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Title:

Simplified measurement technique for rigid-body deformations of two masonry blocks

Authors & affiliations:

G. Stockdale[1], V. Sarhosis[2], G. Milani[1]*
[1]Dept. of Architecture, Built Environment and Construction Engineering, Politecnico di Milano, Piazza Leonardo da Vinci 32, 20133 Italy
[2] School of Engineering and Geosciences, Newcastle University, Newcastle upon Tyne, NE1 7RU, UK
[*] Corresponding author: gabriellee.stockdale@polimi.it

Abstract: (Your abstract must use **Normal style** and must fit in this box. Your abstract should be no longer than 1200 words. The box will 'expand' over 2 pages as you add text/diagrams into it.)

Preparation of Your Abstract

1. The title should be as brief as possible but long enough to indicate clearly the nature of the study. Capitalise the first letter of the first word ONLY (place names excluded). No full stop at the end.
2. Abstracts should state briefly and clearly the purpose, methods, results and conclusions of the work.

Introduction: Clearly state the purpose of the abstract
Methods: Describe your selection of observations or experimental subjects clearly
Results: Present your results in a logical sequence in text, tables and illustrations
Conclusions: Emphasize new and important aspects of the study and conclusions that are drawn from them

Introduction

Recently, the idea has been introduced to look beyond the limiting mechanism of the masonry arch and consider the behaviour of the mechanism itself and the role it can have on the system [1]. Initial investigations into this idea through FE analysis have shown that the arch can be strengthened, and the mechanism can be defined [2,3]. Therefore, the next stage is to begin the experimental investigation, but an efficient and effective measurement technique needs to be established. The purpose of this work is to establish a simplified measurement technique for the common rigid-body displacements between two blocks of a masonry arch in contact.

Identification

The common failure of dry-joint masonry arches is by mechanization. This mechanization occurs through the release of a degree of freedom of motion at four or five block boundaries. Typically, the release is rotational, but it can also be the translational degree of freedom (slip) tangent to the joint. Slip can also transition to rotation after the onset of a mechanization, resulting in the eight possible motions defined in figure 1.

Now consider a 4x4 grid of points applied to the blocks across the joint and identified by lettered rows and numbered columns (figure 2). Considering the change in lengths starting from the original configuration produces

$$\Delta l_{a23} > \Delta l_{b23} > \Delta l_{c23} > \Delta l_{d23} \tag{1}$$

for bottom rotations,

$$\Delta l_{a23} < \Delta l_{b23} < \Delta l_{c23} < \Delta l_{d23} \tag{2}$$

for top rotations and

$$\Delta l_{a23} \approx \Delta l_{b23} \approx \Delta l_{c23} \approx \Delta l_{d23} > 0 \tag{3}$$

for a slip where

$$\Delta l_{i23} = l'_{i23} - l_{i23} \tag{4}$$

for each row. Similarly, the slip-rotation combinations produce

$$\Delta l'_{a23} > \Delta l'_{b23} > \Delta l'_{c23} > \Delta l'_{d23} \tag{5}$$

and

$$\Delta l'_{a23} < \Delta l'_{b23} < \Delta l'_{c23} < \Delta l'_{d23} \tag{6}$$

for slips transformed to bottom and top rotations respectively with

$$\Delta l'_{i23} = l''_{i23} - l'_{i23} \tag{7}$$

again for each row. The same relationships hold for the point pairs *1-3*, *1-4* and *2-4*.

