

Random number generation by differential read of stochastic switching in spin-transfer torque memory

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Abstract—The true random number generator (TRNG) is a key enabling technology for cryptography and hardware authentication, which are becoming essential features in the era of the internet of things (IoT). Here, we present a novel TRNG concept based on the stochastic switching in spin-transfer torque magnetic random access memory (STT-MRAM). The new methodology relies on the STT-MRAM switching variations affecting the current response under applied rectangular or triangular pulses. Random numbers are extracted from the differential read of the integrated current across 2 stochastic switching cycles. The proposed concept passes all tests in the NIST SP-800-22 suite with no post-processing, thus supporting STT-MRAM as a promising technology for data/hardware security in the IoT.

Index Terms—Spin-transfer torque magnetic memory (STT-MRAM), true random number generator (TRNG), magnetotunnel junction (MTJ), switching variability, nonvolatile.

I. INTRODUCTION

THE random number generator (RNG) plays a key role in enabling secure data transmissions [1], [2] by generating random cryptographic keys. The widespread diffusion of internet-based communicating devices and internet of things (IoT) raises the need for compact and reliable RNG circuits, generating random numbers with high entropy and high throughput [3]. In addition, since emerging computing paradigms, such as stochastic [4], [5] and brain-inspired computing [6], [7], require large amounts of random bit sequence [8], they could benefit from a small yet effective random number source. In most cases, random bits are generated via pseudo RNG (PRNG), consisting in software algorithms generating random numbers via a deterministic function of a seed, *e.g.*, the system clock [9]. However, these systems feature limited randomness and can be easily attacked [10]. Data protection against cyberattacks is achieved only with true RNG (TRNG), whose output random sequence comes from inherently-stochastic physical processes [11]. Their unpredictability makes hardware-based TRNG more reliable than software-based PRNG [12], [13]. Possible entropy sources for TRNG are the random telegraph noise (RTN) in dielectrics

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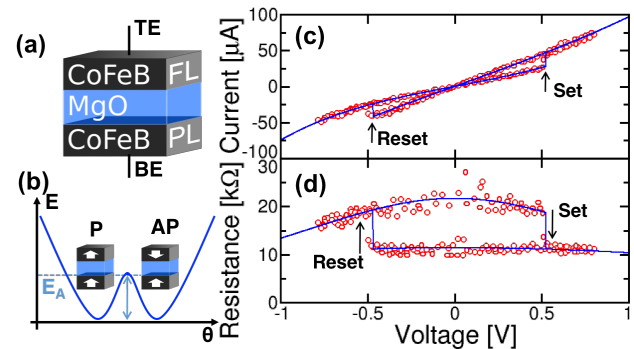


Fig. 1. (a) STT-MRAM device used in this work. (b) Energy as a function of the FL magnetic polarization direction with respect to the PL, evidencing stable P and AP states. (c) Measured and calculated I-V and (d) R-V characteristics, indicating set (AP to P) and reset (P to AP) transitions. The characteristics were obtained under 1 μ s triangular voltage pulses.

[14], [15], quantum processes [11], spintronics phenomena [16], [17] and memristive devices [18]–[20]. In TRNG, the entropy source is usually the intrinsic variability of the switching parameters, such as resistance and set/reset voltages [21], [22]. Both spin-transfer torque magnetic memory (STT-MRAM) [16], [23] and RRAM [18]–[20] have been used to demonstrate working TRNG. A significant drawback in these techniques is their need for a probability tracking scheme to determine operative voltages for uniform TRNG, *i.e.*, equal probability of generating 0 and 1 [20], [24]. Recently, a differential approach using 2 RRAM devices has been proposed to avoid the need for probability tracking and its associated circuit and algorithm overhead [20]. However, such a differential scheme requires a relatively low mismatch between the switching characteristics in the differential pair.

Here, we report a novel methodology for physical unbiased generation of true random numbers based on the stochastic switching time of STT [21]. A differential scheme is adopted by comparing the switching characteristics of the same device over 2 consecutive cycles with either rectangular or triangular pulses. The new RNG concept is finally validated against the NIST SP-800-22 test suite [25], with no post-processing.

