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COMPARATIVE ASSESSMENT OF DIFFERENT CONSTELLATION GEOMETRIES FOR SPACE-BASED APPLICATION

Simeng Huang[1], Camilla Colombo[1], Franco Bernelli Zazzera[1]

[1] Politecnico di Milano, Dep. of Aerospace Science and Technology, Milano, Italy;
Emails: simeng.huang@polimi.it, camilla.colombo@polimi.it, franco.bernelli@polimi.it

This paper conducts a general study of the constellation geometries for two classical constellation patterns with circular orbits in the region of continuous global coverage. The significant properties for constellation design are identified and assessed with a parametrical approach. The comparison of two constellation patterns in terms of the several properties (coverage, launchability, robustness, stationkeeping, build-up, collision avoidance, end-of-life disposal) are presented. Based on the assessments, fitness functions are developed to quantitatively evaluate these properties. Finally, the geometries best suitable for a given mission are derived through the multi-objective optimisation.

Abbreviation

SOC	Street of Coverage
RAAN	Right Ascension of Ascending Node
GPS	Global Positioning System
PU	Pattern Unit
LEO	Low Earth Orbit
MEO	Medium Earth Orbit

Nomenclature

j	Coverage level
N	Number of satellites
P	Number of orbital planes
S	Number of satellites per orbital plane
i	Inclination [deg]
θ	Angular radius of coverage circle [deg]
θ_{min}	Minimum angular radius of coverage circle [deg]
C_j	Half-width of SOC [deg]
$\Delta\Omega_{sj}$	RAAN difference between same-directional orbits in SOC pattern [deg]
$\Delta\Omega_{oj}$	RAAN difference between opposite-directional orbits in SOC pattern [deg]
$\Delta\varphi_{intraj}$	Relative intraplane phase angle in SOC pattern [deg]
$\Delta\varphi_{interj}$	Relative interplane phase angle in SOC pattern [deg]
F	Relative interplane phase angle in Delta pattern [PU]
N_g	number of independent ground tracks
H	altitude [km]
ε	Elevation angle [deg]
R_E	Earth radius [km]
COV	excess coverage
ΔV_1	Velocity change of the first burn [km/s]
ΔV_2	Velocity change of the second burn [km/s]
μ	Gravitational field constant of the Earth [km^3/s^2]
H_0	Initial altitude [km]
H_t	Final altitude after altitude decay [km]
a	Semi-major axis [km]
m/C_DA	Ballistic coefficient [kg/m^2]
ρ	Atmospheric density [kg/m^3]
ρ^*	Reference atmospheric density [kg/m^3]
a^*	Reference semi-major axis [km]

h_s	Scale height [km]
ΔV_{alt}	ΔV budget for altitude maintenance [km/s]
OPP	collision opportunity per year
T	Orbital period [s]
γ_{min}	Minimum angular separation between a pair of interplane satellites [deg]
ΔM	Mean anomaly difference [deg]
$\Delta\Omega$	RAAN difference [deg]
Γ_{min}	Minimum value of γ_{min} [deg]
H_r	Perigee altitude of the re-entry orbit [km]
ΔV_{deo}	ΔV budget for de-orbiting [km/s]
i_{lch}	Inclination of launch site [deg]
MCP_j	Mean value of coverage percentage over one revolution for j-fold coverage [%]

I. INTRODUCTION

As services from space are becoming an asset for life on Earth and the demand for data from space increases, the international interest in satellite constellations is increasingly growing. Generally, the satellite constellations are used for surveillance and reconnaissance, communication, positioning and navigation, and military defence. Companies including OneWeb, Samsung and Space-X, have recently made public their plan to deploy mega constellations of nanosatellites for global internet.

No general rules for constellation design exists, however, there are a series of widely recognised factors that dominate the process of constellation design. Wertz listed the principal and secondary design variables [1], that influence the geometry of a constellation, i.e., constellation pattern, number of satellites, number of orbit planes and orbit elements of the satellites in the constellation. Up to now, a variety of constellation geometries have been proposed to meet a multiplicity of requirements, each one having specific advantages in terms of coverage, access to space, robustness, etc.

Constellation design is a process of trade-off between various performances and system costs, which are strongly affected by the constellation geometry. Therefore, the possible geometries need to be evaluated though a comparative assessment. Lang compared the continuous global coverage constellations in circular orbits in terms of coverage, launch vehicle ability, sparing strategy, crosslinking, space debris mitigation and collision avoidance[2]. Wertz evaluated a number of responsive constellations with respect to coverage, responsiveness, accessibility, range to target and environmental characteristics [3]. Keller examined the geometry of polar and near-polar constellations for the use of global communication [4]. Draim proposed a non-dimensional parameter to assess the coverage performance of elliptical constellations [5]. However, most of those studies only focused on one or few performances, lacking of generalisation.

In this paper, a general study of constellation geometry is conducted to provide a basis for evaluating a constellation design. As the first step of this research, the constellations with circular orbits for continuous global coverage are considered. In this work, several crucial constellation properties are assessed: coverage, launchability, robustness, stationkeeping, build-up of the constellation, collision avoidance manoeuvre requirements, and end-of-life disposal. The assessments are conducted following a parametric approach. Based on the assessment results, each property is quantitatively evaluated by deriving a fitness function. Through multi-objective optimisation, the constellation geometry best suitable for the given mission requirements will be derived.

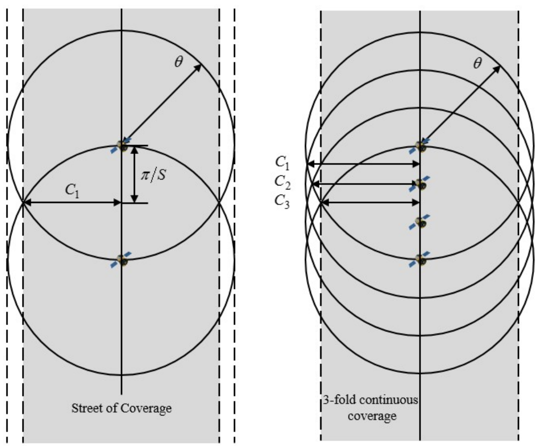
In the first section of this paper, two classical constellation patterns are described and the optimisation approaches to minimum angular radius of coverage circle for both patterns are introduced. In the second section, the constellation properties that dominate the constellation design are identified and parametrically assessed, and the comparison of the optimal coverage geometries for the two patterns are conducted. In the third section, the fitness functions are developed to quantitatively evaluate the assessed properties, and the approach of multi-objective optimisation is used to find out the optimal geometries for a given mission.