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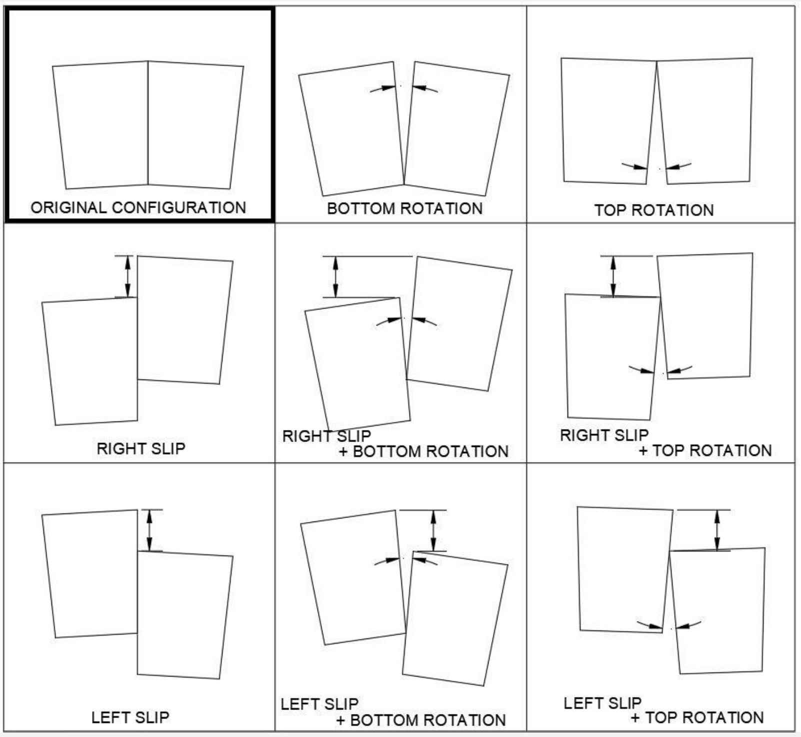

Figure 1 – Defined mechanical behaviors between two rigid blocks in an arch.

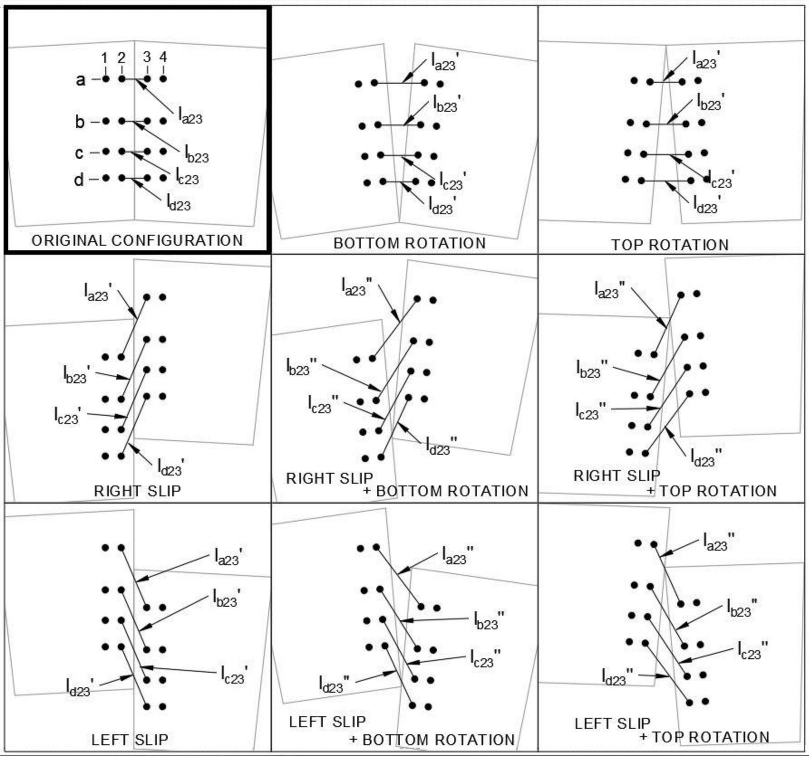

Figure 2 – Defined point grid and the lengths between points 2 and 3 of each row for the defined behaviours.

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Measurements

For rotation measurements, two point sets consisting of three points are examined. The principle idea is triangulation between the points' two configurations and the origin of rotation.

