TITLE PAGE

Citation Format:

Andrea Farina, Tiziano Binzoni, Sanathana Konugolu Venkata Sekar, Antonio Pifferi, Fabrizio Martelli, "Time-resolved analytical model for Raman scattering in a diffusive medium," Proc. SPIE 10412, Diffuse Optical Spectroscopy and Imaging VI, 104120D (28 July 2017); doi: 10.1117/12.2290863.

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DOI abstract link:

https://doi.org/10.1117/12.2290863

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SPIE.

Event: European Conferences on Biomedical Optics, 2017, Munich, Germany

Time-resolved analytical model for Raman scattering in a diffusive medium

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Abstract: In this work an analytical model for the time-resolved signal emitted by a uniformly distributed Raman scatterer in a diffusive parallelepiped is derived and validated with Monte Carlo (MC) simulations.

OCIS codes: 290.5860, 170.5280.

1. Introduction

In the last years Raman scattering, due its high chemical specificity, has been used as a valuable molecular and biochemical marker for biomedical diagnosis and industrial applications [1, 2]. Applications such as the detection of chemically distinct sublayers or tomography have been demonstrated using spatially-resolved or time-gated detection [3, 4], without a deep explanation of the interplay between photon transport and Raman scattering. In classical diffuse optics it has been demonstrated that the time-resolved signal contains information of the tissue at different depths [5]. In particular, using a time-resolved approach, the statistic of photon penetration-depth can be easily assessed as well as the estimation of the probed volume [6]. Thus we expect that using the same approach for Raman, the localization of chemical specimen can be dramatically improved as well. For these motivations, in this work we derive and validate a new analytical expression for the time-resolved reflectance emitted by a uniformly distributed Raman scatterer inside a diffusive parallelepiped.

2. Analytical model

Excitation and Raman emission wavelengths are λ and λ_e, respectively. Optical properties (absorption, reduced scattering, refractive index) for the medium at λ are indicated as μ_a, μ_s', n_i and μ_{ae}, μ_{se}', n_{ie} at λ_e. These optical properties come from the optical properties of the background and from the Raman scattering molecules. We assume that Raman scattering molecules are distributed inside the whole volume of the medium with scattering coefficient μ_{sR} at λ, and μ_{sRe} at λ_e. For simplicity we will assume $\mu_{sRe} = 0$ excluding the possibility of a second Raman scattering event on the same photon. Raman scattering at λ determines an instantaneous re-emission of the scattered light at λ_e. This is equivalent to an absorption effect, i.e., $\mu_a = \mu_{ab} + \mu_{sR}$ and $\mu_s^{'} = \mu_{sb}^{'}$ where μ_{ab} and μ_{sb}' are the optical properties of the background medium.

2.1. *General model*

Light propagation through biological tissue can be described with the time-dependent diffusion equation (DE) [7]. Therefore, the Raman scattering phenomena can be described by two coupled diffusion equations for the time-dependent photon fluence rate at λ and λ_e, $\Phi(\mathbf{r},t)$ and $\Phi_e(\mathbf{r},t)$, that are respectively:

$$\left[\frac{1}{v}\frac{\partial}{\partial t} + \mu_a - D\nabla^2\right]\Phi(\mathbf{r},t) = q(\mathbf{r},t), \tag{1}$$

Diffuse Optical Spectroscopy and Imaging VI, edited by Hamid Dehghani, Heidrun Wabnitz,
Proc. of SPIE-OSA Vol. 10412, 104120D · © 2017 OSA-SPIE
CCC code: 1605-7422/17/$18 · doi: 10.1117/12.2290863

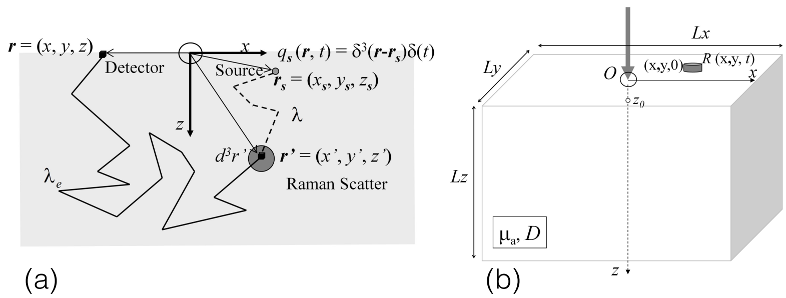

Fig. 1. (a) Source, detector and Raman scattering positions. (b) Reference system used for the derivation of the reflectance coming from a parallelepiped.

