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# A MSD model for coupled analysis of pedestrian-footbridge dynamic interaction

Eleonora Lai, Maria G. Mulas\*

*Department of Civil and Environmental Engineering, Politecnico di Milano, Piazza Leonardo da Vinci 32, Milan 20133, Italy*

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## Abstract

Recent footbridges are characterized by long span, light materials and increasing slenderness which make them more sensitive to dynamic forces induced by pedestrians. While walking, the pedestrian adapts his gait to the bridge motion and interacts with the structure. At contact points, the pedestrian transmits contact forces to the bridge that, in turn, imposes a set of displacements and velocities to the pedestrian's feet. The bridge response accounting for the human-structure interaction depends both on reliable estimates of expected loading scenarios and on the accuracy of the models representing pedestrians.

This work presents a complete framework for the analysis of the human-structure dynamic interaction. To properly describe this phenomenon, the coupled equations of motion for the two mechanical systems are derived. To this aim the standard FE modeling of the structure is retained, while a new bipedal mass-spring-damper model is adopted for the pedestrian. The model is able to reproduce the sequence of single and double support phases typical of the human gait and is excited by both an equivalent bio-mechanical force and the motion at contact points with the bridge. The solution of the coupled formulation is based on a forced uncoupling of the equations, possibly associated to an iterative integration procedure. At each time instant the two systems are analyzed separately: the bridge subjected to vertical contact forces, the pedestrian to an imposed motion at his feet.

The uncoupled strategy of solution is implemented into a research code. In the case study groups of pedestrians cross a lively footbridge, whose modal properties were experimentally identified. The numerical model was developed with ANSYS. To investigate the potential of the proposed approach, numerical analyses have addressed the effect on the bridge response of both the degree of synchronization and the spatial distribution of groups of nine pedestrians. The bridge vertical accelerations reach values out of the range of comfort and show a high dependency on these parameters. The effectiveness of the proposed approach of modeling and analysis is highlighted.

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*Keywords:* human-structure interaction; MSD model; 3D Finite Element footbridge; loading scenarios, synchronization

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\* Corresponding author. Tel.: +39-02-2399-4231;

E-mail address: [mariagabriella.mulas@polimi.it](mailto:mariagabriella.mulas@polimi.it)

## 1. Introduction

Contemporary footbridge design, moved by aesthetic requirements, has led to structures characterized by greater slenderness, low damping and light materials [1]. Their natural frequencies turn out to be into the range of frequencies of human walking [2] and excessive vibration, due to human-induced loads, can lead to discomfort for pedestrians. During footbridge vibration, some form of human-structure interaction (HSI) occurs, playing an important role in the structural dynamic response. A reliable estimate of bridge dynamic response depends on the accuracy of both the expected loading scenarios and pedestrians models [3].

Aim of this work is the study of the dynamic response of a footbridge accounting for HSI in the vertical direction, following the framework adopted in a previous work [4]. A mass-spring-damper (MSD) pedestrian model is developed, that reproduces the correct human gait. The equations of motion of the coupled footbridge-pedestrian system are derived. The solution approach, based on a forced uncoupling of equations, associated to a staggered procedure, is implemented in an ad-hoc numerical code. Numerical analyses of the coupled system are carried out, to evaluate the effect of pedestrians' degree of synchronization and spatial distribution on the bridge response. Analyses with nine pedestrians uniformly distributed in space are carried out to study how transits, with the same spatial distribution but different degree of synchronization, affect the bridge response. Secondly, analyses with three groups of nine pedestrians investigate the effect on the bridge response of the shape of the spatial configuration. Three spatial distributions are considered: single line, uniform distribution and random-like distribution. Conclusions highlight the importance of both the procedure of analysis and the modeling adopted.

