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Evaluation of Damping Loss Factor of Flat Laminates by Sound Transmission

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Abstract

A novel approach to investigate and evaluate the damping loss factor of a planar multilayered structure is presented. A statistical analysis reveals the connection between the damping properties of the structure and the transmission of sound through the thickness of its laterally infinite counterpart. The obtained expression for the panel loss factor involves all the derivatives of the transmission and reflection coefficients of the layered structure with respect each layer damping. The properties of the fluid for which the sound transmission is evaluated are chosen to fulfil the hypotheses on the basis of the statistical formulation. A transfer matrix approach is used to compute the required transmission and reflection coefficients, making it possible to deal with structures having arbitrary stratifications of different layers and also granting high efficiency in a wide frequency range. Comparison with alternative formulations and measurements demonstrates the effectiveness of the proposed methodology.

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1. Introduction

Passive damping treatments are widely used in engineering applications to reduce noise radiation, the amplitude of vibrations and the risk of fatigue failure. In particular, viscoelastic laminates have found application in many areas of structural acoustics due to the high damping levels that can be attained when the cross-sectional properties of the laminate are appropriately chosen. A key requirement for determining the optimal cross-sectional properties of a given laminate is an accurate model of its dynamics.

Typically, at low frequencies, a finite element (FE) model provides a good description of the structural-acoustic behavior of the laminate. A Modal Strain Energy (MSE) analysis on the FE model can provide the loss factor of the structure in terms of the strain energy field of each mode [1]. At higher frequencies, the wavelengths of interest become small with respect to the lateral dimensions of the laminate and then the FE approach becomes impractical. Indeed, Statistical Energy Analysis (SEA) [2] is a more suitable method for estimating the high-frequency responses of a structure under acoustic or mechanical excitation. In order to model a subsystem in SEA, it is necessary to determine the dispersion properties and the Damping Loss Factor (DLF) of each propagating wave type of the subsystem. An approach for evaluating the DLF of a structure is to simplify a real world component down to an equivalent 3-layer beam or plate system. This was first suggested by Ross, Kerwin, Ungar (RKU) [3, 4, 5], involving a fourth order differential equation for a uniform beam under free-wave propagation with the sandwich construction of the 3-layer laminate system represented as an equivalent, frequency dependent, complex stiffness. Several authors have described extensions to RKU analysis by involving different displacement fields to characterize the response of more general laminates [6, 7]. Typically, the assumption of a low-order displacement field is required in order to reduce analytical complexity. While simplified analytical models can provide physical insights into the behavior of certain laminates, the assumed displacement fields can often restrict the types of laminates that can be modeled. Numerical methods to investigate the damping of laminated panels have been developed by several authors [8, 9, 10, 11, 12, 13]. By exploiting a plane wave expansion, the power dissipated by an isotropic poroelastic media within semi-infinite multilayered systems under arbitrary excitation has also been assessed [14]. The loss factor of more general laminates can be explored by involving a one-dimensional FE mesh to describe the cross sectional deformation of a linear viscoelastic laminate, also including a three-dimensional displacement field within the laminate [15]. However, the model is computationally expensive due to the inversion of large matrices as a result of an increasing number of elements in the cross sectional thickness. Regardless of the model adopted to describe the cross sectional deformation, a dispersion problem must be solved by determining, at a specific frequency, ω, and for a specific direction of propagation, a finite number of complex wavenumbers, k, related to the free waves traveling in the structure. The solution of the dispersion problem at discrete frequencies for a specific direction of propagation leads to a $k-\omega$ dispersion diagram where dispersion curves must be identified. Then, the damping loss factor, η_i, for the i-th curve can be evaluated by means of the related eigenvectors. However, the number of curves and their intersections rapidly grow with the frequency, making it more difficult to identify the curves. An alternative approach is to use an exact description of the through-thickness deformation of a laminate by means of a Transfer Matrix Method (TMM) [16]. The characteristic equation that describes free-wave propagation in a laminate can take the form of a nonlinear transcendental eigenvalue problem [17]. However, the computational burden and robustness of the root-tracking algorithms employed to determine dispersion solutions limit the usefulness of the approach.