$$\left[\frac{1}{v_e}\frac{\partial}{\partial t}+\mu_{ae}-D_e\nabla^2\right]\Phi_e(\mathbf{r},t)=q_e(\mathbf{r},t), \tag{2}$$

where $D=1/[3(\mu_s^{'})]$, $D_e=1/[3(\mu_{se}^{'})]$, $v=c/n_i$ and $v_e=c/n_{ie}$, where c is the light speed in vacuum. The term $q(\mathbf{r},t)$ is the real source term at λ, while $q_e(\mathbf{r},t)$ is the virtual source term at λ_e generated by Raman scattering. Thus, equations (1) and (2) are coupled by the excitation fluence, Φ, which determines the source term $q_e(\mathbf{r},t)$ of Eq. (2). The source term, $q_e(\mathbf{r},t)$, at λ_e should account for the number of photons generated at this wavelength per unit volume and time due to Raman scattering events at λ. We can provide the decrement of Φ due to Raman scattering as

$$d\Phi(\mathbf{r},\mu_a,\mu_s^{'},t)=-\mu_{sR}\Phi(\mathbf{r},\mu_a,\mu_s^{'},t)vdt. \tag{3}$$

Consequently, the term $q_e(\mathbf{r},t)$ can be calculated as

$$q_e(\mathbf{r},t)=-d\Phi(\mathbf{r},\mu_a,\mu_s^{'},t)/(vdt)=\mu_{sR}\Phi(\mathbf{r},\mu_a,\mu_s^{'},t). \tag{4}$$

Assuming a unitary source term $q(\mathbf{r},t)=\delta^3(\mathbf{r}-\mathbf{r_s})\delta(t)$, with $\mathbf{r_s}$ position of the source, we can re-write q_e as (see Fig. 1 a)

$$q_e(\mathbf{r}^{'},t')=\mu_{sR}G(\mathbf{r_s},\mathbf{r}^{'},\mu_a,\mu_s^{'},n_i,t^{'}). \tag{5}$$

Substituting Eq. (5) in Eq. (2) and according to the Green's function theorem, the fluence rate at λ_e, $\Phi_e(\mathbf{r},t)$, due to the Raman scattering is given by

$$\Phi_e(\mathbf{r},t)=\mu_{sR}\int_0^t\int_{V'}G(\mathbf{r_s},\mathbf{r}^{'},\mu_a,\mu_s^{'},n_i,t^{'})G_e(\mathbf{r}^{'},\mathbf{r},\mu_{ae},\mu_{se}^{'},n_{ie},t-t^{'})d\mathbf{r}^{'}dt^{'}. \tag{6}$$

Equation (6) is thus the convolution of G with the same Green's function calculated at the site of the Raman scatterers and at the emission time t'. The above solution of Eq. (6) is obtained under the assumption than only single scattering Raman events contribute to the solution.

2.2. *Reflectance from a diffusive parallelepiped*

To model the effect of a laser beam impinging onto the medium, an isotropic source is placed at $\mathbf{r_s}=(0,0,z_0)$ with $z_0=1/\mu_s'$. Equation (6) is then calculated using the Green's function for a diffusive parallelepied (see Fig. 1b) obtained with the eigenfunction method under the extrapolated boundary conditions (EBC) [7]. The Green's function is then:

$$\begin{aligned}&G_{EBC}(\mathbf{r},\mathbf{r}',t)=\tfrac{v2^3}{L_x'L_y'L_z'}\sum_{l,m,n}\cos(K_lx)\cos(K_lx')\cos(K_my)\cos(K_my')\\&\sin[K_n(z+2AD)]\sin[K_n(z'+2AD)]\exp[-(K_l^2+K_m^2+K_n^2)Dv(t-t')-\mu_av(t-t')],\\&\text{with } L_x'=L_x+4AD,\ L_y'=L_y+4AD,\ L_z'=L_z+4AD,\\&K_l=\tfrac{(2l-1)\pi}{L_x'},\ K_m=\tfrac{(2m-1)\pi}{L_y'},\ K_n=\tfrac{n\pi}{L_z'},\ l,m,n=1,2,3,4,5,.....\end{aligned} \tag{7}$$

where the coefficient A is a function of the refractive index of the diffusive (n_i) and external (n_o) medium at λ [7]. Then, making use of Eq. (7) and of the ortho-normality property of the eigenfunctions, the solution of Eq. (6) for the time-resolved fluence rate becomes:

$$\Phi_{eRamanEBC}(\mathbf{r},t) = \frac{\mu_{sR}\, v\, v_e\, 2^3}{L'_x L'_y L'_z} \sum_{l,m,n} \cos(K_l x)\cos(K_m y)$$
$$\times \sin[K_n(z+2A_e D_e)]\sin[K_n(z_0+2AD)][(D_e v_e - Dv)(K_l^2+K_m^2+K_n^2)+(\mu_{ae}v_e-\mu_a v)]^{-1}$$
$$\times\left\{\exp[-(K_l^2+K_m^2+K_n^2)D\,vt-\mu_a vt]-\exp[-(K_l^2+K_m^2+K_n^2)D_e\,v_e t-\mu_{ae}v_e t]\right\}, \tag{8}$$

