Model Predictive Control and Sliding Mode Control for Current Sharing in Microgrids

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***Abstract*— This paper deals with the design of a hierarchical control scheme for complex islanded Alternate Current (AC) microgrids. The proposed solution relies on the combined use of Model Predictive Control (MPC) and Sliding Mode Control (SMC). The model of the microgrid includes several Distributed Generation Units (DGus), affected by unknown load dynamics and modelling uncertainties. Moreover, they are connected according to an arbitrary complex and meshed topology, taking into account the interconnecting line dynamics. The proposed control scheme consists of two control loops. A centralized MPC supervisor generates the voltage reference values for each DGu, while fulfilling input and state constraints on the basis of a reduced order model of the plant. A Suboptimal Second Order Sliding Mode (SSOSM) control is locally designed for each DGu to track, in a decentralized way, the voltage references generated by the supervisor. Simulation results confirm the effectiveness of the proposed control scheme.**

I. INTRODUCTION

Recently, due to the wide diffusion of Renewable Energy Sources (RES) and active participation of consumers to the electric market, one of the most relevant key challenges in power generation and distribution field is the development of resilient and sustainable small-scale power systems that integrate the so-called Distributed Generation units (DGus), storage devices and loads [1]. This challenge can be addressed by exploiting the concept of "microgrids" and "Smart Grids", which are clusters of DGus, loads and storage systems interconnected through power lines [2]. Moreover, they can operate disconnected from the main grid, in the so-called islanded operation mode (IOM) [3].

In a typical microgrid, the presence of new technologies and tools for the smart metering of the processes is mandatory, above all because of the unpredictable behaviours of RES and load dynamics that make the adoption of suitable robust control strategies essential to regulate the electrical signals of the microgrid [4]. Technologies used for power systems have to include protections, data acquisition units and robust control equipments. Automation is widely spread in this systems so that, in the last decades, several solutions have been proposed to cope with the aforementioned problem.

In the literature, controllers of several types, including PI control algorithms [5], $\mathcal{H}_\infty$ controllers [6], or Model Predictive Control [7], formulated both in the so-called Grid Connected Operation Mode (GCOM) and in IOM, have been introduced. Among the controllers, also Sliding Mode Control (SMC) has been applied with satisfactory results to power networks [8]–[13]. SMC is very appreciated for its robustness properties against a wide class of uncertainties and perfectly fits the problem to solve [14]. Moreover, SMC belongs to the class of Variable Structure Control Systems so that it seems perfectly adequate to control the so-called voltage source converter (VSC), that is the interface medium between the grid and the energy source. In fact, power electronic systems represent a typical example in which the discontinuous control is intrinsically provided. The so-called chattering phenomenon is already attenuated by construction thanks to the presence of the VSC output filter [15]. However, in order to obtain more regular modulating signals by increasing the natural relative degree of the auxiliary system, Higher Order Sliding Mode controllers can be applied, as shown in [8], [10], [11].

In reality, apart from the basic control requirements, microgrids often requires consensus algorithm in order to optimize its operation while satisfying some input and state constraints. Model Predictive Control (MPC) [16], [17] is widely used to search for an optimal control solution, while fulfilling the constraints on the basis of a suitable predictor of the plant behaviour.

In this paper, we propose a hierarchical control architecture based on the joint use of MPC and SMC [18]–[20], in order to stabilize the microgrid system and keep the DGus output voltages in a prescribed boundary layer while achieving the so-called current sharing, the latter meaning that the overall load current is equally shared among the DGus [21]. In our proposal, the low level controller implements a decentralized second-order sliding mode control strategy, belonging to the class of Suboptimal algorithms (SSOSM) [22]. This low level controller is used to track the voltage references generated by a MPC supervisor based on a reduced order model of the plant. Finally, a realistic simulation scenario including four DGus in ring topology has been assessed.

The present paper is organized as follows: in Section II the microgrid model is introduced and described. In Section III the control problem is formulated, while in Section IV the proposed hierarchical control scheme is designed. In Section V the simulation results are illustrated. Some conclusions are gathered in Section VI.

