Small oscillations of a Helio-stable solar sail under gravity gradient torque

Narcís Miguel i Baños [1], Martín Lara [2] and Camilla Colombo [1]

[1] Dipartimento di Scienze e Tecnologie Aerospaziali
Politecnico di Milano

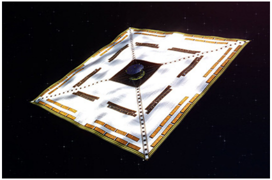
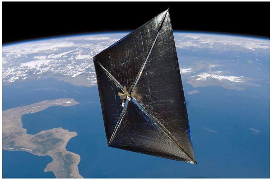
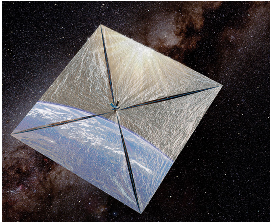
[2] Grupo de Computación Científica
Universidad de La Rioja

SSA-Arizona and DSRI: A Symposium on Astrodynamics

August 28, 2018

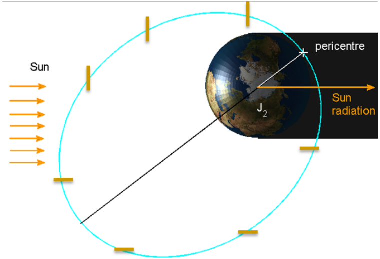
Introduction: Solar sailing

Solar sails are a type of low-thrust propulsion system that takes advantage of the Solar Radiation Pressure (SRP) to accelerate a probe with the aid of a highly reflecting surface.

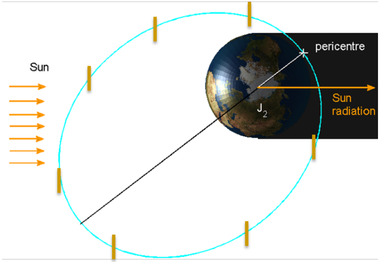
IKAROS (JAXA)	**NanoSail-D2** (NASA)	**LightSail** (Plan Soc)
		
315 kg, 196 m^2	4 kg, 10 m^2	3 kg, 32 m^2

Introduction: SRP and drag acceleration

- There is vast literature on how to use SRP and drag for misson design. The techniques rely strongly on active attitude control either for fixing it or to minimize/maximize SRP effect.
- Examples: Active vs passive deorbiting strategies.



Inwards spiralling
Decrease semi-major axis



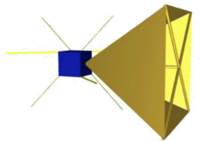
Outwards spiralling
Increse eccentricity

Avoiding attitude control: Auto-stabilizing sail

Q: Can we find a shape of the sail that is auto-stabilizing?

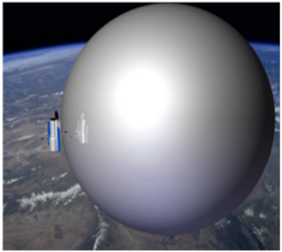
R: Yes!

Quasi-Rhombic Pyramid (QRP) shape
Ceriotti et al. (2014)



STABLE Sun-pointing orientation

Deployable baloon



Goals and assumptions

The Sun-pointing attitude is known to be locally stable, but there is a lack of information on a more global point of view.

GOAL

1. Provide a simplified deterministic model to study attitude stability properties globally, and to be able to predict long-time behaviour.
2. Identify the most relevant physical parameters and perform a sensibility analysis.

ASSUMPTIONS: planar motion

1. Ecliptic obliquity set to 0.
2. Solar radiation pressure constant in a vicinity of the Earth, and its direction is the Sun-Earth vector.

