

# Modeling dependent competing failure processes with degradation-shock dependence

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## Abstract

In this paper, we develop a new reliability model for dependent competing failure processes (DCFPs), which accounts for degradation-shock dependence. This is a type of dependence where random shock processes are influenced by degradation processes. The degradation-shock dependence is modeled by assuming that the intensity function of the nonhomogeneous Poisson process describing the random shock processes is dependent on the degradation processes. The dependence effect is modeled with reference to a classification of the random shocks in three “zones” according to their magnitudes, damage zone, fatal zone, and safety zone, with different effects on the system’s failure behavior. To the best of the authors’ knowledge, this type of dependence has not yet been considered in reliability models. Monte Carlo simulation is used to calculate the system reliability. A realistic application is presented with regards to the dependent failure behavior of a sliding spool, which is subject to two dependent competing failure processes, wear and clamping stagnation. It is shown that the developed model is capable of describing the dependent competing failure behaviors and their dependence.

## Keywords

Dependent competing failure processes; degradation; shock; degradation-shock dependence; Monte Carlo simulation; sliding spool

## Highlights

- A new reliability model is developed for degradation-shock dependence.
- The influence of degradations on random shocks is considered.
- Zone effect of random shocks magnitudes is considered.
- A case study of a sliding spool is conducted.

1 **Acronyms**

2 DCFP Dependent competing failure processes

3 DTS Degradation-Threshold-Shock

4 NHPP Nonhomogeneous Poisson process

5 **Notation**

6  $x(t)$  Degradation at time  $t$

7  $\theta$  Model parameters for the soft failure process

8  $f_{\theta}(\mathbf{x})$  Probability density function of  $\theta$

9  $F_{\theta}(\mathbf{x})$  Cumulative distribution function of  $\theta$

10  $D$  Soft failure threshold

11  $Y_i$  Magnitude of the  $i$  th shock

12  $W_i$  Damage caused by the  $i$  th shock

13  $\alpha$  Shock damage proportional coefficient

14  $f_Y(y)$  Probability density function of  $Y_i$

15  $F_Y(y)$  Cumulative distribution function of  $Y_i$

16  $\lambda(t)$  Intensity of random shocks at time  $t$

17  $\lambda_0$  Initial intensity of random shocks

18  $N(t)$  Number of random shocks that have arrived by time  $t$

19  $N_1(t)$  Number of random shocks that have arrived in the damage zone by time  $t$

20  $N_2(t)$  Number of random shocks that have arrived in the fatal zone by time  $t$

21  $N_3(t)$  Number of random shocks that have arrived in the safety zone by time  $t$

22  $H$  Hard failure threshold due to damage shocks

23  $\gamma_a, \gamma_b, \gamma_c, \gamma_d$  Dependence factors

## 1 **1. Introduction**

2 Industrial components, systems and products are subject to multiple degradation processes and failure modes,  
3 i.e. wear, corrosion, fracture, fatigue, and etc. [1, 2]. Such processes are often “competing” in leading to failure [3]  
4 and may be dependent on each other in a number of ways, in which case they are referred to as dependent competing  
5 failure processes (DCFPs) [4]. Various dependencies have been discussed in literature, e.g., the statistical dependency  
6 described by joint probability distributions [5, 6], copulas [7, 8], and the functional dependency described by failure  
7 propagation and isolations [9-11], etc. In this paper, we consider the dependency between two types of failure  
8 processes: degradation failure processes (or soft failures) and catastrophic failure processes (or hard failures) [12]. In  
9 practice, soft failures are modeled by an evolving degradation process and hard failures are modeled by a random  
10 shock process. Models that consider both degradation and random shocks are called Degradation-Threshold-Shock  
11 (DTS) models [13, 14]. In literature, most DCFPs are modeled by DTS models.

12 An early DTS model was developed by Lemoine and Wenocur [15], where degradation is modeled by a diffusion  
13 process and fatal shocks are modeled by a Poisson process. Soft failures are defined with respect to the degradation  
14 reaching pre-established critical values and hard failures arrive in correspondence of the occurrence of fatal shocks.  
15 Klutke and Yang [16] have developed an availability model for systems subject to degradations and random shocks,  
16 where degradations occur at a constant rate and shocks follow a Poisson process. Marseguerra et al. [17] and Barata  
17 et al. [18] investigate condition-based maintenance optimizations for deteriorating systems subject to degradations  
18 and random shocks. Li and Pham [12, 19] develop a reliability model and an maintenance model for multistate  
19 degraded systems considering two degradation processes and a shock process. In a recent paper by Lin et al. [20], a  
20 DTS model has been developed, where the degradation process is modeled by a continuous-time semi Markov  
21 process and the shock process is modeled using a homogeneous Poisson process.

22 To consider the dependence between degradations and shocks, Peng et al. [4] assume that the arriving shock  
23 would bring a sudden increase to the normal degradation process and develop a probabilistic model to calculate the  
24 system’s reliability. Wang and Pham [21] use a similar approach to model DCFPs and determine the optimal imperfect  
25 preventive maintenance policy. Keedy and Feng [22] apply the model in [4] on a stent, where the degradation process  
26 is modeled by a PoF (Physics of failures) model. Song et al. [23] consider the reliability of a system whose  
27 components are subject to the DCFPs and distinct component shock sets [24].

28 Apart from the dependence considered in [4], Jiang et al. [25] consider a different kind of dependence, where  
29 the threshold of the degradation process is shifted by shocks. Rafiee et al. [26] develop a model in which the

1 degradation rates are shifted by different shock patterns. Wang et al. [27] explore another type of dependence by  
2 assuming that a sudden change to the failure rate is caused by the arriving shocks. Jiang et al. [1] consider dependence  
3 where shocks are divided into different zones, where each different zone has a different effect on the degradation  
4 process.

5 As reviewed above, the majority of existing DCFP models focuses on modeling the influence of random shocks  
6 on degradations (shock-degradation dependence). In engineering practice, however, there are components, systems  
7 and products that are subject to DCFPs in which random shocks are influenced by degradations (degradation-shock  
8 dependence). A typical engineering example is the sliding spool, which is a widely applied component in hydraulic  
9 control systems [28]. A sliding spool is subject to two failure processes: wear, which is modeled as a degradation  
10 process and clamping stagnation, which is modeled by random shocks [29]. As shown in Sasak and Yamamot [30],  
11 the process of clamping stagnation is dependent on the wear process: as the wear process progresses, more wear  
12 debris is generated. The debris will contaminate the hydraulic oil and further, increase the likelihood of clamping  
13 stagnation [31, 32], which causes the degradation-shock dependence.

