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Linear Stability Analysis Of A Full Triga Reactor Plant In A Closed Loop

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ABSTRACT

This work investigates the dynamics of the TRIGA-Mark II located on the premises of Laboratorio Energia Nucleare Applicata (LENA) - Università degli Studi di Pavia; in particular, it focuses on the system stability in a closed loop, with thermal-hydraulics feedback and poison effects on reactivity. The stability analysis is based on the linearized equation system that describes the plant physics, encompassing neutronics, thermal-hydraulics and reactivity feedback, and poison dynamics. The investigation of the linear approximation may provide relevant conclusions about the system stability; in particular, stability against small perturbations may often be deduced [1]. Laplace Transform is applied to the linearized system to derive the transfer function that represents the system differential equations: its poles and zeros effectively define the system response to input perturbations. The system dynamics is studied through the analysis of the poles of the system transfer function and respect to different possible values of the moderator and fuel reactivity feedback coefficients.

1 INTRODUCTION

TRIGA Mark II is a pool-type research reactor, with the core immersed in a demineralised water tank. The water inventory gives a thermal inertia that is a significant contribution to the system stability. Typically, a buoyancy force induces a natural circulation mass flow rate across the core, due to the different water density in the pool: a column of heated water in the core is pushed upwards replacing the cold water at the top of the pool.

The perimeter of this study includes the core and the natural circulation mass flow through it, the reactor pool and the thermal power exchange with the primary cooling loop.

The nominal power in steady state condition is 250 kW with the thermal neutron flux of the order of 10^{13} #n/cm^2. This type of reactor has unique features in terms of safety: the specific composition of fuel (Uranium dispersed in a Zirconium-Hydride matrix), gives a prompt moderating effect due to the presence of Hydrogen in the ZrH lattice. The result is a strong negative reactivity coefficient that contributes to the intrinsic safety of the plant. On the opposite, the moderator has a net positive reactivity coefficient due to non-linear behaviour of the incoherent elastic scattering cross-section of water molecules, over the relevant spectrum. The latter prevails over the water density negative coefficient[1][2].

Three control rods, filled with boron carbide and boron graphite, perform active control of the reactivity. During the reactor life, burn-up of the fuel produces fission products that reduce the neutron population due to a high absorption cross section. Control rods can compensate the effects of neutron poisons until they are completely extracted. ${}^{135}_{54}$Xe and ${}^{149}_{62}$Sm have the highest neutron absorption cross section and the highest fission yield; their dynamics is therefore included in this analysis.

The linearization of the TRIGA-Mark II dynamics has been performed in previous work [3], where the scope of the analysis were the core stand alone. This work extends the perimeter of the analysis to the whole plant system, includes the moderator thermal-hydraulic feedback to the neutronics, that was neglected in preliminary works, as well as poisons accumulation effect to the neutron dynamics. Section 2 presents the non-linear equation system that describes the plant physics. Section 3 presents the linearized system of equations. Section 4 describes the method and develops a stability analysis of the system with respect to the reactivity feedback coefficient of fuel and moderator temperatures (α_f, α_m). For each value of the α_m, the analysis provides a map of the poles of the linear system for different values of the fuel reactivity coefficient α_f.

2 NON-LINEAR DYNAMIC SYSTEM OF TRIGA REACTOR

The dynamic system has 15 state variables (ψ, η_{1-6}, T_m, T_f, T_p, I, Xe, Sm, Pm, U_5) and two external input (CR, P_{ext}), in the following equations governing the system:

$$\frac{d\psi(t)}{dt} = \frac{\rho(t)-\beta}{\Lambda}\psi(t) + \sum_{i=1}^{6}\frac{\beta_i}{\Lambda}\eta_i(t) \tag{1}$$

$$\frac{d\eta_i(t)}{dt} = \lambda_i\psi(t) - \lambda_i\eta_i(t) \qquad i = 1, \dots, 6 \tag{2-7}$$

$$\frac{dT_m}{dt} = \frac{\psi P_0(1-f)}{\tau_m K_0} + \frac{T_f - T_m}{\tau_m} - \frac{\Gamma}{wM_m}(T_m - T_p) \tag{8}$$

$$\frac{dT_f}{dt} = \frac{\psi P_0 f}{\tau_f K_0} - \frac{(T_f - T_m)}{\tau_f} \tag{9}$$