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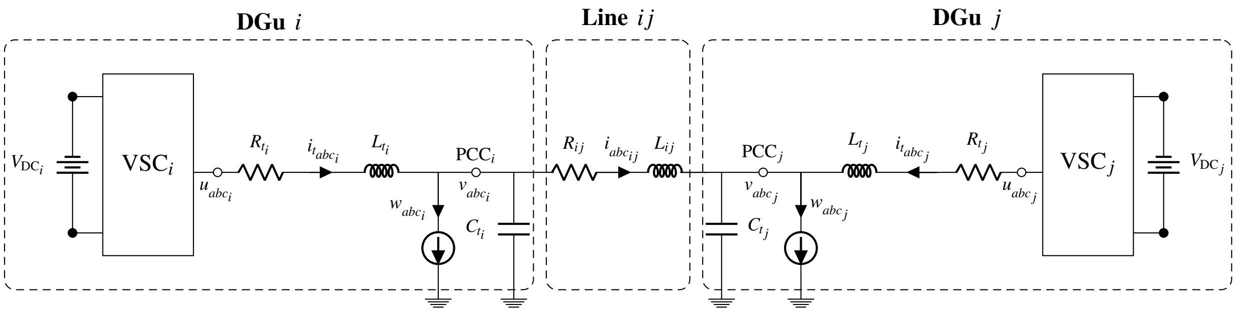

Fig. 1. The considered electrical single-line diagram of a typical islanded AC microgrid composed of two DGUs

II. Microgrid Model

Consider an islanded AC microgrid with n DGus. The network is represented by a connected and undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where the nodes $\mathcal{V} = \{1, ..., n\}$, represent the DGus and the edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V} = \{1, ..., m\}$ represent the distribution lines interconnecting the DGus. The network structure can be represented by its corresponding incidence matrix $D \in \mathbb{R}^{n\times m}$. The ends of edge k are arbitrary labeled with a '+' and a '-'. Then, one has that

$$D_{ik} = \begin{cases} +1 & \text{if } i \text{ is the positive end of } k \\ -1 & \text{if } i \text{ is the negative end of } k \\ 0 & \text{otherwise .} \end{cases}$$

Consider the scheme reported in Figure 1 and assume the system to be symmetric and balanced. For the sake of simplicity, the dependence of the variables on time t is omitted throughout this paper. In the stationary abc-frame, by applying the Kirchhoff's current (KCL) and voltage (KVL) laws, the dynamics equations of the microgrid in IOM are expressed as follow,

$$\begin{cases} \frac{d}{dt} v_{abc} = [C_t]^{-1} i_{t_{abc}} + [C_t]^{-1}[D] i_{abc} - [C_t]^{-1} w_{abc} \\ \frac{d}{dt} i_{t_{abc}} = -[L_t]^{-1}[R_t] i_{t_{abc}} - [L_t]^{-1} v_{abc} + [L_t]^{-1} v_{t_{abc}} \\ \frac{d}{dt} i_{abc} = -[L]^{-1}[D^T] v_{abc} - [L]^{-1}[R] i_{abc} \end{cases}, \quad (1)$$

where $s_{abc} = [s_a^T, s_b^T, s_c^T]^T \in \mathbb{R}^{3n}$, $s_\pi = [s_{\pi_1}, \ldots, s_{\pi_n}]^T \in \mathbb{R}^n$, with $\pi = a, b, c$ and $s \in \{v, i_t, w, v_t\}$, while $i_{abc} = [i_a^T, i_b^T, i_c^T]^T \in \mathbb{R}^{3m}$, $i_\pi = [i_{\pi_1}, \ldots, i_{\pi_m}]^T \in \mathbb{R}^m$. In (1) v_{abc}, $i_{t_{abc}}$, i_{abc}, w_{abc}, and $v_{t_{abc}}$ represent the following three-phase signals: the loads voltages, the currents generated by the DGus, the currents along the interconnecting lines, the currents demanded by the loads, and the VSCs output voltages. Moreover, in system (1) we used $[H]$ to denote the following block diagonal matrix

$$[H] = \begin{bmatrix} H & 0 & 0 \\ 0 & H & 0 \\ 0 & 0 & H \end{bmatrix},$$

where $H \in \{C_t, L_t, R_t, L, R\}$, with C_t, L_t, R_t being $n \times n$ diagonal matrices and L, R being $m \times m$ diagonal matrices, e.g., $R_t = \mathrm{diag}\{R_{t_1}, \ldots, R_{t_n}\}$ and $R = \mathrm{diag}\{R_1, \ldots, R_m\}$, with $R_k = R_{ij}$.

