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Optimal design of sensor networks for damage detection

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Abstract

The structural integrity of buildings and infrastructures can be affected by either environmental conditions or unforeseen external actions. In order to efficiently detect damage, intended as an irreversible degradation of the structural stiffness, many identification algorithms have been proposed in the literature. Nevertheless, a crucial aspect to accurately estimate and locate such damage pertains to the configuration of the deployed structural health monitoring (SHM) system. In addressing this goal, a framework is here proposed for the optimal design of sensor networks, in terms of number, type and spatial deployment of the sensors. The rationale of the method is to simultaneously maximize the information associated with the measurements, and minimize the total cost of the experimental setup; the overarching goal thus lies in the maximization of the information per unit cost, for the efficient allocation of resources. The value of the SHM system is quantified through the Shannon information gain between the a-priori knowledge of the mechanical properties and the values estimated, on the basis of measurements. The types of sensors contained into the overall SHM mix largely affects the estimation accuracy since, as a rule of thumb, the higher the sensor cost, the higher the signal-to-noise ratio and, therefore, the better the attainable estimation. In order to tackle the aforementioned multi objective optimization problem and to derive the associated Pareto front, a-posteriori solution methods relying on evolutionary algorithms are adopted. The proposed method is applied to a shear-type structure, namely the Pirelli tower in Milan, and the relevant multi-criteria optimization solutions are presented.

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Keywords: Structural health monitoring, Sensor network, Optimal sensor placement, Multi-criteria optimization, Damage detection;

1. Introduction

Structural Health Monitoring (SHM) may be exploited to reveal the mechanical properties of structural systems, given a set of measured data. The capability of any SHM procedure for estimating the mechanical parameters depends both on the employed algorithmic tools and the SHM network itself. Therefore, the choice of type, number and positions of the sensors, i.e., the SHM system design, plays a crucial role in maximizing the effectiveness of the monitoring systems. Amongst available possible SHM methods, Bayesian model updating allows to estimate the relevant parameters of the aforementioned model or in other words determine the most plausible structural model, while incorporating the respective uncertainties.

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The design of the SHM sensor network is usually limited to the optimal deployment of the sensors along the structure. Some of the most common Optimal Sensor Placement (OSP) methods may be summarized as [1,2]: the effective independence method (EFI); the effective independence driving-point residue method (EFI-DPR); the kinetic energy method (KEM); the modified variance method (MVM). Alternatively, a method based on topology optimization has been proposed in [3]. Methods of this class are unable to take into account measurement errors and, hence, sensor accuracy. The latter is in fact addressed within a Bayesian context. In relevant works by Papadimitriou [4–6], the OSP has been obtained by minimizing the information entropy [7]. In order to numerically evaluate the objective function, the information entropy has been locally (that is, for a certain set of parameters to be estimated) approximated, by applying the Laplace method of asymptotic expansion.
In this work, a method to efficiently design the sensor network, following a Bayesian experimental design approach, is proposed. Unlike the method of [4], the expected Shannon information gain is computed according to the procedure proposed in [8]. In order to optimally choose the number, position and type of sensors, both the data information and the SHM system cost are accounted for in a multi-objective optimization problem.
The remainder of this paper is organized as follows: firstly, an introduction to the theoretical framework is provided in Section 2, then the results obtained by applying the method to a shear-type building model are discussed in Section 3; finally, some concluding remarks are provided in Section 4.

2. Theoretical framework

2.1. OSP problem formulation

Let the problem variables be defined according to:

- $\boldsymbol{y} \in \mathbb{R}^{n_y}$ is a vectorial random variable that gathers all the measurements obtained from the sensors, where $n_{\boldsymbol{y}}$ is the number of signals;
- $\boldsymbol{\theta} \in \mathbb{R}^{n_\theta}$ is a vectorial random variable that represents the mechanical parameters to be estimated, where $n_{\boldsymbol{\theta}}$ is the number of parameters;
- $\boldsymbol{d} \in \mathbb{R}^{n_d}$ is a vectorial variable that defines the positions of all the sensors in the network. It is assumed that the sensors can only be placed at nodes of a finite element mesh, as follows:

$$\boldsymbol{d} = \{d_1 \ \ \delta_1 \ \ \dots \ \ d_{n_\mathbf{d}} \ \ \delta_{n_\mathbf{d}}\}^T \tag{1}$$

  where d_i is the node label of the corresponding model node, and δ_i is the index of the axis along which the measurement is performed.