## 2. The case study: Seriate Footbridge

The Seriate footbridge in Fig. 1a, 63.75 m long, connects two cycle routes in the Serio Park (Milan, Italy). The width of the timber deck ranges between 2.5m at the entrance and 5m at mid-span. The slightly curved longitudinal steel girders have a rise of 1.3m. The transverse beams of the steel grid are spaced 1m apart and are subdivided in main and secondary elements. The suspension system is composed of 4 steel main pylons, 2 main suspension cables, 42 vertical hangers, 4 backstays cables and 2 stabilizing cables of opposed curvatures. The suspension system is not symmetric neither about the vertical plane crossing the longitudinal bridge axis nor about that crossing the mid-span.

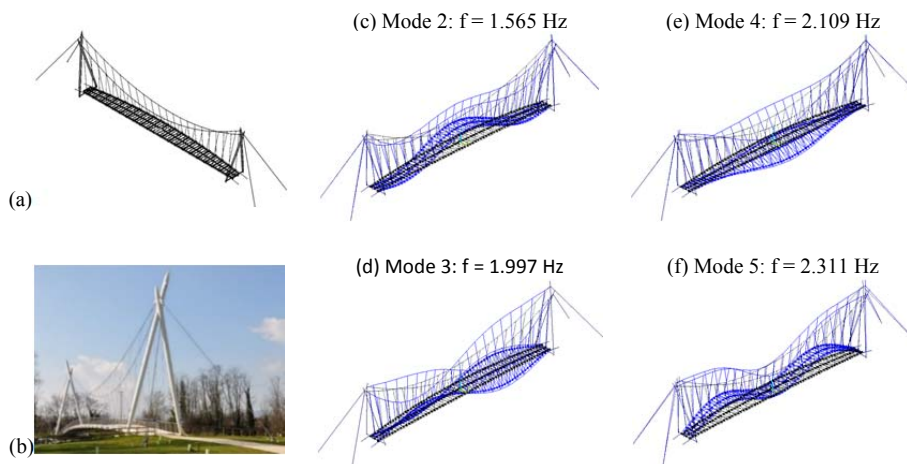


Fig. 1. Seriate Footbridge: (a) overall view, (b) FE model, (c)-(f) 2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup> and 5<sup>th</sup> natural frequencies and mode shapes (after [5]).

The Finite Element model of the bridge (Fig. 1b) was set up within the ANSYS framework, based on the original drawings. The cables geometric stiffness and the variation of configuration associated to dead loads are accounted for in a preliminary non-linear static analysis. Once the model correctly reproduces the design value of tension in cables and the deformed geometry, the modal analysis is performed. The modal properties of the footbridge were

identified during an experimental campaign. The 2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup>, 5<sup>th</sup> modal shapes and frequencies are depicted in Fig. 1c-f. The agreement between experimental and numerical value is fully satisfactory [5].

### 3. Pedestrian MSD model

To study the HSI in the vertical direction, the pedestrian is modeled through a recently proposed bipedal mechanical system [4]. The model simulates the human locomotion, characterized by the alternate sequence of Single Support Phase (SSP) and Double Support Phase (DSP) of the typical human gait cycle (Fig. 2a). SSP begins when the leading foot hits the ground and the trailing foot is off the ground, moving through the air towards the next position. DSP starts when the trailing foot hits the ground becoming the leading foot and both feet are in contact with the bridge deck. The pedestrian model is a SDOF mass-spring-damper mechanical system. Its equation of motion takes into account the interaction with the bridge and a fictitious biomechanical force  $F_b$  acting on it. The MSD system has two spring-damper legs (Fig. 2b), capable to reproduce the correct human gait (SSP and DSP in a sequence). The spring and damper parameters,  $k_h$  and  $c_h$  respectively, are computed as a function of the pedestrian weight, height and walking speed (Kim & Park [6]). The leg parameters remain constant during SSP and vary linearly during DSP, but their sum remain equal to the constant SSP value, preserving the linear behavior of the model. The mass, lumped at the center of gravity of the body, oscillates in the vertical direction only. The force  $F_b$  that excites the MSD system depends on  $k_h$ ,  $c_h$  and the walking frequency. By solving an inverse problem, in this work  $F_b$  is derived as the force that, applied on the system moving on a rigid surface, transmits a given ground reaction force through the model feet. Further details on the MSD model are found in [4].