Each three-phase variable of (1) can be transferred to the rotating dq-frame by applying the Clarke's and Park's transformations. Then, the so-called state-space representation of the whole system (1) can be expressed as

$$\begin{cases} \dot{V}_d = \omega_0 V_q + C_t^{-1} I_{t_d} + C_t^{-1} D I_d - C_t^{-1} W_d \\ \dot{V}_q = -\omega_0 V_d + C_t^{-1} I_{t_q} + C_t^{-1} D I_q - C_t^{-1} W_q \\ \dot{I}_{t_d} = -L_t^{-1} V_d - L_t^{-1} R_t I_{t_d} + \omega_0 I_{t_q} + L_t^{-1} V_{t_d} \\ \dot{I}_{t_q} = -L_t^{-1} V_q - \omega_0 I_{t_d} - L_t^{-1} R_t I_{t_q} + L_t^{-1} V_{t_q} \\ \dot{I}_d = -L^{-1} D^T V_d - L^{-1} R I_d + \omega_0 I_q \\ \dot{I}_q = -L^{-1} D^T V_q - \omega_0 I_d - L^{-1} R I_q \end{cases}, \quad (2)$$

where $x = [V_d^T, V_q^T, I_{t_d}^T, I_{t_q}^T, I_d^T, I_q^T]^T \in \mathbb{R}^{4n+2m}$ is the state variables vector, $\mu = [V_{t_d}^T, V_{t_q}^T]^T \in \mathbb{R}^{2n}$ is the input vector, $w = [W_d^T, W_q^T]^T \in \mathbb{R}^{2n}$ is the disturbance vector. Then, the previous system, in a more compact form, can be written as

$$\dot{x} = Ax + B\mu + B_w w \quad (3)$$

where $A \in \mathbb{R}^{(4n+2m)\times(4n+2m)}$ is the dynamics matrix of the microgrid, $B \in \mathbb{R}^{(4n+2m)\times(2n)}$, and $B_w \in \mathbb{R}^{(4n+2m)\times(2n)}$.

To permit the controller design in the next sections, the following assumption is required on states and disturbances.

Assumption 1 *All the state variables are measurable, while the current load disturbances W_d and W_q are unknown but bounded, of class* C *and Lipschitz continuous.*

III. Problem Formulation

Before introducing the control problem that will be solved in the paper, for the readers' convenience, some basic notions on DGus are presented. In islanded operation mode, the voltage and frequency at the point of common coupling (PCC) could deviate significantly from the desired values, due to the power mismatch between the DGu and the load. Therefore, each DGu has to provide voltage and frequency regulation. Specifically, in this paper the frequency is controlled in open-loop by equipping each DGu with an internal oscillator that provides the Park's transformation angle $\theta(t) = \int_{t_0}^{t} \omega_0 d\tau$, with $\omega_0 = 2\pi f_0$, f_0 being the nominal frequency. Moreover, in the rotating dq-frame, the generated active and reactive powers can be expressed as

$$P_i = \frac{3}{2}(V_{d_i} I_{t_{d_i}} + V_{q_i} I_{t_{q_i}}), \quad Q_i = \frac{3}{2}(V_{q_i} I_{t_{d_i}} - V_{d_i} I_{t_{q_i}}). \quad (4)$$

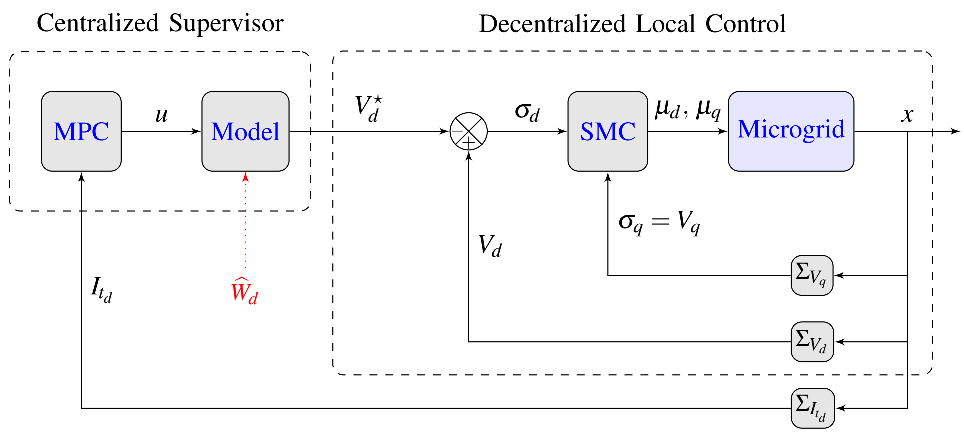

Fig. 2. Hierarchical MPC/SMC control scheme, where Σ_S is an appropriate matrix that selects the state S from the overall system state x.