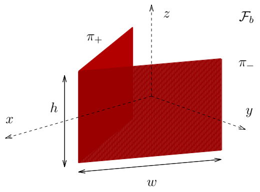
This reduces to a 6D problem: 2D for attitude, 6D for orbit, when adding SRP, J_2 and atmospheric drag accelerations

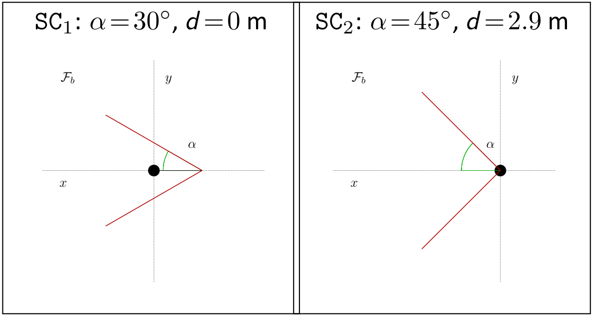
Geometry of the sail structure

Since we restrict ourselves to planar motion, we consider a Sail structure consisting of two panels. The bus is added in the bisecting plane of the two panels. Denote the aperture angle α and the distance between centres of mass of the sail and bus d.

$\mathcal{F}_b$: the body frame. A, B and C inertia moments of the spacecraft.

3D view





Attitude dynamics

The rotation dynamics of the spacecraft is fully explained using a single Euler angle, $\varphi \in [0, 2\pi)$ and the Euler equations reduce to $\ddot{\varphi} = \mathbf{M}_3 / C$, where $\mathbf{M}_3$ are the torques:

1. SRP: Torque due to each panel:

$$\mathbf{M}^{\pm}_{\text{SRP},3} = \frac{A_s}{M} \frac{p_{\text{SR}}}{2} \left(a_{1,1}(\eta)\sigma_1\sigma_2 \pm a_{2,0}(\eta)\sigma_1^2 \pm a_{0,2}(\eta)\sigma_2^2 \right)$$

2. Atmospheric drag: Torque due to each panel:

$$\mathbf{M}^{\pm}_{\text{drag},3} = \frac{A_s}{M} \frac{\rho v_{\text{rel}}^2 C_D}{4} \left(b_{1,1}\nu_1\nu_2 \pm b_{2,0}\nu_1^2 \pm b_{0,2}\nu_2^2 \right),$$

   where $b_{1,1} = a_{1,1}(0)$, $b_{2,0} = a_{2,0}(0)$, and $b_{0,2} = a_{0,2}(0)$.

3. Gravity gradient (GG): Assuming symmetric bus

$$\mathbf{M}_{\text{GG},3} = \frac{3\mu}{r_{\text{E}}^3} (B - A)\, \gamma_1\gamma_2$$

where $\mathbf{u}_S = (\sigma_1, \sigma_2, \sigma_3)^\top$, $\mathbf{u}_E = (\gamma_1, \gamma_2, \gamma_3)^\top$ and $\mathbf{u}_{\text{rel}} = (\nu_1, \nu_2, \nu_3)^\top$ are sunlight, position of the spacecraft and relative velocity wrt atmosphere in $\mathcal{F}_b$.

Attitude dynamics for SRP and GG torques

Attitude dynamics for SRP and GG torques

Consider fixed Keplerian orbit dynamics, circular aparent motion of the Sun, anomaly λ, the adimensionsl attitude dynamics of SRP and gravity-gradient torques in a rotating frame are given by

$$\tilde{\lambda}' = t_\star n_\odot, \qquad \tilde{M} = t_\star n, \qquad \tilde{\varphi}' = \tilde{\Phi},$$

$$\tilde{\Phi}' = \left\{ \begin{array}{lll} \sigma_1\sigma_2 - \frac{a_{2,0}(\eta)}{a_{1,1}(\eta)}\sigma_1^2 - \frac{a_{0,2}(\eta)}{a_{1,1}(\eta)}\sigma_2^2 & \text{if} & \tilde{\varphi} \in (-\pi+\alpha, -\alpha) \\ 2\sigma_1\sigma_2 & \text{if} & \tilde{\varphi} \in (-\alpha, \alpha) \\ \sigma_1\sigma_2 + \frac{a_{2,0}(\eta)}{a_{1,1}(\eta)}\sigma_1^2 + \frac{a_{0,2}(\eta)}{a_{1,1}(\eta)}\sigma_2^2 & \text{if} & \tilde{\varphi} \in (\alpha, \pi-\alpha) \\ 0 & & \text{otherwise} \end{array} \right\} \leftarrow \boxed{\text{SRP}}$$