II. STT-MRAM SWITCHING CHARACTERISTICS

We considered STT-MRAM devices based on a magnetotunnel junction (MTJ) structure with perpendicular magnetic anisotropy (PMA), as shown in Fig. 1(a) [26]. The MTJ consists of CoFeB pinned layer (PL) and free layer (FL), acting as bottom electrode (BE) and top electrode (TE),

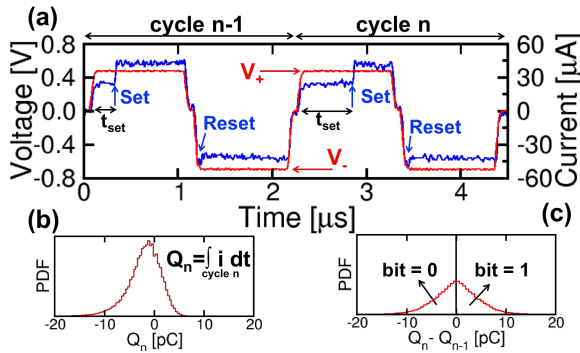


Fig. 2. (a) Measured rectangular voltage pulses and current response for 2 consecutive cycles $n-1$ and n , (b) PDF of the integrated current Q_n and (c) PDF of differential charge $\Delta Q_n = Q_n - Q_{n-1}$. The pulse sequence includes positive and negative rectangular pulses for stochastic set and reset transitions, respectively, as evidenced by the abrupt steps in the current response. The random bit is assigned from the value of ΔQ_n in (c).

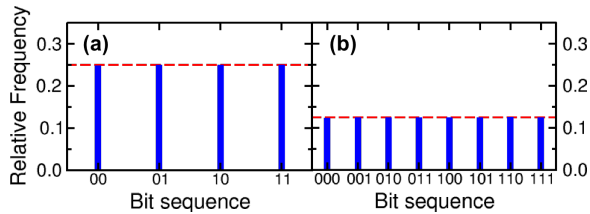


Fig. 3. Relative occurrence of 2-bit (a) and 3-bit sequences (b) for $V_+ = 0.5$ V and $V_- = 0.7$ V. Data show uniform distribution of sequences, with equal probabilities of 0.25 and 0.125 for the 2 and 3-bit sequences, respectively.

respectively, separated by a crystalline MgO dielectric layer. As schematically shown by the energy diagram of Fig. 1(b), the device has 2 stable states, where the magnetic polarization in the FL is either parallel (P) to the PL, corresponding to a low resistance of the MTJ, or antiparallel (AP) state to the PL, corresponding to a high resistance of the MTJ [26], [27]. Fig. 1(c) shows the measured current-voltage (I - V) characteristics, which was obtained by applying triangular positive/negative voltage pulses to the TE with pulse-width $1 \mu\text{s}$, and collecting the current response from an oscilloscope [26]. Fig. 1(d) shows the corresponding resistance-voltage (R - V) characteristics, demonstrating AP-to-P (set) transition at the positive set voltage V_{set} , and P-to-AP (reset) transition at the negative reset voltage V_{reset} . Calculations based on a compact model for STT-MRAM are reported in the figure, showing a good agreement with data [26].

III. STT-MRAM BASED TRNG

The switching event in STT-MRAM is inherently-stochastic, *i.e.*, V_{set} and V_{reset} changes from cycle to cycle [21], [22]. The statistical variation was explained in terms of a thermally-assisted magnetization reversal [28], where the transition from P to AP (or vice versa) arises from a random thermal fluctuation of the FL polarization within the potential well of Fig. 1(b), eventually causing the stochastic transition over the energy barrier E_A . As a result, for any applied voltage $V_A > 0$, there exists a statistical distribution of set times t_{set} , namely the time delay between the application of V_A and the set