Then, in order to decouple the active and reactive power control, the quadrature voltage component V_q is regulated to zero, such that the active and reactive powers in (4) become

$$P_i = \frac{3}{2} V_{d_i} I_{t_{d_i}}, \quad Q_i = -\frac{3}{2} V_{d_i} I_{t_{q_i}}, \tag{5}$$

which depend only on the direct and quadrature current component, respectively.

Assuming that the quadrature voltage component V_q is steered to zero, relying on (5), we introduce the following definition of "active current load sharing".

Definition 1 *Direct current sharing is achieved if the overall direct component of load current is equally shared among the DGus, i.e.,*

$$I_{t_d} = \mathbb{1}_n I^*_{t_d}, \quad I^*_{t_d} = \frac{1}{n} \mathbb{1}_n^T W_d \in \mathbb{R} \tag{6}$$

where $\mathbb{1}_n \in \mathbb{R}^n$ is the vector containing all ones.

Now we are in a position to formulate the control problem. *Let Assumption 1 hold. Given system* (1)-(3)*, design a hierarchical control scheme capable of guaranteeing that: i) V_q is steered to zero in a finite time in spite of the uncertainties; ii) V_d is regulated in order to achieve direct current sharing while fulfilling input and state constraints; iii) given constant references and disturbances, the overall microgrid system is asymptotically stable.*

IV. The Proposed Hierarchical Control Scheme

In this section, the proposed hierarchical control scheme illustrated in Figure 2 is described. It consists of two control loops. The outer one is based on a centralized MPC supervisor, while a decentralized SSOSM control algorithm is locally used to solve the aforementioned voltage tracking control problem. Since the SSOSM component is very simple from a computational viewpoint, it is locally implemented so that it can run at a higher rate to allow one to develop the MPC controller at slower rate relying on a simplified model.

A. SSOSM Decentralized Local Control

Consider the state-space model (3) and set the so-called sliding variables vector as

$$\begin{aligned} \sigma_d &= V_d - V_d^\star \\ \sigma_q &= V_q - V_q^\star, \end{aligned} \tag{7}$$

where $V_d^\star$ and $V_q^\star$ are the reference values.

In order to design the controller, the following assumption is required on the generation of reference values.

Assumption 2 *The voltage reference $V_d^\star$ is of class C^2 and with first time derivative Lipschitz continuous, while, in order to decouple the active and reactive power control, $V_q^\star = 0$.*

Let r be the relative degree of the system, i.e., the minimum order r of the time derivative $\sigma^{(r)}$ of the sliding variable in which the control μ explicitly appears. With reference to (7), one can verify that r is equal to 2, so that a second order sliding mode (SOSM) control naturally applies [22]. According to the SOSM control theory, the so-called auxiliary variables $\xi_{1_\nu} = \sigma_\nu$ and $\xi_{2_\nu} = \dot{\sigma}_\nu$, with the subscript $\nu = d, q$, have to be defined and the corresponding auxiliary systems can be expressed as

$$\begin{cases} \dot{\xi}_{1_\nu} = \xi_{2_\nu} \\ \dot{\xi}_{2_\nu} = f_\nu(x, w) + g_\nu \mu_\nu \end{cases}, \tag{8}$$

where μ_ν are the control inputs previously defined, and ξ_{2_ν} is assumed to be unmeasurable. More specifically, one has that