Rotations – Point Set 1

Consider three points separated by a straight joint line and with points 2 and 3 aligned perpendicular to the joint which undergoes a rotation of 2α at point P and results in the displaced states 1', 2' and 3' for points 1, 2 and 3 respectively (figure 3). This produces various triangular relationships between the points in both states as can be seen in figure 3. These relationships include a point's displacement, δi, and the radius from the rotation point, Li,

$$L_i = \frac{\delta_i}{2\sin\left(\frac{\alpha}{2}\right)} \tag{8}$$

Note that the length between points 2-3 remains constant, but the change in the other two lengths can be expressed as

$$\begin{aligned} \Delta V_{12} &= l'_{12}\sin(\pi + \alpha - \theta'_2) - l_{12}\sin(\pi - \theta_2) = (\delta_2 - \delta_1)\cos(\alpha) \\ \Delta H_{12} &= l'_{12}\cos(\pi + \alpha - \theta'_2) - l_{12}\cos(\pi - \theta_2) = (\delta_1 + \delta_3)\sin(\alpha) \end{aligned} \tag{9}$$

and

$$\begin{aligned} \Delta V_{13} &= l'_{13}\sin(\alpha + \theta'_3) - l_{13}\sin(\theta_3) = (\delta_3 - \delta_1)\cos(\alpha) \\ \Delta H_{13} &= l'_{13}\cos(\alpha + \theta'_3) - l_{13}\cos(\theta_3) = (\delta_1 + \delta_3)\sin(\alpha) \end{aligned} \tag{10}$$

where ΔHij and ΔVij are the perpendicular and parallel deformations with respect to the joint line and between points i and j respectively. If the initial and final configuration lengths are known, then angles θi and θi' can be determined through the law of cosines. The rotation angle can also be determined by

$$\alpha = \frac{\theta_3 - \theta'_3}{2} \tag{11}$$

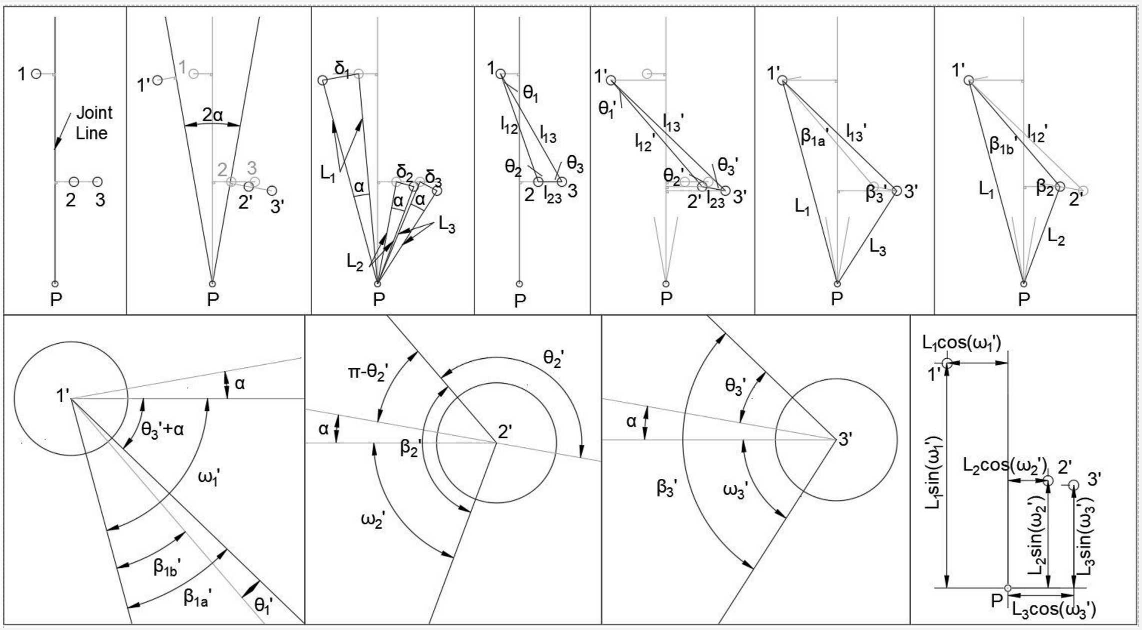

Figure 3 – Geometries and configurations for point set 1.