The scope of this work consists in defining the DLF of a planar structure, averaged among all dispersion curves, by avoiding both the solution of the dispersion problem and the modal approach. We are avoiding the solution of a dispersion problem because i) identify dispersion curves at high frequency could be prohibitive and ii) take into account the damping of all the propagating waves may be impractical. On the other hand, we are discarding the modal approach because i) it could be computationally prohibitive even at relatively low frequencies and ii) materials characterized by frequency dependent properties cannot be easily taken into account. A theory producing the DLF of a multilayered planar structure and overcoming the limitations of the above discussed approaches is proposed. A statistical analysis reveals the connection between the damping properties of the structure and the transmission of sound through the thickness of its laterally infinite counterpart. The incident diffuse acoustic field prescribed by the statistical approach to evaluate the sound transmission ensures the excitation of all the propagating waves contributing to the damping of the medium, thus providing a mean loss factor for the structure. A TMM is used to evaluate the required transmission and reflection coefficients, making it possible to deal efficiently with structures having generic stratifications, possibly including in-plane periodic layers [18]. The wave approach on the basis of the TMM also avoids the need to set a specific kinematic model for the laminate, thus yielding high accuracy.

The DLF of a multilayered planar structure is derived in Section 2 by means of a statistical analysis on the sound transmission through the thickness of the structure. A number of applications are then discussed and compared with alternative formulations and measurements.

2. Layered Systems

Let us consider a layered structure in which the i-th layer is characterized by hysteretic damping through the loss factor $\eta_i(\omega)$. The time-averaged power dissipated by the i-th layer, Π_i, when the structure is subjected to harmonic excitation at angular frequency ω, can be expressed as [2]

$$\Pi_i = \omega E_i \eta_i \ , \tag{1}$$

where E_i is the time-averaged total energy stored in the layer. The DLF of the layered structure, $\eta_s(\omega)$, concerns the overall time-averaged power dissipated by the structure, Π_{diss}, when a diffuse reverberant field exists within it, and can be expressed as [2]

$$\eta_s(\omega) = \frac{\Pi_{\text{diss}}}{\omega E_s} = \frac{\sum_{i=1}^{N} E_i \eta_i}{\sum_{i=1}^{N} E_i} \ , \tag{2}$$

where the total dissipated power, Π_{diss}, is the sum of the power dissipated by the N layers in the medium, and the total panel energy, E_s, is the sum of the energies in all layers. We propose to derive the total energy stored in each layer of a planar structure, E_i, by means of the transmission and reflection coefficients of the laterally infinite counterpart of the structure. Such a purpose draws legitimacy from the idea that the phenomenon of sound transmission through the thickness of the structure hides and carries the very same information as the dispersion problem for the medium. Such information are exposed by means of a statistical analysis of the sound transmission through the structure. The adopted statistical approach is here reliable at any frequency since an infinite extent is considered for the structure.

2.1. Statistical Approach

Sound transmission through the thickness of a planar structure can be investigated by placing the structure between two rooms. In the context of SEA, two energy paths can be identified between the rooms. The first one links the rooms without involving the resonance of the interposed wall, and depends only on the specific mass of the wall, the so-called non-resonant path. A second path treats the interposed structure as a subsystem, so involving its strain energy, the so-called reverberant path. The non-resonant path is therefore neglected in the following since it is not sensitive to the panel properties we are looking for, *i.e.* the energy field within the panel. The conditions under which such a choice may be effective are investigated afterwards (Section 2.2).