14 Compared to shock-degradation dependence, there are fewer models considering the degradation-shock  
15 dependence. In [33], Fan et al. consider the degradation-shock dependence by assuming that the magnitude of random  
16 shocks is increased by the degradation processes. Bagdonavicius et al. [34] have developed a model to consider the  
17 degradation-shock dependence among a linear degradation process and several extreme shocks. In their model, the  
18 intensities of the random shocks are assumed to depend only on the degradation level. Ye et al. [35] model the  
19 degradation-shock dependence by assuming that the destructive probability of a shock is determined by the remaining  
20 hazard of the degradation process. In [13, 36, 37], the intensity of the random shock is assumed to be a piecewise  
21 function of the degradation level.

22 The aforementioned models only consider a relatively simple type of random shock: the extreme shocks (also  
23 referred to as traumatic shocks), where the arrival of a shock in the critical region causes the immediate failure of the  
24 product [38]. In practice, however, more complex shock patterns, such as cumulative shocks [39],  $\delta$ -shocks [38],  
25 mixed shocks [40], run shocks [41], are frequently encountered. Since the 1990s, works on imperfect fault coverage  
26 models [42-44] categorize the effects of a component fault into three types: single-point failure, which causes  
27 immediate system failure, permanent coverage, which causes cumulative damages to the system and transient  
28 restoration, which has no effect on the system. Based on a similar consideration, Jiang et al. discuss a shock pattern  
29 called zone shocks, where shocks with different magnitudes might have different impacts on the failure behavior of

1 the product [1]. Zhou [45] reports an example of the zone shocks based on experimental results of a sliding spool:  
2 the sliding spools exhibit different failure behaviors when they are tested under different levels of shock magnitudes  
3 (measured by sizes of wear debris). To the best of our knowledge, the zone shocks process has only been considered  
4 in the shock-degradation dependence model [1]. However, after a careful literature review, we find that there are no  
5 existing models that consider both the degradation-shock dependence and the zone shocks.

6 In this paper, we develop a new DCFP model which considers degradation-shock dependence and zone shocks.

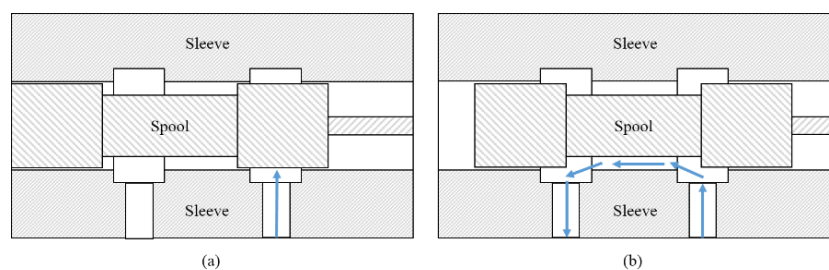
7 The developed model contributes to the existing scientific literature and to engineering practice in two aspects:

- 8 ● it allows for explicit modeling of the degradation-shock dependence, where random shocks are influenced  
9 by degradation processes and
- 10 ● it allows for the consideration of zone shocks in the degradation-shock dependence model, where shocks  
11 with different magnitudes have different impacts on the components failure behaviors.

12 The remainder of the paper is organized as follows. Sect. 2 introduces the engineering motivation of the present  
13 research. In Sect. 3, the new reliability model is developed to describe failure behaviors of systems subject to  
14 competing soft failures and hard failures involving degradation-shock dependence. In Sect. 4, the developed model  
15 is applied to model the dependent failure behavior of a sliding spool subject to wear and clamping stagnation. Finally,  
16 the paper is concluded with a discussion on potential future works in Sect. 5.

## 17 2. Engineering problem setting

18 The model developed in this paper is motivated by the actual engineering problem of modeling the reliability of  
19 a sliding spool. Sliding spools are critical control components in hydraulic control systems [28]. As illustrated in  
20 Figure 1, a sliding spool is composed of a spool and a sleeve, where the spool slides in the sleeve to control hydraulic  
21 oil flows [46].



22  
23 Figure 1 Illustration to a sliding spool: (a) Closed position; (b) Open position [46]

24 A sliding spool is subject to two failure mechanisms [29]. One is wear between the spool and the sleeve, and the  
25 other is clamping stagnation (also referred to as hydraulic locking or sticking), in which the spool is stuck in the

1 sleeve. In practice, wear can be modeled by a physical deterioration model, for example, the linear Archard model  
2 suggested by Liao [29]. The modeling of clamping stagnation, however, is more complex because the factors  
3 contributing to clamping stagnation are much more varied. According to the survey by Sasak and Yamamot [30], one  
4 of the major causes of clamping stagnation is the sudden appearance of pollutant in the hydraulic oil, which can be  
5 modeled by a random shock model. The pollutant may come inside the hydraulic system from the outside  
6 environment or be generated by the hydraulic system itself. One significant source of internal pollutant is the wear  
7 debris generated due to the wear of the sliding spool. Therefore, the random shock process is dependent on the wear  
8 process: As the wear process progresses, more wear debris is generated. The debris will contaminate the hydraulic  
9 oil and further, increase the likelihood of clamping stagnation [31, 32]. This kind of dependence, caused by the  
10 influence of degradation on shocks, needs to be considered when developing the reliability model of the spool valve.

11 Furthermore, experimental results show that the most harmful effect of shocks is generated by wear debris,  
12 whose sizes are either close to or much smaller than the clearance of the sliding spool [30, 45, 47]. This is because  
13 the clamping stagnation is caused by two failure mechanisms, immediate stagnation and cumulative stagnation [45].  
14 When a particle with a size close to the clearance is generated, it causes immediate stagnation of the sliding spool  
15 [45], as shown in Figure 2 (a). If particle sizes are smaller than the clearance, the particles can enter the clearance  
16 with the hydraulic oil and form filter cakes cumulatively [48]. When the filter cakes become large enough, cumulative  
17 stagnation occurs, as shown in Figure 2 (b). According to Zhou [45], particles whose sizes are greater than the  
18 clearance have little effect on clamping stagnation, because they are blocked outside the clearance. Hence, based on  
19 their magnitudes, the random shocks affecting clamping stagnation can be classified into three zones with different  
20 effects on the clamping stagnation. In other words, the sliding spool is subject to zone shocks.

