# A b&b approach to schedule a no-wait flow shop to minimize the residual work content under uncertainty

Simone Dolceamore[1], Marcello Urgo[1]

Mechanical Engineering Department, Politecnico di Milano, Italy
simone.dolceamore@mail.polimi.it, marcello.urgo@polimi.it

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## 1 Introduction and problem statement

In recent years, approaches providing robust schedules have been increasing their importance in the production scheduling research area. The pursued objective is to obtain schedules being insensitive - as much as possible - to disturbing factors, protecting the decision-maker against the impact of unfavorable uncertain events. In this paper we address the scheduling of a set of jobs $\mathcal{J}$ in a paced assembly line in presence of uncertainty affecting the availability of production resources. The proposed approach takes inspiration from the assembly process in the aircraft manufacturing industry. Each job $j$ has to be processed in the assembly line made up of $M$ stations. Being paced, the line is characterized by a cycle time, i.e., at a given time, all the parts move to the next station simultaneously. Hence, within the cycle time, a given deterministic amount of work has to be accomplished in each station. The availability of production resources, i.e., the available working hours of the workers during each cycle time, is modeled as a stochastic variable. The manufacturing system described is a permutation flow-shop with no-wait property (Emmons and Vairaktarakis (2013)). The proposed approach address the definition of a robust scheduling for the assembly line aiming at minimizing the conditional value-at-risk ($CVaR$) of the *residual work content*, i.e. the amount of workload that cannot be completed during the cycle time in the stations, due to a lack of available resources. A branch & bound approach is developed to solve the described problem to optimality. The objective function used, the $CVaR$ is a measure of risk widely used in the financial research, e.g. in portfolio optimization (Rockafellar and Uryasev (1999), Rockafellar and Uryasev (2002)). This class of risk measure has been already taken into consideration for scheduling approaches (Tolio, T. *et al.* (2011), Sarin, S. C. *et al.* (2014)). Specifically, the permutation flow-shop scheduling problem (with or without no-wait property) has been addressed in a considerably large number of papers, e.g., a branch & bound approach is developed by (Kim (1995)) with the objective of minimizing total tardiness, whereas several mixed integer formulations and an implicit enumeration approach are proposed in (Samarghandi and Behroozi 2017) and (Samarghandi and Behroozi (2016)). Nevertheless, the proposed scheduling problem has not been addressed in previous researches.

## 2 Description of the approach

The proposed branch & bound framework relies on a sequential definition of the schedule. At each level $l$ of the associated tree $l \in \mathcal{J}$, a partial solution provides the sequence of the first $l$ jobs scheduled, while the remaining $J - l \in \mathcal{J} \setminus S$ jobs are the candidates to be scheduled next in the sequence. Hence, each node of the tree has as many child nodes as the jobs to schedule, each of them representing a partial solution where a different jobs is added to the partial sequence. The solution tree is explored adopting a depth-first strategy selecting the most promising branches in terms of the best lower bound. At each node, a lower and an upper bound on the target performance (the residual work content) are calculated to determine the most promising branches and prune the dominated ones. The contribution to the objective function of already scheduled jobs is easily calculated. Being the system a permutation flow-shop, once a job is scheduled in the first station, the cycle times where it will be processed by the following stations are automatically determined. Then, considering a single resource with availability $A_c$ for each cycle time period $c$, the sequencing of that job $j$ also entails a resource consumption $R_{jc}$. If a job $j$ is scheduled to enter the first station of the line in period $p$, its contribution to the objective function is:

$$RWC_{S\cup\{j\}} = \underset{c}{\ast}(A_c * R_{j,c}), \quad \forall \quad j \in \mathcal{J} \setminus S, \quad c = p, \dots p + M - 1 \tag{1}$$

where $*$ is the convolution operator.

The lower bound distribution of the residual work content caused by an unscheduled job $i \in \mathcal{J} \setminus S + \{j\}$ can be estimated through the scheduling of a dummy job $\tilde{i_1}$, having the lowest resource request among the ones of the $J - l$ unscheduled jobs. This contribution can be estimated according to Eq. 2.

$$RWC^{LB}_{S+\{j\}\cup i} = \underset{c}{\ast}(A_c * R_{\tilde{i_1},c}), \quad \forall \quad i \in S + \{j\} \setminus \mathcal{J}, \quad c = p, \dots, p + M - 1 \tag{2}$$

In an dual way, the upper bound distribution of the residual work content caused by an unscheduled job $i \in \mathcal{J} \setminus S + \{j\}$ can be estimated scheduling a dummy job $\tilde{i_2}$ having highest among the resource request of the $J - l$ unscheduled jobs (Eq. 3).