II. CONSTELLATION PATTERNS

In this section, two classical constellation patterns with circular orbits for continuous global coverage are described, and the optimisation approaches to minimum angular radius of coverage circle for both patterns are introduced. In these two patterns, all of the orbital planes have the same altitude and inclination so that the perigee and nodal shifts caused by Earth's oblateness are same for all of the satellites in constellation.

II.I. Street of Coverage Pattern

The SOC (Street of Coverage) pattern was developed based on the SOC concept, firstly published by Lüders and having been widely used by many constellation designers [6]. The geometries of SOC concept for single and multiple coverage are shown in Fig. 1.



a) Single coverage. **b) Multiple coverage.**

Fig. 1: Geometries of SOC concept. a) Single coverage, b) Multiple coverage.

As shown in Fig. 1, for a single orbital plane with S evenly distributed satellites, if the satellite separation is less than $2\theta/j$, where θ is the angular radius of coverage circle and j is the coverage level, there will be a narrower swath, referred to as SOC, in which the coverage is continuous. The relationship between S, θ and the half-width of SOC for j-fold continuous coverage C_j, is given by [7]:

$$\cos C_j = \frac{\cos\theta}{\cos(j\pi/S)} \quad (1)$$

In 1977, Beste proposed the polar SOC pattern [8]. He discovered that the adjacent planes moving in same direction were less overlapped than those moving in opposite directions. In polar SOC pattern, the orbital planes are unevenly spaced over half of the equator. Compared with the previous work by Lüders, Beste reduced the number of satellites by 15% for 1-fold continuous global coverage. Fig. 2 shows the polar SOC pattern in polar view. It can be observed that all satellites transit across the equator line northward in one side and southward in the other side. Therefore, there are two sides at which satellites of adjacent planes move in same and opposite directions.

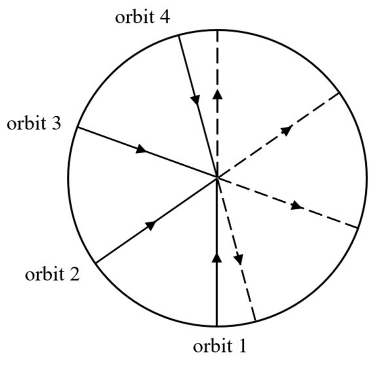
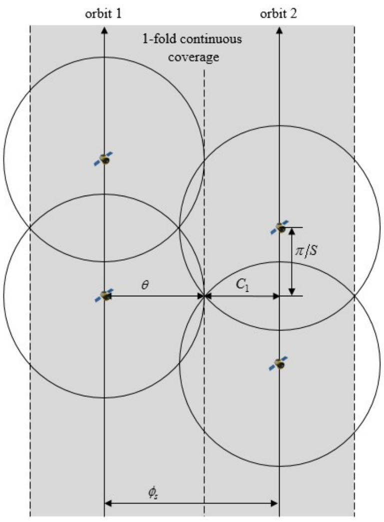
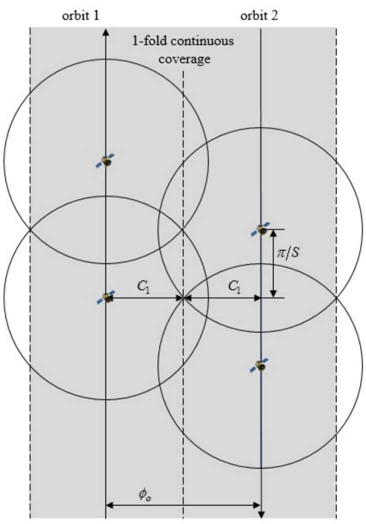


Fig. 2: Polar view of polar SOC pattern.

Fig. 3 illustrates the coverage in adjacent planes in polar SOC pattern for 1-fold continuous coverage. For orbits moving in the same direction, as shown in Fig. 3a), the relative positions of satellites are invariant. Therefore, the dip of coverage circle in one orbit can always be offset by the bulge of coverage circle in the other orbit if the satellites are properly distributed. In this way, the coverage overlaps are minimised. However, for orbits moving in the opposite directions, as shown in Fig. 3b), the relative positions of satellites change over time. To ensure the continuous coverage at any moment, the orbital separation has to be narrowed, although leading to larger coverage overlaps.



a) Same direction.



b) Opposite directions.

Fig. 3: Coverage in adjacent planes. a) Same direction, b) Opposite directions.

However, the polar SOC pattern is not suitable for practical applications because of the collision hazard at the poles; in addition, the best coverage offered by the polar SOC pattern is at poles while, for telecommunication mission, the Earth population is mainly distributed at middle latitudes. Therefore, Yuri proposed the inclined SOC pattern [9]. The inclined pattern is an extension of the polar pattern by transforming the original polar constellations to a new class of inclined constellations, including the polar ones.

The orbital separation and satellites distribution of SOC pattern at the inclination i for j-fold continuous global coverage are given by [9]

$$\Delta\Omega_{sj} = 2\sin^{-1}\left\{\frac{\sin\left[\left(C_j+\theta\right)/2\right]}{\sin i}\right\}$$
$$\Delta\Omega_{oj} = \cos^{-1}\left[\frac{\cos\left(C_j+C_1\right)+\cos^2 i}{\sin^2 i}\right] \qquad (2)$$

$$\Delta\varphi_{intraj} = 2\pi/S$$
$$\Delta\varphi_{interj} = j\pi/S - 2\cos^{-1}\left\{\frac{\cos\left(\Delta\Omega_{sj}/2\right)}{\cos\left[\left(C_j+\theta\right)/2\right]}\right\} \qquad (3)$$

where, $\Delta\Omega_{sj}$ and $\Delta\Omega_{oj}$ are RAAN (Right Ascension of Ascending Node) differences of same-directional orbits and opposite-directional orbits, $\Delta\varphi_{intraj}$ and $\Delta\varphi_{interj}$ are the relative intraplane and interplane phase angles. Moreover, there is a constraint between $\Delta\Omega_{sj}$ and $\Delta\Omega_{oj}$, given by

$$\left(P-1\right)\Delta\Omega_{sj} + \Delta\Omega_{oj} \geq \pi \qquad (4)$$

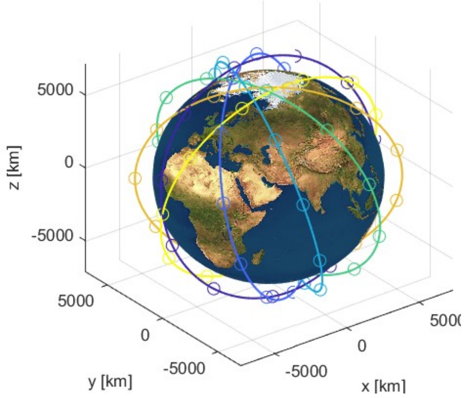
For SOC pattern, there exists a series of alternate geometries that can meet the coverage requirement for a given number of satellites. Take the example of 66 satellites constellation with 6 orbital planes for 1-fold continuous global coverage. The geometrical parameters are listed in Table 1, where θ_{min} is the minimum angular radius of coverage circle.