Focusing on the reverberant path, the power balance of a panel perturbed by incident acoustic power, Π_{inc}, can be expressed as

$$\Pi_{\text{tra}}(\omega, \eta_1, \ldots, \eta_N) + \Pi_{\text{ref}}(\omega, \eta_1, \ldots, \eta_N) + \sum_{i=1}^{N} \Pi_i(\omega, E_i, \eta_i) = \Pi_{\text{inc}} \ , \qquad (3)$$

where the transmitted power, Π_{tra}, and the reflected power, Π_{ref}, depend on the damping of all layers. Therefore, the power balance for the panel, Eq. (3), can be written in normalized form:

$$\tau_d(\omega, \eta) + r_d(\omega, \eta) + \frac{\omega}{\Pi_{\text{inc}}} \sum_{i=1}^{N} E_i \eta_i = 1 \ , \qquad (4)$$

where $\tau_d = \Pi_{\text{tra}}/\Pi_{\text{inc}}$ is the power transmission factor, $r_d = \Pi_{\text{ref}}/\Pi_{\text{inc}}$ is the power reflection factor and vector η collects the damping factors. In the following analysis each layer energy, E_i, is evaluated assuming a laminate with null damping. In other words, the dynamics of the structure is evaluated by employing the kinematics related to the undamped counterpart of the structure. This assumption implies that the cross sectional displacement field of a given propagating wave is not significantly sensitive to the damping. It should be noted that this assumption is implicit in previous studies which assume a fixed displacement field for the cross section that is independent of damping, *e.g.* RKU and MSE. Therefore, by linearizing Eq. (4) around the undamped condition, $\eta = \mathbf{0}$, with respect to each layer damping, η_i, and invoking the conservative power balance ($\tau_d|_{\eta=\mathbf{0}} + r_d|_{\eta=\mathbf{0}} = 1$), we obtain the following set of N uncoupled equations:

$$\delta\eta_i \left[\frac{\partial \tau_d}{\partial \eta_i} + \frac{\partial r_d}{\partial \eta_i} + \frac{\omega}{\Pi_{\text{inc}}} \left(E_i + \sum_{j=1}^{N} \frac{\partial E_j}{\partial \eta_i} \eta_j \right) \right]_{\eta=\mathbf{0}} = 0 \ . \qquad (5)$$

Since Eq. (5) has to hold for any arbitrary damping perturbation, $\delta\eta_i$, the desired expression for the energy of the i-th layer of the panel is obtained:

$$E_i = -\frac{\Pi_{\text{inc}}}{\omega}\left(\frac{\partial \tau_d}{\partial \eta_i} + \frac{\partial r_d}{\partial \eta_i}\right)_{\eta=\mathbf{0}} . \tag{6}$$

Finally, the expression for the ensemble average DLF, Eq. (2), becomes

$$\eta_s = \frac{\sum_{i=1}^{N} F_i \eta_i}{\sum_{i=1}^{N} F_i} , \tag{7}$$

where

$$F_i = \left(\frac{\partial \tau_d}{\partial \eta_i} + \frac{\partial r_d}{\partial \eta_i}\right)_{\eta=\mathbf{0}} \tag{8}$$

is the frequency dependent *loss function* for the i-th layer.

2.2. Weak coupling and non-resonant path

The expression for the ensemble average DLF, Eq. (7), is derived under i) the hypothesis of negligibility of the non-resonant path in the power transmission and ii) the SEA hypothesis concerning the weak coupling between subsystems [19] ($\eta_{ij} \ll \min(\eta_i, \eta_j)$). The only way to fulfil these hypotheses is to properly choose the properties of the fluid for which the sound transmission is evaluated. In particular, the non-resonant path in the sound transmission is related to the mass-law contribution, which is predominant below the acoustic coincidence. Moreover, a strong coupling between the two semi-infinite fluids (rooms) is due to coincidence phenomena. As a result, moving the coincidence region to low frequencies, well below the frequency range of interest, ensures both a negligibility of the non-resonant contribution to the sound transmission and a weak coupling between the rooms. To this end, the speed of sound, c, must be small enough to fulfil the above discussed hypotheses at the minimum frequency at which the DLF is desired. Additionally, it can be observed that for a diffuse field at a given frequency, ω, the modulus of the projection of the incident wave on the interface, $k_t = \sqrt{k_x^2 + k_y^2} = \omega \sin(\theta)/c$, where θ defines the wave elevation, spans as $0 \leq k_t < \omega/c$. As a consequence, the speed of sound, c, must be set as small as possible to ensure the excitation of all the propagating waves contributing to the energy field within the medium. Moreover, the limit of the mechanical impedance of a thin plate can be expressed in terms of its mass per unit area, m, and flexural rigidity, B, as [16]