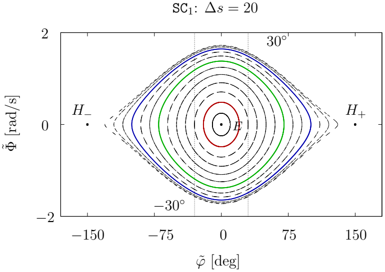
$$+ \quad t_\star^2 \frac{3\mu}{r^3} \frac{D(\alpha, d)}{C} \gamma_1\gamma_2. \quad \leftarrow \boxed{\text{gravity-gradient}}$$

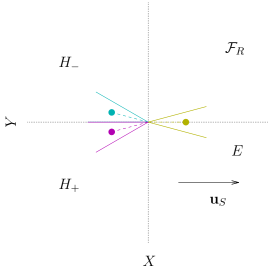
To be understood as

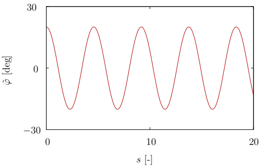
$$\tilde{\Phi}' = \left\{ \begin{array}{ll} \sigma_1\sigma_2 - \frac{a_{2,0}(\eta)}{a_{1,1}(\eta)}\sigma_1^2 - \frac{a_{0,2}(\eta)}{a_{1,1}(\eta)}\sigma_2^2 & \text{if only left panel faces sunlight} \\ 2\sigma_1\sigma_2 & \text{if both panels face sunlight} \\ \sigma_1\sigma_2 + \frac{a_{2,0}(\eta)}{a_{1,1}(\eta)}\sigma_1^2 + \frac{a_{0,2}(\eta)}{a_{1,1}(\eta)}\sigma_2^2 & \text{if only right panel faces sunlight} \end{array} \right\}$$

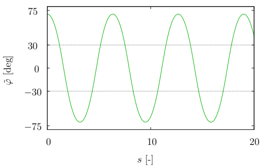
Phase space: unperturbed motion

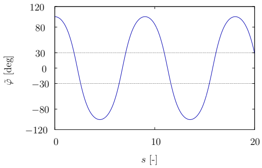
The phase space of the unperturbed motion resembles that of a pendulum. In fact, in $|\tilde{\varphi}| < \alpha$ it is a mathematical pendulum. It has three equilibria: two unstable $H_{\pm}$ and a stable one, E, the sun-pointing direction.









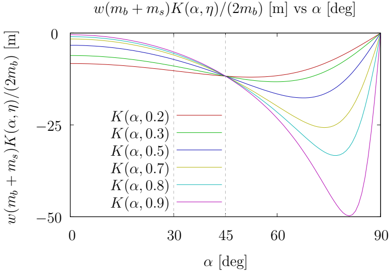


Stability of the Sun-pointing attitude

Necessary stability condition: $a_{1,1}(\eta) > 0$. Conditions on d and α, not on A/M.

Given α aperture angle $\Rightarrow$ constraint on minimum d

$$a_{1,1}(\eta) > 0 \;\Leftrightarrow\; d > \frac{wM}{2mb} K(\alpha,\eta), \quad K(\alpha,\eta) = \frac{\eta\cos(3\alpha) - \cos\alpha}{2\eta\cos(2\alpha) + \eta + 1}.$$



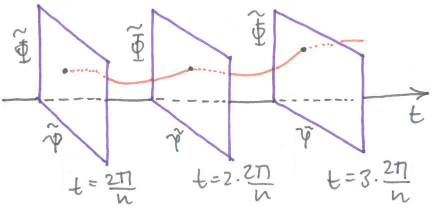
Note that since $\eta \in (0,1)$ and $\alpha \in (0,\pi/2)$, $K(\alpha,\eta) < 0$. Hence the bus can be behind the centre of mass of the sail!.