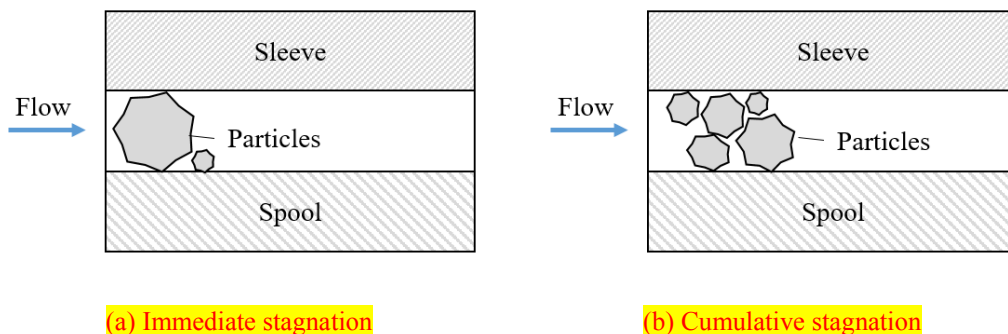


Figure 2 Two failure mechanisms leading to clamping stagnation [45]

21 Hence, to accurately describe the failure behavior of components like the sliding spool, a new model for DCFPs  
22 is needed, which allows for: (1) the consideration of the degradation-shock dependence, where random shocks are

influenced by degradation processes and (2) the modeling of zone shocks, where random shocks with different magnitudes have different effects on the products' failure behaviors. This is the motivation of the new DCFP model, which will be developed in Sect. 3.

### 3. DCFP model considering degradation-shock dependence

#### 3.1 System descriptions

As shown in Figure 3, we consider a DCFP that comprises two failure processes: soft failures due to a degradation process (Figure 3 (a)) and hard failures due to random shocks. According to their magnitudes, random shocks are divided into three zones: damage zone, fatal zone and safety zone [1]. As shown in Figure 3 (b-1), a shock in the damage zone generates a cumulative damage to the system; as shown in Figure 3 (b-2), a shock in the fatal zone causes immediate failure of the system; a shock in the safety zone has no effect on the system's failure behavior. Note that the three shock zones in Figure 3 are shown for illustrative purposes only. In different applications, case-specific shock zones can be defined based on the magnitude of the shocks.

Degradation-shock dependence exists among the failure processes, that is, the arrival rate of the random shocks is dependent on the degradation levels, as illustrated in Figure 3. Failures occur whenever one of the three events happens:

- the degradation process reaches its threshold (denoted by  $T_1$  in Figure 3);
- the cumulative damage resulting from the shocks in the damage zone exceeds its threshold (denoted by  $T_2$  in Figure 3);
- a shock in the fatal zone occurs (denoted by  $T_3$  in Figure 3).

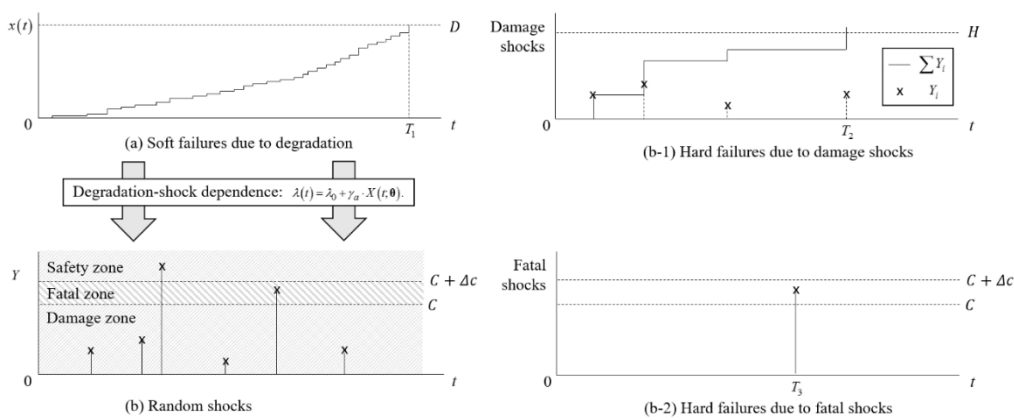


Figure 3 Soft failures due to degradation and hard failures due to random shocks

1 Additional assumptions of this model include:

2 (1) The degradation process can be modeled by the general path model proposed by Lu and Meeker [49]:

$$3 \quad x(t) = X(t; \boldsymbol{\theta}), \quad (1)$$

4 where  $x(t)$  is the degradation measure and  $X(\cdot)$  is the deterministic degradation path, which can be  
5 obtained from knowledge of physics of failure [50]. Vector  $\boldsymbol{\theta}$  in (1) contains a set of random parameters  
6 that represent the influence of uncertainties. It should be noted that the general path model in (1) is a  
7 general degradation model, for each specific case, the form of  $X(\cdot)$  in (1) could be specified to describe  
8 the case-specific degradation mechanism.

9 (2) The random shocks can be modeled by a nonhomogeneous Poisson process (NHPP) with intensity function  
10  $\lambda(t)$ , so that the arrival shocks prior to  $t$ , denoted by  $N(t)$ , follow a Poisson distribution with mean

$$11 \quad a(t) = \int_0^t \lambda(u) du \quad [51]:$$

$$12 \quad P(N(t) = n) = \frac{e^{-a(t)} a(t)^n}{n!}. \quad (2)$$

13 (3) The magnitude of each shock, denoted by  $Y_i$ , is an independent and identically distributed (i.i.d.) random  
14 variable.

15 (4) The damage caused by the  $i$ th shock in damage zone, denoted by  $W_i$ , is proportional to its magnitude  $Y_i$   
16 with coefficient  $\alpha$ :

$$17 \quad W_i = \alpha Y_i. \quad (3)$$

18 (5) To consider the degradation-shock dependence, we introduce a dependence factor  $\gamma_a$  and assume:

$$19 \quad \lambda(t) = \lambda_0 + \gamma_a \cdot X(t; \boldsymbol{\theta}). \quad (4)$$

20 Remark 1: the description of the shock damage in the form of equation (3) belongs to a type of commonly used  
21 damage model, which assumes that the shock damage is linearly dependent on the shock magnitude. Similar damage  
22 models can be found in [1, 52].