Equations 8 through 12 allow the radii from the rotation to each point to be expressed through the length changes and rotation angle

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$$L_1 = \frac{\Delta H_{13} - \Delta V_{13}\tan(\alpha)}{4\sin(\alpha)\sin\left(\frac{\alpha}{2}\right)}$$
$$L_2 = \frac{\Delta H_{12} + \Delta V_{12}\tan(\alpha)}{4\sin(\alpha)\sin\left(\frac{\alpha}{2}\right)} \quad (12)$$
$$L_3 = \frac{\Delta H_{13} + \Delta V_{13}\tan(\alpha)}{4\sin(\alpha)\sin\left(\frac{\alpha}{2}\right)}$$

The angle decomposition of points 1', 2' and 3' as seen in figure 3 produce

$$\omega_1' = \theta_3' + \alpha + \beta_{1a}'$$
$$\omega_2' = \beta_2' - \pi + \theta_2' - \alpha \quad (13)$$
$$\omega_3' = \beta_3' - \theta_3' - \alpha$$

and thus, the cartesian style description between the points and the origin of rotation are defined.

Rotations – Point Set 2

Following the same methodology as for point set 1, figure 4 shows the geometries and cartesian style coordinates of the final point configuration against the origin of rotation for point set 2.

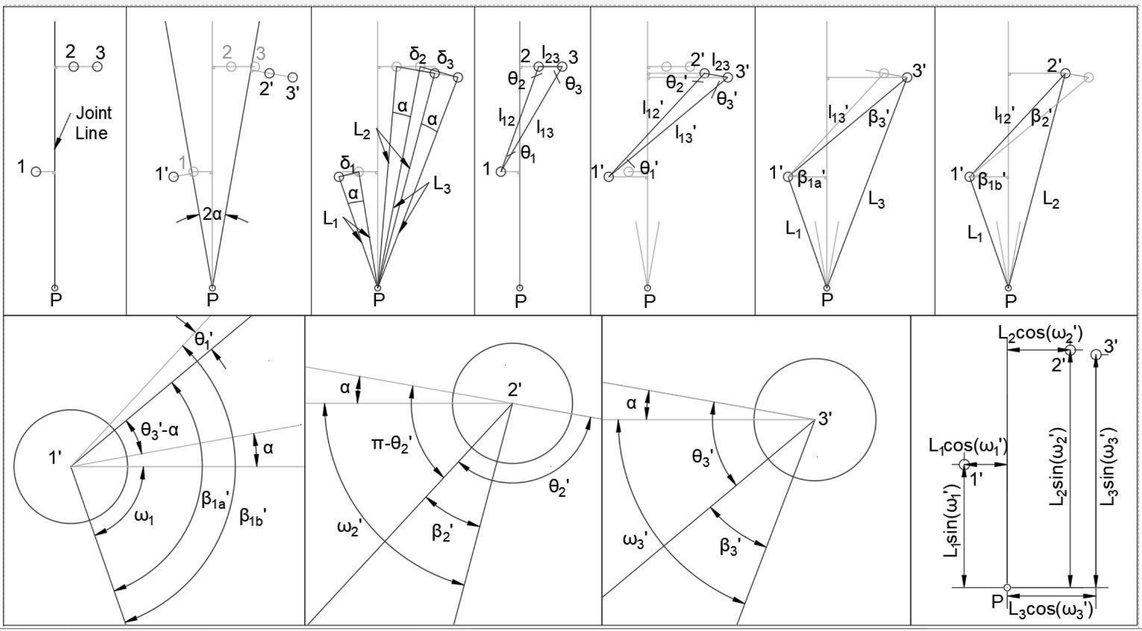

Figure 4 – Geometries and configurations for point set 2.