$$\frac{dT_p}{dt} = \frac{\Gamma c_m}{wc_pM_p}\left(T_m - T_p\right) - \frac{P_{ext}}{c_pM_p} \tag{10}$$

$$\frac{dI(t)}{dt} = y_I\sigma_{fiss}U_5\,\psi\phi_0 - \lambda_I I \tag{11}$$

$$\frac{dXe(t)}{dt} = y_{Xe}\sigma_{fiss}U_5\,\psi\phi_0 + \lambda_I I - (\sigma_\alpha^{Xe}\psi\phi_0 + \lambda_{Xe})Xe \tag{12}$$

$$\frac{dPm(t)}{dt} = y_{Pm}\sigma_{fiss}U_5\,\psi\phi_0 - \lambda_{Pm}Pm \tag{13}$$

$$\frac{dSm(t)}{dt} = \lambda_{Pm}Pm - \sigma_\alpha^{Sm}\psi\phi_0 Sm \tag{14}$$

$$\frac{dU_5(t)}{dt} = -\sigma_\alpha^{fuel}\phi_0\psi U_5 \tag{15}$$

[1] The reactivity variation as a function of water temperature has been simulated in the MCNP model developed for the full power TRIGA reactor, in a temperature range of 21-77°C; reactivity values are positive and growing with a good 2nd-degree polynomial approximation (R^2=0.99894).

The constants are defined as follows:

$\tau_m = \frac{M_m c_m}{K_0}$; $\tau_f = \frac{C_{f0}}{K_0}$; $\Phi_0 = \frac{P_0}{E_{fiss} V_{fuel} U_5^0 \sigma_{fiss}}$; C_{f0}=[750+1.55*($T_{f\,0}$-25)]*nfe ;

Equations (1-7) describe a point reactor kinetics model, with one energy group and six delayed neutron precursors groups [4]. Equation (8) is the governing equation of the moderator temperature, with the heat transfer from the fuel to the moderator and from the moderator to the pool. Equation (9) describes the heat transfer from the fuel to the moderator; equation (10) describes the heat transfer from the pool to the cooling system; equations (11-14) describes the poison concentration dynamics depending on the neutron flux and equation (15) describes the dynamics of the U-235 concentration.

The model parameters are given in Table 1.

Table 1: Model parameters

Group		β_i/β	$\lambda_i(s^{-1})$
1		0.042	3.01
2		0.115	1.14
3		0.396	0.301
4		0.196	0.111
5		0.219	0.0305
6		0.033	0.0124
Λ	Mean neutron generation time		60 μs
β	Delayed neutron fraction		730 x 10^{-5} $\Delta K/K$
c_m	Specific heat capacity of moderator in core		4178.4 J kg^{-1} K^{-1}
M_m	Moderator total mass		22.7 kg
M_p	Water mass in pool		1.787e+4 kg
w	Weighting factor for computation of moderator average temperature in core		0.5
f	Fraction of power deposited in fuel		1
nfe	Number of fuel elements		80
y_I	Fission product yield, Iodine		0.0639
y_{Xe}	Fission product yield, Xenon		0.00237
y_{Pm}	Fission product yield, Promethium		0.01071
σ_α^{Xe}	Cross section at E = 0.025 ev, Xenon		2.65e+6 b
σ_α^{Sm}	Cross section at E = 0.025 ev, Samarium		4.1e+6 b
σ_α^{fuel}	Cross section at E = 0.025 ev, fuel		680.8 b
σ_{fiss}	Fission cross section at E = 0.025 ev		582.2 b
λ_I	Decay constant, Iodine		2.87e-5 s^{-1}
λ_{Xe}	Decay constant, Xenon		2.09e-5 s^{-1}
λ_{Pm}	Decay constant, Promethium		3.63e-6 s^{-1}
E_{fiss}	Fission energy		3.2e-11 J/#fiss
V_{fuel}	Volume of fuel		2.8e+4 cm^3
U_5^0	Value of fuel volumetric density		9.813e+19 #at/cm^3
δ_p^0	Density of inlet water at steady state		993.1 kg m^{-3}
υ	Moderator thermal expansion coefficient		245 x 10^{-6} °C^{-1}
g	Gravitational acceleration		9.81 m s^{-2}
L	Core height		0.7224 m
α_2	Factor for friction along core channels		0.1287 $kg^{-1}m^{-1}$