23 Remark 2: the assumption that the intensity of the NHPP is a linear function of the degradation level is adapted  
24 from the assumption proposed by Mercer [53]. In Mercer's model, the traumatic failure intensity is of the form  
25  $\lambda(t) + c \cdot Z(t)$ , where  $Z(t)$  is a degradation function describing the piecewise constant degradation process. Also,



1 note that  $\lambda_0$  in (4) denotes the initial intensity of the NHPP.

### 2 3.2 Modeling zone shocks

3 Once a random shock arrives, we should first classify it into one of the three zones according to its magnitude.

4 Let  $P_1, P_2, P_3$  denote the probabilities that the  $i$ th shock belongs to the damage, fatal and safety zone, respectively.

5 It is clear that  $P_1 + P_2 + P_3 = 1$ . Because the magnitude of each shock is an i.i.d. random variable, the value of  $P_1, P_2$

6 and  $P_3$  can be easily determined from the distribution of  $Y_i$ .

7 Let us define  $N(t)$  as the number of shocks which has arrived prior to time  $t$  and  $N_1(t), N_2(t), N_3(t)$  be  
8 the number of shocks that have fallen into the damage, fatal and safety zone prior to time  $t$ , respectively. According

9 to [54], if a NHPP with intensity  $\lambda(t)$  is randomly split into two sub-processes with probabilities  $P_1$  and  $P_2$ ,

10 where  $P_1 + P_2 = 1$ , then, the resulting sub-processes are independent NHPPs with intensities  $P_1\lambda(t)$  and  $P_2\lambda(t)$ .

11 The conclusion can be easily generalized to the three-sub-processes case, which leads to the following proposition:

12 Proposition 1: The number of shocks falling into damage and fatal zones, denoted by  $N_1(t)$  and  $N_2(t)$   
13 respectively, follow NHPPs with intensity functions  $P_1\lambda(t)$  and  $P_2\lambda(t)$ .

14 From Proposition 1, the probability mass function of  $N_1(t)$  and  $N_2(t)$  can be derived:

$$\begin{aligned}
 P(N_1(t) = n) &= \frac{e^{-P_1 a(t)} (P_1 a(t))^n}{n!}, \\
 P(N_2(t) = n) &= \frac{e^{-P_2 a(t)} (P_2 a(t))^n}{n!},
 \end{aligned}
 \tag{5}$$

16 where  $a(t) = \int_0^t \lambda(u) du$ . Proposition 1 and equation (5) will be used in the next section to develop the reliability

17 model.

### 18 3.3 Reliability modeling

19 It can be seen from Figure 3 that a reliable system at time  $t$  should satisfy the following conditions: (1) the  
20 degradation process does not exceed its threshold  $D$  before  $t$ ; (2) no shocks in the fatal zone occur before  $t$  and  
21 (3) the cumulative damage resulting from shocks in the damage zone does not exceed its threshold  $H$  before  $t$ .

22 Therefore, we can express the reliability at time  $t$  as:

$$R(t) = \sum_{i=0}^{\infty} P\left(X(t; \mathbf{0}) < D, \sum_{j=1}^{N_1(t)} W_j < H, N_2(t) = 0 \mid N_1(t) = i\right) \cdot P(N_1(t) = i).
 \tag{6}$$

1 Equation (6) comes from a direct application of the total probability formula.

2 For ease of illustration, we define three events  $E_1, E_2, E_3$  and let them denote the events  
3  $X(t; \boldsymbol{\theta}) < D, \sum_{j=1}^{N_1(t)} W_j < H, N_2(t) = 0$ , respectively. Because the degradation-shock dependence is considered (see  
4 (4)),  $E_1, E_2, E_3$  are dependent. However, once the value of  $\boldsymbol{\theta}$  is fixed, the three events become conditional-  
5 independent [50]. Therefore, we can apply the total probability formula on  $\boldsymbol{\theta}$  in (6), which leads to:

$$6 \quad R(t) = \sum_{i=0}^{\infty} \int_{\boldsymbol{\theta}} P(E_1, E_2, E_3 | N_1(t) = i, \boldsymbol{\theta} = \mathbf{x}) \cdot P(N_1(t) = i | \boldsymbol{\theta} = \mathbf{x}) \cdot f_{\boldsymbol{\theta}}(\mathbf{x}) d\mathbf{x}, \quad (7)$$

7 where  $f_{\boldsymbol{\theta}}(\mathbf{x})$  is the probability density function of  $\boldsymbol{\theta}$ . Considering the fact that  $E_1, E_2, E_3$  are conditional-  
8 independent given  $\boldsymbol{\theta}$ , (7) becomes:

$$9 \quad R(t) = \sum_{i=0}^{\infty} \int_{\boldsymbol{\theta}} P(E_1 | A, B) \cdot P(E_2 | A, B) \cdot P(E_3 | A, B) \cdot P(A | B) \cdot f_{\boldsymbol{\theta}}(\mathbf{x}) d\mathbf{x}, \quad (8)$$

10 where event  $A$  denotes  $N_1(t) = i$  and event  $B$  denotes  $\boldsymbol{\theta} = \mathbf{x}$ . To use (8) to calculate the reliability, expressions  
11 for  $P(E_1 | A, B)$ ,  $P(E_2 | A, B)$ ,  $P(E_3 | A, B)$  as well as  $P(A | B)$  should be determined.

12 To determine  $P(E_1 | A, B)$ , first note that it denotes the probability that  $X(t; \boldsymbol{\theta}) < D$  given that  $\boldsymbol{\theta} = \mathbf{x}$ . Because  
13  $X(\cdot)$  is a deterministic degradation path model and  $\boldsymbol{\theta}$  is the only random variable [48], we have

$$14 \quad P(E_1 | A, B) = \begin{cases} 1 & \text{if } X(t; \mathbf{x}) < D, \\ 0 & \text{if } X(t; \mathbf{x}) \geq D. \end{cases} \quad (9)$$

15 The probability  $P(E_2 | A, B)$  is expressed as

$$16 \quad P(E_2 | A, B) = P\left(\sum_{j=1}^{N_1(t)} W_j < H \mid N_1(t) = i, \boldsymbol{\theta} = \mathbf{x}\right), \quad (10)$$

17 which can be further expanded once the distribution of  $Y_i$  is determined.