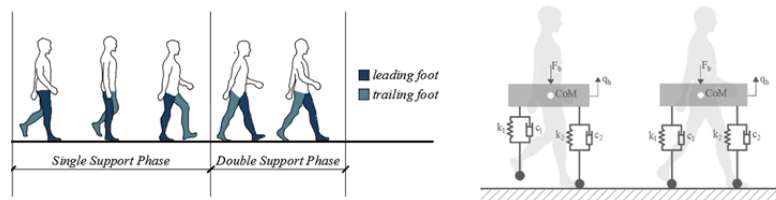


Fig. 2. Pedestrian model. (a) Human gait cycle, SSP and DSP sequence. (b) MSD model: one leg (left) and both legs (right) in contact [4].

### 4. Numerical analysis of dynamic interaction

The analytical procedure for the coupled analysis of the pedestrian-footbridge interaction, including the correct human locomotion, is implemented in a research code. The program reads in input both the bridge structural matrices (derived from the ANSYS 3D bridge model) and the pedestrian dynamic properties needed to assembly the MSD matrices or to compute the feet forces. The FE element approach for the bridge modeling adopted in the uncoupled solution strategy enables to consider both different pedestrian's models (force or mechanical system) and any type of rectilinear bridges. The code can consider groups of pedestrians having different spatial configurations, degree of synchronization, dynamic properties and step frequencies. Each pedestrian can walk freely following a rectilinear trajectory parallel to the bridge axis, which could not coincide with a line of mesh nodes.

#### 4.1. Pedestrian traffic and human-human synchronization

Numerical code can consider either a single pedestrian or a group of pedestrians. In both cases the pedestrian's trajectory is a straight line parallel to the bridge axis, but not necessarily a line of mesh nodes. The mutual positions of pedestrians in a group can be distributed according to three spatial configurations, all relying on a rectangular grid: uniform grid (Fig. 3a), "chessboard" (Fig. 3b) and "random-like" distribution (Fig. 3c). In the last case the grid lines can be not equally spaced and the position of each pedestrian, with a discrete uniform probability density, can be in each node of the grid. Pedestrians move with the same step velocity and their mutual (deterministic) distances do not change during the analysis.

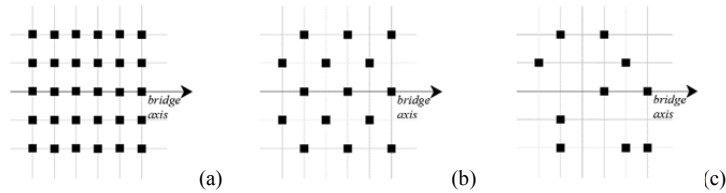


Fig. 3 – Pedestrians’ spatial configuration: (a) uniform grid; (b) “chessboard” grid; (c) “random-like” grid.

Pedestrians’ synchronization refer to the forces transmitted by each pedestrian. Three models of synchronization among pedestrians are implemented: full, partial and no synchronization. A full synchronization takes place among  $n$  pedestrians that, having the same gait properties, i.e. step velocity and frequency, are walking in phase, having entered the bridge with the same arrival time, the same support phase and the same foot at contact. In the case the pedestrians have also the same dynamic properties, they transmit the same values of force amplitudes (Fig. 4a) at each instant and can be considered perfectly correlated. In the second model, the pedestrians on the same transverse row have the same properties and the same phase, which are different from those of pedestrians of the other rows (Fig. 4b). Therefore, in this case the pedestrians on the same row transmit the same force, while pedestrians on different rows transmit different force values. Finally, it is possible to represent a group of  $n$  pedestrians without any synchronization (Fig. 4c). This is the case of out-of-phase pedestrians that, at each instant and for the whole time history, apply different forces. The lack of synchronization can be due either to differences in arrival times and support phase when entering the bridge or to different step frequencies or both.