The meaning of the variables in the system (1-15) is the following:

ψ; η_{1-6}	= normalized variables for neutrons and six delayed neutron precursors
T_m, T_f, T_p	= average temperature of moderator, fuel and pool respectively [°C]
I, Xe, Sm, Pm, U_5	= concentration of Iodine, Xenon, Samarium, Promethium and U-235 [#at $(cm^3)^{-1}$]
ρ	= reactivity [K K^{-1}]
P_0; P_{ext}	= core power at steady state; thermal power transferred to the pool by the primary cooling circuit [W]
τ_m, τ_f	= fuel and moderator time constants [s]
K_0	= global heat transfer coefficient fuel-moderator at steady state [W/K]
Γ	= moderator mass flow rate in core (natural circulation) [Kg s^{-1}]
Φ_0	= neutron flux at steady state [#n $(cm^2)^{-1}$ s^{-1}]
C_{f0}	= thermal capacity of fuel at steady state [J $°C^{-1}$]

In particular, the function that gives the thermal capacity of Zirconium Hydride (C_{f0}) is provided by General Atomics and depends on the fuel temperature in °C; at steady state it is equal to 7.6e+4 J/°C. Neutron flux at steady state (Φ_0) can be calculated from the power, using the reaction rate; at steady state it is 4.77e+12 #n/cm^2 s.

The parameter K_0 represents the whole process of heat exchange between fuel and moderator and is equal to 2.2e+3 at steady state. In particular, the heat transfer coefficient between cladding and coolant is modelled by the Dittus-Boelter [5] correlation. In spite of its application to different physical situations (turbulent flow in narrow channels), this correlation has proven valid to describe the heat transfer process in the TRIGA whole core volume, characterized by a transition flow regime [3]. The coefficient thus obtained is able to average all the phenomena involved, among which the sub-cooling boiling in the most inner channel.

The mass flow rate of moderator in core is triggered and sustained by natural circulation, according to $\Gamma = \sqrt{\frac{\delta_p g L v}{\alpha_2 w}(T_m - T_p)}$. Its value calculated at steady state is 9.3 kg/s.

Average temperature of the moderator in the core is calculated as a weighted average between the temperatures at the inlet and outlet, according to the $T_m = wT_{out} + (1-w)T_p$, where the pool water temperature representing the moderator temperature at the core inlet.

The system reactivity $\rho(t)$ may be expressed as in (16):

$$\rho = CR + \alpha_m 10^{-5}(T_m - T_m^0) + \alpha_f 10^{-5}\left(T_f - T_f^0\right) + c[\sigma_\alpha^{Xe} Xe(t) + \sigma_\alpha^{Sm} Sm(t)] \quad (16)$$

with:

α_m= reactivity feedback coefficient of moderator temperature (pcm/°C)

α_f= reactivity feedback coefficient of fuel temperature (pcm/°C)

Control rods reactivity insertion, CR, is an external input to the system dynamics. Reactivity feedback coefficient of moderator temperature, α_m, and fuel temperature, α_f, are the parameters investigated in this work for the stability analysis. The values of the former has been calculated in [6]; the values of the latter are estimated in [3] for some different values of core power.

Poison anti-reactivity is given by a normalized reactivity variation coefficient, *c*, multiplied by the poison concentration and their respective cross sections. Experimental data of the TRIGA in Pavia allow to calculate *c*; preliminary works by Politecnico di Milano give the following result: *c* = -7.786 cm.

3 TRIGA SYSTEM LINEARIZATION

The TRIGA system linearization is performed at nominal power conditions (P_0 = 250 kW). Stationary values of state variables are given in Table 2 and comes from the solution of equation system (1-15) with left hand side = 0 (time derivative of state variable = 0), assuming steady state value for moderator temperature in core (T_m) and for U_5. Neutron and precursors normalized density at steady state is = 1 by definition. The value of inputs at steady state is such that:

- thermal power extracted from the pool by the cooling system is equal to the stationary core power (P_{ext}^0 = P_0 = 250kW);
- poison anti-reactivity is balanced by control rods insertion (the system is critical)

$$CR_0 = c[\sigma_\alpha^{Xe} Xe(t) + \sigma_\alpha^{Sm} Sm(t)] = 0.01588 \frac{\Delta K}{K} \quad (17)$$

Table 2: Value of state variables at steady state

ψ; η_{1-6}	1
T_m	41.0 °C
T_f	153.4 °C
T_p	37.78 °C
I	6.07e+14 #at/cm^3
Xe	5.39e+14 #at/cm^3
Sm	1.49e+16 #at/cm^3
Pm	9.05e+14 #at/cm^3
U_5	9.813e+19 #at/cm^3

The expressions for ρ and Γ in the non-linear system (1-15) and the variables are re-written in terms of their variation respect to the steady state value ($x = x_0 + \delta x$).

The linearization of the equation system is obtained neglecting the bilinear terms, according to perturbation theory:

$$\frac{d\delta\psi}{dt} = \left(\frac{-\beta}{\Lambda}\right)\delta\psi + \frac{10^{-5}}{\Lambda}\alpha_m\delta T_m + \frac{10^{-5}}{\Lambda}\alpha_f\delta T_f + \frac{c\sigma_\alpha^{Xe}}{\Lambda}\psi_0\delta Xe + \frac{c\sigma_\alpha^{Sm}}{\Lambda}\psi_0\delta Sm + \sum_{i=1}^{6}\frac{\beta_i}{\Lambda}\delta\eta_i\,, \quad (18)$$

$$\frac{d\delta\eta_i(t)}{dt} = \lambda_i\delta\psi(t) - \lambda_i\delta\eta_i(t) \qquad i = 1, \dots, 6\,, \quad (19\text{-}25)$$

$$\frac{d\delta T_m}{dt} = \frac{P_0(1-f)}{\tau_m K_0}\delta\psi - \frac{\delta T_m}{\tau_m} + \frac{\delta T_f}{\tau_m} - \frac{3}{2}\frac{1}{M_m w}\sqrt{\frac{\delta_p g L v}{\alpha_2 w}} * (T_m - T_p)^{\frac{1}{2}}\delta T_m +$$

$$+ \frac{3}{2}\frac{1}{M_m w}\sqrt{\frac{\delta_p g L v}{\alpha_2 w}} * (T_m - T_p)^{\frac{1}{2}}\delta T_p\,, \quad (26)$$

$$\frac{d\delta T_f}{dt} = \frac{P_0 f}{Cf_0}\delta\psi + \frac{K0(\delta T_m)}{Cf_0} - \frac{K0(\delta T_f)}{Cf_0}\,, \quad (27)$$

$$\frac{d\delta T_p}{dt} = \frac{3}{2}\frac{c_m}{w c_p M_p}\sqrt{\frac{\delta_p g L v}{\alpha_2 w}} * (T_m - T_p)^{\frac{1}{2}}\delta T_m - \frac{3}{2}\frac{c_m}{w c_p M_p}\sqrt{\frac{\delta_p g L v}{\alpha_2 w}} * (T_m - T_p)^{\frac{1}{2}}\delta T_p\,, \quad (28)$$

$$\frac{d\delta I(t)}{dt} = y_I\sigma_{fiss}\phi_0(U_5^0\delta\psi + \psi\,\delta U_5) - \lambda_I\delta I\,, \quad (29)$$

$$\frac{d\delta Xe(t)}{dt} = y_{Xe}\sigma_{fiss}\phi_0(U_5^0\delta\psi + \psi\,\delta U_5) + \lambda_I\delta I - \sigma_\alpha^{Xe}\phi_0(Xe\delta\psi + \psi\,\delta Xe) - \lambda_{Xe}\delta Xe\,, \quad (30)$$

$$\frac{d\delta Pm(t)}{dt} = y_{Pm}\sigma_{fiss}\phi_0(U_5^0\delta\psi + \psi\,\delta U_5) - \lambda_{Pm}\delta Pm\,, \quad (31)$$

$$\frac{d\delta Sm(t)}{dt} = \lambda_{Pm}\delta Pm - \sigma_\alpha^{Sm}\phi_0(Sm\delta\psi + \psi\,\delta Sm)\,, \quad (32)$$

$$\frac{d\delta U_5(t)}{dt} = -\sigma_\alpha^{fuel}\phi_0(U_5^0\delta\psi + \psi\,\delta U_5)\,. \quad (33)$$