18 The probability  $P(E_3 | A, B)$  is the probability that no fatal shocks occur given  $\boldsymbol{\theta} = \mathbf{x}$  and  $N_1(t) = i$ . From  
19 equation (4) and Proposition 1, we have

$$20 \quad P(E_3 | A, B) = \exp\left\{-P_2 \int_0^t (\lambda_0 + \gamma_a \cdot X(u; \mathbf{x})) du\right\}, \quad (11)$$

21 where  $P_2$  is the probability that the arrival shock belongs to the fatal zone.

22 Finally, from (5),  $P(A | B)$  is given by:

$$P(A|B) = \frac{e^{-P_1\Lambda(t;\mathbf{x})} (P_1\Lambda(t;\mathbf{x}))^i}{i!}, \quad (12)$$

where  $P_1$  is the probability that the arrival shock belongs to the damage zone and  $\Lambda(t;\mathbf{x})$  is given by:

$$\Lambda(t;\mathbf{x}) = \int_0^t (\lambda_0 + \gamma_a \cdot X(u;\mathbf{x})) du. \quad (13)$$

The reliability model developed in (8) is an extension to the model in [1], obtained by considering the degradation-shock dependence described by (4). If we let  $\gamma_a = 0$ , which indicates that there is no dependency between the degradation process and the shock process, the reliability model in (8) degenerates to

$$R(t) = \sum_{i=0}^{\infty} P\left(X(t) < D, \sum_{j=1}^{N_i(t)} W_j < H\right) \frac{e^{-\lambda(P_1+P_2)t} (\lambda P_1 t)^i}{i!}. \quad (14)$$

Equation (14) is in accordance with the model developed in [1], where  $P\left(X(t) < D, \sum_{j=1}^{N_i(t)} W_j < H\right)$  corresponds to the  $\int_0^H f_{x_s}(x_s | t, n) dx_s$  in [1].

### 3.4 Reliability model quantification by Monte Carlo simulation

Due to the complexity of (8), the analytical solution of the reliability is hard to obtain. A numerical approach based on Monte Carlo simulation is, then, developed to compute the system reliability, by the following pseudo-code:

---

#### Algorithm 1 Monte Carlo simulation for DCFP model considering degradation-shock dependence

---

**Set**  $t, M, n$ .

**For**  $j = 1 : n$ ,

Sample a  $\theta_j$  from  $F_{\theta}$ .

Calculate  $x(t)$  by (1).

Calculate  $P^j(E_1|A,B)$  and  $P^j(E_3|A,B)$  by (9) and (11), respectively.

Sample a set of  $Y$ ,  $\{Y_1, Y_2, \dots, Y_M\}$  from  $F_Y$ . Pick those belonging to the damage zone and reassemble them as a new set  $\{Y_1, Y_2, \dots, Y_m\}$ .

**For**  $i = 1 : m$ ,

Calculate  $P_i^j(A|B)$  by (12) and (13).

Calculate  $\sum_{Y_k \in \{Y_1, \dots, Y_i\}} \alpha Y_k$  by adding the first  $i$  elements in  $\{Y_1, Y_2, \dots, Y_m\}$ .

Calculate  $P_i^j(E_2|A,B)$  by (15):

---

$$P(E_2|A,B) = \begin{cases} 1 & \text{if } \sum_{Y_k \in \{Y_1, \dots, Y_i\}} \alpha Y_k < H, \\ 0 & \text{if } \sum_{Y_k \in \{Y_1, \dots, Y_i\}} \alpha Y_k \geq H. \end{cases} \quad (15)$$

**End For**

Calculate  $R_j(t)$  by (16):

$$R_j(t) = \sum_{i=0}^m P^j(E_1|A,B) P_i^j(E_2|A,B) P^j(E_3|A,B) P_i^j(A|B). \quad (16)$$

**End For**

Calculate  $R(t)$  by (17):

$$R(t) = \frac{1}{n} \sum_{j=1}^n R_j(t). \quad (17)$$

1

2 The computational complexity of the simulation method is  $O(M \times n \times n_i) \times O_{obj}$ , where  $n_i$  is the evaluated  
 3 number of  $t$ ,  $O_{obj}$  is the computational complexity of each evaluation of the objective function, i.e., the  
 4  $P^j(E_1|A,B)P_i^j(E_2|A,B)P^j(E_3|A,B)P_i^j(A|B)$  in (16).

5 To implement the algorithm in practice, the values for  $M, n$  and  $n_i$  need to be set. The values of  $M$  and  
 6  $n$  determine the accuracy of the reliability estimation at each time point  $t$ . Therefore, they should be determined  
 7 based on the requirements on the confidence bands. It is well known that the error of Monte Carlo simulation is  
 8 proportional to  $\frac{1}{\sqrt{M \times n}}$ . Therefore, the appropriate sample sizes could be determined by trial-and-error: determine  
 9 an initial sample size and check if the confidence band satisfies the requirement; then, adjust the sample sizes based  
 10 on the proportional relation between sample sizes and simulation errors. The value of  $n_i$ , reflects the closeness of  
 11 the point wise approximation values of the reliability function, output by the algorithm, to the true values: the larger  
 12  $n_i$ , the better the approximation. Proper balance is needed between the accuracy of the approximation and the  
 13 computational complexity of the algorithm.

## 14 4. Applications

### 15 4.1 Model development

16 In this section, the developed DCFP model is applied to describe the failure behavior of a sliding spool. The  
 17 case study is artificially constructed, based on a realistic physical background, to illustrate the proposed modeling  
 18 framework. As discussed in Sect. 2, the sliding spool is subject to two dependent failure processes, wear and clamping

1 **stagnation.** According to a study of physics-of-failures [29], the wear of the sliding spool increases smoothly for most  
 2 of its lifetime, and therefore the wear process can be modeled by a linear model:

$$3 \quad x(t) = X(t; \varphi, \beta) = \varphi + \beta t, \quad (18)$$

4 where the initial value  $\varphi$  is a constant and the degradation rate  $\beta$  is assumed to be a normal random variable.