$$\lim_{c\to 0} Z_p = \mathrm{j}\omega \lim_{c\to 0}\left(m - \frac{Bk_t^4}{\omega^2}\right) = -\mathrm{j}\omega^3 B \frac{\sin^4(\theta)}{c^4} \ , \tag{9}$$

where the panel mass and, consequently, the non-resonant contribution disappear. Instead, the choice of the fluid density, ρ, is less critical. In fact, a low speed of sound of the surrounding fluid yields to $Z = \rho c \ll Z_p$, so granting a weak coupling between the structure and the fluid and, consequently, between the rooms, regardless of the chosen density, ρ. The expression for the *loss functions*, Eq. (8), can therefore be modified as

$$F_i(\omega) = \lim_{c\to 0}\left(\frac{\partial \tau_d(\omega,\rho,c)}{\partial \eta_i} + \frac{\partial r_d(\omega,\rho,c)}{\partial \eta_i}\right)_{\eta=\mathbf{0}} \quad \forall \rho \in \mathbb{R}^+ \ , \tag{10}$$

where the limit ensures fulfilment of the hypotheses involved to derive Eq. (7) in the frequency range of interest.

2.3. Evaluation of the transmission and reflection coefficients

The diffuse transmission factor, τ_d, and reflection factor, r_d, can be defined by expressing the diffuse acoustic field in the reverberant room as a combination of plane waves traveling in all the possible directions [16]. At a given frequency, ω, each plane wave impinging upon the flat structure is defined by its amplitude, I, azimuth, α, and elevation, $\pi/2 - \theta$. Both a transmitted wave and a reflected wave therefore propagate from the medium and their amplitudes, T and R, depend on the properties of the barrier. Assuming a complete ($0 \leq \theta \leq \pi/2$, $0 \leq \alpha < 2\pi$) and unitary ($I = 1 \ \forall \ \theta, \alpha$) diffuse field, the classical expressions for the power transmission and reflection factors [16] can be simplified as

$$\tau_d(\omega) = \frac{1}{\pi} \int_0^{2\pi} \int_0^{\pi/2} |T(\omega, \theta, \alpha)|^2 \cos(\theta) \sin(\theta) \mathrm{d}\theta \mathrm{d}\alpha \ , \tag{11}$$

and

$$r_d(\omega) = \frac{1}{\pi} \int_0^{2\pi} \int_0^{\pi/2} |R(\omega, \theta, \alpha)|^2 \cos(\theta) \sin(\theta) \mathrm{d}\theta \mathrm{d}\alpha \ . \tag{12}$$

A practical and efficient tool for evaluating the transmission and reflection coefficients, T and R, of planar, stratified media is the TMM. This approach easily allows for multilayers made from a combination of elastic, porous and fluid layers. It assumes the multilayer of infinite extent and uses a representation of plane wave propagation in different media in terms of transfer matrices. The transfer matrix of a layered medium is obtained from the transfer matrices of individual layers by imposing continuity conditions at interfaces. Enforcing the impedance condition of the surrounding fluid, at both the excitation and the termination sides, allows calculation of the transmission coefficient, T, and the reflection coefficient, R. This methodology is explained in detail in chapter 11 of Ref. [16]. In the frame of linear vibroacoustics, the wave approach on the basis of the TMM provides accuracy and efficiency in defining the sound transmission through planar structures with infinite extent, flat interfaces and homogeneous layers. However, the last two limitations can be overcome by involving a FE model for the periodic unit cell of each heterogeneous layer [18].