4 LINEAR STABILITY ANALYSIS

4.1 Method

The system (20-35) can be expressed as in (36):

$$\frac{d\delta X}{dt} = f(\delta X(t), \delta u(t)) = A\delta X(t) + B\delta u(t) \quad (34)$$

with: **A** = $\left.\frac{\partial f}{\partial X}\right]_{X_0,u_0}$ and u_0=[$CR_0; P_{ext}^0$]

The matrix **A** of the dynamics provides all the necessary information on the system stability, since its eigenvalues represent the poles of the transfer function G(s) that links the Laplace transform of the system output with the Laplace transform of the input. The poles of G(s) are actually the roots of the determinant of the matrix (sI_n – A).

The linear stability analysis is performed with reference to different possible values of the reactivity feedback coefficient of moderator and fuel temperatures (α_m, α_f respectively). For a given value of α_m, α_f varies in the range [-20 ; 0] pcm/°C and all the poles fo the system are calculated. Values considered for α_m are [-2 ; 4 ; 10]. If and only if all the poles have negative real part, then the system will be asymptotically stable.

4.2 Results

The analysis shows that the TRIGA system is asymptotically stable to perturbations (control rods insertion or change in the cooling circuit capability) when the value of α_f is negative and α_f has higher absolute value than α_m. In these cases the real part of the poles is always negative (Fig. 1 and 3; real part of the closest pole to zero is -3.25e-9 s^{-1}); this means that the feedback of the fuel temperature is able to overcome the unstable dynamics of a positive coefficient of moderator temperature. The negative reactivity introduced by the fuel temperature increase prevails over the positive contribution given by the water temperature rise and brings the core to stability. Figures 2 and 4 show that the opposite cases, where the value of α_f is negative, but has absolute value ≤ α_m; the unstable dynamics of moderator prevails, as confirmed by the positive real part of some poles. Figure 5 shows the case of a stable dynamics of both moderator and fuel temperature (negative $\alpha_m$and α_f coefficients): poles have negative real part for any negative value of α_f. For the completeness of the analysis, Fig. 6 shows a case where instability is produced by a positive o zero value of α_f combined with a negative value of α_m (with higher absolute value than α_f): some poles have positive real part.