Table 1: Geometrical parameters of 66 satellites SOC constellation with 6 orbital planes for 1-fold continuous global coverage.

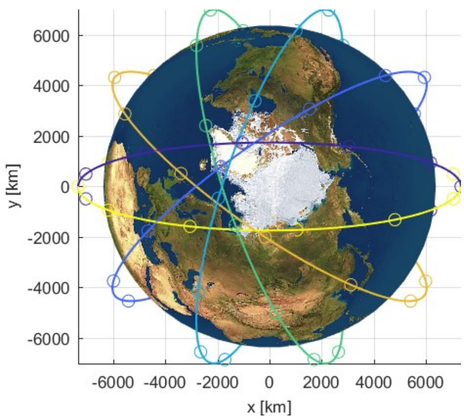
Parameter	Value/Range
N	66

P	6
S	11
i	76.29 to 90 deg
θ_{min}	21.23 to 19.91 deg
$\Delta\Omega_{s1}$	35.99 to 31.40 deg
$\Delta\Omega_{o1}$	0.02 to 22.99 deg
$\Delta\varphi_{intra1}$	32.73 deg
$\Delta\varphi_{inter1}$	7.62 to 16.36 deg

As shown in Table 1, the minimum inclination happens when $\Delta\Omega_{oj}$ equals to zero. Fig. 4 shows the constellation at the minimum inclination.



a) 3D view.



b) Polar view.

Fig. 4: 66 satellites SOC constellation with 6 orbital planes at the inclination of 76.29deg for 1-fold continuous global coverage. a) 3D view, b) Polar view.

II.II. Delta Pattern

Delta pattern was first proposed by J. Walker and has been widely used in many practical applications, such as the GPS (Global Positioning System) constellation. Differently from the SOC pattern, orbital planes in Delta pattern are evenly spaced over the equator. Satellites in each orbital plane are also evenly distributed. Therefore, the geometry of Delta pattern is completely symmetrical.

The geometry of Delta pattern is determined and designated by i, N, P, and relative interplane phase angle F, written in shorthand notation as i: $N/P/F$. The specific angular unit for Delta pattern is PU (Pattern Unit), defined by

$$1\ \text{PU} = 2\pi/N \tag{5}$$

F can be any integer from 0 to (P - 1) PU because the constellation geometry repeats in an interval of P PU.

Fig. 5 shows the 54.57 deg: 24/6/4 Delta constellation for 4-fold continuous global coverage.

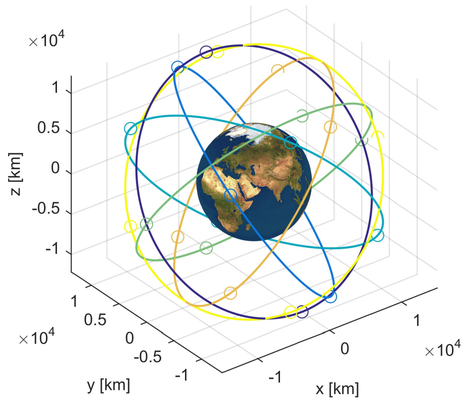


Fig. 5: 54.57deg: 24/6/4 Delta constellation for 4-fold continuous global coverage.

II.III. Optimisation Approaches to θ_{min}

For a given geometry, i.e., fixed values of N, P, F (only for Delta pattern), and i, there exists a range of θ meeting the coverage requirement. The minimum value of θ, represented by θ_{min}, is the parameter which is usually regarded as the coverage efficiency of a constellation. Therefore, the optimisation process to obtain θ_{min} is a necessary step in constellation design.

SOC Pattern

For SOC pattern, the optimisation problem of θ_{min} is formulated as

$$
\begin{aligned}
\text{Min} \quad & f(\theta) = (P-1)\Delta\Omega_{sj} + \Delta\Omega_{oj} - \pi \\
\text{Subject to} \quad & j\pi/S \le \theta \le \pi/2 \\
& C_j + \theta \le 2i \\
& C_j + C_1 \ge \pi - 2i \\
& (P-1)\Delta\Omega_{sj} + \Delta\Omega_{oj} \ge \pi
\end{aligned} \tag{6}
$$

Taking the example of 66 satellites SOC constellation for 1-fold continuous global coverage, Fig. 6 shows the value of θ_{min} as a function of the inclination for different number of orbital planes. It can be seen that the value of θ_{min} decreases with the inclination and the number of orbital planes. For a given number of orbital planes, the minimum value of θ_{min} always happens at the inclination of 90 deg.

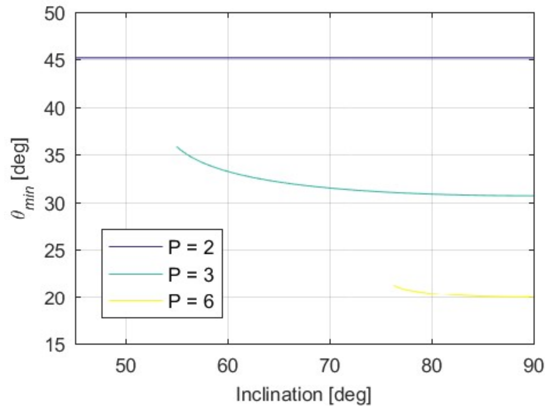


Fig. 6: θ_{min} of 66 satellites SOC constellation for 1-fold continuous global coverage.

<u>Delta Pattern</u>

Many work were dedicated in finding θ_{min} with different approaches [10] [11]. Up to now, the most efficient approach was proposed by Lang [12]. He assumed that the Earth's rotational rate was the same as the satellites orbital rotational rate in order to reduce the amount of calculations. Calculated number of independent ground tracks N_g, the Earth surface is divided into $4N_g$ equal regions and only one region needs to be checked. By calculating the angular distance between all of the terrestrial points and the satellites over one revolution, the minimum value of θ can be found.

Taking the example of the 24 satellites Delta constellation for 4-fold continuous global coverage, Fig. 7 shows the value of θ_{min} as a function of the inclination and F for different number of orbital planes.