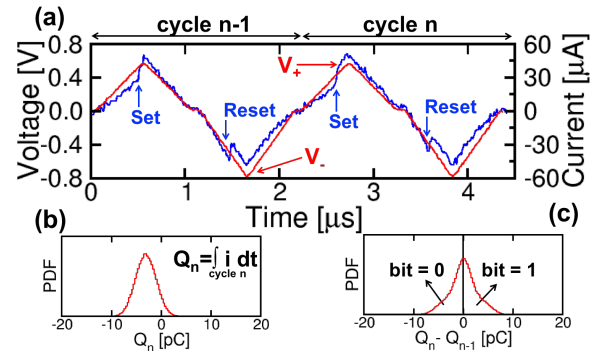


Fig. 4. (a) Measured triangular voltage pulses and current response for 2 consecutive cycles $n-1$ and n , (b) PDF of the integrated current Q_n and (c) PDF of differential charge $\Delta Q_n = Q_n - Q_{n-1}$. The pulse sequence includes positive and negative triangular pulses for stochastic set and reset transitions, respectively, as evidenced by the abrupt steps in the current response. The random bit is assigned from the value of ΔQ_n in (c).

transition [21]. Similarly, there is a distribution of reset times t_{reset} measuring the time delay between the application of a voltage $V_A < 0$ and the reset transition.

The variation of switching voltage and/or time was used as the entropy source in previous TRNG concepts [16], [17]. However, these schemes adopt a probability tracking approach to ensure TRNG uniformity. Differential schemes can generate random numbers with no need for probability tracking from the competition between 2 devices, although suffering from the device-to-device mismatch of the switching voltage [20].

To solve this issue, here we adopt a differential scheme, where the device responses over 2 consecutive cycles are compared to yield a random bit. Fig. 2(a) shows the applied voltage and the current response in 2 consecutive cycles. In each cycle, a positive stochastic pulse of voltage V_+ is applied, followed by a negative deterministic pulse of voltage V_- . Both pulses have a duration $t_p = 1 \mu\text{s}$. Note that the positive current response during the cycle $n - 1$ differs from cycle n , namely, the set time t_{set} during cycle $n - 1$ is shorter than in cycle n as a result of the stochastic AP - P transition. The random bit is generated by comparing the integrated current in the 2 successive cycles of the same STT-MRAM device. Fig. 2(b) shows the experimental probability distribution function (PDF) of the integrated current $Q_n = \int i dt$. The measured difference $\Delta Q_n = Q_n - Q_{n-1}$ shows a symmetric PDF in Fig. 2(c), with equal portions of positive and negative ΔQ_n . Therefore, ΔQ_n is chosen as the statistical variable for generating the random bit, with the bit assigned to 0 or 1 for $\Delta Q_n < 0$ or $\Delta Q_n > 0$, respectively. The symmetric PDF in Fig. 2(c) ensures the uniformity of the generated bit with equal probabilities of the bit being 0 or 1. To further support the TRNG uniformity, Fig. 3 shows the probability of the occurrence of 2-bit sequences (a) and 3-bit sequences (b), indicating equal probabilities of 0s and 1s. Similar TRNG concepts with MTJ rely on whitening algorithms, such as the Von Neumann correction [23] or the XOR operation [8], to obtain a sufficiently uniform bit stream. On the other hand, no post-processing technique is used in the present concept, thus contributing to minimizing the energy and area overhead of the TRNG.

TABLE I

NIST TEST RESULTS FOR A SEQUENCE OF MORE THAN 1 Mb OBTAINED FROM RECTANGULAR AND TRIANGULAR PULSES FOR DIFFERENT APPLIED VOLTAGES. SEQUENCES WERE DIVIDED IN 55 SEGMENTS. TEST ARE PASSED IF $P_T > 0.0001$ AND PROPORTION $\geq 52/55$.