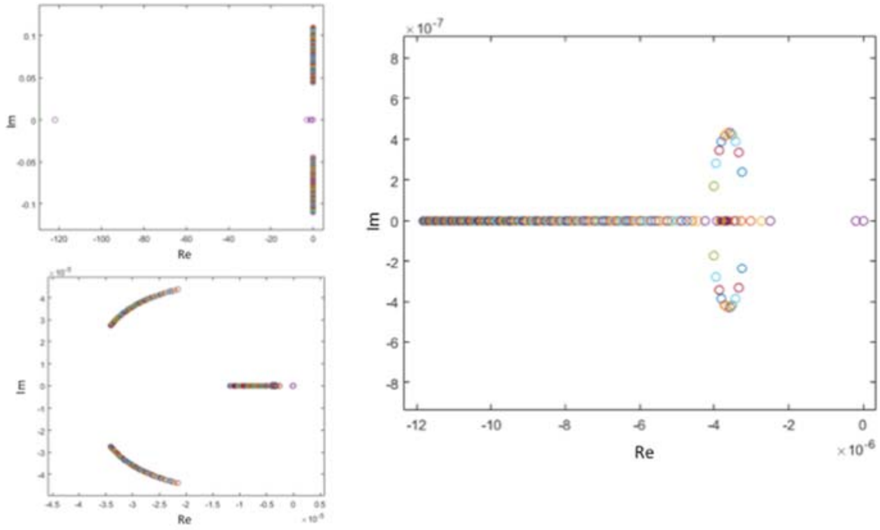

Figure 1: Poles with α_m= 4 pcm/°C and α_f=[-20;-5]pcm/°C – System stable

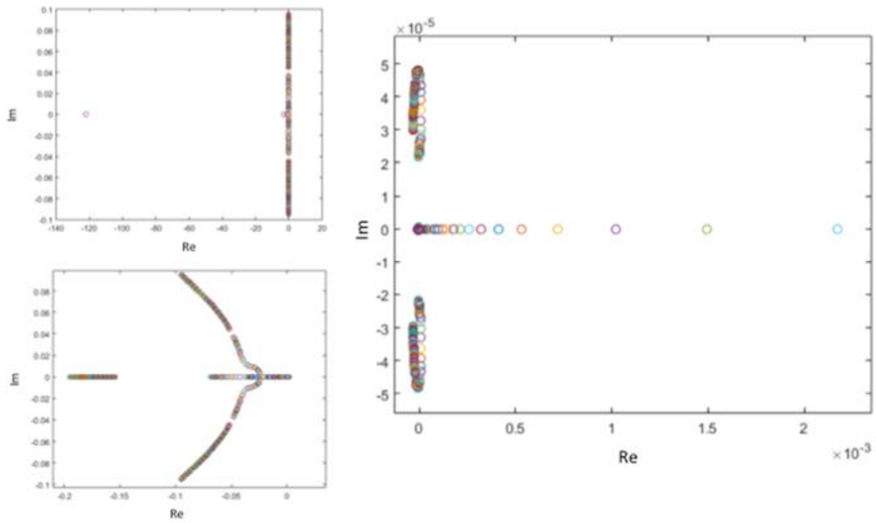

Figure 2: Poles with α_m= 4 pcm/°C and α_f=[-4;0] pcm/°C – Sistem unstable

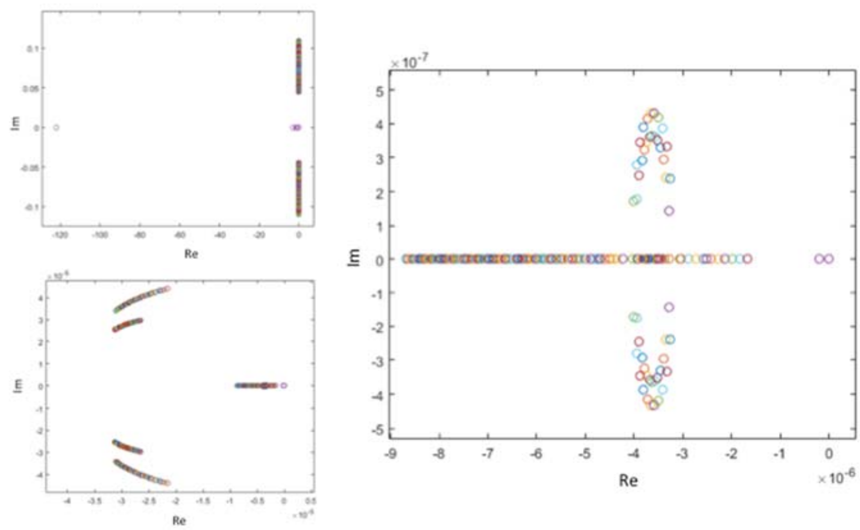

Figure 3: Poles with α_m= 10 pcm/°C and α_f=[-20;-11] – System stable

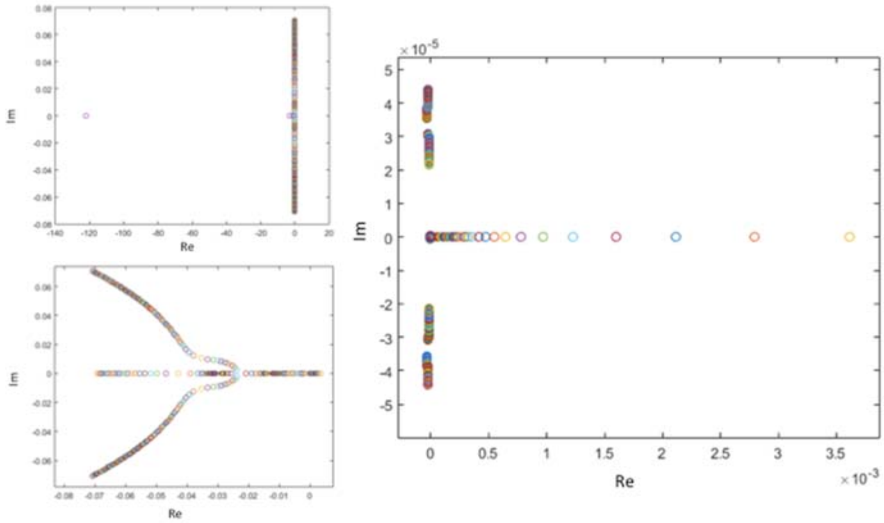

Figure 4: Poles with α_m= 10 pcm/°C and α_f=[-10;0] pcm/°C – System unstable

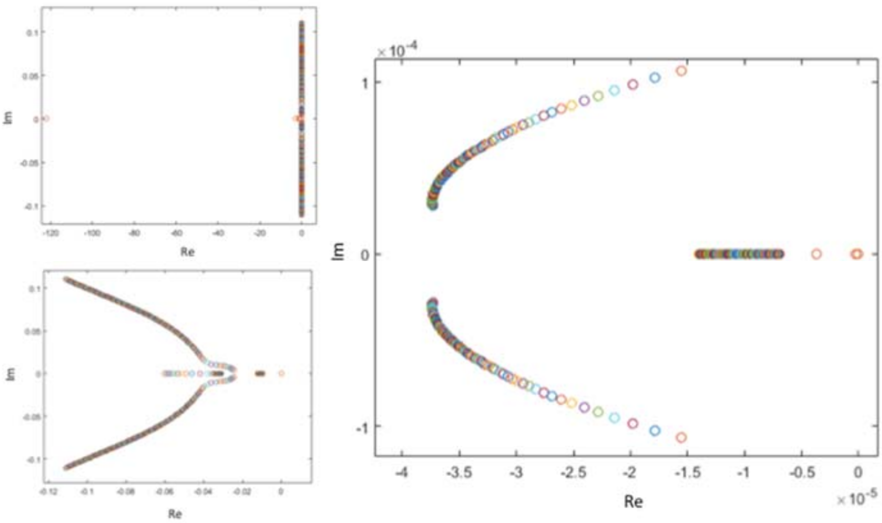

Figure 5: Poles with α_m= -2 pcm/°C and α_f=[-20;-1] pcm/°C – System stable

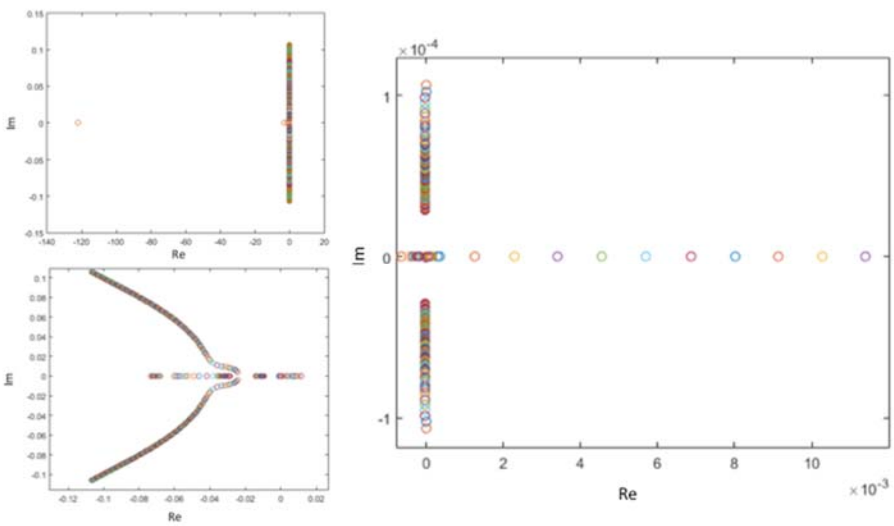

Figure 6: Poles with α_m= -2 pcm/°C and α_f=[0;+1] pcm/°C – System unstable

5 CONCLUSION

A TRIGA reactor plant system has been modelled by its governing differential equation system. The equations have been linearized to study the system stability.

The matrix of the dynamics provides the information needed to calculate the poles of the system transfer function. If and only if all the poles have negative real part, then the system will be asymptotically stable.

The analysis has been preformed for different possible values of the reactivity feedback coefficient of moderator and of fuel temperature (α_m and α_f respectively). It has been found that, for the asymptotic stability of the system, α_f must be negative and, in case α_m is positive, α_f must have absolute value strictly higher than α_m.

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