Even though the TMM is exact from a mathematical point of view, some researchers found divergences in its results for high frequencies and/or large layer thicknesses. The reason of this divergence has been ascribed to a bad numerical evaluation of the involved exponential terms because of the finite arithmetic. An alternative approach to determine the acoustic reflection and transmission coefficients of multilayered panels which avoids exponential terms is proposed in Ref. [20]. However, since no numerical issues were found for the treated laminates in the frequency range explored, the standard TMM [16] was used in the present work.

3. Validation Examples

The derivatives of the transmission and reflection coefficients required to compute *loss functions*, F_i, are evaluated by means of finite differences. A perturbing damping factor of 10^{-6} usually ensures satisfactory precision and avoids numerical issues. As prescribed by Eq. (10), the speed of sound of the fluid is reduced starting from a guess, c_0, until every *loss function*, F_i, converges in the whole frequency range explored. A fluid density $\rho = 1.225\ \mathrm{kgm}^{-3}$ is used for all applications. At fixed fluid conditions and for each frequency of interest, ω_j, $N+1$ evaluations of the transmission and reflection coefficients are needed to evaluate all the *loss functions*, $F_i(\omega_j, \rho, c)$, where N is the number of layers. In case of structures characterized by symmetric stacking, the number of required analyses can be reduced by exploiting the symmetry of the sound transmission ($F_i = F_{N+1-i}$).

3.1. Damping of a sandwich panel with soft core

The first application involves a sandwich panel with a 1 mm-thick soft core ($\rho = 1425$ kgm^{-3}, $E = 4.186$ MPa, $\nu = 0.495$) and aluminum ($\rho = 2700$ kgm^{-3}, $E = 71$ GPa, $\nu = 0.3$) 1 mm-thick skins. The configuration is typical of the application of viscoelastic materials with constraint layer and enables comparison of the present theory with the RKU method [3] for the evaluation of damping of a three-layer structure. In the RKU method, the contribution made by core damping to the total damping of the structure can be evaluated by setting a unitary core damping and null skin damping. Figures 1 and 2 show the effects of the sound speed, c, and the damping perturbation employed for the finite differences, $\delta\eta$, on the core *loss function*, F_{core}, scaled with respect to $F_s = 2F_{\text{skin}} + F_{\text{core}}$. A good degree of agreement can be observed, in the frequency range explored, between the estimation acquired from the RKU method and the result obtained by means of the proposed methodology with $c = 25$ ms^{-1} and $\delta\eta = 10^{-6}$. Higher values of sound speed prevent convergence at low frequencies, and a damping perturbation lower than 10^{-6} can imply numerical issues, especially at low frequencies. The value of sound speed which grants the convergence of the DLF at a specific frequency depends on the stacking properties of the laminate since it is related to coincidence phenomena. On the contrary, the discussion about the damping perturbation, $\delta\eta$, has general validity. Therefore, all the subsequent applications employ a damping perturbation $\delta\eta = 10^{-6}$.

3.2. Damping of a sandwich panel with honeycomb core

The second application involves a sandwich panel [10] made of aluminum ($\rho = 2700$ kgm^{-3}, $E = 71$ GPa, $\nu = 0.3296$) with isotropic 0.6 mm-thick