Phase space: perturbed motion. How to

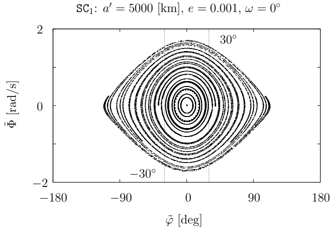
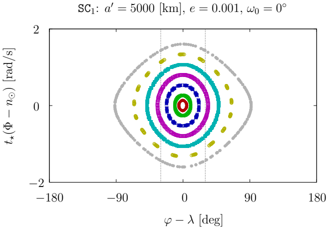
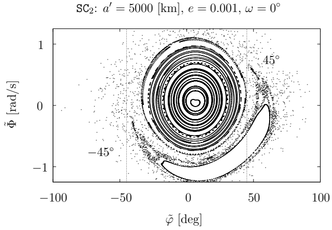
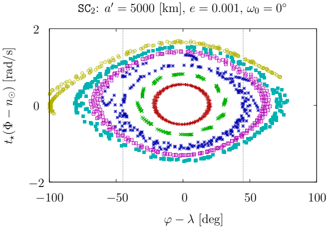
The perturbed motion is a quasi-periodic perturbation: there are two additional frequencies: $n, n_\odot$.

Since $n_\odot \ll n$, one is led to study the dynamics of the *return-to-perigee map* (RTPM): Start motion in the perigee, plot every $f = 2\pi k$, $k \in \mathbb{N}$ or equivalently $t = 2\pi/n \times k$, $k \in \mathbb{N}$.

Some orbits have been propagated starting at the perigee for different initial configurations in $(\tilde{\varphi}, \tilde{\Phi})$. Here we show results for a fixed Keplerian orbit, with altitude 5000 km (low MEO), $e = 0.001$.



Results: deterministic vs complete

	Deterministic model	Complete ORB+ATT
SC_1		
SC_2		

Attitude dynamics for DRAG and GG torques

Attitude dynamics for DRAG and GG torques

The atmospheric drag case plus gravity gradient perturbation can be approached similarly, but now the only-drag attitude dynamics depends on the orbit explicitly via the air density ρ and the relative velocity v_{rel}. Proceeding analogously as for the SRP case the attitude dynamics read:

$$
\begin{aligned}
\hat{\varphi}' &= \hat{\Phi}, \\
\hat{\Phi}' &= \left\{ \begin{array}{ll}
\rho v_{\text{rel}}^2 \left(\nu_1\nu_2 - \frac{b_{2,0}}{b_{1,1}}\nu_1^2 - \frac{b_{0,2}}{b_{1,1}}\nu_2^2\right) & \text{if} \quad \hat{\varphi} \in (-\pi+\alpha, -\alpha) \\
\rho v_{\text{rel}}^2 (2\nu_1\nu_2) & \text{if} \quad \hat{\varphi} \in (-\alpha, \alpha) \\
\rho v_{\text{rel}}^2 \left(\nu_1\nu_2 + \frac{b_{2,0}}{b_{1,1}}\nu_1^2 + \frac{b_{0,2}}{b_{1,1}}\nu_2^2\right) & \text{if} \quad \hat{\varphi} \in (\alpha, \pi-\alpha) \\
0 & \qquad \text{otherwise}
\end{array} \right\} \\
&\quad - t_{\star\star}^2 \frac{d}{dt}\left(\frac{1}{1+e\cos f}\frac{df}{dt}\right) + t_{\star\star}^2 \frac{3\mu}{r^3}\frac{D(\alpha,d)}{C}\gamma_1\gamma_2 .
\end{aligned}
$$

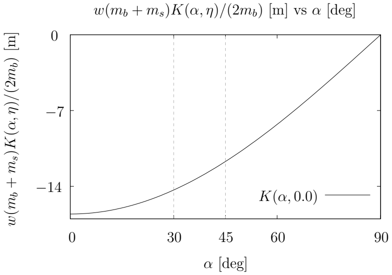
Each line corresponding to only left, both or only right panels facing the atmosphere.