The change in lengths are

$$\Delta V_{12} = l_{12}'\sin(\pi - \alpha - \theta_2') - l_{12}\sin(\pi - \theta_2) = (\delta_1 - \delta_2)\cos(\alpha)$$
$$\Delta H_{12} = l_{12}'\cos(\pi - \alpha - \theta_2') - l_{12}\cos(\pi - \theta_2) = (\delta_1 + \delta_2)\sin(\alpha) \quad (14)$$
$$\Delta V_{13} = l_{13}'\sin(\theta_3' - \alpha) - l_{13}\sin(\theta_3) = (\delta_1 - \delta_3)\cos(\alpha)$$
$$\Delta H_{13} = l_{13}'\cos(\theta_3' - \alpha) - l_{13}\cos(\theta_3) = (\delta_1 + \delta_3)\sin(\alpha)$$

with

$$\alpha = \frac{\theta_3' - \theta_3}{2} \quad (15)$$

Therefore, the point radii are

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$$L_1 = \frac{\Delta H_{13}+\Delta V_{13}\tan(\alpha)}{4\sin(\alpha)\sin\left(\frac{\alpha}{2}\right)}$$
$$L_2 = \frac{\Delta H_{12} 1 \Delta V_{12}\tan(\alpha)}{4\sin(\alpha)\sin\left(\frac{\alpha}{2}\right)}$$
$$L_3 = \frac{\Delta H_{13}-\Delta V_{13}\tan(\alpha)}{4\sin(\alpha)\sin\left(\frac{\alpha}{2}\right)} \qquad (16)$$

with angles

$$\omega_1' = \beta_{1a}' - \theta_3'$$
$$\omega_2' = \beta_2' + \pi - \theta_2' - \alpha$$
$$\omega_3' = \beta_3' + \theta_3' - \alpha \qquad (17)$$

Slip Displacements

For slip measurements, three conditions exist for the point set analysis as can be seen in figure 5. The first two conditions start with the same point sets as for the rotations (figure5a-b), and the third condition involves slip displacements that transform point set 1 into point set 2 (figure 5c). The only displacements are along the joint and thus the point displacements δ1 and δ2 are directly equal to the slip displacement δS. Therefore, the first two conditions (figure 5a and 5b) produce

$$\delta_S = \Delta V_{12} = l_{12}'\sin(\pi - \theta_2') - l_{12}\sin(\pi - \theta_2)$$
$$\delta_S = \Delta V_{13} = l_{13}'\sin(\theta_3') - l_{13}\sin(\theta_3) \qquad (18)$$

and the third condition where points 2 and 3 cross point 1 produces

$$\delta_S = \Delta V_{12} = l_{12}'\sin(\pi - \theta_2') + l_{12}\sin(\pi - \theta_2)$$
$$\delta_S = \Delta V_{13} = l_{13}'\sin(\theta_3') + l_{13}\sin(\theta_3) \qquad (19)$$