with $L'_x = L_x + 4AD, L'_y = L_y + 4AD, L'_z = L_z + 4AD,$
$K_l = \frac{(2l-1)\pi}{L'_x},\ K_m = \frac{(2m-1)\pi}{L'_y},\ K_n = \frac{n\pi}{L'_z}, l,m,n = 1,2,3,4,5,....$

where the coefficient A_e is a function of the refractive index of the diffusive (n_{ie}) and external (n_o) medium at λ_e [7]. Finally the time-resolved Raman reflectance is calculated using the Fick's law:

$$R_{eRamanFick}(x,y,t) = \mathbf{J}_{eRaman}(x,y,z=0,t)\cdot(-\hat{k}) = D_e \frac{\partial \Phi_{eRaman}(x,y,z=0,\lambda_e,t)}{\partial z}, \tag{9}$$

that by using Eq. (8) becomes

$$R_{eRamanFick}(x,y,t) = \frac{D_e \mu_{sR}\, v\, v_e\, 2^3}{L'_x L'_y L'_z} \sum_{l,m,n} \cos(K_l x)\cos(K_m y) K_n$$
$$\times \cos[K_n(2A_e D_e)]\sin[K_n(z_0+2AD)][(D_e v_e - Dv)(K_l^2+K_m^2+K_n^2)+(\mu_{ae}v_e-\mu_a v)]^{-1}$$
$$\times\left\{\exp[-(K_l^2+K_m^2+K_n^2)D\,vt-\mu_a vt]-\exp[-(K_l^2+K_m^2+K_n^2)D_e\,v_e t-\mu_{ae}v_e t]\right\}, \tag{10}$$

with $L'_x = L_x + 4AD, L'_y = L_y + 4AD, L'_z = L_z + 4AD,$
$K_l = \frac{(2l-1)\pi}{L'_x},\ K_m = \frac{(2m-1)\pi}{L'_y},\ K_n = \frac{n\pi}{L'_z}, l,m,n = 1,2,3,4,5,.....$

3. Comparison between model and Monte Carlo simulations

To validate Eq.(10) an existing MC code based on CUDA framework [8] has been adapted to work in time-domain and to implement the Raman scattering event. In the code the probability of an absorption, scattering or Raman scattering event has been evaluated as $p_i = \frac{\mu_i}{\sum \mu_i}$ where i represents the different events and μ_i the interaction coefficients. For simplicity an isotropic scattering has been considered. A good agreement has been found between MC and the analytical model over a wide range of optical properties and source-detector separations. In Fig.2 we show a particular case. The well known discrepancy at early times is due to the diffusion approximation.

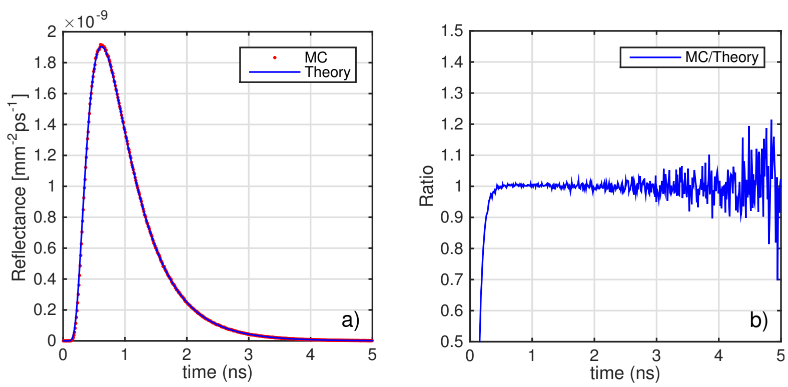

Fig. 2. a) Simulated and theoretical reflectance at 20 mm source-detector separation. μ_{ab}=5e-3 mm^{-1}, μ_{abe}=7e-3 mm^{-1}, μ'_{sb}=μ'_{sbe}=1 mm^{-1}, μ_{sR}=1e-3 mm^{-1}, n_i=n_{ie}=1.4. (b) Ratio between the two curves.

4. Conclusions

A new solver for the time-resolved signal, coming from distributed Raman scatterers inside a diffusive medium with parallelepiped geometry, has been derived under the diffusion approximation and validated with MC simulations. Diffuse Raman signal is often overwhelmed by a background fluorescence signal. A further refinement of the model discussed in this work, taking into account the fluorescence emission, is available in ref [9].

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