Stability of the velocity-directed attitude

Atmospheric drag attitude has to be studied together with the orbit dynamics, but there is also a necessary stability condition related to that of SRP: $b_{1,1} > 0$. These are conditions on d and α, not on A/M:

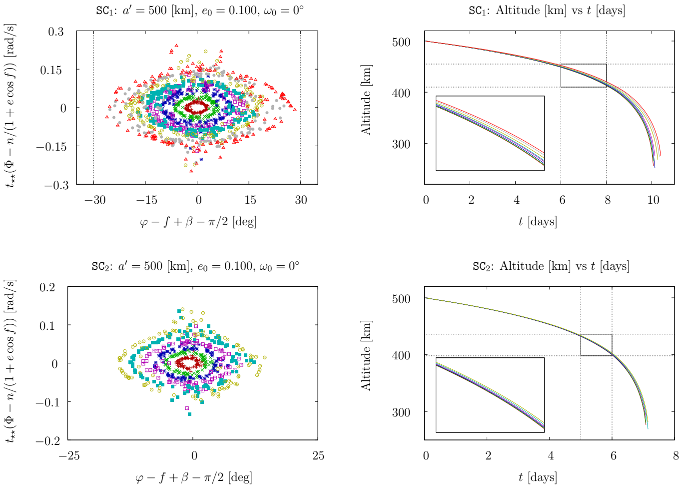
$$b_{1,1} = a_{1,1}(0) > 0 \;\Leftrightarrow\; d > \frac{wM}{2mb} K(\alpha, 0) = -\frac{wM}{2mb} \cos\alpha$$

Again, there are stable positions of the bus behind the centre of mass of the sail!.



Results: complete orbit+attitude

Deorbiting from 500 km of altitude without attitude control.



Oscillations around the sun-pointing attitude

Attitude dynamics, fixed Keplerian orbit, 2 panels

The dynamics of the proposed solar sail when both panels face the sunlight can be written, in adimensional variables (dropping the $\tilde{\cdot}$ notation)

$$\left\{ \begin{array}{rcl} \varphi' & = & \Phi, \\ \Phi' & = & -\sin(2\varphi) + \delta(1 + e\cos f)^3 \sin\left(2(f + \phi - \varphi)\right), \end{array} \right. \qquad |\varphi| < \alpha,$$

where

$$\delta = \frac{P(\alpha, d)}{a^3(1 - e^2)^{3/2}}.$$

Here $P(\alpha, d)$ accounts for the dependencies on physical parameters, and we have used

$$\begin{array}{rcl} 2\sigma_1\sigma_2 & = & -\sin(2\varphi), \\ 2\gamma_1\gamma_2 & = & \sin\left(2(f + \phi - \varphi)\right), \quad \phi = \omega - \lambda. \end{array}$$

The variables a, e, ω, f are Keplerian elements and λ is the anomaly of the apparent motion of the Sun.

Hamiltonian structure

The previous is a quasi-periodically perturbed mathematical pendulum that has the conserved quantity

$$\mathcal{H} = \frac{\Phi^2}{2} - \frac{1}{2}\cos(2\varphi) + \frac{1}{2}\delta(1 + e\cos f)^3 \cos\left(2(f + \phi - \varphi)\right), \qquad |\varphi| < \alpha.$$

From $\mathcal{H}$ the dynamics of the sail is recovered via the usual

$$\Phi' = \frac{\partial \mathcal{H}}{\partial \varphi}, \quad \varphi' = -\frac{\partial \mathcal{H}}{\partial \Phi}.$$

Note that for $\delta = 0$, the system is integrable and the dynamics is librational around the sun-pointing direction and contained in

$$|\varphi| < \alpha, \qquad |\Phi| < \sqrt{1 - \cos(2\alpha)}.$$

Motivation

For the two studied sail configurations the dynamics in $|\varphi| < \alpha$ seems to be close to the integrable case $\delta = 0$. Hence perturbation techniques may be applicable to describe the dynamics close to the sun-pointing direction.

APPLICATION: To be used in a hybrid orbit and attitude propagator, as suggested by Hatten & Russell (2017).