Applied Pulse	Rect. $V_+ = 0.43V$		Rect. $V_+ = 0.57V$		Rect. $V_+ = 0.67V$		Tri. $V_+ = 0.5V$		Tri. $V_+ = 0.57V$		Tri. $V_+ = 0.73V$	
	P_T value	Prop.	P_T value	Prop.	P_T value	Prop.	P_T value	Prop.	P_T value	Prop.	P_T value	Prop.
Frequency	0.014550	55/55	0.437274	55/55	0.021999	50/55	0.401199	55/55	0.334538	55/55	0.401199	54/55
Block Frequency	0.002203	53/55	0.048716	55/55	0.000000	53/55	0.000216	52/55	0.010988	52/55	0.004629	53/55
Cumul. Sums - 1	0.000026	55/55	0.401199	55/55	0.000883	50/55	0.334538	55/55	0.162606	55/55	0.304126	53/55
Cumul. Sums - 2	0.003996	55/55	0.014550	55/55	0.000253	50/55	0.834308	55/55	0.090936	54/55	0.437274	54/55
Runs	0.000000	45/55	0.002559	55/55	0.000648	50/55	0.000274	54/55	0.304126	53/55	0.678686	55/55
Longest Run	0.474986	55/55	0.042808	54/55	0.474986	55/55	0.062821	54/55	0.554420	55/55	0.162606	55/55
Rank	0.000026	52/55	0.304126	55/55	0.025193	54/55	0.048716	54/55	0.719747	54/55	0.554420	55/55
FFT	0.224821	52/55	0.759756	55/55	0.514124	54/55	0.275709	54/55	0.162606	54/55	0.062821	55/55
Non-overl. templ.	45/148	51/48	148/148	145/148	55/148	8/148	148/148	143/148	148/148	148/148	148/148	147/148
Approx Entropy	0.145326	52/55	0.071177	55/55	0.002203	46/55	0.025193	55/55	0.012650	53/55	0.025193	55/55
Serial - 1	0.024550	52/55	0.145326	55/55	0.000082	50/55	0.401199	55/55	0.719747	53/55	0.595549	54/55
Serial - 2	0.946308	55/55	0.304126	55/55	0.003996	55/55	0.514124	55/55	0.304126	53/55	0.334538	55/55

The same methodology is shown in Fig. 4, for triangular applied pulses instead of rectangular pulses. The current response in Fig. 4(a) shows variable set and reset currents. Integrating the current leads to the stochastic charge Q_n in Fig. 4(b), while Fig. 4(c) shows the symmetric distribution of the charge difference ΔQ_n , supporting TRNG with triangular pulses. Note that the TRNG pulse-width can be much faster than $1 \mu s$ used in Figs. 2 and 4, since sub-nanosecond switching was reported for STT-MRAM [29]. A shorter time might allow for a higher TRNG throughput and a smaller Q_n , hence a correspondingly smaller area of the integrating capacitor. Also, thanks to the differential read, our concept is reasonably immune from any external biasing effect, such as the application of an external magnetic field or a change in temperature. In fact, the external bias would only affect the switching threshold, but not its cycle-to-cycle variability, which is the key entropy source in the proposed TRNG.

IV. RANDOMNESS ANALYSIS

To validate a TRNG concept, evaluation of the randomness is mandatory. To this purpose, a industry standard statistical test suite for randomness qualification, namely the NIST SP-800-22 test [25], was applied to our TRNG scheme for a wide variety of pulse amplitude V_+ , for both rectangular and triangular pulses. Tab. I shows the detailed NIST test output for 3 different voltages for both rectangular and triangular pulses. Although overall results indicate a good randomness, rectangular pulses show failure of some tests at relatively low or high voltages, with best results being achieved within a narrow V_+ window. On the other hand, randomness of the triangular case remains high for all the applied voltages. Fig. 5 summarizes the NIST test results by showing the pass rate for the non-overlapping template test. Results indicate the generally good performance with the triangular pulse scheme, and the existence of an optimum window for the rectangular pulse scheme. Results in Fig. 5 can be explained by the dependence on V_A of the stochastic parameters t_{set} and V_{set} for rectangular and triangular pulses, respectively. For a rectangular pulse, the set time t_{set} can be written as [30]:

$$t_{set} = \tau_0 \exp\left(\Delta\left(1 - \frac{V}{V_0}\right)\right), \quad (1)$$

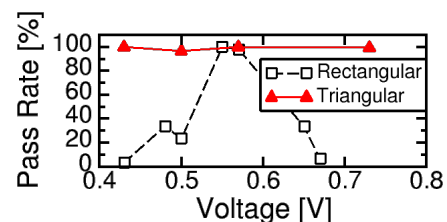


Fig. 5. Pass rate of the non-overlapping template NIST test as a function of pulse voltage for rectangular and triangular pulses. The pass rate is referred to a total of 148 tests. Rectangular pulses show an operation window around 0.6 V, whereas triangular pulses show voltage-independent high randomness.

