TRAJECTORY DESIGN IN HIGH-FIDELITY MODELS

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ABSTRACT

The tools developed at Politecnico di Milano for high-fidelity trajectory design are presented. These include: ULTIMAT (Ultra Low Thrust Interplanetary Mission Analysis Tool) for design and feasibility assessment of limited control authority missions in highly unstable regions; LT2O (Low-Thrust Trajectory Optimizer) for the indirect optimization of multi-revolution low-thrust transfers in multi-body models; DIRETTO (DIREct collocation Tool for Trajectory Optimization) for the direct optimization of impulsive and low-thrust trajectories; GRATIS (GRAvity TIdal Slide) for the computation and manipulation of stable sets, from which ballistic capture orbits can be derived.

Index Terms— High-fidelity modeling, Trajectory design, Trajectory optimization, Low-thrust transfer, Ballistic capture, Lagrange point orbits, CubeSat, LUMIO

1. INTRODUCTION

The design of space missions is generally driven by severe requirements on the Delta-v budget. Navigation is also becoming more and more challenging, asking for the satisfaction of stringent conditions characterized by unprecedented accuracy. As a consequence, an increased complexity in the trajectory design is needed, ultimately leading to employing high-fidelity models already in the early stages of trajectory design.

Flying in highly nonlinear gravity fields allows exploiting unique features, such as libration point orbits, ballistic captures, and low-energy transfers. These features are achieved by exploiting the sensitivity in initial conditions of highly nonlinear environments, and open up new scenarios for spacecraft characterized by very limited thrust authority.

However, the dynamics being highly sensitive to initial conditions makes the preliminary trajectory design more challenging, and thus dedicated solutions need to be devised. In the reminder of this paper, a short overview of the tools currently being developed at Politecnico di Milano's Department of Aerospace Science and Technology will be given.

2. ULTIMAT

ULTIMAT, short for Ultra Low Thrust Interplanetary Mission Analysis Tool, is an engineering tool initiated under ESA Contract[1], the aim of which is performing impulsive trajectory optimization into highly nonlinear models, where the design is constrained from the very-limited control authority, e.g., tiny Δv budgets for LISA Pathfinder. Moreover, the tool performs a number of hierarchical tasks ranging from the preliminary geometrical checks to detailed navigation analyses.

ULTIMAT is a command-line tool made up of 300 files that add up to $\approx 25,000$ lines of code. ULTIMAT is written in Matlab, it is fully integrated with JPL's SPICE Toolkit[2], and its core propagators have been validated against NASA's GMAT. ULTIMAT is currently developed, updated, and maintained through GIT.

ULTIMAT is composed by two main modules: the *Design module* and the *Assessment module*.

The Design Module implements state-of-the-art trajectory optimization techniques implemented in near real-world models. It is suitable to design trajectories for very-limited control authority spacecraft characterized by severe Δv budgets, where perturbations are exploited, not counteracted. The Design Module allows the user to

- Find periodic or quasi-periodic orbits using a Differential Correction Scheme;
- Perform impulsive trajectory optimization into highly nonlinear models, by using a multiple-burn multiple-shooting strategy;
- Transform an impulsive solution into its finite-burn equivalent, compatible with the engine ultra-low control capabilities;
- Refine the solution for consistency with later stages by tuning the thrust pointing angles;
- Perform Station Keeping Monte-Carlo analysis based on either target points method or the Floquet modes cancellation strategy.

[1]ESA Contract No. 4000118201/16/F/MOS (ITT 8601 - Feasibility of ultra low thrust transfers in L_1, L_2, Sun, Earth & Moon Systems.

[2]SPICE is NASA's Observation Geometry and Information System for Space Science Missions [1, 2]. https://naif.jpl.nasa.gov/naif.

The Assessment Module is aimed at assessing the *flyability* of designed transfers by performing a number of hierarchical tasks ranging from the preliminary geometrical checks to detailed navigation analyses. The Assessment Module allows the user to

- Pre-process the solution by evaluating mission requirements against eclipse windows duration, solar conjunctions, elevation angles, occultations, and linking constraints;
- Compute the visibility windows from a given set of ground stations and simulate the radiometric measurements (range and range rate);
- Perform a design sensitivity analysis against variations of low-thrust maneuver timing, duration, magnitude, and pointing angles;
- Carry out a batch Orbit Determination (OD) and Covariance Analysis (CA) through either extended Kalman filter (EKF) or square-root information filter (SRIF) that include dynamic correlated Gauss-Markov process noise and considered parameters;
- Quantify the Navigation Cost (NC) required to track a given baseline solution through a first-order approach.