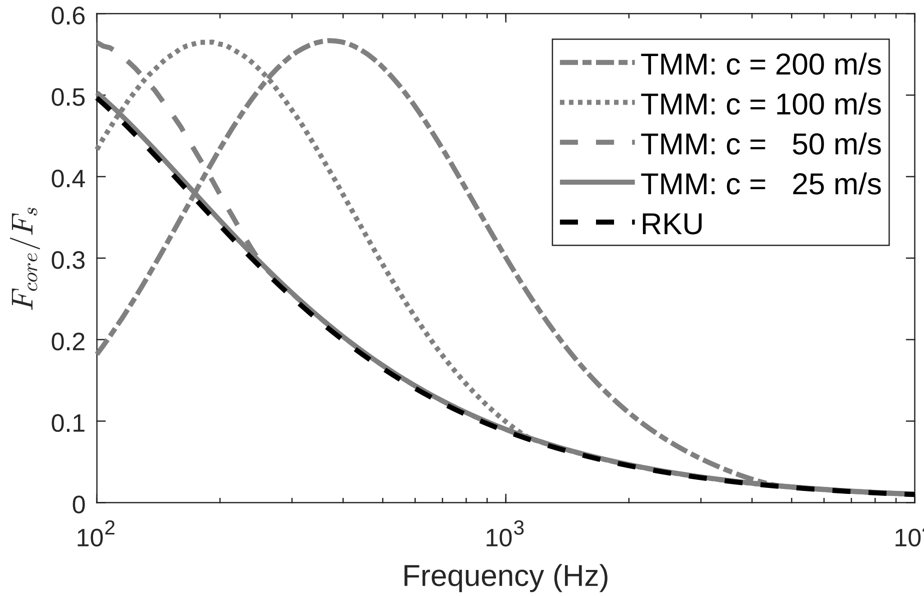

Figure 1: Core contribution to the damping of a sandwich panel ($\delta\eta = 10^{-6}$)

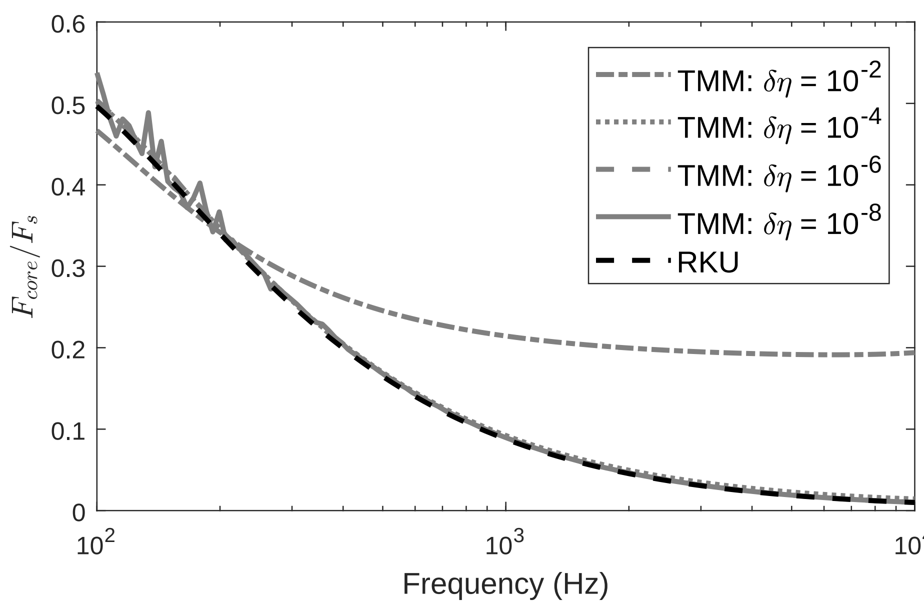

Figure 2: Core contribution to the damping of a sandwich panel ($c = 25$ ms^{-1})

skins and a 15 mm-thick honeycomb core made of hexagonal cells with a foil thickness of 0.0508 mm and a side length of 5.5 mm. The equivalent material properties for the core are obtained by means of a homogenization technique [21]. The DLF of the panel is computed according to Eq. (7) for a particular distribution of damping through thickness. As proposed by Cotoni et al [10], the internal damping loss factor of the core is kept constant at $\eta_{\text{core}} = 2\%$ while the damping of the skins takes on the values $\eta_{\text{skins}} = 1\%, 3\%, 5\%$. The predicted loss factors are shown in Figure 3 as functions of frequency. A speed of sound $c = 40$ ms^{-1} grants the convergence of the DLF in the whole frequency range explored. The results according to Nilsson [22] are plotted as a reference. They were obtained by substituting the undamped wavenumber into the expression of the strain energy and taking the ratio of the imaginary part over the real part. It can be seen that the damping loss factor depends on which part of the composite undergoes the most deformation. At low frequencies, the wave motion is essentially governed by the extensional motion of the skins, and the resulting loss factor is close to the skin loss factor. At high frequencies, the shear of the core governs wave motion, and the damping loss factor gets close to that of the core. This behavior is ruled by the *loss functions* F_{skin} and F_{core}.