where V_A and τ_0 are constants, V is the applied voltage ($V = V_A$), and Δ is the thermal stability factor. Given the exponential dependence in Eq. (1), and since the switching time t_{set} should be comparable to the pulse width t_p in Fig. 2, there is a relatively narrow window of V_A for which the entropy is high. On the other hand, for a triangular pulse with ramped voltage according to $V = 2V_A t/t_p$, the set voltage can be estimated by integrating the switching probability according to $\int 1/t_{set} dt = 1$, with t_{set} given by Eq. (1). The set voltage along the triangular pulse is thus given by [28], [31]:

$$V_{set} \approx V_0 \ln \frac{t_0 V_A}{V_0 t_p}. \quad (2)$$

indicating a logarithmic dependence of V_{set} on the maximum applied voltage V_A . This explains the much smaller dependence of entropy on V_A with respect to the rectangular pulse in Fig. 5.

V. CONCLUSIONS

A new TRNG based on the stochastic switching time in a STT-MRAM is presented. Purely white random bits are generated from the comparison of the current response over 2 consecutive set/reset cycles. Statistical NIST tests demonstrate the excellent randomness of the output bit-stream, with no need for whitening processes. For triangular pulses, no tracking process is also needed. Due to the simple implementation, high endurance, high scalability and CMOS compatibility of the STT-MRAM, the proposed TRNG is a promising tool for data and hardware security in IoT systems.