ULTIMAT has been used to design and validate the ultra-low thrust transfer from Sun–Earth L_1 to the Sun–Earth gravitational Saddle Point (SP) for the possible mission extension of LISA Pathfinder. Figures 1-13 show an output example for the design and feasibility assessment for the LPF-to-SP transfer. Figure 1 shows the trajectory in a rotating frame. Distances from Earth and from the Moon are shown in Figure 2 and Figure 3, respectively, whereas the declination angle and the Sun-Earth-Spacecraft angle profiles are shown in Figures 4 and 5. The navigation analysis has been carried out assuming range and range-rate measurements from the Cebreros ESA ground station, and the trajectory ground-track is shown in Figure 7. Lisa Pathfinder optimal Saddle Point transfer shows good visibility windows and daily coverage performances respect to Cebreros with at least 8 hours of continuous tracking possibility (see Figure 9. The range and range-rate (Doppler) measurement plan is shown in Figures 10-11 for the entire transfer duration.

An important parameter for the navigation is the achievable knowledge of the spacecraft state at the SP arrival, as it determines if the spacecraft is truly able to reach the SP region with the required confidence level. This is reported in Figure 12 for position and in Figure 13 for velocity. Note how the total accuracy is always around the order of 1 km, due to the effects of process and measurements noises. Moreover, it is worth highlighting that the uncertainties on the thrust magnitude and direction do not produce big changes in the achievable knowledge level, as typically observed during thrust legs, mainly due to their limited duration.

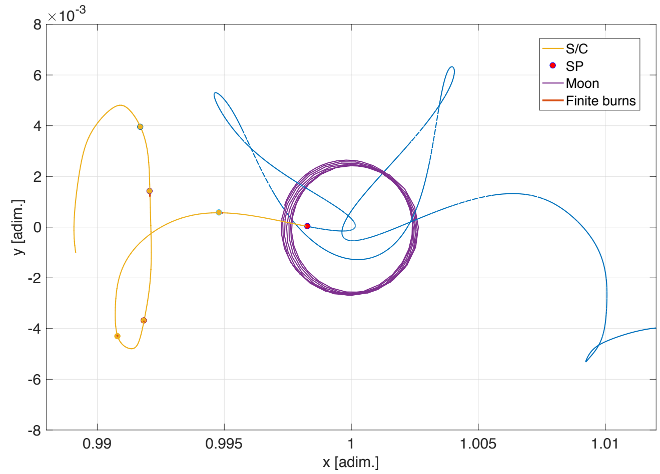


Fig. 1. LPF sample SP transfer. Departing date: 2017 June 21 10:54:42.408 TDB; arrival date: 2018 January 15 10:32:24.516 TDB.

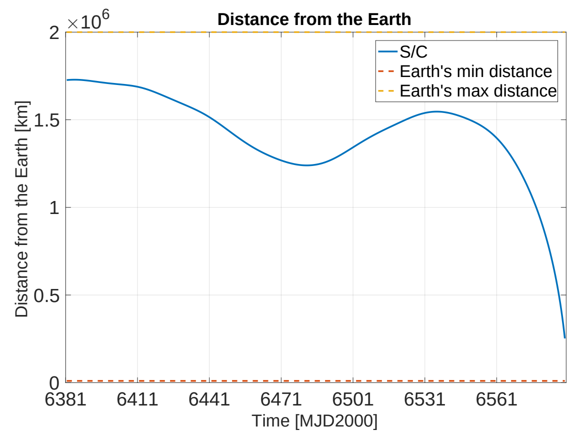


Fig. 2. Distance from Earth of the LPF solution.

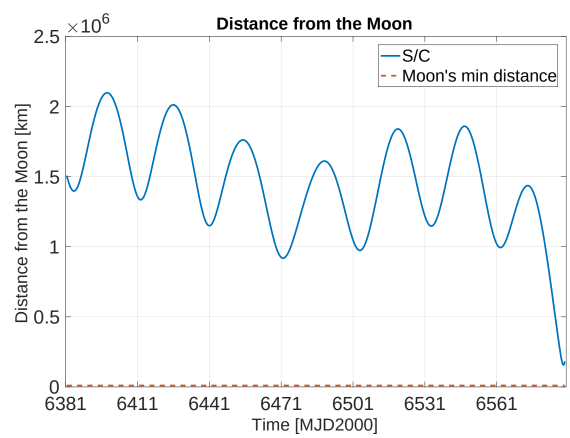


Fig. 3. Distance from Moon of the LPF solution.

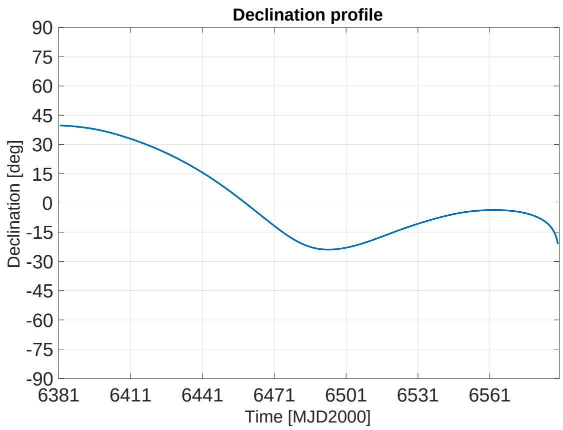


Fig. 4. Declination angle profile of the LPF solution.