3.3. Damping of laminates with multiple viscoelastic inclusions

The last application involves some of the specimens tested in [23]. The laminates considered are made of 0.5 mm-thick aluminum plates ($\rho = 2780$ kgm^{-3}, $E = 73.1$ GPa, $\nu = 0.33$) separated by 0.31 mm-thick foils made of styrene butadiene rubber ($\rho = 1450$ kgm^{-3}, $\nu = 0.49$). The identification of the viscoelastic properties of the rubber leads, in the frequency range

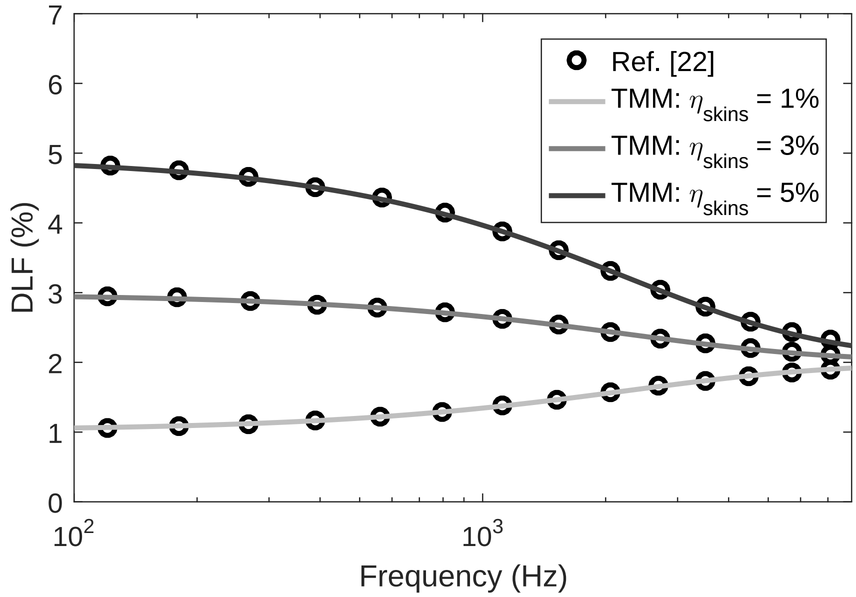

Figure 3: Loss factor of a sandwich panel with honeycomb core ($\eta_{\text{core}} = 2\%$)

100-2500 Hz, to the following approximations for the real part of the shear modulus

$$\Re(G) = [2.1282 \log(f) - 5.5217] \text{ MPa} , \tag{13}$$

and damping

$$\eta = [1.8487 \log(f) - 5.1500] \ \% . \tag{14}$$

The DLF measured for three different batches (#15, #9 and #10 with 2, 3 and 5 viscoelastic inclusions respectively [23]) are shown in Figures 4, 5 and 6 along with the results obtained with a *General Laminate* model [15], implemented in the ESI VAOne code to predict subsystem properties in the frame of an SEA, the results obtained in terms of MSE [1], and the results obtained with the proposed methodology (TMM). Comparisons are satisfactory among all methods, thus proving that boundary effects are negligible.