REFERENCES

- [1] S. K. Mathew, S. Srinivasan, M. A. Anders, H. Kaul, S. K. Hsu, F. Sheikh, A. Agarwal, S. Satpathy, and R. K. Krishnamurthy, "2.4 Gbps, 7 mW all-digital PVT-variation tolerant true random number generator for 45 nm CMOS high-performance microprocessors," *IEEE Journal of Solid-State Circuits*, vol. 47, no. 11, pp. 2807–2821, Oct. 2012, DOI: 10.1109/JSSC.2012.2217631.
- [2] A. J. Menezes, P. C. Van Oorschot, and S. A. Vanstone, *Handbook of applied cryptography*. CRC press, 1996.
- [3] S. Ghosh, "Spintronics and Security: Prospects, Vulnerabilities, Attack Models, and Preventions," *Proceedings of the IEEE*, vol. 104, no. 10, pp. 1864–1893, Sep. 2016, DOI: 10.1109/JPROC.2016.2583419.
- [4] A. Alaghi and J. P. Hayes, "Survey of stochastic computing," *ACM Transactions on Embedded Computing Systems (TECS)*, vol. 12, no. 2s, p. 92, 2013 DOI: 10.1145/2465787.2465794.
- [5] J. S. Friedman, L. E. Calvet, P. Bessière, J. Droulez, and D. Querlioz, "Bayesian inference with Muller C-elements," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 63, no. 6, pp. 895–904, Jun. 2016 DOI: 10.1109/TCSI.2016.2546064.
- [6] P. A. Merolla, J. V. Arthur, R. Alvarez-Icaza, A. S. Cassidy, J. Sawada, F. Akopyan, B. L. Jackson, N. Imam, C. Guo, Y. Nakamura, et al., "A million spiking-neuron integrated circuit with a scalable communication network and interface," *Science*, vol. 345, no. 6197, pp. 668–673, Aug. 2014 DOI: 10.1126/science.1254642.
- [7] G. Pedretti, V. Milo, S. Ambrogio, R. Carboni, S. Bianchi, A. Calderoni, N. Ramaswamy, A. S. Spinelli, and D. Ielmini, "Stochastic learning in neuromorphic hardware via spike timing dependent plasticity with RRAM synapses," *IEEE Journal on Emerging and Selected Topics in Circuits and Systems*, vol. 8, no. 1, pp. 77–85, Mar. 2018 DOI: 10.1109/JETCAS.2017.2773124.
- [8] D. Vodenicarevic, N. Locatelli, A. Mizrahi, J. S. Friedman, A. F. Vincent, M. Romera, A. Fukushima, K. Yakushiji, H. Kubota, S. Yuasa, et al., "Low-energy truly random number generation with superparamagnetic tunnel junctions for unconventional computing," *Physical Review Applied*, vol. 8, no. 5, p. 054045, Nov. 2017 DOI: 10.1103/PhysRevApplied.8.054045.
- [9] G. Alvarez and S. Li, "Some basic cryptographic requirements for chaos-based cryptosystems," *International Journal of Bifurcation and Chaos*, vol. 16, no. 08, pp. 2129–2151, 2006, DOI: 10.1142/S0218127406015970.
- [10] J. Kelsey, B. Schneier, D. Wagner, and C. Hall, "Cryptanalytic attacks on pseudorandom number generators," in *Fast Software Encryption*, pp. 168–188, Springer, Oct. 1998, DOI: 10.1007/3-540-69710-1-12.
- [11] N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, "Quantum cryptography," *Reviews of modern physics*, vol. 74, no. 1, p. 145, Mar. 2002, DOI: 10.1103/RevModPhys.74.145.
- [12] B. Jun and P. Kocher, "The Intel random number generator," *Cryptography Research Inc. white paper*, 1999.
- [13] S. Sahay and M. Suri, "Recent trends in hardware security exploiting hybrid CMOS-resistive memory circuits," *Semiconductor Science and Technology*, vol. 32, no. 12, p. 123001, Oct. 2017, DOI: 10.1088/1361-6641/aa8f07.
- [14] R. Brederlow, R. Prakash, C. Paulus, and R. Thewes, "A low-power true random number generator using random telegraph noise of single oxide-traps," in *Solid-State Circuits Conference, 2006. ISSCC 2006. Digest of Technical Papers. IEEE International*, pp. 1666–1675, IEEE, Sep. 2006, DOI: 10.1109/ISSCC.2006.1696222.
- [15] C.-Y. Huang, W. C. Shen, Y.-H. Tseng, Y.-C. King, and C.-J. Lin, "A contact-resistive random-access-memory-based true random number generator," *IEEE Electron Device Letters*, vol. 33, no. 8, pp. 1108–1110, Jun. 2012, DOI: 10.1109/LED.2012.2199734.
- [16] A. Fukushima, T. Seki, K. Yakushiji, H. Kubota, H. Imamura, S. Yuasa, and K. Ando, "Spin dice: A scalable truly random number generator based on spintronics," *Applied Physics Express*, vol. 7, no. 8, p. 083001, Jul. 2014, DOI: 10.7567/APEX.7.083001.
- [17] S. Chun, S.-B. Lee, M. Hara, W. Park, and S.-J. Kim, "High-density physical random number generator using spin signals in multidomain ferromagnetic layer," *Advances in Condensed Matter Physics*, vol. 2015, Jan. 2015, DOI: 10.1155/2015/251819.