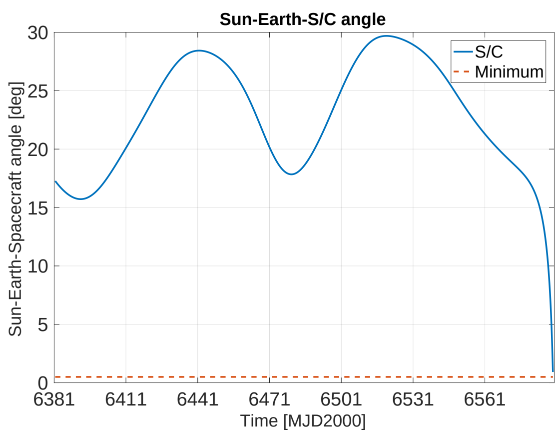


Fig. 5. Sun-Earth-Spacecraft angle profile of the LPF solution.

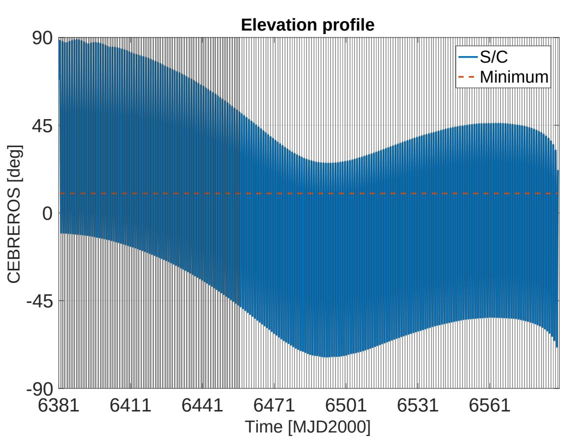


Fig. 6. Elevation angle profile of the LPF solution.

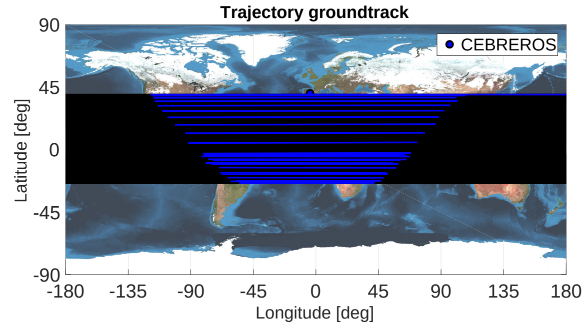


Fig. 7. Trajectory groundtrack of the LPF solution.

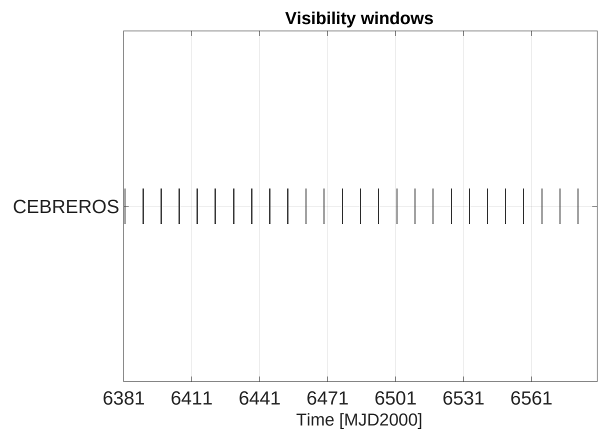


Fig. 8. Visibility windows Gantt chart.

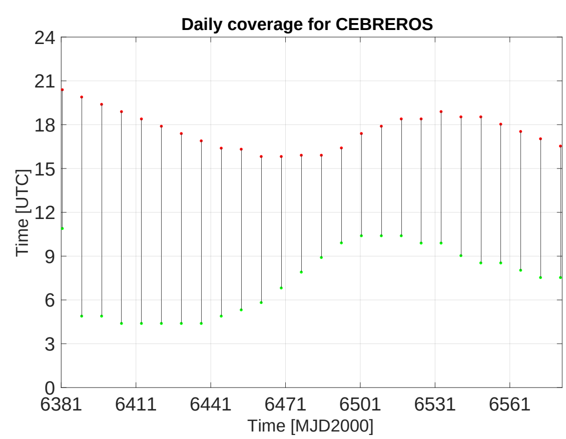


Fig. 9. Daily coverage from Cebreros ground station.