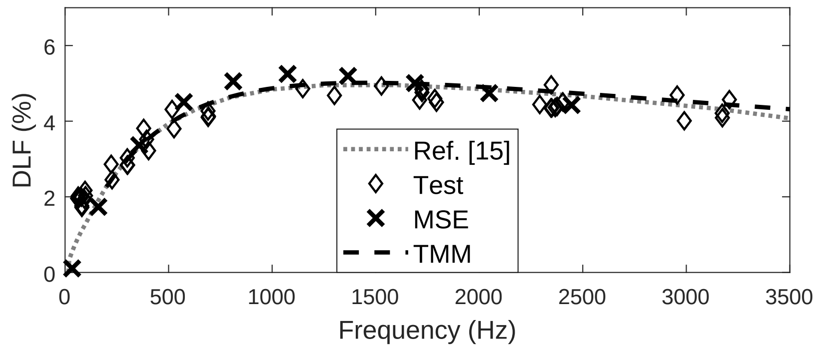

Figure 4: Damping of a laminate with 2 viscoelastic inclusions

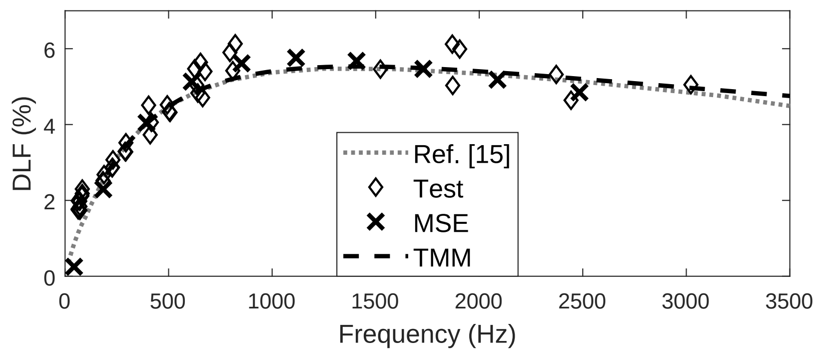

Figure 5: Damping of a laminates with 3 viscoelastic inclusions

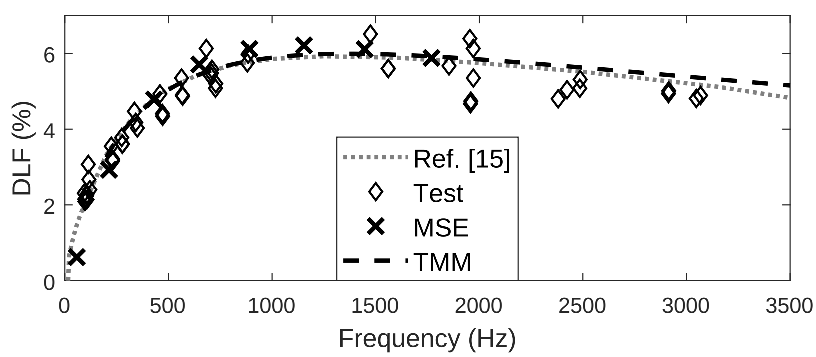

Figure 6: Damping of a laminates with 5 viscoelastic inclusions

4. Conclusions

A connection is identified between the sound transmission through the thickness of a planar layered structure and its DLF. Complex dynamics involved in dissipative mechanisms are assessed by means of a statistical analysis of the sound transmission. The exposed theory reveals the influence of each layer on the ensemble average loss factor of a structure. A *loss function* in the frequency domain is assigned to each layer, making it possible to build the DLF of the whole structure once individual damping properties are assigned to each layer.

Good agreement with respect to the RKU method was observed for a three-layered structure in terms of the influence of the core damping on the global damping of the structure. The effects of the speed of sound of the fluid for which the sound transmission is evaluated and of the damping perturbation employed to evaluate the finite differences have also been addressed. Results on a sandwich panel with honeycomb core highlight the role of *loss functions* in defining the ensemble average loss factor of a layered structure. The comparison with the DLF measured for some laminates with multiple viscoelastic inclusions demonstrates the effectiveness of the proposed methodology even at low-medium frequencies in the case of complex layouts.

Ultimately, the proposed methodology may represent a reliable tool for investigating the DLF of a layered structure. In particular, the so-called *loss functions* may guide an optimization process for the stacking of a layered panel, *e.g.* when the optimal location of a damping material must be determined. Moreover, the transfer matrix approach adopted for evaluating the required transmission and reflection coefficients provides efficiency and accuracy.

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