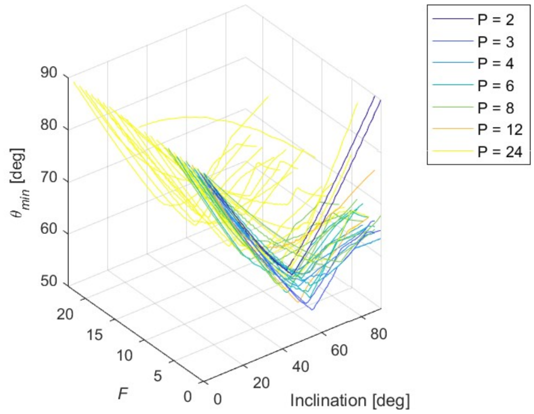


Fig. 7: θ_{min} of 24 satellites Delta constellation for 4-fold continuous global coverage.

<u>Definition of Constellation Geometry</u>

The constellation geometry is the geometrical information describing the absolute and relative positions for all of the satellites in a constellation. As mentioned in Sec. I, the constellation geometry consists of the constellation pattern, the number of satellites, the number of orbital planes, the orbital separation, the satellites distribution, etc. The geometry of a constellation can be determined with a number of parameters, summarised in Table 2.

Table 2: Critical geometrical parameters.

Geometrical Parameters	Symbol	Pattern
Number of satellites	N	SOC, Delta
Number of orbital planes	P	SOC, Delta
Relative interplane phase angle	F	Delta
Inclination	i	SOC, Delta
Minimum angular radius of coverage circle	θ_{min}	SOC, Delta
Elevation angle	ε	SOC, Delta

As shown in Table 2, Delta pattern has one more design variable than SOC pattern.

III. CONSTELLATION PROPERTIES ASSESSMENT

In this section, the significant constellation properties that drive the constellation design are identified and parametrically assessed, and the comparison of the optimal coverage geometries for SOC and Delta pattern is conducted.

III.I. Coverage

Coverage is the principal property of constellations. The coverage geometry of a single satellite is shown in Fig. 8.

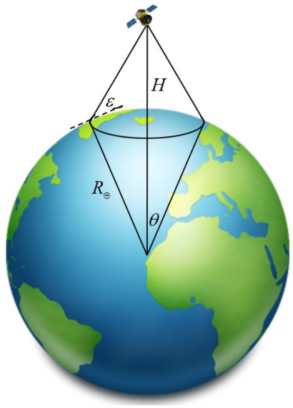


Fig. 8: Coverage geometry of a single satellite.

The relationship between the angular radius of coverage circle θ, the altitude H and the elevation angle ε is given by

$$\frac{\cos(\theta+\varepsilon)}{\cos\varepsilon}=\frac{R_E}{H+R_E} \tag{7}$$

where R_E is the Earth radius.

For a fixed altitude, a lower value of θ will allow a larger value of ε, easing the problem of terrain obstruction due to atmosphere. Conversely, for a fixed value of ε, a lower value of θ will allow a lower altitude, in some sense saving the system costs. In a word, the lower value of θ, the better coverage property.

Optimal Coverage Geometry

The optimal coverage geometry is the geometry having the minimum value of θ_{min} for a given number of satellites. It represents the best geometry in terms of coverage property. Fig. 9 shows the minimum value of θ_{min} as a function of the number of satellites for 1-fold to 4-fold continuous global coverage. It is observed that the Delta pattern is generally superior to the SOC pattern except cases with less than 20 satellites for 1-fold continuous global coverage.

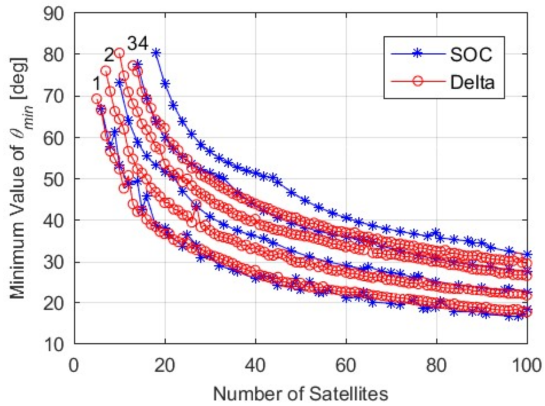


Fig. 9: Minimum value of θ_{min} for 1-fold to 4-fold continuous global coverage.

Excess Coverage

The common approach to assess the coverage property for continuous coverage constellations is the excess coverage, i.e., the ratio of the total available coverage to the required coverage. For a constellation of N satellites for j-fold continuous coverage, the excess coverage is given by [7]

$$COV=\frac{N(1-\cos\theta_{min})}{2j} \tag{8}$$

The physical insight into the excess coverage is the redundancy of the satellites utility if the satellites are perfectly distributed. For example, COV = 2 means that the number of satellites is twice as much as that would be needed.

III.II. Launchability

The launchability property is one of the major cost drivers in the constellation design. If we neglect the influence of the spacecraft mass the spacecraft mass, the key parameters that determine the launchability property is the altitude and inclination of constellations [13].

Altitude

A specific coverage requirement can be achieved by less satellites at a higher altitude or by more satellites at a lower altitude. Therefore, there is a trade-off between the number of satellites and the altitude. Fig. 10 shows the altitude of optimal coverage geometry as a function of the number of satellites for 1-fold to 4-fold continuous global coverage. In this plot, the value of ε is 10 deg. For other values of ε, the relationships of the altitude and number of satellites remain the same.

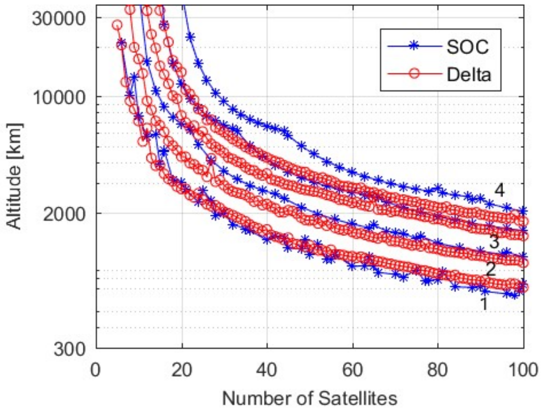


Fig. 10: Altitude of optimal coverage geometry for 1-fold to 4-fold continuous global coverage (ε = 10 deg).

Inclination

Basically, the payload capability of launch vehicle decreases as the constellation inclination increases above the inclination of launch site.

Fig. 11 shows the minimum inclination of SOC pattern as a function of the number of satellites for 1-fold continuous global coverage. In the SOC pattern, the minimum inclination happens at the geometry with the minimum number of orbital planes, indicating that the geometry with less orbital planes can offer a wider selection of inclination.