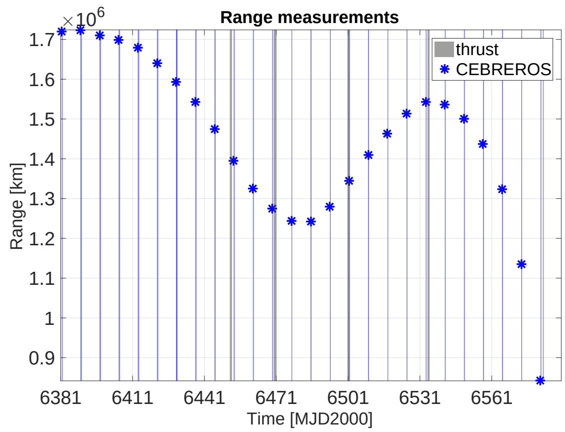


Fig. 10. Range measurements.

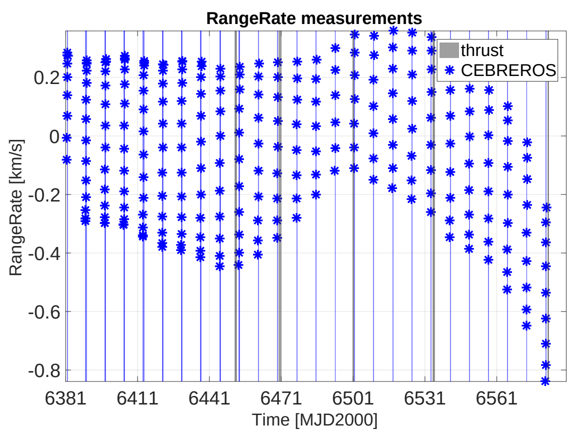


Fig. 11. Doppler measurements.

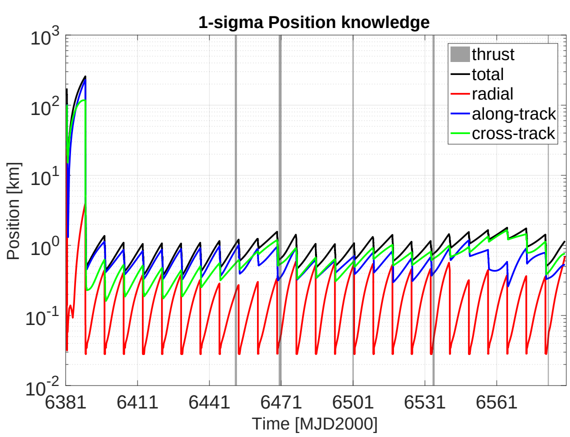


Fig. 12. Achievable position knowledge.

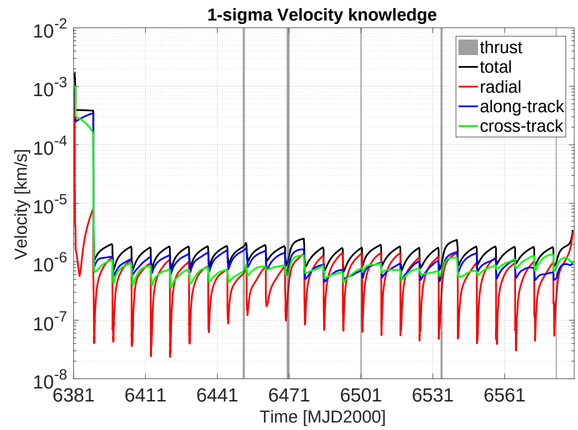


Fig. 13. Achievable velocity knowledge.

For more information on ULTIMAT refer to [3, 4, 12, 13, 14].

3. LT2O

LT2O, or Low-Thrust Trajectory Optimizer, is an internally-funded project to conduct low-thrust trajectory optimization of many-spiral transfers with indirect methods. The tool implements sophisticated hybrid techniques for low-thrust trajectory optimization, e.g., thrust- or orbital parameters-continuation, a smoothing technique, analytical derivatives, and an accurate switching detection technique that allow conducting end-to-end optimization of up to hundreds revolutions transfers. LT20 handles time-, radiation-, energy-, and fuel-optimal problems in a MATLAB-native environment, and it implements the following dynamics models

- Two-body model in Cartesian coordinates (with and without J2 perturbation)
- Two-body model in Modified Equinoctial Elements (MEE)
- Restricted three- and four-body problems in Cartesian coordinates.