$$\begin{aligned} f_d(x,w) \quad &= -\left(\omega_0^2 I_{n\times n} + C_t^{-1} L_t^{-1} + C_t^{-1} D L^{-1} D^T\right) V_d \\ &\quad - C_t^{-1} L_t^{-1} R_t I_{t_d} + 2\omega_0 C_t^{-1} I_{t_q} \\ &\quad - C_t^{-1} D L^{-1} R I_d + 2\omega_0 C_t^{-1} D I_q \\ &\quad - C_t^{-1} \dot{W}_d - \omega_0 C_t^{-1} W_q - \ddot{V}_d^\star \\ f_q(x,w) \quad &= -\left(\omega_0^2 I_{n\times n} + C_t^{-1} L_t^{-1} + C_t^{-1} D L^{-1} D^T\right) V_q \\ &\quad - 2\omega_0 C_t^{-1} I_{t_d} - C_t^{-1} L_t^{-1} R_t I_{t_q} \\ &\quad - 2\omega_0 C_t^{-1} D I_d - C_t^{-1} D L^{-1} R I_q \\ &\quad + \omega_0 C_t^{-1} W_d - C_t^{-1} \dot{W}_q \\ g_d = g_q \quad &= C_t^{-1} L_t^{-1}, \end{aligned} \tag{9}$$

are allowed to be uncertain but bounded, i.e.,

$$|f_{\nu_i}(\cdot)| \le F_{\nu_i}, \quad G_{\min_{\nu_i}} \le g_{\nu_{ii}} \le G_{\max_{\nu_i}}, \quad i = 1,\dots,n, \tag{10}$$

with F_{ν_i}, $G_{\min_{\nu_i}}$ and $G_{\max_{\nu_i}}$, $\nu = d,q$, being known positive constants. Note that, it is reasonable to assume that such bounds exist. In fact, the functions f_ν depend on electric signals related to the finite power of the system, while $g_{\nu_{ii}}$ are uncertain constant values. In practical cases, these bounds can be estimated relying on data acquisition in different test conditions and are therefore assumed known.

The i-th control law, μ_{ν_i}, which is proposed to steer $\xi_{1_{\nu_i}}$ and $\xi_{2_{\nu_i}}$, $i = 1,\dots,n$, to zero in a finite time in spite of the uncertainties, in analogy with [22], can be expressed as follows

$$\mu_{\nu_i} = -\alpha_{\nu_i} M_{\max_{\nu_i}} \operatorname{sgn}\left(\xi_{1_{\nu_i}} - \tfrac{1}{2}\xi_{1_{\max_{\nu_i}}}\right), \tag{11}$$

with bounds

$$M_{\max_{\nu_i}} > \max\left(\frac{F_{\nu_i}}{\alpha^*_{\nu_i} G_{\min_{\nu_i}}}; \frac{4F_{\nu_i}}{3G_{\min_{\nu_i}} - \alpha^*_{\nu_i} G_{\max_{\nu_i}}}\right) \tag{12}$$

$$\alpha^*_{\nu_i} \in (0,1] \cap \left(0, \frac{3G_{\min_{\nu_i}}}{G_{\max_{\nu_i}}}\right). \tag{13}$$

B. MPC Centralized Supervisor

By virtue of the presence of the SSOSM controller that, according to Assumption 2, regulates the voltage V_q to zero in a finite time, the MPC module can be designed on the basis of a simplified model with a beneficial effect in terms of computational burden, i.e.,

$$\begin{cases} \dot V_d = C_t^{-1} I_{t_d} + C_t^{-1} D I_d - C_t^{-1} \widehat{W}_d \\ \dot I_{t_d} = -L_t^{-1} V_d - L_t^{-1} R_t I_{t_d} + \omega_0 I_{t_q} + L_t^{-1} u_d \\ \dot I_{t_q} = -\omega_0 I_{t_d} - L_t^{-1} R_t I_{t_q} + L_t^{-1} u_q \\ \dot I_d = -L^{-1} D^T V_d - L^{-1} R I_d + \omega_0 I_q \\ \dot I_q = -\omega_0 I_d - L^{-1} R I_q \\ y = V_d \end{cases}, \tag{14}$$

where $u = [u_d, u_q]^T$ is the input vector generated by the MPC supervisor, while $\widehat{W}_d$ is the load estimate, which can be obtained by estimating $\dot V_{d_i}$ in a finite time via the Levant's differentiator

$$\begin{cases} \dot{\hat z}_{1_i} = -\lambda_{0_i} |\hat z_{1_i} - z_{1_i}|^{1/2} \operatorname{sgn}(\hat z_{1_i} - z_{1_i}) + \hat z_{2_i} \\ \dot{\hat z}_{2_i} = -\lambda_{1_i} \operatorname{sgn}(\hat z_{1_i} - z_{1_i}) \end{cases}, \tag{15}$$