When the attitude dynamics can be considerd a perturbation problem each step of the integrator would consist of

1. A step of the orbit propagation: from $(a, e, f, \omega)_{\text{old}}$ at t to $(a, e, f, \omega)_{\text{new}}$ at $t + h$, h: stepsize.
2. Assuming (a, e, f, ω) fixed, after a triple averaging procedure on $\mathcal{H}$, one obtains the frequency ρ''' of φ'''. Prediction: $\varphi'''_{\text{new}} = \varphi'''_{\text{old}} + h'''\rho'''$[1].

[1] Triple prime: variables after triple averaging

Step -1: Suitable variables for integrable part

The Unperturbed Hamiltonian, $\delta = 0$:

$$\mathcal{H} = \frac{\Phi^2}{2} - \frac{1}{2}\cos(2\varphi) = -\frac{1}{2} + \frac{\Phi^2 + \sqrt{2}^2\varphi^2}{2} + \sum_{k\geq 2}\frac{(-1)^k}{(2k)!}(2\varphi)^{2k}.$$

Introducing Poincaré variables $\Phi = \sqrt{2\omega_0\Theta}\cos\theta, \varphi = \sqrt{2\Theta/\omega_0}\sin\theta,\ \ \omega_0 = \sqrt{2}$ and neglecting $-1/2$, $\mathcal{H}$ reads

$$\mathcal{H} = \sqrt{2}\Theta + \sum_{k\geq 2}\frac{(-1)^k}{(2k)!}(2\Theta)^k \sin^{2k}\theta.$$

This is suitable for treatment as perturbation series: Introduce ``book-keeping" parameter $\sigma = \mathcal{O}(\Theta)$ with numerical value $\sigma = 1$ and rearrange

$$\mathcal{H} = \sum_{k\geq 0}\frac{\sigma^k}{k!}H_k \leftarrow$$

Lie-Deprit method to get rid of dependency on θ up to order N of convenience

Step 0: Arrangement of the Hamiltonian

Study of the full problem

1. Introduce conjugate momenta to τ (time) and λ: T and Λ, where

$$\frac{d\tau}{dt} = \frac{\partial \mathcal{H}}{\partial T} = 1, \qquad \frac{dT}{dt} = -\frac{\partial \mathcal{H}}{\partial \tau} = -\frac{\partial \mathcal{H}}{\partial f}\frac{\partial f}{\partial \tau}.$$

   ``Extended Hamiltonian": $\tilde{\mathcal{H}} = \mathcal{H} + T + n_{\odot}\Lambda$.

2. Expand perturbation term using

$$\cos\left(2(f + \phi - \varphi)\right) = \cos\left(2(f+\phi)\right)\cos(2\varphi) + \sin\left(2(f+\phi)\right)\sin(2\varphi)$$

   and write it in Poincaré variables.

3. Find a suitable arrangement as before,

$$\tilde{\mathcal{H}} = \sum_{k\geq 0} \frac{\sigma^k}{k!}\tilde{H}_k, \quad \sigma = \mathcal{O}(\Theta^?),$$

   for systematic treatment.

Step 0: Arrangement of the Hamiltonian

- After expanding $\sin(2\varphi)$ and $\cos(2\varphi)$ half-integer powers of Θ appear, hence, here $\delta = \mathcal{O}(\Theta^{1/2})$.
- We have freedom to assume $\delta = \mathcal{O}(\Theta^{2\nu})$.

$$
\begin{aligned}
H_0 &= \sqrt{2}\Theta, \quad H_1 = T, \quad H_2 = n_\odot \Lambda - \frac{2}{3}\Theta^2 \sin^4\theta, \quad H_3 = 0, \\
H_l &= K_c \frac{(-1)^{l_1}}{(2l_1)!} 2^{5l_1/2-1}\Theta^{l_1}\sin^{2l_1}\theta + \frac{(-1)^{l_2+1}}{(2l_2)!}2^{5l_2/2-1}\Theta^{l_2}\sin^{2l_2}\theta, \quad \text{even} \quad l \\
H_l &= K_s \frac{(-1)^{l_3}}{(2l_3+1)!}2^{5l_3/2+1/4}\Theta^{l_3+1/2}\sin^{2l_3+1}\theta, \quad \text{odd} \quad l,
\end{aligned}
$$