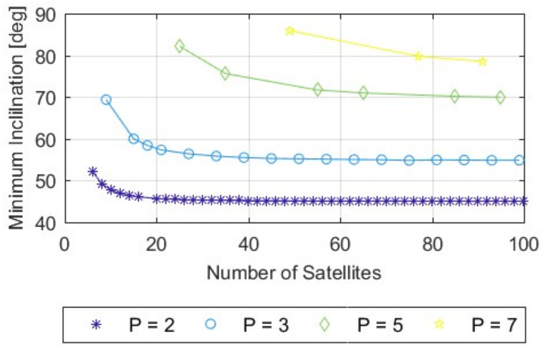


Fig. 11: Minimum inclination of SOC pattern for 1-fold continuous global coverage.

Fig. 12 shows the inclination of optimal coverage geometry for Delta pattern as a function of the number of satellites. It is observed that the inclination of the optimal coverage geometry generally increases with the number of satellites. Reminding that the minimum value of θ_{min} decreases with the number of satellites (shown in Fig. 9). As a consequence thereof, the inclination increases to ensure the coverage at poles.

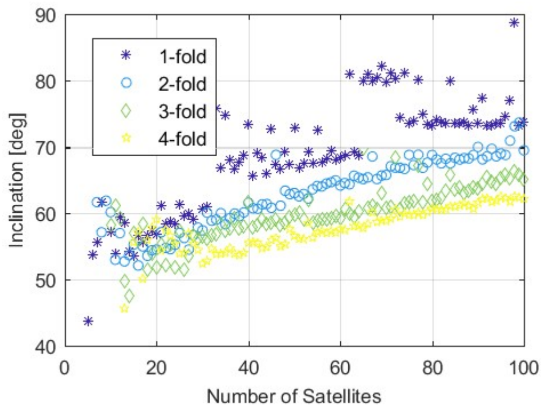


Fig. 12: Inclination of optimal coverage geometry for Delta pattern.

III.III. Robustness

The robustness property is the ability of constellations to keep the properties, especially the coverage property, when a failure happens. In this study, the robustness property is assessed by the coverage percentage.

Coverage percentage

The coverage percentage is the percentage of Earth surface that is covered by the satellites. The coverage percentage for different fold of coverage can be obtained by checking the number of satellites that are visible to each terrestrial point after calculating the angular distance between all of the terrestrial points and satellites.

As an example, Fig. 13 shows the coverage percentage of 54.57 deg: 24/6/4 Delta constellation for 4-fold continuous global coverage over one revolution.

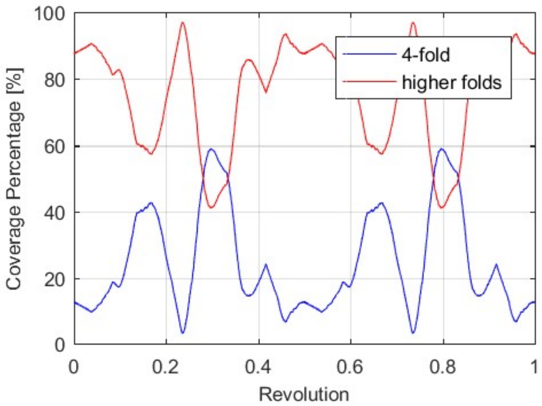


Fig. 13: Coverage percentage of 54.57deg: 24/6/4 Delta constellation.

If the terrestrial region is offered extra folds of coverage, the coverage property in that region will be maintained when failure happens. Therefore, the lower coverage percentage of the required coverage level implies the stronger robustness. As shown in Fig. 13, the strongest robustness happens at about the 0.25 and 0.75 revolution.

III.IV. Stationkeeping

The dominant perturbations for most constellations are the atmospheric drag and Earth's oblateness. For circular-orbit constellations with identical inclination and common altitude, the perigee and nodal shifts caused by the Earth oblateness are same for each satellite, thus the relative positions of satellites with respect to each other are stable. Therefore, the only parameter to be maintained is the decaying altitude caused by the atmospheric drag.

Altitude Maintenance

Assuming that the altitude maintenance is conducted via 2-burn Hohmann transfer, the fundamental equations are given by

$$\Delta V_1 = \sqrt{\mu}\left(\sqrt{\frac{2}{H_t+R_E}-\frac{2}{H_0+H_t+2R_E}}-\sqrt{\frac{1}{H_t+R_E}}\right)$$
$$\Delta V_2 = \sqrt{\mu}\left(\sqrt{\frac{1}{H_0+R_E}}-\sqrt{\frac{2}{H_0+R_E}-\frac{2}{H_0+H_t+2R_E}}\right) \quad (9)$$

where, ΔV_1 and ΔV_2 are the velocity changes for each burn, μ is the gravitational field constant of the Earth, H_0 is the initial altitude, and H_t is the final altitude after altitude decay. The value of H_t is derived by integrating the differential of semi-major axis a ($a = H + R_E$):

$$\delta a = -\frac{C_D A}{m}\rho\sqrt{\mu a}\mathrm{d}t \quad (10)$$

where, $m/C_D A$ is the ballistic coefficient and ρ is the atmospheric density, given by the exponential model:

$$\rho = \rho^* \exp\left(-\frac{a-a^*}{h_s}\right) \quad (11)$$

where, ρ^* is the reference atmospheric density, a^* is the reference semi-major axis and h_s is the scale height.

For a constellation of N satellites, the total ΔV for altitude maintenance is given by:

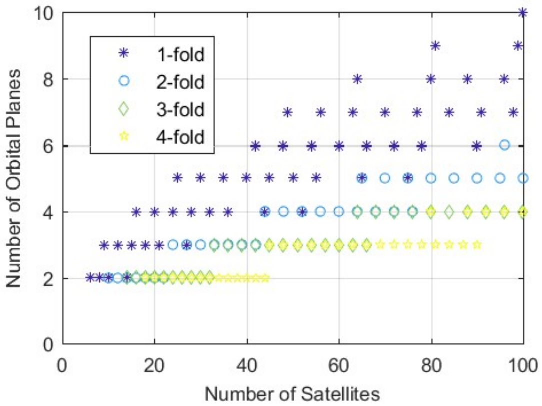
$$\Delta V_{alt} = N\left(\Delta V_1 + \Delta V_2\right) \quad (12)$$

III.V. Build-Up

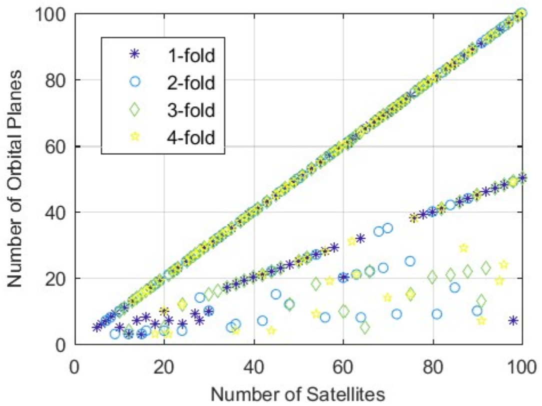
Constellations can be built up in a variety of ways. The build-up process is unique and strongly related to the number of satellites that the launch vehicle is able to place on orbit by a single launch.