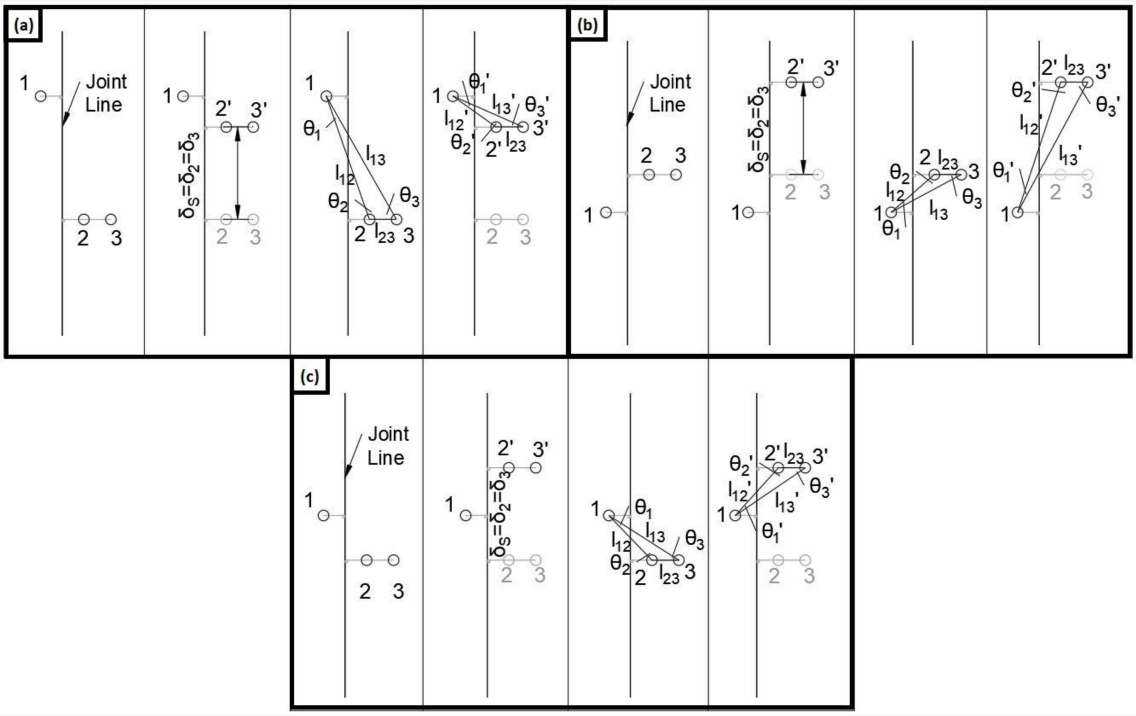

Figure 5 – Geometries and configurations due to slip for (a) point set 1, (b) point set 2 and (c) the transition from point set 1 to 2.

Slip + Rotation

The combination of slip and then rotation can be measured in a two-step process with the original configuration of the point set for rotations being the final configuration from the slip.

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Conclusions

The purpose of this work is to establish a simplified measurement technique for rigid-body displacements between two blocks in contact. The displacements, or rotations and hinge point can be determined solely by knowing the lengths between the three points of point set before and after the displacement and the type of motion. These measurements are constructed in the reference frame of the joint line and rotation point. Therefore, by identifying the mechanical behaviour between two rigid blocks with a 4x4 point grid and equations 1 through 7, the measurement equation sets and coordinate system can be established in the global reference frame.

Concerning rotations and slip, the 4x4 point grid (see figure 2) produces 24 distinct measurements of the rotation angle and hinge location or displacements with point *1* placed at the points of columns 1 and 2 of the grid. Then repeating the process mirrored, that is with point *1* on columns 3 and 4, doubles the number of measurements to 48. Therefore, taking the average of these 48 measurements increases the accuracy by a factor of 6.9. Note that the 24 counts removed the set where points *1*, *2* and *3* are on the same row to avoid conflicts in set identification.

Finally, it must be noted that for the condition of slip plus rotation there exists the potential that the hinge point exists within the boundaries of the deformed grid. This will result in some invalid measurements, but length changes from equation 7 will be less than or equal to zero and thus an exclusion parameter can be established. The next step is to establish the algorithm and incorporate into point tracking software.

References

[1] Stockdale G (2016) Reinforced stability-based design: a theoretical introduction through a mechanically reinforced masonry arch. In Int. J. Masonry Research and Innovation 1(2): 101-142

[2] Stockdale G and Milani G (2017) FE Model Predicting the Load Carrying Capacity of Progressive FRP Strengthening of Masonry Arches Subjected to Settlement Damage. Key Engineering Materials, Vol. 747, pp. 128-133

[3] Stockdale G and Milani G (2017) FE model predicting the increase in seismic resistance induced by the progressive FRP strengthening on already damaged masonry arches subjected to settlement. International Conference of Computational Methods in Sciences and Engineering. AIP Conference Proceedings 1096, 090002, Thessaloniki, Greece vol. 1906, 090002, DOI 10.1063/1.5012359