5 **Based on the analysis in Sect. 2, clamping stagnation is modeled by the zone shocks discussed in Sect. 3.2. The**  
 6 **arrival of the particles is assumed to follow a nonhomogeneous Poisson process with intensity function  $\lambda(t)$ . The**  
 7 **size of the  $i$  th particle getting into the clearance of the sliding spool, denoted by  $Y_i$ , is assumed to be an i.i.d. random**  
 8 **variable with a probability distribution  $f_Y(y)$ . Based on the failure mechanisms of clamping stagnation discussed**  
 9 **in Figure 2,  $Y_i$  is classified into three zones according to its magnitude:**

- 10 ● if  $Y_i < C$ , the shock belongs to the damage zone;
- 11 ● if  $C \leq Y_i < C + \Delta C$ , the shock belongs to the fatal zone;
- 12 ● if  $Y_i \geq C + \Delta C$ , the shock belongs to the safety zone,

13 where  $C$  is the clearance of the sliding spool and  $\Delta C$  is the critical particle size. The probabilities that the  $i$  th  
 14 shock belongs to the damage, fatal and safety zones, denoted by  $P_1, P_2, P_3$ , respectively, are calculated by:

$$15 \quad \begin{aligned} P_1 &= \int_0^C f_Y(y) dy, \\ P_2 &= \int_C^{C+\Delta c} f_Y(y) dy, \\ P_3 &= \int_{C+\Delta c}^{\infty} f_Y(y) dy. \end{aligned} \quad (19)$$

16 Moreover, degradation-shock dependence exists between the wear and the clamping stagnation, because wear  
 17 processes generate wear debris that accelerates the arrival rate of the particles which cause the clamping stagnation  
 18 [30, 31]. In our developed model, the degradation-shock dependence is modeled by (4). Because the wear process is  
 19 modeled by (18), (4) becomes:

$$20 \quad \lambda(t) = \lambda_0 + \gamma_a \cdot (\varphi + \beta t). \quad (20)$$

21 Substituting (18)-(20) into (8), the reliability of the sliding spool under the two dependent failure processes can be  
 22 calculated.

## 23 **4.2 Reliability evaluation**

24 The numerical algorithm presented in Sect. 3.4 is used to evaluate the reliability of the sliding spool based on  
 25 the developed model. In this application, the values for the model parameters are estimated by domain experts. For

1 illustrative purposes, we assume the expert estimations are accurate and given in Table 1. In practice, parameters  
 2 related to degradation processes and random shocks, for example, the  $\varphi$  and  $\beta$  in Table 1, could be estimated  
 3 from test data.

4 Table 1 Model parameters of the reliability model for the sliding spool

Parameter/distribution	Description	Values
$\varphi$	The initial wear magnitude	0 mm
$\beta$	The wear rate	$N(1 \times 10^{-4}, 1 \times 10^{-5})$ mm/s
$D$	The wear threshold	5 mm
$F_Y(y)$	The CDF for particle diameters	$N(1.2, 0.2)$
$\alpha$	The shock damage proportional coefficient	1
$H$	The cumulative clamping stagnation threshold	7.5 mm
$[C, C + \Delta c]$	The range for fatal shocks	[1.5, 1.6]mm
$\lambda_0$	The initial intensity of random shocks	$2.5 \times 10^{-5}/s$
Parameters for simulation	$n = 1000, M = 200, n_i = 1000, t = [0, 6 \times 10^4]s$	
Dependence factors	$\gamma_a = 0, 10^{-5}, 10^{-4}, 10^{-3}$	

5

### 6 4.3 Results and discussions

7 We compute the reliability of the sliding spool under different levels of dependence ( $\gamma_a = 0, 1 \times 10^{-5}, 1 \times 10^{-4},$   
 8  $1 \times 10^{-3}$ ), to investigate the failure behavior of the sliding spool. Results are given in Figure 4. In general, the  
 9 reliability of the sliding spool increases with the decrease of  $\gamma_a$ . As expected, when  $\gamma_a = 0$ , indicating that there  
 10 is no dependence between the two failure processes,  $R(t)$  takes the largest values. This is mainly due to the  
 11 dependence mechanism between wear and clamping stagnation. As the wear process progresses, more wear debris is  
 12 generated and, the arrival rate of particles causing clamping stagnation increases. In this way, clamping stagnation is  
 13 aggravated by wear. Therefore, positive dependence exists between the two failure mechanisms, as described by (20).  
 14 Proactively, this suggests that if we can mitigate the dependence between the two failure processes, the reliability of  
 15 the sliding spool could be enhanced. In practice, filters can be installed to stop the debris generated by wear processes  
 16 from entering the hydraulic oil. In this way, the dependence between wear and clamping stagnation can be controlled,  
 17 so that the reliability of the sliding spool can be increased.

18 Furthermore, it can be seen from Figure 4 that when  $\gamma_a$  is relatively small, the reliability changes slowly at

1 first, then declines sharply at about  $4 \times 10^4 s$ . As  $\gamma_a$  increases, the reliability curve decreases more smoothly. When  
2  $\gamma_a$  becomes relatively large ( $\gamma_a = 10^{-3}$ ), the declination of the reliability curve becomes more rapid. The main  
3 physical and mathematical reasons of the phenomenon are:

4 (1) A small  $\gamma_a$  indicates that the two failure mechanisms are almost independent. The sliding spool would  
5 experience two distinct failure behaviors. When  $t$  is small, because the wear process is far from its failure  
6 threshold, failures due to wear seldom occur. Hence, the failure behavior is primarily dominated by clamping  
7 stagnation. Because the rate of the clamping stagnation is relatively small, the sliding spool will have high  
8 reliability, which is indicated by the first part of the reliability curve.

9 As  $t$  increases, the wear level approaches its threshold. Therefore, failures are more likely to be caused by the  
10 wear processes. On the other hand, because  $\gamma_a$  is small, from (4), the rate of the clamping stagnation remains  
11 almost unchanged. Hence, when  $t$  is large, the failure behavior of the spool is primarily determined by wear,  
12 which leads to the second part of the reliability curve starting at around  $t = 4 \times 10^4 s$ .

13 (2) When  $\gamma_a$  is relatively large, the wear process has considerable influence on the shock process. Even when  $t$   
14 is small, the failure probability of the sliding spool is relatively high due to the positive dependence between the  
15 two failure processes. Hence, the difference between the two failure behaviors in the independent case becomes  
16 small. Therefore, the reliability curve declines more smoothly comparing to the independent case.