In LT20, the optimal low-thrust transfer problem is transformed to a two-point boundary value problem (TPBVP) by the well-known Pontryagin's minimum principle, and the resulted trajectory optimization is solved by shooting methods. These attempts to identify a suitable initial condition to satisfy the TPBVP. Furthermore, the performance of shooting method is enhanced by using 1) analytic derivatives, 2) thrust magnitude continuation, 3) energy-to-fuel homotopy, 4) orbital parameters continuation, and 5) switching detection technique. By exploiting LT2O, long-lasting, multi-spiral trajectories can be computed, and bang-bang optimal control

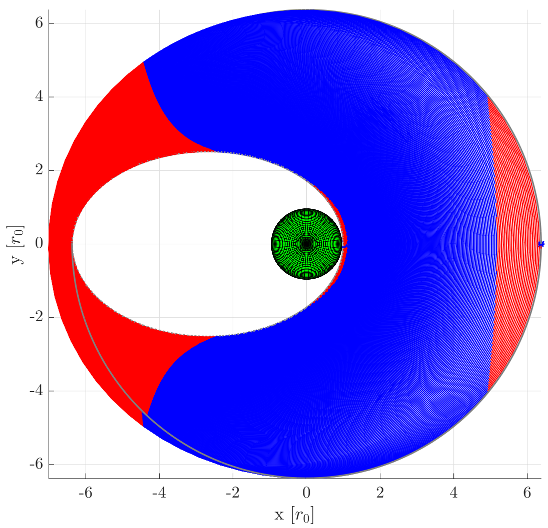


Fig. 14. GTO to GEO fuel-optimal transfer, $T/m = 2.5 \times 10^{-4} m/s^2$, $I_{sp} = 3000$ s and transfer time ≈ 150 days.

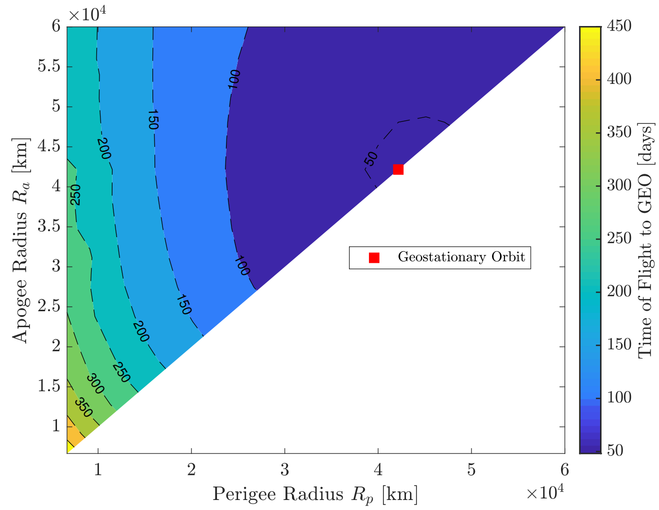


Fig. 15. Low-thrust transfer to GEO, $T/m_0 = 1 \times 10^{-4}\ m/s^2$ and I_{sp} = 2000 s.

profile might characterize the solution, which are not likely to be obtained by those direct or indirect methods that lack accurate techniques, such as analytic derivatives and continuation. LT2O has been successfully used to solving the following challenging design problems

1. Low-thrust fuel- and time-optimal transfers to the Geostationary orbit from variable injection orbits, see Figures 14–15;
2. End-of-life disposal maneuvers for next generation Galileo satellites, see Figure 16;
3. Transfers from GTO to Earth-Moon L1 halo orbits with 500 revolutions, see Figure 17.
4. Long-lasting low-thrust fuel-optimal transfer from the Earth to NEO, see Figure 18.

For more information on LT2O refer to [18, 16, 5].

4. DIRETTO

DIRETTO (DIREct collocation tool for Trajectory Optimization) tool uses direct transcription method to solve the optimal control problem in which state and control variables are discretized and the optimal control problem is converted to a non-linear programming (NLP) problem.

The dynamics of the general optimal trajectory design problem are let to incorporate a number of constant parameters as well as the initial and final conditions that are prescribed within lower and upper bounds. Additionally, the