Up to now, the technology of placing multiple satellites into a single orbit through a single launch is mature. However, due to technical issues, it is still inefficient to place multiple satellites into multiple orbits by a single launch. Therefore, the number of orbital planes is an important parameter in terms of the build-up period.

Fig. 14 shows the number of orbital planes of optimal coverage geometry for SOC and Delta pattern. It is observed that the number of orbital planes for SOC pattern is significantly less than Delta pattern. Therefore, the SOC pattern might be favoured in terms of the build-up period. The physical insight behind Fig. 14 is the geometry characteristics of each pattern. For SOC pattern, according to Eq. (6), there is a strong relation between θ and P, so that the value of P is always limited by θ. While for Delta pattern, because of its completely symmetrical geometry, the value of P can be as large as the number of satellites.



a) SOC pattern.



b) Delta pattern.

Fig. 14: Number of orbital planes of optimal coverage geometry. a) SOC pattern, b) Delta pattern.

III.VI. Collision Avoidance

Any collision in the constellation will lead to a chain reaction in which a debris cloud will remains, consequently increasing the possibility of the subsequent collisions. Therefore, the constellation must be designed for collision avoidance. In this study, the collision avoidance property is assessed by the collision opportunity and minimum angular separation.

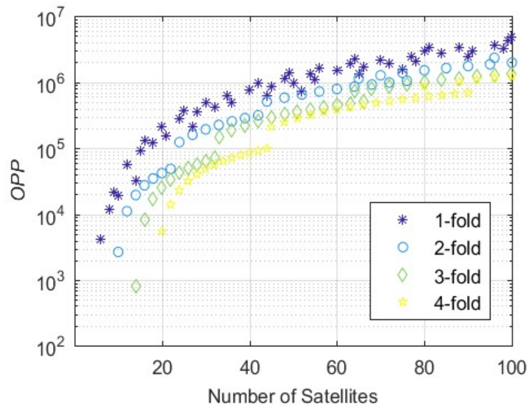
Collision Opportunity

Collision opportunity is defined as an incident in which one satellite passes through the orbital plane of another satellite [13]. For a constellation of N satellites with P orbital planes, the collision opportunities per year is defined by

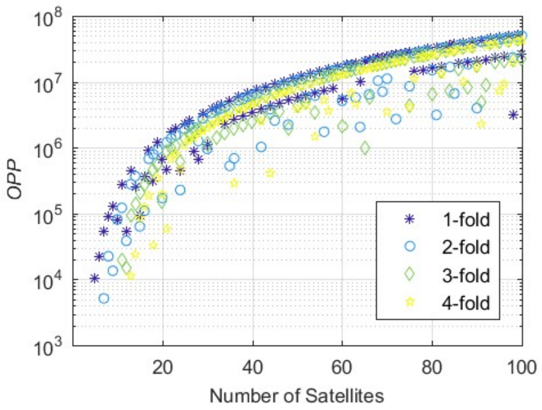
$$OPP = 2N\left(P-1\right)\cdot\frac{31536000}{T} \quad (13)$$

where, T is the orbital period and 31,536,000 is the number of seconds in one year.

Fig. 15 shows the collision opportunities of optimal coverage geometry for SOC and Delta pattern as a function of the number of satellites. Apparently, the value of OPP exponentially increases with the number of satellites. Comparing the two patterns, the collision opportunities of SOC pattern is generally one-tenth of Delta pattern, because the number of orbital planes for SOC pattern is always less than Delta pattern (shown in Fig. 14).



a) SOC pattern.



b) Delta pattern.

Fig. 15: Collision opportunities of optimal coverage geometry. a) SOC pattern, b) Delta pattern.

Minimum Angular Separation

The minimum angular separation is the minimum angular distance between a pair of interplane satellites in the constellation. For circular orbits with the same altitude and inclination, the minimum angular separation is given by [14]

$$\cos\gamma_{min} = \cos^2\alpha - \sin^2\alpha\cos\beta \tag{14}$$

where

$$\alpha = \Delta M + 2\tan^{-1}\left[\tan(\Delta\Omega/2)\cos i\right] \tag{15}$$

$$\cos\beta = \cos^2 i + \sin^2 i\cos\Delta\Omega \tag{16}$$

γ_{min} is the minimum angular separation between a pair of interplane satellites, ΔM is the mean anomaly difference and $\Delta\Omega$ is the RAAN difference.

By computing the values of γ_{min} for all pairs of interplane satellites in the constellation, the minimum value of γ_{min}, represented by Γ_{min}, can be found. From the collision avoidance point of view, constellations with large values of Γ_{min} would be favoured.

For Delta pattern, the value of Γ_{min} generally decreases with the number of satellites. Fig. 16 shows Γ_{min} of the optimal coverage geometry for Delta pattern as a function of the number of satellites.

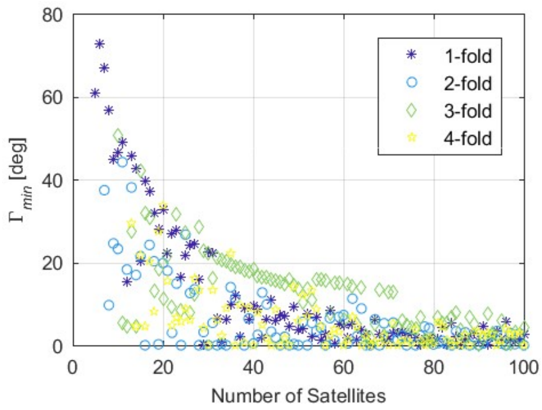


Fig. 16: Γ_{min} of optimal coverage geometry for Delta pattern.

For SOC pattern, the value of Γ_{min} is not only related to the number of satellites, but also strongly dependent on the number of orbital planes and the inclination. As an example, Fig. 17 shows Γ_{min} of 66 satellites SOC pattern as a function of the inclination for different number of orbital planes. It is observed that the collisions might happen in constellations with more than 2 orbital planes and definitely happen at the inclination of 90 deg. Therefore, SOC constellations with inclined orbits and less orbital planes would be favoured.

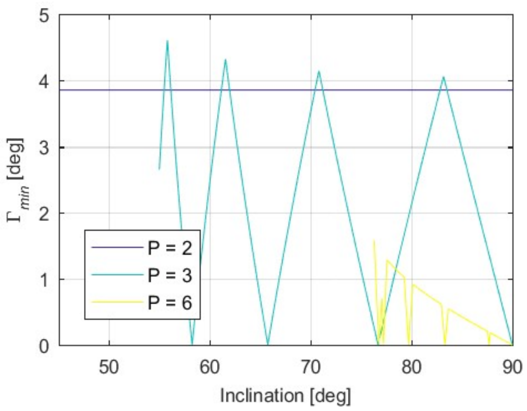


Fig. 17: Γ_{min} of 66 satellites SOC constellation.