where $\hat z_{1_i}, \hat z_{2_i}$ are the estimated values of $V_{d_i}, \dot V_{d_i}$, respectively, and $\lambda_{0_i} = 1.5\Lambda_i^{1/2}$, $\lambda_{1_i} = 1.1\Lambda_i, \Lambda_i > 0$, is a possible choice of the differentiator parameters [23]. Then, one has

$$\widehat{W}_d = -C_t \hat z_2 + I_{t_d} + D I_d. \tag{16}$$

Note that the MPC controller is designed on the discrete time version of the reduced order system (14) and the feedback is provided with sampling time T.

The adopted MPC controller consists in solving the so-called Finite-Horizon Optimal Control Problem (FHOCP), that is minimizing, at any sampling time instant t_k, a suitably cost function with respect to the control sequence $\mathbf{u}_{[t_k,t_{k+N-1}|t_k]} := [u_0(t_k), u_1(t_k), \dots, u_{N-1}(t_k)]$, with $N \ge 1$ being the prediction horizon. In this paper, the main objective is to obtain current sharing among the DGus of the microgrid. As illustrated in the overall control scheme in Figure 2, after the generation of the optimal control sequence $\mathbf{u}^{\mathrm{o}}_{[t_k,t_{k+N-1}|t_k]} := [u^{\mathrm{o}}_0(t_k), u^{\mathrm{o}}_1(t_k), \dots, u^{\mathrm{o}}_{N-1}(t_k)]$, the latter is fed into the reduced order model (14) in order to generate the voltage references $V_d^\star$ for the local SSOSM controllers.

Remark 1 *Note that, according to Assumption 2, local controllers require smooth $V_d^\star$ signals. Then, in order to satisfy Assumption 2, local controllers can be enhanced with pre-filters.*

Let $\mathcal{L} = D\mathcal{Q}D^T$ denote the weighted Laplacian matrix associated with the network graph. The cost function in question to minimize with respect to $\mathbf{u}_{[t_k,t_{k+N-1}|t_k]}$ is a quadratic function as

$$J(I_{t_d}(t_k), \mathbf{u}_{i_{[t_k,t_{k+N-1}|t_k]}}, N) = \sum_{j=0}^{N-1} I^T_{t_d}(t_{k+j}) \mathcal{L} I_{t_d}(t_{k+j}) + u^T(t_{k+j}) \mathcal{R} u(t_{k+j}), \tag{17}$$

where $\mathcal{L}$ is the semi-positive definite Laplacian matrix, while $\mathcal{R}$ is the matrix of the input weights. The cost function (17) is subject to the hard constraints represented by the dynamics of the discrete time version of (14), and inequalities constraints on states and input variables, i.e.,

$$V_d(t_{k+j}) \in \mathcal{V}_d \tag{18}$$

$$u(t_{k+j}) \in \mathcal{U} \tag{19}$$

with $j = 1,\dots,N-1$.

Then, according to the Receding Horizon strategy, the applied piecewise-constant control law is the following

$$u_i(t) = \kappa_{\mathrm{MPC}}(I_{t_d}(t_k)),\ t \in [t_k, t_{k+1}) \tag{20}$$

where $t_{k+1} - t_k = T$ is the MPC sampling time, and

$$\kappa_{\mathrm{MPC}}(I_{t_d}(t_k)) := u^{\mathrm{o}}_0(t_k) \tag{21}$$

with $u^{\mathrm{o}}_{i_0}(t_k)$ the first value at t_k of the optimal control sequence obtained by solving the FHOCP.

Remark 2 *Note that for a connected and undirected graph the null space of the Laplacian matrix is $\mathcal{N}(\mathcal{L}) = \{\alpha \mathbb{1}_n,\ \alpha \in \mathbb{R}\}$, so that, in the cost function (17), the term $I^T_{t_d}\mathcal{L}I_{t_d}$ is equal to zero if and only if $I_{t_d} = \alpha\mathbb{1}_n$. Then, considering that V_q is steered to zero via the local SSOSM control, from (2) one can compute the steady state value of I_{t_d}, i.e.,*

$$\begin{aligned} 0 &= \alpha\mathbb{1}_n + DI_d - W_d \\ 0 &= \alpha\mathbb{1}_n^T\mathbb{1}_n + \mathbb{1}_n^T D I_d - \mathbb{1}_n^T W_d. \end{aligned} \tag{22}$$

Since $\mathbb{1}_n^T D = 0$, one can verify that $\alpha = \frac{1}{n}\mathbb{1}_n^T W_d$, i.e., according to Definition 1, direct current sharing is achieved.