$$RWC^{UB}_{S+\{j\}\cup i} = \underset{c}{\ast}(A_c * R_{\tilde{i_2},c}), \quad \forall \quad i \in S + \{j\} \setminus \mathcal{J}, \quad c = p, \dots, p + M - 1 \tag{3}$$

Finally, the lower and upper bounds of the considered node can be calculated as:

$$RWC^{LB} = \underset{i}{\ast} RWC^{LB}_{S+\{j\}\cup i} * \underset{j}{\ast} RWC_{S\cup\{j\}}, \quad \forall j \in S \setminus \mathcal{J}, i \in S + \{j\} \setminus \mathcal{J} \tag{4}$$

$$RWC^{UB} = \underset{i}{\ast} RWC^{UB}_{S+\{j\}\cup i} * \underset{j}{\ast} RWC_{S\cup\{j\}}, \quad \forall j \in S \setminus \mathcal{J}, i \in S + \{j\} \setminus \mathcal{J} \tag{5}$$

Grounding on these calculations, the lower and upper bounding distributions for the residual work content can be calculated in each node. Furthermore, these distributions can also support the calculation of the lower and upper bound of a function of the risk associated to the resource consumption, e.g., the $CVaR$, with the aim at assessing the robustness of the solution. Notice that, Eq. 4 and 5 provides effective bounds for the $CVaR$ only in case the resource requirements of the jobs are deterministic. In this particular case, the convolution operator merely shifts the availability distributions without re-shaping it. This ensures the conditional value-at-risk of residual work content being a regular objective function.

Figure 1 further depicts the branching scheme adopted, as well as the computation of the bounds for the $CVaR$. Blue and black cumulative distribution functions represent the lower and upper bound distributions respectively. Nodes with a lower bound of the $CVaR$ higher than the incumbent $CVaR$ are pruned.

## 3 Testing and Industrial Application

The developed branch-and-bound approach has been implemented in C++ using the BoB++ library. Computational experiments have been performed on 8 parallel threads

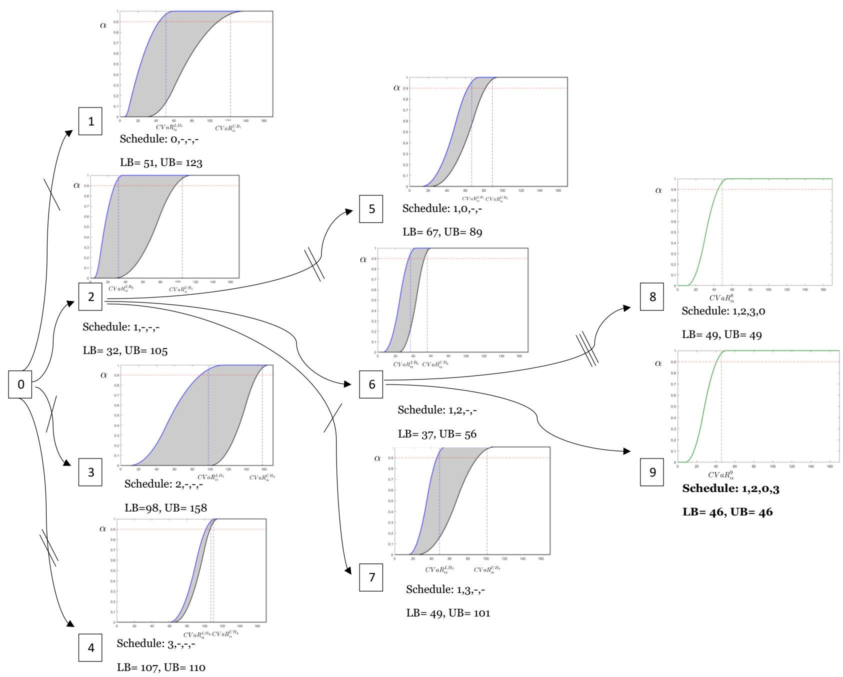


**Fig. 1.** Branching scheme and bounds computation

on an Intel Four-Core i7 Processor 7700-HQ@3.4GHz and 16 GB of DDR4 SDRAM. The performance of the algorithm has been analyzed in terms of the time to find an optimal solution and the fraction of nodes explored solving 9-jobs instances sampled from a pool of 68 real orders. The testing instances have been constructed as follows:

1. the resource requirement of a job $j$ in station $m$ is deterministic. In fact, at the time the assembling of an aircraft is scheduled, order specifications are known and fixed;
2. the resource availability in station $m$ in time cycle $c$ is a discrete triangular distribution, whose maximum value matches the planned ideal amount of workforce while minimum and the mode model the variability caused by absenteeism or other lacks of personnel;
3. the risk level used for the $CVaR$ is set to 10%, this value depends on the risk aversion of the planner, since it defines the quantile of the tail whose expected value must be minimized.

