r r_\odot}$$

$$H_\odot = H_\odot(a, e, i, \Omega, \omega, M, \theta_\odot; n_\odot, a_\odot)$$

We average in closed form over the satellite's mean anomaly

$$\bar{H}_\odot = \bar{H}_\odot(a, e, i, \Omega, \omega, -, \theta_\odot; n_\odot, a_\odot)$$

Ecliptic reduction

Reduction of the Moon's perturbing effect

$$H_{☾} = -\beta \frac{n_{☾} a_{☾}^3}{r_{☾}} \left(\frac{r}{r_{☾}} \right)^2 P_2(cos\psi_{☾})$$

where

$$\cos(\psi_{☾}) = \frac{\xi\xi_{☾} + \eta\eta_{☾} + \zeta\zeta_{☾}}{rr_{☾}}$$

$$H_{☾} = H_{☾}(a, e, i, \Omega, \omega, M, \Omega_{☾}, \theta_{☾}; \beta, n_{☾}, a_{☾}, i_{☾}, \epsilon)$$

We average in closed form over the satellite's mean anomaly

$$\bar{H}_{☾} = \bar{H}_{☾}(a, e, i, \Omega, \omega, -, \Omega_{☾}, \theta_{☾}; \beta, n_{☾}, a_{☾}, i_{☾}, \epsilon)$$

Ecliptic reduction

We average one more time again in closed form, over the Moon's mean anomaly

$$\bar{\bar{H}}_{☾} = \bar{\bar{H}}_{☾}(a, e, i, \Omega, \omega, -, \Omega_{☾}, -; \beta, n_{☾}, a_{☾}, i_{☾}, \epsilon, \eta_{☾})$$

The full system is

$$\bar{\bar{H}} = \bar{H} + \bar{H}_{\odot} + \bar{\bar{H}}_{☾}$$

and is still of **2.5** degrees of freedom

$$\bar{\bar{H}} = \bar{\bar{H}}(a, e, i, \Omega, \omega, -, \Omega_{☾}, \theta_{\odot}; \mu, J_2, R_{\oplus}, \epsilon, n_{\odot}, a_{\odot}, n_{☾}, a_{☾}, \eta_{☾})$$

However, in this representation, the time-dependencies appear coupled with the satellite's **ecliptic node**.

Ecliptic reduction

Therefore, we can proceed with a further **elimination of the ecliptic node**. This is accomplished by working in a suitable **rotating frame** and is a valid operation when the perturbations are **of the same order**, i.e. for **distant** Earth's satellites.

$$\bar{\bar{H}}_{J_2} = \frac{J_2 R_\oplus^2 \mu (3\cos^2 i - 1)(3\sin^2\epsilon - 2)}{8a^3\eta^3}$$

$$\bar{\bar{H}}_{\odot} = a^2 n_\odot^2 \left(-\frac{15}{16} e^2 \cos 2\omega \sin^2 i + \frac{1}{16}(2+3e^2)(3\sin^2 i - 2) \right)$$

$$\bar{\bar{\bar{H}}}_{\leftmoon} = -\frac{a^2 n_{\leftmoon}^2 \beta (3\cos^2 i_{\leftmoon} - 1)((2+3e^2)(3\cos^2 i - 1) + 15e^2\sin^2 i \cos 2\omega)}{32\eta_{\leftmoon}^2}$$

Ecliptic reduction

The reduction on the ecliptic results in a **1 D.O.F** Lidov-Kozai type Hamiltonian

$$\bar{\bar{\bar{H}}} = \frac{A}{\eta^3}(2 - 3\sin^2 i) + B((2 + 3e^2)(2 - 3\sin^2 i) + 15e^2 \sin^2 i \cos 2\omega)$$

where

$$A = -\frac{J_2 R_\oplus \mu}{8a^3}(2 - 3\sin^2 \epsilon)$$

and

$$B = -\frac{1}{16}\left(n_\odot^2 + \frac{n_{☾}^2}{\eta_{☾}}\beta\frac{3\cos^2 i_{☾} - 1}{2}\right)a^2$$