V. Simulation Results

In this section, the proposed control scheme is evaluated in simulation through the realistic model of an AC islanded

TABLE I
ELECTRICAL PARAMETERS OF MICROGRID IN FIG. 3

DGus	Filter Parameters		Shunt capacitance	Load Currents		References
	R_{t_i} [mΩ]	L_{t_i} [mH]	C_{t_i} [μF]	W_{d_i} [A]	W_{q_i} [A]	$V^{\star}_{q_i}$ [V]
DGu_1	40.2	9.5	62.86	50	-20	0
DGu_2	38.7	9.2	62.86	100	-15	0
DGu_3	34.6	8.7	62.86	40	-10	0
DGu_4	31.8	8.3	62.86	80	-18	0

TABLE II
ELECTRICAL PARAMETERS OF THE DISTRIBUTION LINES

Line impedance Z_{ij}	R_{ij} [Ω]	L_{ij} [μH]
Z_{12}	0.25	1.2
Z_{23}	0.27	1.3
Z_{34}	0.24	1.8
Z_{14}	0.26	2.1

microgrid with nominal frequency $f_0 = 60\,\text{Hz}$, and consisting only of four DGus (n =4) for the sake of clarity. Note that the proposed approach has a more general validity. The DGus are in a ring topology (m =4), as illustrated in Figure 3. The incidence matrix $D \in \mathbb{R}^{4\times4}$, which describes the power network topology can be expressed as

$$D = \begin{bmatrix} -1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix},$$

while the electrical parameters of each DGu and of the interconnecting distribution lines are reported in Table I and in Table II, respectively. The VSC control amplitude $M_{\max}$, for all the decentralized controllers, is set equal to 1000, in order to consider a DC renewable energy voltage source $V_{DC} = 1000\,\text{V}$. Then, the sliding mode control law, switching

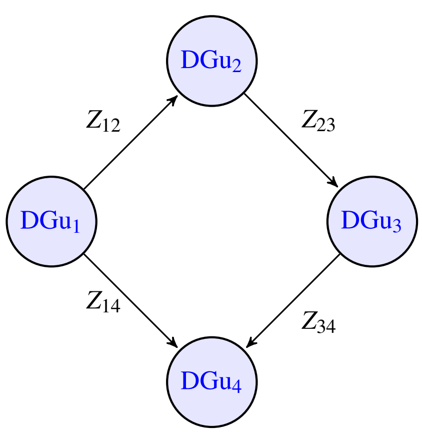

Fig. 3. Scheme of the considered microgrid composed of 4 DGus. The arrows indicate the positive direction of the currents through the power network

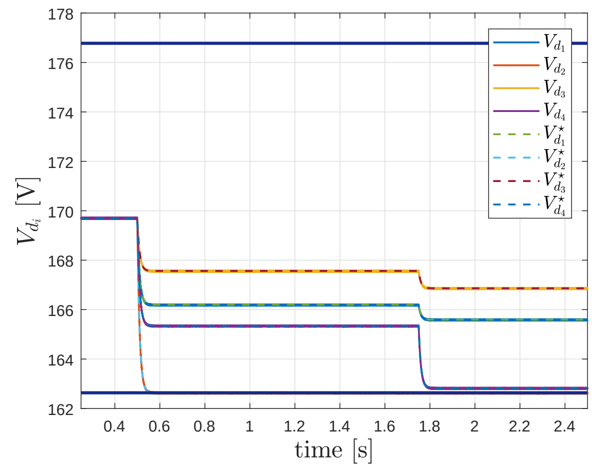

Fig. 4. d-components of the loads voltages and reference values generated by the MPC centralized supervisor