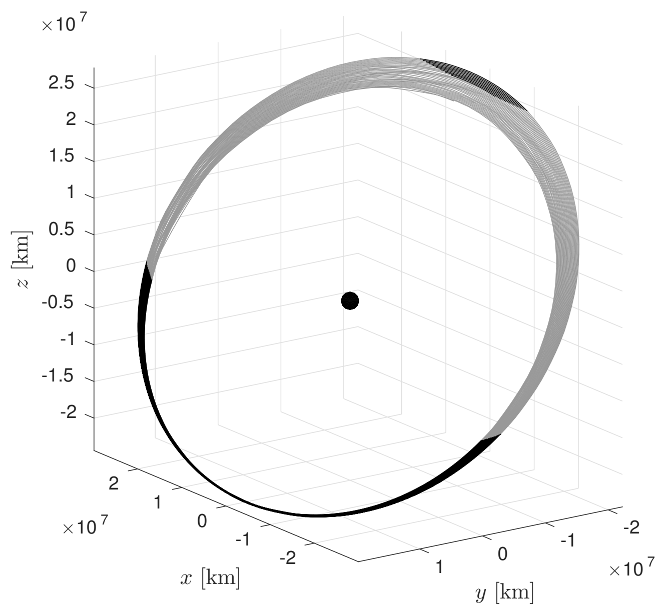
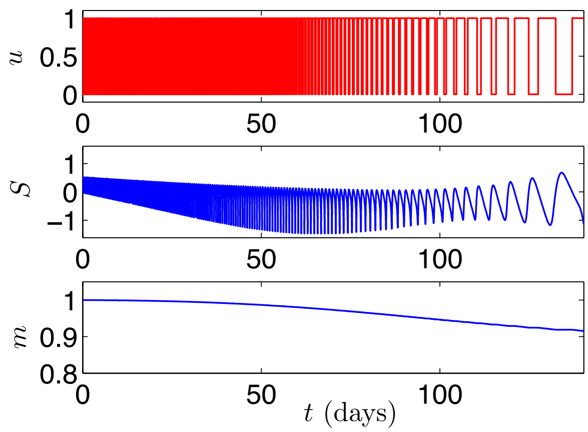
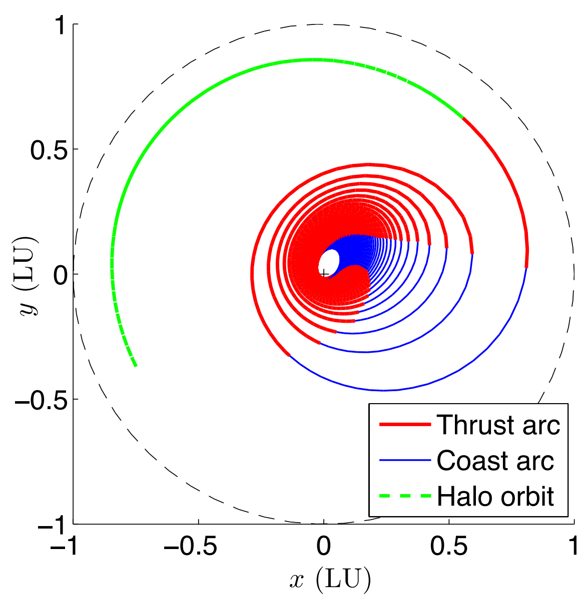


Fig. 16. End-of-Life disposal of a Galileo satellite via a fuel-optimal strategy, $T/m_0 = 2.2 \times 10^{-4} m/s^2$, specific impulse of 4000 s, and transfer time ≈ 19 days.

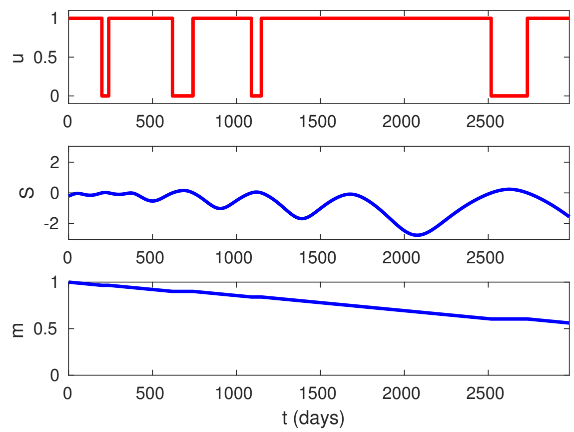


(a) Throttle factor, switching function, and mass.

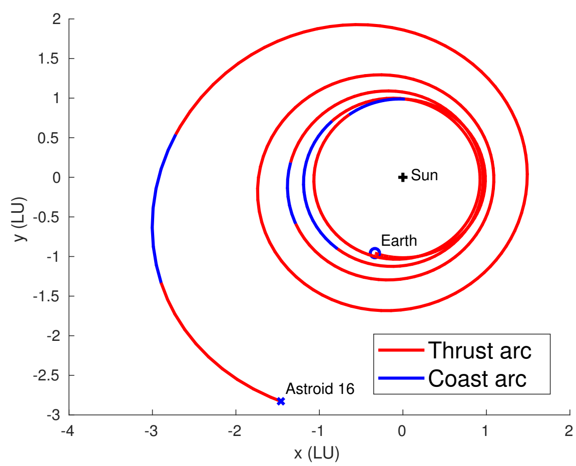


(b) Earth-centred orbit.

Fig. 17. GTO-L1 halo low-thrust fuel-optimal transfer with 150 switches, 500 revolutions, $T/m_0 = 4 \times 10^{-4} m/s^2$, $I_{sp} = 3000$s; cartesian coordinates are used.



(a) Throttle factor, switching function, and mass.



(b) Sun-centred orbit.