17 (3) When  $\gamma_a$  becomes even larger, the reliability of the sliding spool decreases rapidly with time. This is because  
18 strong dependence exists between wear and clamping stagnation. Therefore, even at the early stages of the  
19 spool's life cycle, a lot of spools might fail due to the high risks of clamping stagnation, which results from the  
20 strong positive dependence on the wear process.

21

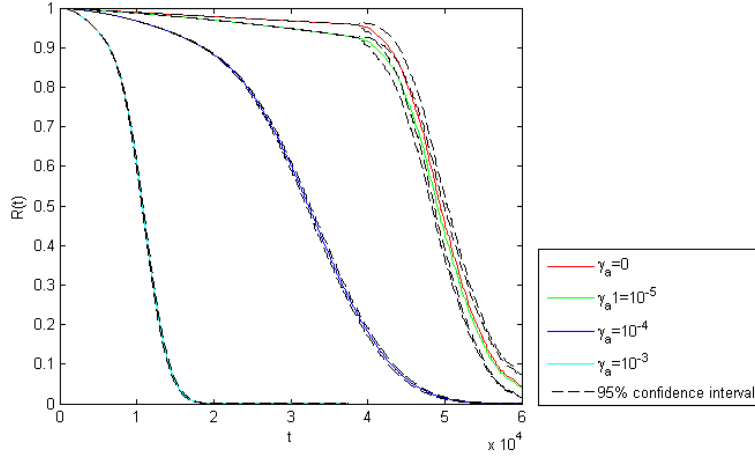


Figure 4 Reliability under different values of the dependence factor,  $\gamma_a$

#### 4.4 Some extensions

In real cases, the dependency behaviors might be more complicated. For example, as the debris accumulates, more wear and debris are expected. As a consequence, the intensity of the shock arrival process might not only depend on the degradation level, but also on the accumulated shock damage  $\sum W$ . Furthermore, the size of the wear debris might also be affected by the degradation levels. In addition, the degradation rate also depends on the accumulated shock damage, i.e., the shock-degradation dependence. In this section, we show how to extend the developed model to capture more complex dependency behaviors.

Except for the assumptions we made in Sect. 4.1, three additional assumptions are made:

- the intensity function of the arrival process of the wear debris, i.e., the  $\lambda(t)$  in (20), depends on the degradation level  $x(t)$  and the current cumulative shock damage  $\sum W$ :

$$\lambda(t) = \lambda_0 + \gamma_a x(t) + \gamma_b \sum W, \quad (21)$$

where  $\gamma_a, \gamma_b$  are dependence factors and  $\lambda_0$  is the initial intensity of the arrival process.

- the mean of the debris size distribution  $F_Y(y)$ , denoted by  $\mu_Y(t)$ , is affected by the degradation level  $x(t)$ :

$$\mu_Y(t) = \mu_{Y_0} + \gamma_c \cdot x(t), \quad (22)$$

where  $\gamma_c$  is the dependence factor, and  $\mu_{Y_0}$  is the initial value of  $\mu_Y(t)$ .

- the degradation rate, i.e., the  $\beta$  in (18), depends on the current cumulative shock damage  $\sum W$ :



$$\beta = \beta_0 + \gamma_d \cdot \sum W \quad (23)$$

where  $\gamma_d$  is the dependence factor.

The first two assumptions consider degradation-shock dependence, where the intensity of the shock arrival process and the distribution of shock magnitudes are both influenced by the degradation process. The last assumption describes shock-degradation dependence, where the degradation rate varies with the accumulated damages of the random shocks.

Monte Carlo method (with a sample size of  $10^4$ ) is used to investigate the effect of  $\gamma_b, \gamma_c$  and  $\gamma_d$  on the spool valve's reliability. The detailed procedures are given in the Appendix. As shown in Figure 5, when  $\gamma_b$  varies from 0 to  $10^{-3}$ , the predicted reliability decreases significantly. This can be explained by the fact that  $\gamma_b$  measures the effect of cumulative shock damage  $\sum W$  on the intensity function of the arrival process of the wear debris, as shown in (21), and on the system's degradation rate, as shown in (23). As  $\sum W$  increases, on one hand, more shocks occur, which increases the probability of a hard failure; on the other hand, the system degrades more rapidly, which accelerates the arrival of a soft failure. Therefore,  $\gamma_b$  might has a significant effect on the system reliability.

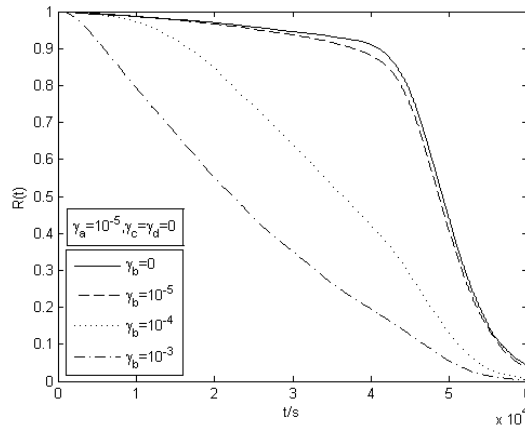
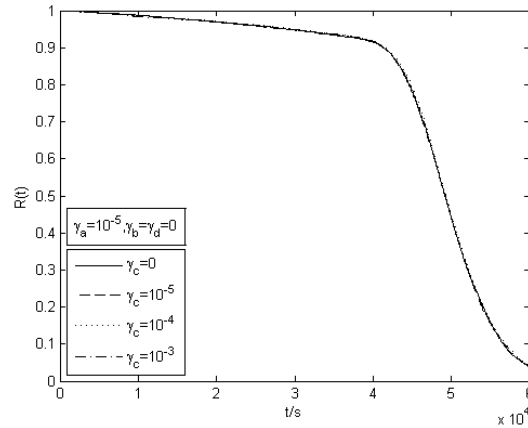