According to [9], a common objective function for experimental design problems reads:

$$U(\boldsymbol{d}) = \int_{\mathcal{Y}} \int_{\boldsymbol{\Theta}} u(\boldsymbol{d}, \boldsymbol{y}, \boldsymbol{\theta}) p(\boldsymbol{\theta}, \boldsymbol{y}|\boldsymbol{d}) d\boldsymbol{\theta} d\boldsymbol{y} \tag{2}$$

where $p(\boldsymbol{\theta}, \boldsymbol{y}|\boldsymbol{d})$ is the joint probability density function of the mechanical parameters $\boldsymbol{\theta}$ and the measurements $\boldsymbol{y}$, conditioned on the design variable $\boldsymbol{d}$; $\mathcal{Y}$ and $\boldsymbol{\Theta}$ are, respectively, the supports of the likelihood function $p(\boldsymbol{y}|\boldsymbol{d})$ and the prior probability distribution $p(\boldsymbol{\theta})$. As suggested in [10], the utility function $u(\boldsymbol{d}, \boldsymbol{y}, \boldsymbol{\theta})$ is the relative entropy (Kullbach-Leibler divergence) between the prior and the posterior probability distributions. With this choice, the objective function defined in Eq. (2) is termed the expected Shannon information gain between the prior $p(\boldsymbol{\theta})$ and the posterior $p(\boldsymbol{\theta}|\boldsymbol{y})$ probability distributions of the parameters. This function is interpreted as an index of the utility of the measured data with respect to the parameters to be estimated. Following [8], the objective function is rearranged, through the Bayes' rule, as:

$$U(\boldsymbol{d}) = \int_{\mathcal{Y}} \int_{\boldsymbol{\Theta}} \{\ln[p(\boldsymbol{y}, \boldsymbol{\theta}|\boldsymbol{d})] - \ln[p(\boldsymbol{y}|\boldsymbol{d})]\} p(\boldsymbol{y}, \boldsymbol{\theta}|\boldsymbol{d}) p(\boldsymbol{\theta}) d\boldsymbol{\theta} d\boldsymbol{y} \tag{3}$$

The likelihood function $p(\boldsymbol{y}|\boldsymbol{\theta}, \boldsymbol{d})$ is the probability density of the measured data $\boldsymbol{y}$, for a specific set of mechanical parameters $\boldsymbol{\theta}$ and design variable $\boldsymbol{d}$. In order to compute this latter probability, a forward model that links input and output variables is required; for general nonlinear settings, this model is defined as:

$$\boldsymbol{y} = \boldsymbol{G}(\boldsymbol{d}, \boldsymbol{\theta}) + \boldsymbol{\epsilon} \tag{4}$$

$\boldsymbol{\epsilon} \in \mathbb{R}^{n_{sens}}$ being the model prediction error, accounting for uncertainties due to both modeling and measurement error. The measurement error is assumed to be a zero mean Gaussian noise with probability distribution $p_{\boldsymbol{\epsilon}} = \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma})$ and covariance matrix $\boldsymbol{\Sigma} = \sigma^2 \boldsymbol{I}$, where the standard deviation σ is related to the sensor accuracy. In the following optimization process, we instead disregard the model error.
The OSP problem is addressed by finding the sensors spatial configuration $\boldsymbol{d}^*$ for which the expected Shannon information gain is maximized, according to:

$$\boldsymbol{d}^* = \arg \max_{\boldsymbol{d} \in \mathcal{D}} U(\boldsymbol{d}) \tag{5}$$

where $\mathcal{D}$ is the space of all possible network topologies.