- [18] Z. Wei, Y. Katoh, S. Ogasahara, Y. Yoshimoto, K. Kawai, Y. Ikeda, K. Eriguchi, K. Ohmori, and S. Yoneda, "True random number generator using current difference based on a fractional stochastic model in 40-nm embedded ReRAM," in *Electron Devices Meeting (IEDM), 2016 IEEE International*, pp. 4–8, IEEE, Dec. 2016, DOI: 10.1038/s41467-017-00869-x.
- [19] S. Balatti, S. Ambrogio, Z. Wang, and D. Ielmini, "True random number generation by variability of resistive switching in oxide-based devices," *IEEE Journal on Emerging and Selected Topics in Circuits and Systems*, vol. 5, no. 2, pp. 214–221, Jun. 2015, DOI: 10.1109/JETCAS.2015.2426492.
- [20] S. Balatti, S. Ambrogio, R. Carboni, V. Milo, Z. Wang, A. Calderoni, N. Ramaswamy, and D. Ielmini, "Physical unbiased generation of random numbers with coupled resistive switching devices," *IEEE Transactions on Electron Devices*, vol. 63, no. 5, pp. 2029–2035, May 2016, 10.1109/TED.2016.2537792.
- [21] A. F. Vincent, N. Locatelli, J.-O. Klein, W. S. Zhao, S. Galdin-Retailleau, and D. Querlioz, "Analytical macrospin modeling of the stochastic switching time of spin-transfer torque devices," *IEEE Transactions on Electron Devices*, vol. 62, no. 1, pp. 164–170, Jan. 2015, DOI: 10.1109/TED.2014.2372475.
- [22] S. Ambrogio, S. Balatti, A. Cubeta, A. Calderoni, N. Ramaswamy, and D. Ielmini, "Statistical Fluctuations in HfO_x Resistive-Switching Memory: Part I-Set/Reset variability," *IEEE Transactions on Electron Devices*, vol. 61, no. 8, pp. 2912–2919, Aug. 2014, DOI: 10.1109/TED.2014.2330200.
- [23] W. H. Choi, Y. Lv, J. Kim, A. Deshpande, G. Kang, J.-P. Wang, and C. H. Kim, "A magnetic tunnel junction based true random number generator with conditional perturb and real-time output probability tracking," in *Electron Devices Meeting (IEDM), 2014 IEEE International*, pp. 12–5, IEEE, Dec. 2014, DOI: 10.1109/IEDM.2014.7047039.
- [24] Y. Qu, J. Han, B. F. Cockburn, W. Pedrycz, Y. Zhang, and W. Zhao, "A true random number generator based on parallel STT-MTJs," in *2017 Design, Automation & Test in Europe Conference & Exhibition (DATE)*, pp. 606–609, IEEE, Mar. 2017, DOI: 10.23919/DATE.2017.7927058.
- [25] A. Rukhin, J. Soto, J. Nechvatal, M. Smid, and E. Barker, "A statistical test suite for random and pseudorandom number generators for cryptographic applications," tech. rep., NIST, Gaithersburg, MD, USA, Special Publication 800-22, Sep. 2001.
- [26] R. Carboni, S. Ambrogio, W. Chen, M. Siddik, J. Harms, A. Lyle, W. Kula, G. Sandhu, and D. Ielmini, "Understanding cycling endurance in perpendicular spin-transfer torque (p-STT) magnetic memory," in *Electron Devices Meeting (IEDM), 2016 IEEE International*, pp. 21–6, IEEE, 2016, DOI: 10.1109/IEDM.2016.7838468.
- [27] D. Apalkov, B. Dieny, and J. Slaughter, "Magnetoresistive Random Access Memory," *Proceedings of the IEEE*, vol. 104, no. 10, pp. 1796–1830, Oct. 2016, DOI: 10.1109/JPROC.2016.2590142.
- [28] Z. Li and S. Zhang, "Thermally assisted magnetization reversal in the presence of a spin-transfer torque," *Physical Review B*, vol. 69, no. 13, p. 134416, Apr. 2004, DOI: 10.1103/PhysRevB.69.134416.
- [29] G. Jan, L. Thomas, S. Le, Y.-J. Lee, H. Liu, J. Zhu, J. Iwata-Harms, S. Patel, R.-Y. Tong, S. Serrano-Guisan, et al., "Achieving sub-ns switching of STT-MRAM for future embedded LLC applications through improvement of nucleation and propagation switching mechanisms," in *VLSI Technology, 2016 IEEE Symposium on*, pp. 1–2, IEEE, Jun. 2016 DOI: 10.1109/VLSIT.2016.7573362.
- [30] R. Heindl, W. H. Rippard, S. E. Russek, M. R. Pufall, and A. B. Kos, "Validity of the thermal activation model for spin-transfer torque switching in magnetic tunnel junctions," *Journal of Applied Physics*, vol. 109, p. 073910, Apr 2011 DOI: 10.1063/1.3562136.
- [31] C. Cagli, F. Nardi, and D. Ielmini, "Modeling of set/reset operations in NiO-based resistive-switching memory devices," *IEEE Transactions on electron devices*, vol. 56, no. 8, pp. 1712–1720, Aug. 2009 DOI: 10.1109/TED.2009.2024046.