III.VII. End-of-Life Disposal

According to the international regulation, the dead satellites must be removed from orbit within 25 years of their end of life. Therefore, the end-of-life disposal of constellations is a critical issue.

Generally, there are two approaches to end-of-life disposal: de-orbiting satellites in LEO (Low Earth Orbit) and moving satellites in MEO (Medium Earth Orbit) to graveyard orbits. The ΔV budget for end-of-life disposal for constellation is not only determined by the altitude, but also the number of the satellites. De-orbiting a constellation of 100 satellites at a lower altitude might not be more efficient than de-orbiting a constellation of 5 satellites at a higher altitude. Considering that the selection of the graveyard orbit is relatively flexible, thus only the first approach is discussed in this study for the purpose of consistency.

De-orbiting

The ΔV budget for a constellation of N satellites to transfer from the initial orbits to the re-entry orbits is given by

$$\Delta V_{deo} = N\sqrt{\mu}\left(\sqrt{\frac{1}{H_0+R_E}} - \sqrt{\frac{2}{H_0+R_E} - \frac{2}{H_0+H_r+2R_E}}\right) \quad (17)$$

where H_r is the perigee altitude of the re-entry orbit.

IV. MULTI-OBJECTIVE OPTIMISATION

In this section, the fitness functions to quantitatively evaluate the assessed properties are developed. A multi-objective optimisation method is used to find out the optimal geometries for a given mission.

IV.I. Fitness Functions

To evaluate the properties assessed in Sec. III, a set of fitness functions are developed:

$$\begin{aligned} J_1 &= COV \\ J_2 &= i/i_{lch} \\ J_3 &= MCP_j \\ J_4 &= -10/\log_{10}(\Delta V_{alt}) \\ J_5 &= \log_{10}(OPP)/8 - \Gamma_{min}/90 \\ J_6 &= \mathrm{P}/10 \\ J_7 &= \log_{10}(\Delta V_{deo})/10 \end{aligned} \quad (18)$$

where, i_{lch} is the inclination of launch site and MCP_j is the mean value of coverage percentage over one revolution for j-fold continuous global coverage.

All functions are to be minimised through multi-objective optimisation. Specifically, only the inclination is taken into the consideration of J_2. According to the assessments in Sec. III, the selection of altitude is a trade-off process with respect to N and ΔV budgets for altitude maintenance and de-orbiting, and the value of altitude has been included in the evaluation of other properties.

IV.II. Design Variables

The design variables for the optimisation of the constellation geometrical parameters in the present work are the sequence number of integer parameters (represented by $\#_{int}$ hereinafter) and the inclination.

As mentioned in Sec. II.III, if the value of ε is fixed, the geometrical parameters that determine the constellation geometry are N, P, F (only for Delta pattern), i and θ_{min}. θ_{min} can be derived through optimisation for given values of N, P, F and i. Therefore, the parameters to be optimised are N, P, F and i.

Note that N, P and F are integers; P must be a divisor of N and the value of F must be between 0 and (P - 1). Therefore, the design variable is selected as $\#_{int}$ to reduce the number of variables in the optimisation programme and to simplify the optimisation process. The values of $\#_{int}$ and the corresponding integer parameters for 6 satellites constellation are listed in Table 3.

Table 3: $\#_{int}$ and integer parameters for 6 satellites constellation.

Pattern	$\#_{int}$	Integer Parameters
SOC	1	$N = 6$, $P = 2$
SOC	2	$N = 6$, $P = 3$
Delta	1	$N = 6$, $P = 2$, $F = 0$
Delta	2	$N = 6$, $P = 2$, $F = 1$
Delta	3	$N = 6$, $P = 3$, $F = 0$
Delta	4	$N = 6$, $P = 3$, $F = 1$
Delta	5	$N = 6$, $P = 3$, $F = 2$

Delta	6	$N = 6, P = 6, F = 0$
Delta	7	$N = 6, P = 6, F = 1$
Delta	8	$N = 6, P = 6, F = 2$
Delta	9	$N = 6, P = 6, F = 3$
Delta	10	$N = 6, P = 6, F = 4$
Delta	11	$N = 6, P = 6, F = 5$

IV.III. Mission Scenarios

In the present work, the space-based application considered is a remote sensing mission. The mission conditions and the design variables for the given mission are listed in Table 4 and Table 5 respectively. The altitude maintenance is assumed to conduct once per year.

Table 4: Conditions for remote sensing mission.

Parameter	Symbol	Value/Range
Altitude	H	300 to 2000 km
Ballistic coefficient	m/C_DA	100 kg/m^2
Coverage level	j	1
Elevation angle	ε	10 deg
Inclination of launch site	i_{lch}	5.4 deg
Number of satellites	N	5 to 100
Perigee altitude of re-entry orbit	H_r	75 km

Table 5: Design variables for remote sensing mission.

Pattern	Variable	Range	Variable Type
SOC	$\#_{int}$	1 to 152	Integer
SOC	Inclination	5.4 to 90 deg	Real
Delta	$\#_{int}$	1 to 7897	Integer
Delta	Inclination	5.4 to 90 deg	Real

IV.IV. Results and Discussion

SOC Pattern

Fig. 18 to Fig. 21 compare the optimal coverage geometries with the optimal geometries obtained through multi-objective optimisation for SOC pattern in terms of different properties.

It is observed from Fig. 18 that the optimal coverage geometries possess better coverage and end-of-life disposal properties for given numbers of satellites as those two properties are proportional to the value of θ_{min}.

While regarding other properties, the optimal geometries obtained through multi-objective optimisation perform better. In Fig. 19, the improvements of robustness and stationkeeping properties are at the price of the increase of θ_{min}, consistent with the conclusion drawn from Fig. 18. Fig. 20 shows that the launchability and build-up properties are improved, indicating that the values of i and P are lower. Therefore, the collision avoidance property, which is strongly related to i and P (derived from Eqs. (13) to (16)), is consequently improved, as shown in Fig. 21.

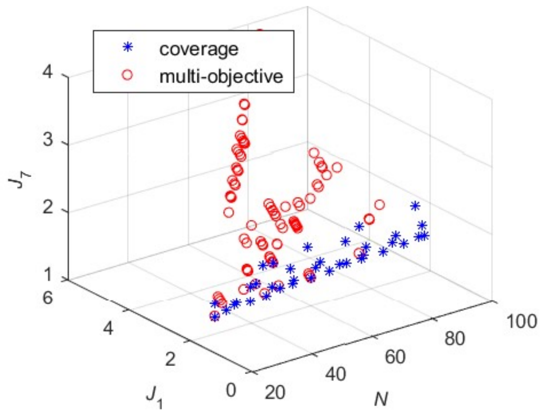


Fig. 18: Coverage and end-of-life disposal properties for SOC pattern.