2.2. Cost - Utility optimization

As specified in Eq. (4), measured data depends on the structural response and on the measurement noise. Therefore, according to Eq. (3), the expected Shannon information depends on the positions $\boldsymbol{d}$ of the sensors, on their number n_{sens} and on sensor accuracy (measured through σ), hence $U = U(\boldsymbol{d}, n_{sens}, \sigma)$. For a thorough SHM system design, OSP is not sufficient: the complete parameter set $(\boldsymbol{d}, n_{sens}, \sigma)$ should be determined.
To better understand how U changes at varying parameters $(\boldsymbol{d}, n_{sens}, \sigma)$, let us define the function $\bar{U} = U(\boldsymbol{d}^*, n_{sens}, \sigma)$, where $\boldsymbol{d}^*$ is the optimal sensor configuration obtained from the optimization in Eq. (5). Since every choice of (n_{sens}, σ) corresponds to a unique OSP solution $\boldsymbol{d}^*$, the function $\bar{U} = \bar{U}(n_{sens}, \sigma)$ can be considered to depend on (n_{sens}, σ) only. As analytically demonstrated in [4], if the number of sensors n_{sens} is increased, $\bar{U}$ increases as well as more information becomes available for estimating the mechanical parameters $\boldsymbol{\theta}$. On the other hand, by increasing σ, that is by decreasing the sensor accuracy, the measurements get more noisy and $\bar{U}$ decreases.
In the ideal case where costs and technological constraints are neglected, the information gain can be indefinitely increased by adding more sensors or by increasing their accuracy. In this case, the upper bound would be represented by the value $\bar{U}(n_{dof}, 0)$, namely when all the model nodes are occupied by noise-free sensors. Of course, this design solution has no practical meaning, besides any cost issue, because neither OSP nor Bayesian methods would be required. In real applications, the following constraints have to be taken into account:

1. observability constraint: $n_{sens} > n_{obs}$, where n_{obs} is the minimum number of sensors required to guarantee observability and identifiability of the parameters $\boldsymbol{\theta}$, see [11,12];
2. technological constraint: $\sigma > \sigma_{min}$, where σ_{min} is related to the most accurate sensor available on the market, for a chosen physical quantity to be observed;
3. budgetary constraint: $C(n_{sens}, \sigma) \leq B$, where B is the maximum amount of economic resources that can be allocated for the SHM sensor network.

A reasonable assumption, here employed, is that the cost C of the SHM system is independent of, or negligibly depend on the spatial configuration of the network. Accordingly, the OSP optimization statement can be generalized as follows:

$$(\boldsymbol{d}^*, n^*_{sens}, \sigma^*) = \arg \max_{\substack{\boldsymbol{d} \in \mathcal{D} \\ n_{sens} \in [n_{obs}, +\infty] \\ \sigma \in [\sigma_{min}, +\infty]}} [U(\boldsymbol{d}, n_{sens}, \sigma)] \qquad \text{subject to constraint 3.} \tag{6}$$

This formulation provides the best spatial configuration, type and number of sensors that guarantee the maximum information, given a certain budget. While this optimization approach corresponds to the maximum allocation of resources, it does not guarantee the maximum efficiency; following a common approach in economics [13], an alternative formulation can be proposed as:

$$(\boldsymbol{d}^*, n^*_{sens}, \sigma^*) = \arg \max_{\substack{\boldsymbol{d}\in\mathcal{D} \\ n_{sens}\in[n_{obs},+\infty] \\ \sigma\in[\sigma_{min},+\infty]}} \left[\frac{U(\boldsymbol{d}, n_{sens}, \sigma)}{C(n_{sens}, \sigma)} \right] \qquad \text{subject to constraints 3.} \tag{7}$$

In this case, the optimal design corresponds to the maximization of the information per monetary unit (measure units are [nat/€], where [nat] stand for natural unit of information), leading to the most efficient design solution.

2.3. Numerical solution

The forward model $\boldsymbol{G}(\boldsymbol{d}, \boldsymbol{\theta})$ in Eq. (4) is an operator defined in $\mathbb{R}^{n_d} \times \mathbb{R}^{n_\theta} \rightarrow \mathbb{R}^{n_{sens}}$ and, for a quasi-static structural case numerically modeled, it is given by:

$$\boldsymbol{G}(\boldsymbol{d}, \boldsymbol{\theta}) = \boldsymbol{L}(\boldsymbol{d})\boldsymbol{K}(\boldsymbol{\theta})^{-1}\boldsymbol{F} \tag{8}$$

where $\boldsymbol{L} \in \mathbb{R}^{n_{sens} \times n_{dof}}$ is a Boolean matrix selecting the actually measured model responses; $\boldsymbol{K} \in \mathbb{R}^{n_{dof} \times n_{dof}}$ is the stiffness matrix; and $\boldsymbol{F} \in \mathbb{R}^{n_{dof}}$ represents the load vector. Using Eq. (4), the likelihood function in Eq. (3) can be computed as follows:

$$p(\boldsymbol{y}|\boldsymbol{\theta}, \boldsymbol{d}) = p_\epsilon(\boldsymbol{y} - \boldsymbol{G}(\boldsymbol{d}, \boldsymbol{\theta})) \tag{9}$$