Study of the reduced model

We introduce the non-singular elements

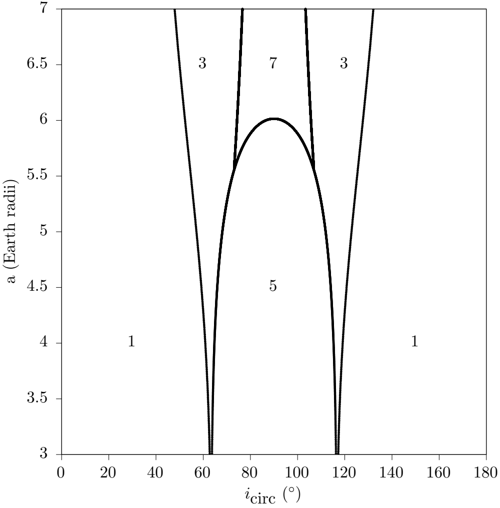
$$k = e\cos\omega,\ h = e\sin\omega$$

and the equations of motion are

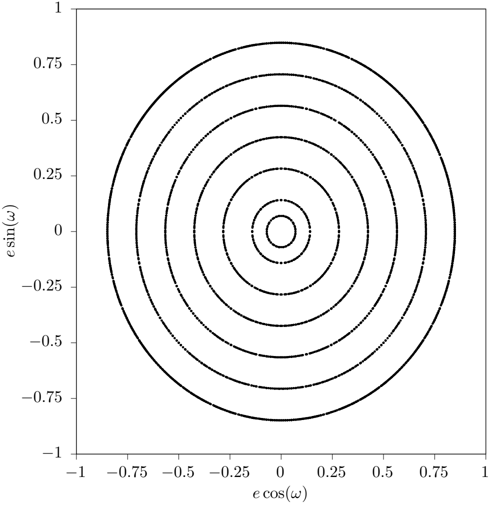
$$\frac{dk}{dt} = -\frac{\sqrt{1-h^2-k^2}}{na^2}\frac{dV(k,h)}{dh}$$
$$\frac{dh}{dt} = \frac{\sqrt{1-h^2-k^2}}{na^2}\frac{dV(k,h)}{dk}$$

- Equilibrium points: $dk/dt = dh/dt = 0$
- Stability determined from the eigenvalues of the linearised system
- Parameter space of (a, i_{circ})