Figure 5 Effect and sensitivity of  $\gamma_b$  on the predicted reliability

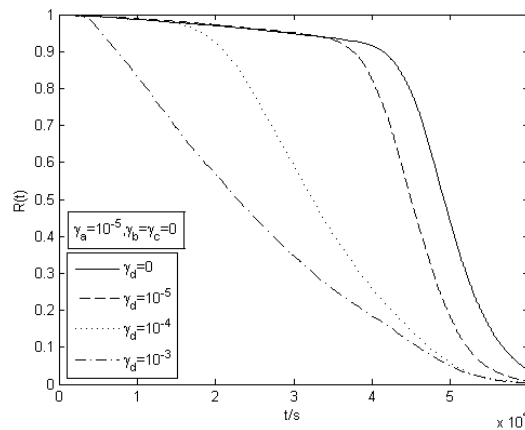
As shown in Figure 6, when  $\gamma_c$  varies from 0 to  $10^{-3}$ , the predicted reliability remains almost unchanged. This can be explained by the fact that  $\gamma_c$  measures the effect of the degradation magnitude  $x(t)$  on the mean of the debris size distribution  $\mu_Y(t)$ , as shown in (22). In this case, the initial value of  $\mu_Y(t)$  was in the damage shock zone; as  $x(t)$  increases,  $\mu_Y(t)$  moves first into the fatal shock zone, and then into the safety shock zone.

1 When  $x(t)$  is relatively small, its effect on  $\mu_Y(t)$  is negligible. However, when  $x(t)$  is large enough, since  
 2  $\mu_Y(t)$  has already entered the safety shock zone, most random shocks would also be in the safety shock zone. The  
 3 failure process in this period is dominated by the degradation process. As a result, the system's reliability is not  
 4 sensitive to  $\gamma_c$  and, therefore, the effect of  $\gamma_c$  can be ignored in modeling the sliding spool.



5  
6 **Figure 6 Effect and sensitivity of  $\gamma_c$  on the predicted reliability**

7 As shown in Figure 7, when  $\gamma_d$  varies from 0 to  $10^{-3}$ , the predicted reliability decreases significantly. This  
 8 can be explained by the fact that  $\gamma_d$  measures the effect of cumulative shock damage  $\sum W$  on the degradation  
 9 rate  $\beta$ , as shown in (23). As  $\gamma_d$  increases, the degradation process becomes more rapid, according to (23), which,  
 10 then, creates more random shocks due to the degradation-shock dependence (Eq. (20)). The system's reliability is  
 11 significantly reduced by the mutually enhanced dependences.



12  
13 **Figure 7 Effect and sensitivity of  $\gamma_d$  on the predicted reliability**

14 **5. Conclusions**

15 In this paper, a new reliability model is developed for dependent competing failure processes. **In the developed**

1 model, failure of a component could be caused by either “soft” failure (described by a degradation model) or “hard”  
2 failure (described by a shock model), and the degradation-shock dependence exists among the failure processes  
3 (where the intensity of the shock process is dependent on the degradation process and the effect of dependence is  
4 classified into different zones according to the magnitude of the random shocks). Since degradation-shock  
5 dependence is considered, the shock process is influenced by the degradation process but the degradation process is  
6 not influenced by the shock process.

The developed model contributes to existing ones in two aspects. First, it  
7 considers a new type of dependence, the degradation-shock dependence, where random shocks are influenced by  
8 degradation processes. Second, it combines zone shocks into the degradation-shock dependence, where random  
9 shocks are classified into different zones according to their magnitudes and different zones have different effects on  
10 the failure behavior. The developed model is applied to describe the dependent failure behavior of an actual product,  
11 a sliding spool, which is subject to wear and clamping stagnation.

12 In the future, additional work will be done to further improve the developed model. First of all, in this paper we  
13 only consider a simple dependence scenario, where linear dependence exists between the intensity function and the  
14 degradation processes. In the future, the influence of other dependence relations, possibly nonlinear, will also be  
15 considered. Second, in this paper we only consider the degradation-shock dependence. In the future, the combination  
16 of two types of dependencies will be considered. Third, in this paper we assume that the parameters of the model are  
17 perfectly known. In practice, however, many times precise values of the parameters are not available due to limited  
18 data and knowledge. Hence, the model is subject to epistemic uncertainty, whose effect will also be studied.

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- 47

**Appendix**


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**Algorithm 2** Monte Carlo simulation for DCFP model considering multiple dependences in Sect. 4.4

---

**Step 1:** Set samplesize,  $i = 0, x = 0, \sum W = 0, \tau = 0$ ;

**Step 2:** If  $i > \text{samplesize}$ , stop;

else generate a  $\beta_s$  from  $F_\beta$ ;

**Step 3:** Update the intensity function  $\lambda(t) = \lambda_0 + \gamma_a \left( x + (\beta_s + \gamma_d \cdot \sum W)(t - \tau) \right) + \gamma_b \sum W$ ;

Generate an arrival time  $\Delta\tau$  from a NHPP with intensity function  $\lambda(t)$  using thinning method [51];

Set  $T_{\max}$ ; Calculate  $\lambda_M = \lambda(T_{\max})$ ;  $tt = \tau$ ;

While 1

$\Delta\tau \leftarrow$  Generate a random number from  $\exp(\lambda_M)$ ;  $tt = tt + \Delta\tau$ ;

If  $tt < T_{\max}$ ,  $p_{acc} = \lambda(tt) / \lambda_M$ ; Accept  $\Delta\tau$  with probability  $p_{acc}$ ; Return  $\Delta\tau = tt - \tau$ ;

Else  $tt = T_{\max}$ ; Increase the value of  $T_{\max}$ ; Calculate  $\lambda_M = \lambda(T_{\max})$ ; Continue;

Endwhile

$\tau = \tau + \Delta\tau$ ;

**Step 4:**  $x = x + (\beta_s + \gamma_d \cdot \sum W)\Delta\tau$ ;

If  $x > D$ ,  $i = i + 1, t_f(i) = (D - x) / (\beta_s + \gamma_d \cdot \sum W) + \tau$ , go to Step 2;

Else go to Step 5;

**Step 5:**  $\mu_Y = \mu_{Y_0} + \gamma_c \cdot x$ ;

Generate  $Y \sim \text{Normal}(\mu_Y, \sigma_Y^2)$ ;

If  $Y < C$ , go to Step 6;

Else if  $Y < C + \Delta C$ ,  $i = i + 1, t_f(i) = \tau$ , go to Step 2;

**Step 6:**  $\sum W = \sum W + \alpha \cdot Y$ ;

If  $\sum W \geq H$ ,  $i = i + 1, t_f(i) = \tau$ , go to Step 2;

Else go to Step 3;

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