The algorithm was able to find the optimal solution in 8264.15 seconds on average, ranging from a minimum of 7803.20 to a maximum of 8819.61. The average number of evaluated nodes was 280721 over a total of 623547, with an average pruning efficiency of about 55%. The main cause of the relatively long computational times is due to the modest variability in terms of workload requirements among the considered orders, because their assembly process is composed of more or less 90% of mounting and testing operations for structural components that are common to all the orders, while customization activities have a lower impact in terms of equivalent man hours. Nevertheless this is partially due to the oversimplification of the assembly process to a single type of resource and, hence, reducing the impact of the uncertainty affecting the availability of specific resources. More-

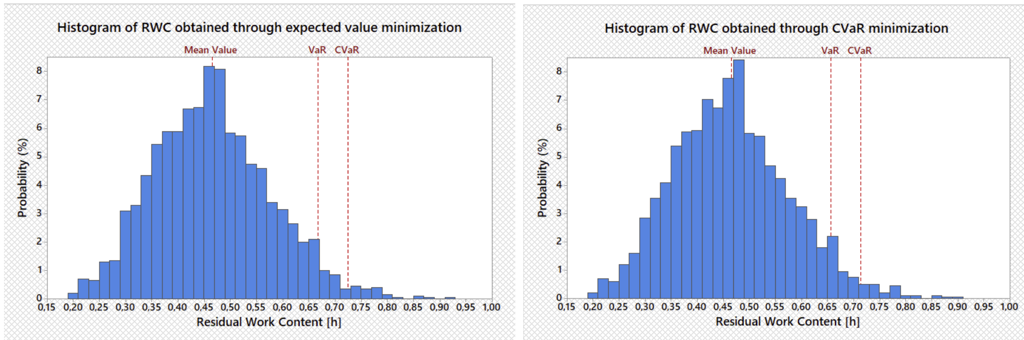


**Fig. 2.** Distribution of the residual work content obtained with the minimization of the $CVaR$ (right) and the expected value (left).

over, due to the convolution operations, the amplitude of the support of the distributions has a strong influence on the time needed to accomplish calculations within a single node.

An additional analysis was carried out to compare the proposed approach against scheduling to minimize the expected value fo the residual work content ($RWC$). An example is provided in Figure 2 showing the histogram of the $RWC$ in the case of the minimization of the expected value (left) and the $CVaR$ (right). Although the expected value in both the cases is almost identical, the $CVaR$ is rather different (0.73 against 0.70) clearly showing that minimizing the $CVaR$ actually reduce its value in the optimal solution. Moreover the distribution on the right shows a low occurrence probability for the highest values of the $RWC$, thus demonstrating the capability of the approach to protect the schedule against the worst cases.

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## References

Emmons, H. and Vairaktarakis, G., 2013, Flow Shop Scheduling: Theoretical Results, Algorithms, and Applications, *International Series in Operations Research & Management Science*, Vol. 182, Springer.

Rockafellar, R. T. and Uryasev, S. 2002, Conditional value-at-risk for general loss distributions, *Journal of Banking & Finance*, Vol. 26, pp. 1443-1471.

Rockafellar, R. T. and Uryasev, S. 1999, Optimization of Conditional Value-at-Risk, *Journal of Risk*, Vol. 2, pp. 21-41.

Tolio, T., Urgo, M., and Váncza, J., 2011, Robust production control against propagation of disruptions, *CIRP Annals - Manufacturing Technology*, Vol. 60, pp. 489-492.

Sarin, S. C., Sherali, H. D. and Liao, L., 2014, Minimizing conditional-value-at-risk for stochastic scheduling problems, *Journal of Scheuling*, Vol. 17, pp. 5-15.

Kim, Y.-D., 1995, Minimizing total tardiness in permutation flowshops, *European Journal of Operational Research*, Vol. 85, pp. 541-555.

Samarghandi, H and Behroozi, M., 2016, An Enumeration Algorithm for the No-Wait Flow Shop Problem with Due Date Constraints, *IFAC - PapersOnLine*, Vol. 49-12, pp. 1803-1808.

Samarghandi, H and Behroozi, M., 2017, On the exact solution of the no-wait flow shop problem with due date constraints, *Computers and Operations Research*, Vol. 81, pp. 141-159.