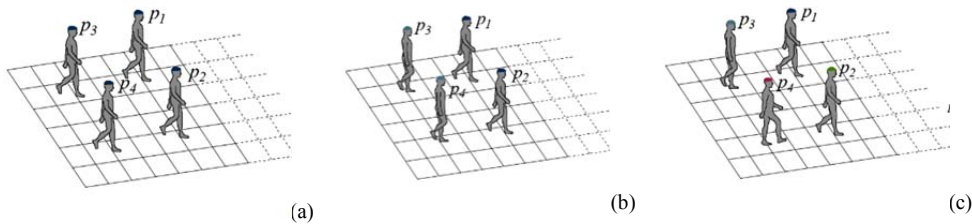


Fig. 4 – Pedestrians’ synchronization: (a) full; (b) partial; (c) no synchronization.

### 5. Numerical results

With the staggered procedure adopted to integrate the forcedly uncoupled equations of motion, the two systems are analyzed separately at each time step  $\Delta t$ . At contact point the bridge is subjected to vertical forces transmitted by the pedestrians that, in turn, are subjected to the bridge motion. The Rayleigh damping matrix is computed for a 1% damping, a value experimentally identified, on the first and fourth mode. The Newmark’s coefficient  $\beta$  and  $\gamma$  are respectively 0.25 and 0.5 (constant acceleration method);  $\Delta t$  is 0.005s, smaller than 1/10 of the period of the 14<sup>th</sup> mode. All the pedestrians considered in the analyses have a common weight (700 N), height (1.70 m) and walk at a velocity of 1.3 m/s, the mean value of a normal distribution [7]. The spring and damper parameters,  $k_h$  and  $c_h$ , of each MSD model are respectively  $2.23 \cdot 10^4$  N/m and 90.94 N·s/m [6].

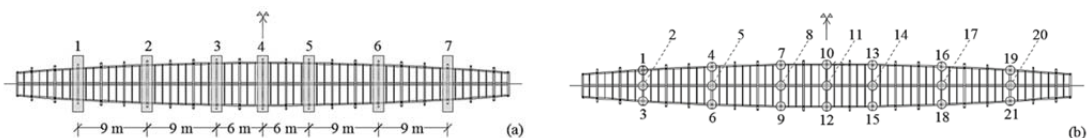


Fig. 5. Bridge response evaluation: (a) cross sections and (b) points where vertical acceleration is recorded [4].

The relevant response parameters are the time histories of vertical accelerations evaluated in seven transverse sections of the deck for three points (Fig. 5). Numerical analyses aim to evaluating the effect, on the bridge response, of both the degree of synchronization and the spatial distribution of a group of pedestrians.

### 5.1. Nine pedestrians with different degrees of synchronization

The vertical maximum accelerations produced by three group of 9 pedestrians (Fig. 3a), moving with different degrees of synchronization (full, partial and no synchronization) on a uniform grid, are analyzed. Each group is composed of three transverse rows of equal pedestrians, spaced 2m apart. The center of gravity of the group is on the longitudinal bridge axis. Pedestrians have a common step frequency of 2.0 Hz, near resonance with the 3<sup>rd</sup> mode, and are excited by the same biomechanical force. The degree of synchronization is governed by the walking phases (see §4.1.). In the unsynchronized case, pedestrian phases are distributed according to a uniform probability density function.

The comparison of vertical maximum accelerations (Fig. 6a) shows that, as expected, pedestrians perfectly correlated (full synchronization) produce the largest vertical accelerations. On the contrary, pedestrians walking without synchronization induce the lowest values. The values obtained with partially synchronized pedestrians are intermediate between the upper and lower ones, but closer to the upper values. The step frequency of pedestrians, close to the bridge 3<sup>rd</sup> natural frequency, induces a resonant phenomenon. The largest values of maximum accelerations are detected at the bridge ends, coherently with the 3<sup>rd</sup> modal shape.