Fig. 18. Long-lasting low-thrust fuel-optimal transfer from the Earth to the Asteroid 16, $T/m_0 = 8.7 \times 10^{-3} m/s^2$, $I_{sp} = 2640$ s, launch date: 2020/06/01, arrival date: 2028/08/01; Cartesian coordinates are used.

solution of the optimal trajectory problem can be subject to certain path constraints. The fundamental problem is to determine the control vectors such that a performance index, such as consumed fuel or time of flight, is minimized. Non-linear programming problem is a decisional problem concerning a scalar objective function and a vector of constraints. Contrary to the the optimal control problem, there are no dynamics involved in an NLP problem.

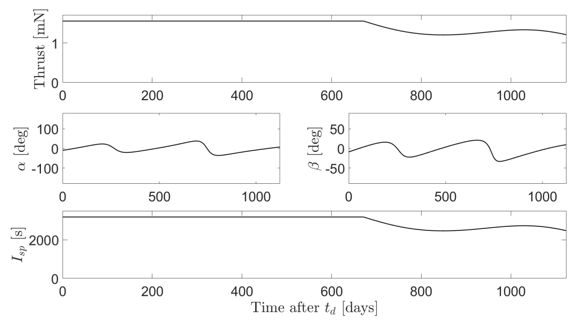
In the direct approach, the solution to the optimal control problem is connected strongly to the numerical integration of the differential equations. The dynamics are handled in such a way that the equations of motion of the spacecraft are transcribed into a finite set of equality constraints. Depending upon the numerical scheme, the optimal control problem can be solved within the degree of accuracy of that scheme. The time domain is discretized into a set of nodes and the discretization step size can be uniform or non-uniform. The discretized states and control variables are then treated as a set of NLP variables. The differential equations are replaced by a finite set of defect constraints derived by the numerical integration scheme (collocation method). Gradients of the objective function and the constraints are assembled and then supplied to an NLP solver.

Collocation is used to transcribe the differential dynamic constraints into a set of algebraic constraints. Polynomials upto a certain degree with a number of points in the time domain (collocation points) are chosen and the they are enforced to satisfy the equations of motion at those collocation points. DIRETTO implements three different collocation methods, which vary in the way the state and control variables are discretized and how the dynamic constraints are satisfied. They are Hermite-Simpson (low-order), Gauss-Lobatto (variable & high order), and Pseudospectral methods.

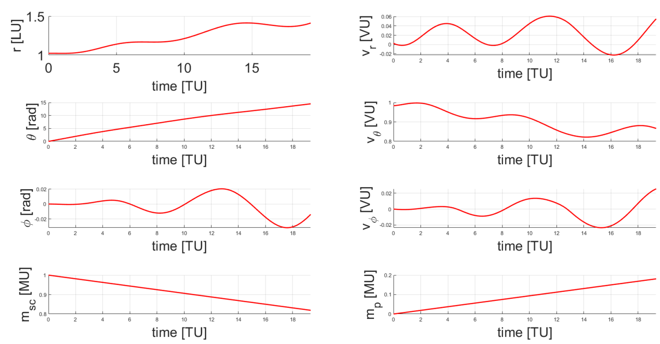
Since solving an optimal trajectory design problem with a direct method could yield a large NLP problem, measures pertaining to scaling (adimensionalization of distance, time, and velocity units), sparse matrix techniques, and differentiation for fast optimization are implemented in DIRETTO to avoid computational issues.

DIRETTO has been successfully used in solving low-thrust Earth–Mars transfers with ballistic capture for interplanetary CubeSats. The solution incorporates variable thrust and specific impulse with the Sun-spacecraft distance and low-thrust engine models. Some results retrieved using DIRETTO are illustrated in Figures 19–20. The context of the problem is a 16U interplanetary CubeSat that escapes from Earth using chemical propulsion and then performs a low-thrust deep-space cruise to Mars after escape. The optimization involves the heliocentric transfer and ballistic capture point targeting.

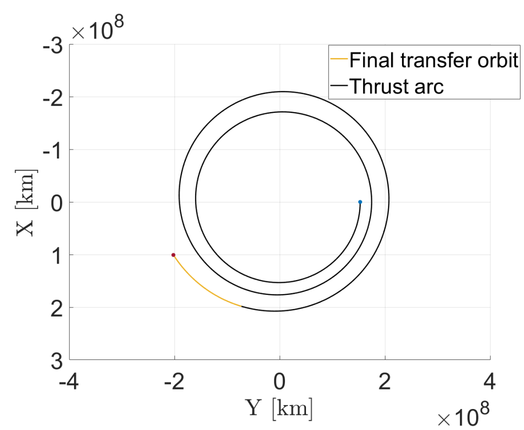
For more information on DIRETTO refer to [17, 15].