where

$$
K_{\{c,s\}} = \delta(1+e\cos f)^3\{\cos, \sin\}\left(2(f+\phi)\right),
$$

and

$$
l_1 = \frac{l+2}{2}, \quad l_2 = \frac{l-2\nu}{2}, \quad \text{and} \quad l_3 = \frac{l-1-2\nu}{2}.
$$

Steps 1,2,3: Average wrt θ,τ (or f), λ

Up to order N we average wrt θ, τ (or f), λ in three different applications of the Lie-Deprit method. Note this implies assuming no resonances!. Notation: variables and functions involved, $\cdot' \to \cdot'' \to \cdot'''$.

Some remarks:

- θ After averaging resp. θ all odd terms of the previous arrangement vanish, but K_s does not disappear! For computational reasons: re-arrange to fill blank spots.
- τ Put T at order 0. Recall $df/dt = n(1 + e\cos f)^2/(1-e^2)^{3/2}$. If $e < \sqrt{2} - 1$ it is convenient to use $(1 + e\cos f)^{-2} = \sum_{k\geq 0}(-1)^k(k+1)e^k\cos^k f$. Also to expand $(1-e^2)^{3/2}$ around $e = 0$.
- λ Put $n_\odot \Lambda$ at order 1. Each step then consists of computing

$$H_l''' = \frac{1}{2\pi}\int_0^{2\pi} \tilde{H}_l'' \, d\lambda, \qquad W_{l-1}'' = \frac{1}{n_\odot l}\int \left(\tilde{H}_l'' - H_l'''\right) d\lambda,$$

where $\tilde{H}_l''$ is the lth term in the intermediate Hamiltonian of the procedure.

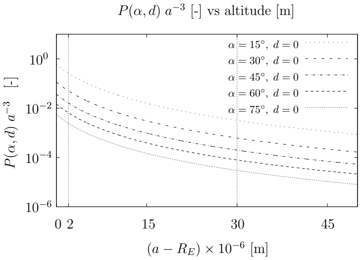
Computational aspects

First tests can be done in `Mathematica`. But since we are dealing with perturbation series, one may have to go to high orders so it is better to have ways to

1. Handle periodic functions: Fourier series symbolic manipulator.
   1.1 Averages are stored as independent term.
   1.2 Derivatives and Integrals consist of shifting and indices and multiplying by scalars.
   1.3 etc.
2. Handle polynomials of n variables. Here $n \geq 4$.

Good, free solution: `PARI/gp` calculator.

A perturbation problem?



Magnitude of δ as a function of altitude

Note that the value of δ has to be small compared to θ so that this procedure is applicable.

1. Advantage: this method provides the averaged frequency ρ as polynomial in Θ''', δ and e $\rightarrow$ Easy to evaluate.
2. Disadvantage: High orders far from sun-pointing direction.

Summary and conclusions

In this work we have considered a simplified QRP sail structure and we have been able to

1. Provide a deterministic model for the attitude dynamics considering SRP + gravity gradient torques.
2. Detect regions of stable motion around the sun-pointing orientation.
3. Validate the results with a complete orbit and attitud propagator taking into account SRP and J_2 accelerations.
4. Translated the study to the drag + gravity-gradient torque attitude dynamics, directly on the orbit+attitude complete model.
5. Provided a perturbation scheme to study the frequency of oscillations close to sun-pointing attitude.

As Future work:

1. Translating this work to the 3D QRP of *Ceriotti et al. (2014)*, maybe assuming spin around some axis of inertia as *Felicetti et al. (2016)*.

Thanks a lot for your attention!

The research leading to these results has received funding from the Horizon 2020 Program of the European Union's Framework Programme for Research and Innovation (H2020-PROTEC-2015) under REA grant agreement number 687500 – ReDSHIFT.