Fig. 6b/c/d show the time history of vertical acceleration at node 11 (mid-span node) for the cases of full, partial and no synchronization, respectively. The bridge responses for the three levels of synchronization have a common pattern but different amplitude. Full and partial synchronization produce a similar response, with the same level of acceleration, close to  $1\text{m/s}^2$ . The response due to not synchronized pedestrians is significantly lower.

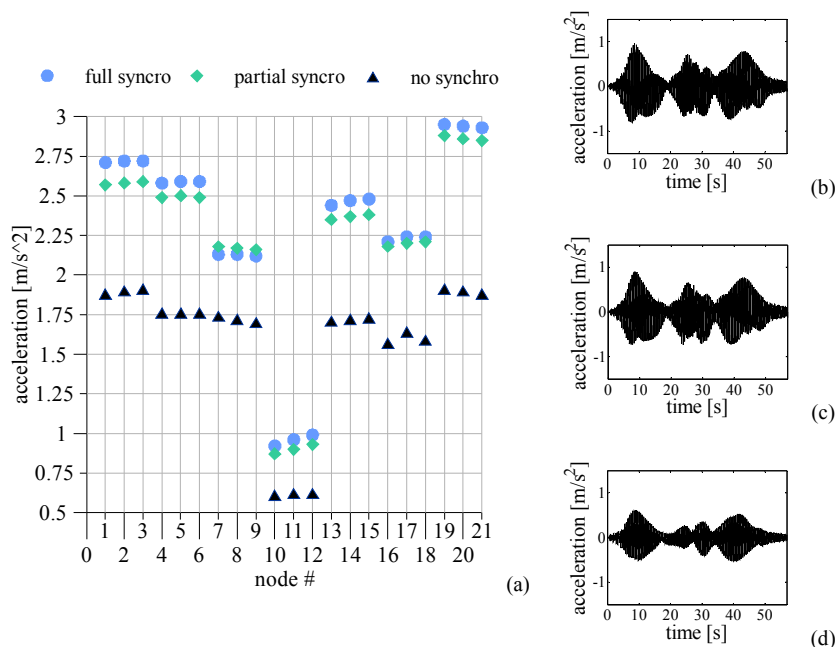


Fig. 6. Nine pedestrians with different degree of synchronization: (a) maximum vertical acceleration at each node. Mid-span acceleration time-history: (b) full synchronization, (c) partial synchronization, (c) no synchronization.

5.2. Nine pedestrians with different spatial distribution

Three groups of 9 people are considered: (a) in a row along the longitudinal bridge axis, spaced 1 m apart; (b) in an uniform distribution (Fig. 3b), with three transverse rows spaced 4 m apart; (c) in a *random-like* distribution (Fig. 3c). In this case the group’s center of gravity is not on the bridge axis and the 9 pedestrians are subdivided into 5 horizontal rows, spaced 2 m apart. In all cases the distance between the first and last pedestrians’ row is 8 m. The pedestrians have the same step frequency of 1.7 Hz and are excited by the same biomechanical force, but have different phases, distributed according to a uniform probability density function. Thus, they are not synchronized. The frequency value is intermediate between the 2<sup>nd</sup> and 3<sup>rd</sup> mode frequency and close to the mean value identified by Pachi [7].

Fig. 7a shows the maximum accelerations in the vertical direction at each node in Fig. 5b for the three cases at study. The uniform and the random-like configurations produce the largest and the lowest values, respectively. The accelerations produced by 9 people walking in a single line along the bridge axis are in an intermediate range. At each section, the time histories of accelerations have a common pattern. The comparison among the 21 time history of accelerations (not shown here for the sake of brevity) shows that both the single line distribution and the uniform one induce a maximum acceleration at a node belonging to the last section, node 19 and 20, respectively. Fig. 7b/c/d depicts the time-history of acceleration at node 11, obtained with a single line, a uniform and a random-like configuration, respectively. The beat phenomenon appearing in Fig. 7d is less significant in Figs. 7b and 7c.