(a) Thrust and Specific Impulse

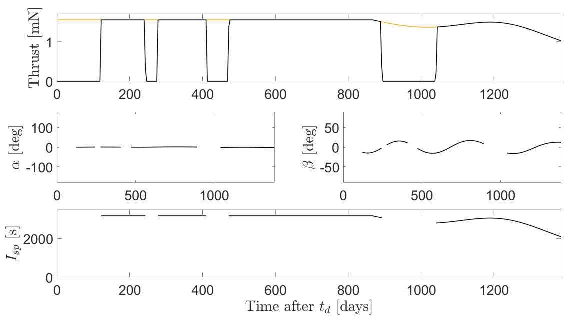


(b) States and Controls

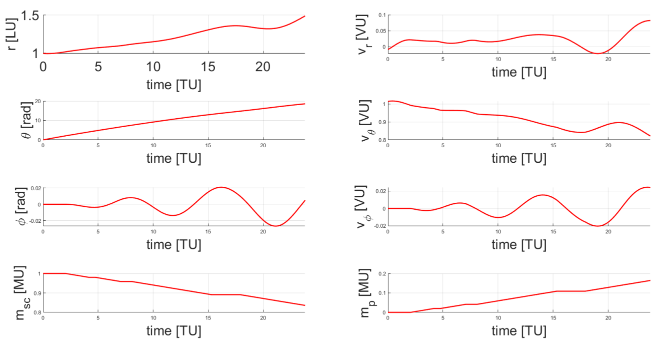


(c) Trajectory

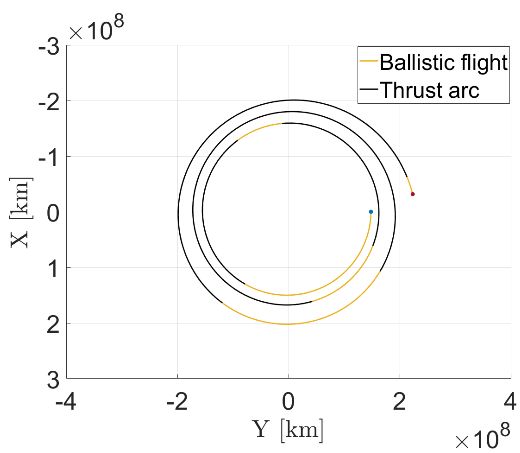
Fig. 19. Variations of thrust, specific impulse, states, control vectors, and heliocentric trajectory with ballistic capture for a time-optimal solution



(a) Thrust and Specific Impulse



(b) States and Controls



(c) Trajectory

Fig. 20. Variations of thrust, specific impulse, states, control vectors, and heliocentric trajectory with ballistic capture for a fuel-optimal solution.

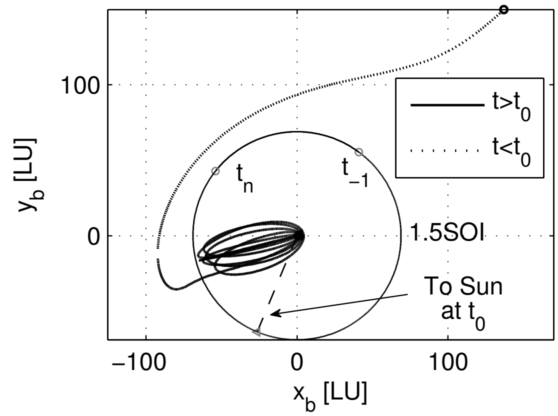


Fig. 21. A sample ballistic capture orbit at Mercury.

5. GRATIS

GRATIS, short for GRAvity TIdal Slide, is an internally-developed software tool for the computation, manipulation and extraction of the stable sets associated to the algorithmic definition of ballistic capture. In essence, GRATIS is used to design the final branch of a possible transfer culminating in ballistic capture about a celestial body.

Ballistic capture is a phenomenon by which a spacecraft can both approach a celestial body and start revolving around it, without needing to manoeuvre in between. The mechanism being highly sensitive, a high-fidelity model has to be implemented. The latest version of GRATIS feature a Restricted n-Body Problem (RnBP) where all the planets and their satellite systems are accounted for. In addition, Solar Radiation Pressure (SRP) as well as the central body non-spherical gravity are also included in the model.

Figure 21 shows an example of what GRATIS can do: candidate ballistic capture orbits are generated by manipulating, i.e, intersecting, a number of stable sets generated by grid sampling and integration in the real system. Trajectory are labelled according to their forward and backward behavior and stored for later use. Depending on the prescription of stability number (number of revolutions in the post-capture phase), orbits are retrieved and analyzed. Only those that pass a filter (geometric and energetic) are reported for later use. These can be, e.g., targeted by using a low-thrust trajectory obtained either by DIRETTO or LT2O.

For more information on GRATIS refer to [11, 6, 7, 10, 8, 9]

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