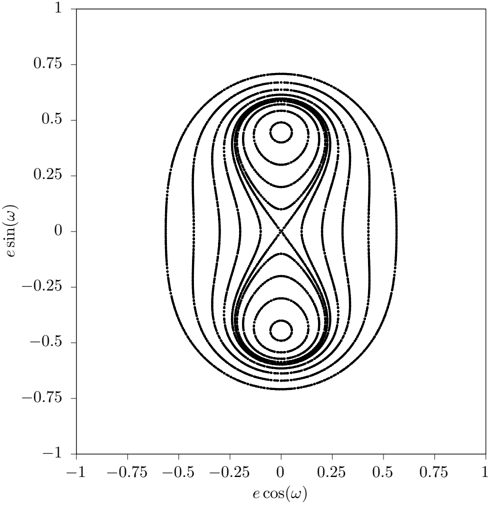
Bifurcation diagram



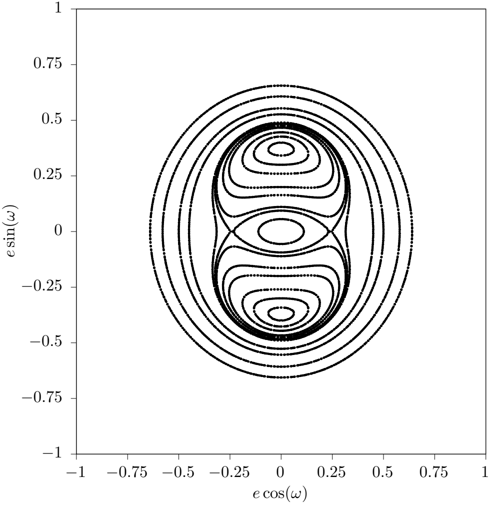
Phase-space study



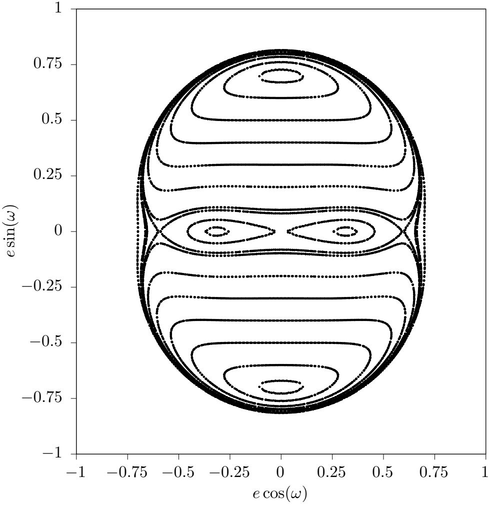
Phase-space study



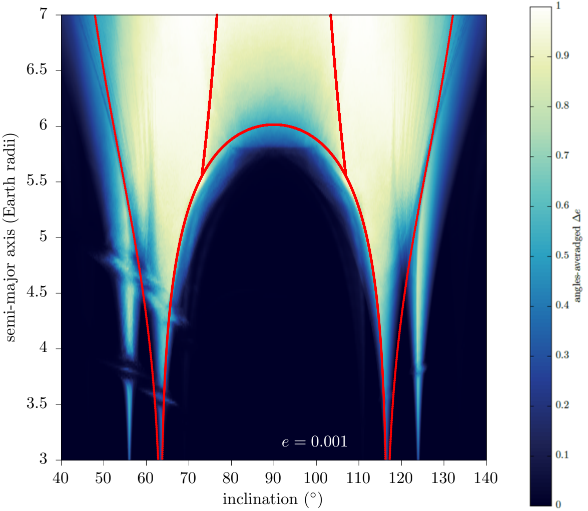
Phase-space study



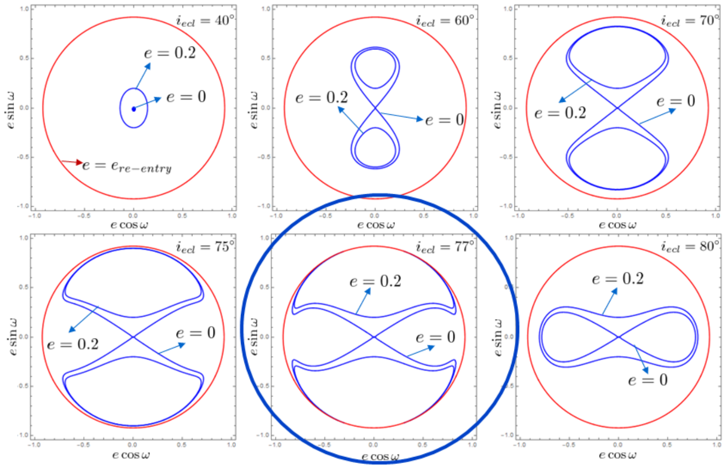
Phase-space study



Bifurcation diagram vs numerical simulations



Disposal design



Conclusion

- We have reduced the problem of high Earth satellites using an ecliptic representation
- The resulting 1 D.O.F system describes the in-plane stability
- We studied the reduced phase-space by computing the equilibrium points and their stability
- We have calculated the bifurcation diagram

Further work:

- Recover the short-periodic terms
- Add more perturbations, J_2^2 and up to P_4 for the Moon
- Study the equilibria and their bifurcation on a sphere
- Exploit the reduced dynamics for preliminary mission design

Thank you for your attention!

Ackowledgements

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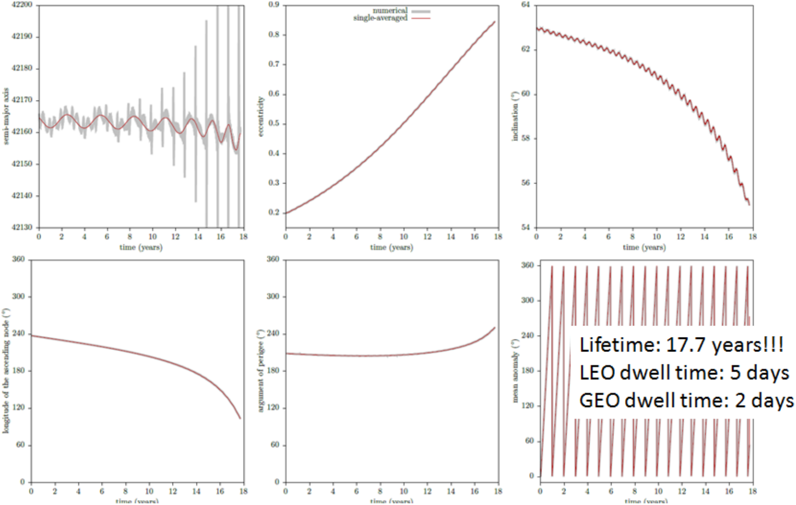
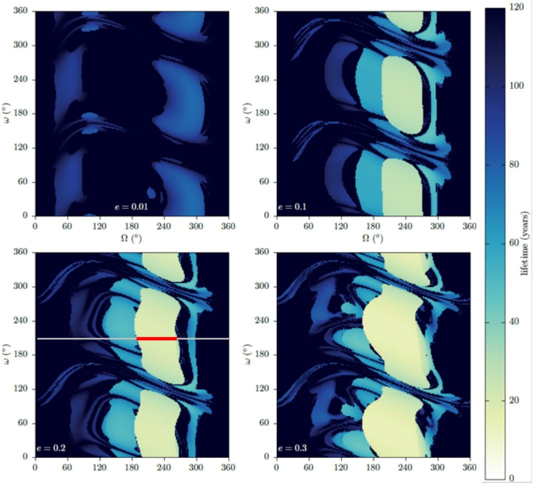
Fast re-entering orbits



High Earth Orbits lifetimes

$a = R_{GEO}$
$i = 63°$



Effective cleansing mechanism