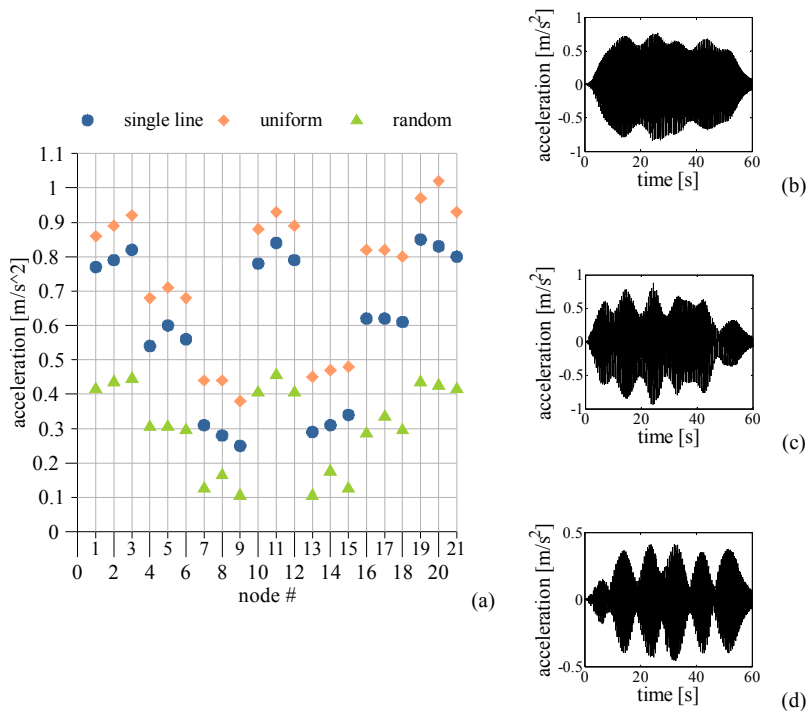


Fig. 7. Nine pedestrians with different spatial configurations: (a) maximum vertical acceleration at each node. Time-history of mid-span vertical acceleration for the configurations: (b) single-line, (c) uniform, (c) random-like.

6. Conclusions

In this work the coupled response of a lively footbridge to walking pedestrians is numerically computed with an uncoupled solution procedure. The pedestrian is modeled as a mechanical bipedal model, that simulates the human

locomotion and applies forces to the footbridge through the feet. Numerical analyses of the bridge crossed by different groups of pedestrians are performed to evaluate the effect on the bridge response of the degree of synchronization among pedestrians and of their spatial distribution. The results lead to the following conclusions: (a) as a general trend, perfectly correlated people induce accelerations almost double those induced by uncorrelated pedestrians; (b) a step frequency tuned with a bridge frequency produces remarkable accelerations, which exceed those obtained with a not tuned step frequency by a factor larger than two; (c) the spatial distribution of pedestrians affects the bridge response. A uniform distribution produces the maximum vertical accelerations, a random-like distribution the minimum values, while the accelerations induced by a single line of pedestrians have intermediate values. Since the level of bridge vibration is out of the range of comfort according to the European guidelines HiVoSS [8], the importance of both the procedures of analysis and the modeling adopted is underlined. Different trajectories can be simulated on the 3D bridge geometry. The pedestrian model introduces the mechanical properties of the human body in the analysis, modifying the bridge dynamic properties. The generality of the uncoupled solution procedure, where each system is integrated separately, allows for the adoption of different structural systems, pedestrian models and loading scenarios. Thus, the high potential of the proposed approach is highlighted.

### Acknowledgements

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