In order to solve the optimization problem in Eq. (5), the integrals providing the expected utility $U(\boldsymbol{d})$ have to be computed. Since they cannot be analytically handled, an approximation strategy is required. In [4], an asymptotic approximation, valid for a large datasets has been proposed relying on the Laplace method: such approach allows to obtain an analytical expression of $U(\boldsymbol{d})$, and hence to reduce the associated computational cost. The disadvantage is that the approximation has to be locally computed for a specific set $\boldsymbol{\theta}_0$ of parameters. To overcome this problem and also extend the formulation to non-linear systems, the approach proposed in [8] is applied, and the relevant integrals are approximated through a double Monte Carlo (MC) integration, according to:

$$U(\boldsymbol{d}) \approx \frac{1}{N_{out}} \sum_{i=1}^{N_{out}} \left\{ \ln\left[p(\boldsymbol{y}^i|\boldsymbol{\theta}^i, \boldsymbol{d})\right] - \ln\left[\frac{1}{N_{in}} \sum_{j=1}^{N_{in}} p(\boldsymbol{y}^i|\boldsymbol{\theta}^j, \boldsymbol{d})\right]\right\} \tag{10}$$

where N_{out} and N_{in} are the number of samples to be drawn in the outer and inner sums respectively; the samples $\boldsymbol{\theta}^i$, $\boldsymbol{\theta}^j$ and $\boldsymbol{y}^i$ are drawn from the probability distributions $p(\boldsymbol{\theta})$ and $p(\boldsymbol{y}|\boldsymbol{\theta}^i, \boldsymbol{d})$; The numerical computation of the objective function according to Eq. (10) would require to evaluate the likelihood function and the system response $N_{in} \times N_{out}$ times. To considerably reduce the relevant computational costs, the forward model in Eq. (8) is replaced by a cheaper, surrogate one [14] based on Polynomial Chaos Expansion (PCE). The PCE surrogate aims at reproducing the map between the joint input variable $\boldsymbol{x} = \begin{bmatrix}\boldsymbol{\theta} & \boldsymbol{d}\end{bmatrix}^T$ and the output variable $\boldsymbol{y}$ as:

$$\boldsymbol{y} \cong \sum_{\alpha\in\mathcal{A}} y_\alpha \boldsymbol{\psi}_\alpha(\boldsymbol{x}) \tag{11}$$

where: $\boldsymbol{\psi}_\alpha$ are multivariate polynomials of order p: $\boldsymbol{\alpha} \in \mathbb{N}^{n_{PCE}}$ is a multi-index associated with $\boldsymbol{\psi}_\alpha$; and $y_\alpha \in \mathbb{R}$ is the corresponding coefficient. A set of n^{PCE} joint input samples $\boldsymbol{x}$ is first drawn from the prior distribution $p(\boldsymbol{\theta})$ and the uniform distribution $\mathcal{U}(\mathcal{D})$, where $\mathcal{D}$ is the space of all the possible nodal positions; then, the corresponding output samples are computed through the full model $\boldsymbol{G}(\boldsymbol{d}, \boldsymbol{\theta})$; finally, the PCE bases and coefficients are computed according to [15,16]. The optimization problem in Eq. (5) is next solved by applying the Covariance Matrix Adaptation Evolution Strategy (CMA-ES) algorithm [17], which is particularly suitable for noisy objective functions, characterized by several local maxima. For further details on the procedure, readers are referred to [18]. Via adoption of the numerical procedure here explained, it is possible to compute the objective functions of both formulations (6) and (7) and hence optimally design the SHM system.

3. Results

The optimization procedure of Section 2 is applied for optimally designing a monitoring system for the Pirelli tower in Milan. The building features 39 storeys and its total height is about 130 m. The floor dimensions are approximately 70 x 10 m and the structure is made of CIP reinforced concrete. The structure is assumed to behave elastically, with lumped masses at each storey. We aim to assess now the effects on the objective functions $\bar{U}$ and $\frac{\bar{U}}{C}$ of the design parameters. We assume that the sensors network cost is calculated according to the following hypothetical simple rule:

$$C(n_{sens}, \sigma) = C_0 + c(\sigma)\, n_{sens} \tag{12}$$

where C_0 represents the fixed network cost and $c(\sigma)$ is the unitary cost per sensor.
In Fig. 1 (a) the contour plot of the objective function $\bar{U}$ (defined in Section 2.2) is reported in [nat] at varying n_{sens} and σ. As expected, $\bar{U}$ is shown to increase with increasing sensor number and precision. The red line in the graph represents the budgetary constraint threshold, which is calculated by imposing $B = 5000$ €, that is 10 times the unitary cost c. In Fig. 1 (b), a compact diagram to compare different design solutions is proposed. Each line represents a Pareto front for a fixed number of sensors. The x-axis represents the cost savings percent and it is expressed by $\frac{B-C(n_{sens},\sigma)}{B}$. By decreasing the SHM system cost (or increasing the saving with respect to the budget), lower information can be obtained. The regions on the left of each Pareto front correspond to sub-optimal spatial configurations $\boldsymbol{d}_{sub}$, for which $U(\boldsymbol{d}_{sub}, n_{sens}, \sigma) < U(\boldsymbol{d}^*, n_{sens}, \sigma)$.

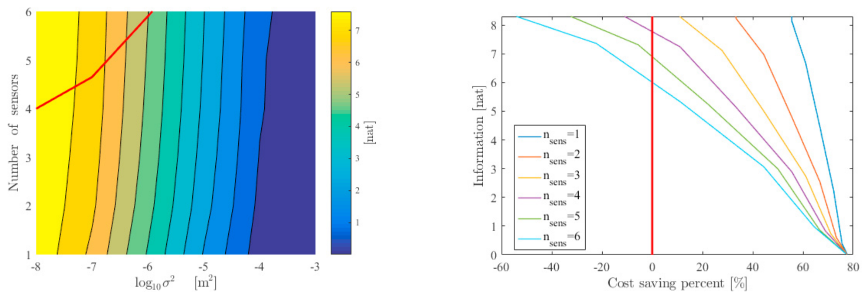

Fig. 1. (a) Contour plot of $\bar{U}(n_{sens}, \sigma)$, red line represents the equation $B = C(n_{sens}, \sigma)$, with $C_0 = 500$ €, $B = 5000$ €; (b) Pareto front of the functions $\bar{U}(n_{sens}, \sigma)$ and $[B - C(n_{sens}, \sigma)]/B$, red line represents the budget constraint $B = C(n_{sens}, \sigma)$.

For the same problem, Fig. 2 provides the contour plot of the objective function defined in Eq. (7). Two different hypotheses are considered: low fixed cost in Fig. 2 (a), high fixed cost in Fig. 2 (b). As a low number of sensors corresponds to a more efficient allocation of the resources, the optimal solution is essentially ruled by constraint 1 in Section 2.2, with n_{obs} that prescribes a lower bound on the function value. Variation of the fixed network cost leads to different design solutions: for low C_0, it is more efficient to employ cheap, low accurate sensors; for high C_0, very accurate sensors should be chosen.

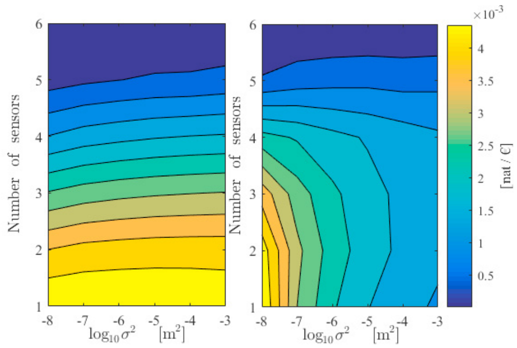

Fig. 2. Contour plot of $\frac{\bar{U}(n_{sens},\sigma)}{C(n_{sens},\sigma)}$. (a) $C_0 = 500$ €; (b) $C_0 = 10000$ €.

4. Conclusions

In the present work, a comprehensive framework to optimally design SHM sensors networks for Bayesian model updating has been presented. Firstly, the Bayesian experimental design framework for optimal sensor placement has been explicitly described. Then a numerical, Monte-Carlo based procedure to solve the optimization problem has been presented. The OSP problem has been generalized to take into account the number of sensors to be deployed and their accuracy. Two possible optimization strategies have been described, and the relative optimization constraints have been discussed. The proposed procedure have been applied to an actual-scale case study, i.e. the Pirelli tower. As results illustrate, the optimal design, that is the choice of spatial configuration $\boldsymbol{d}$, sensors accuracy σ and number of sensors n_{sens}, depends on both the mechanical system and the assumed cost model. The proposed framework allows to quantitatively take into account both of these factors.

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