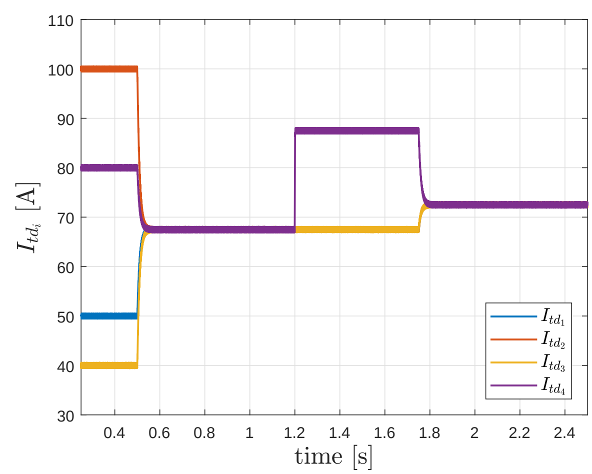

Fig. 5. d-components of the generated currents

between ± 1000, emulates the VSC behaviour.

The performances of the considered microgrid are assessed by taking into account unknown load dynamics and piece-wise constant d-component voltage reference values, generated by the centralized MPC sypervisor. More specifically, the weighted Laplacian matrix is set equal to $\mathcal{L} = DQD^T$, with $\mathcal{Q} = 10^5 I_{m\times m}$, while $\mathcal{R} = 10^{-5} I_{n\times n}$, $I_{n\times n}$ being the $n \times n$ identity matrix. The prediction horizon is $N = 5$, with

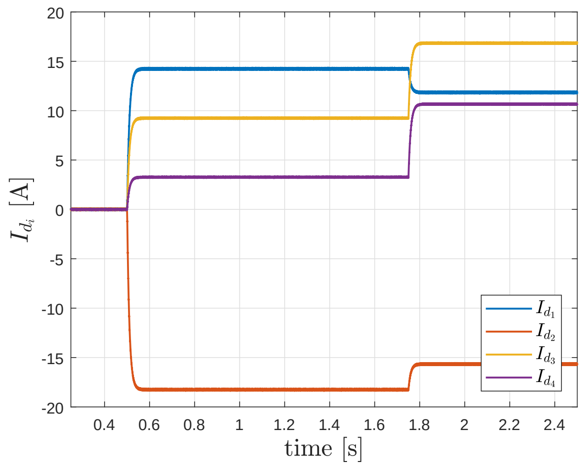

Fig. 6. d-components of the exchanged currents through the interconnecting lines

sampling time $T = 0.25\,\mathrm{s}$. Finally, input and state constraints are set equal to $u_{\min} = -1000 \cdot \mathbb{1}_{2n}$, $u_{\max} = 1000 \cdot \mathbb{1}_{2n}$, and $V_{d_{\min}} = 115\sqrt{2} \cdot \mathbb{1}_n$, $V_{d_{\max}} = 125\sqrt{2} \cdot \mathbb{1}_n$, $\mathbb{1}_n$ being the unit vector with n components, respectively.

In Figure 4 the time evolution of the d-components of the loads voltages is reported. One can observe in Figure 4 that the local SSOSM controllers guarantee finite time voltage tracking performance with respect to the corresponding direct reference values generated by the MPC component. Moreover, note that all the voltage constraints are suitably fulfilled as it is evident in the case of V_{d_2}. The quadrature voltage V_q is steered to zero in a finite time allowing one to use the reduced order system (14) to solve the FHOCP via the MPC supervisor. In Figure 5 the direct components of the generated currents are represented. One can observe that the control objective of current sharing is completely satisfied and, when the load variation $\Delta W_{d_4} = 20\,\mathrm{A}$ occurs at the time instant $t = 1.2\,\mathrm{s}$, the controlled system reacts to establish the current sharing again. In Figure 6 the time evolution of the direct components of the exchanged currents through the distribution lines interconnecting the DGus is illustrated.

VI. Conclusions

In this paper, a hierarchical control architecture based on the joint use of MPC and SMC, is designed in order to stabilize islanded AC microgrids with arbitrary topology and affected by unavoidable modelling uncertainties. The control objective is indeed to keep the DGus output voltages in a prescribed boundary layer while achieving the so-called current sharing among the DGus. The low control level uses SSOSM control strategy in order to track, in a decentralized way, the voltage references generated by a MPC supervisor. The performances of the proposed algorithm have been evaluated in simulation considering a microgrid with four DGus in a ring topology.

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