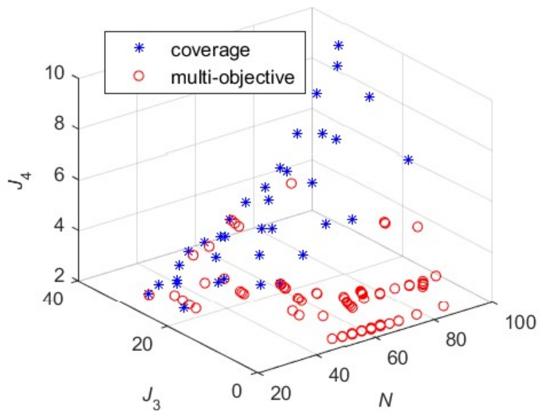


Fig. 19: Robustness and stationkeeping properties for SOC pattern.

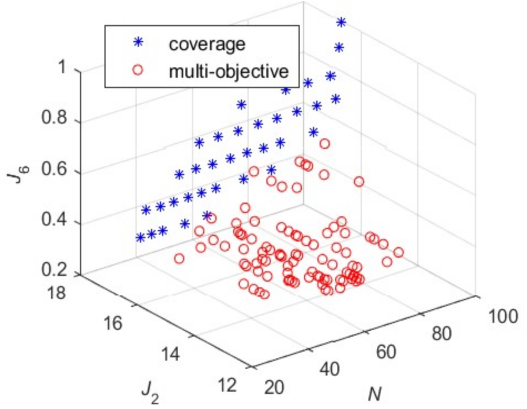


Fig. 20: Launchability and build-up properties for SOC pattern.

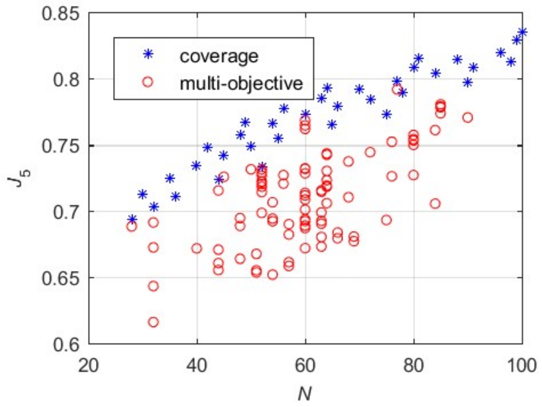


Fig. 21: Collision avoidance property for SOC pattern.

<u>Delta Pattern</u>

Fig. 22 to Fig. 25 compare the optimal coverage geometries with the optimal geometries obtained through multi-objective optimisation for Delta pattern in terms of different properties.

The same conclusions of SOC pattern is drawn from Fig. 22 and Fig. 24 for Delta pattern in terms of the properties of coverage, end-of-life disposal, robustness and stationkeeping. From Fig. 23 it is observed that the launchability and collision avoidance properties are on the same levels for both types of geometries due to the fact that the values of inclination are not greatly reduced. However, the build-up property, which might be the most severe drawback for Delta pattern, is greatly improved.

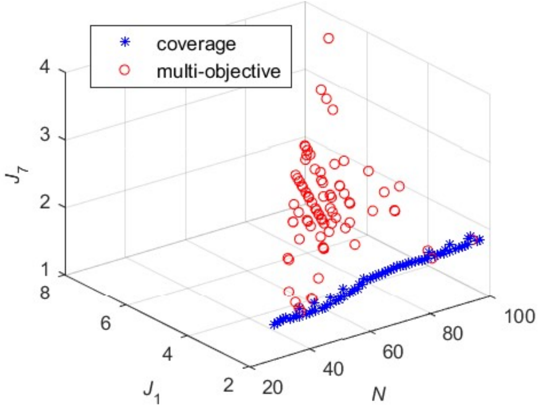


Fig. 22: Coverage and end-of-life disposal properties for Delta pattern.

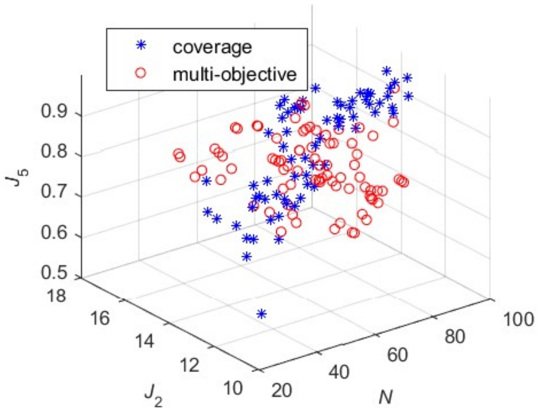


Fig. 23: Launchability and collision avoidance properties for Delta pattern.

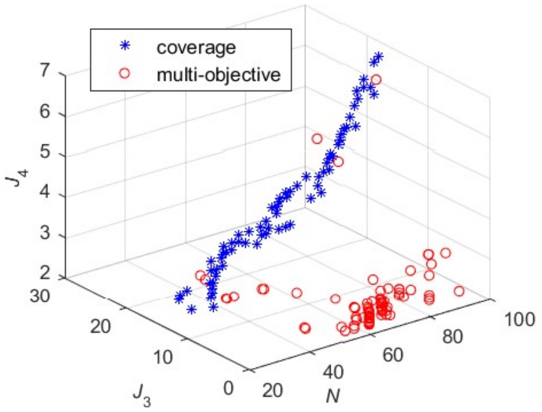


Fig. 24: Robustness and stationkeeping properties for Delta pattern.

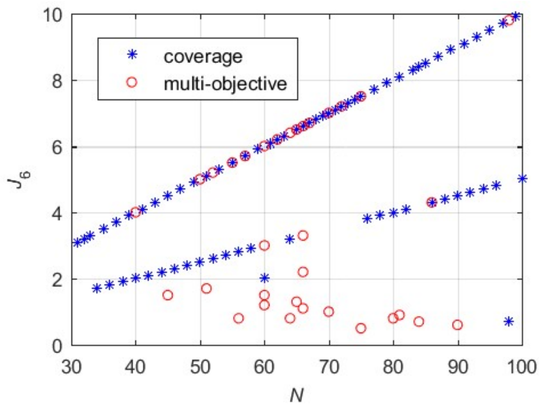


Fig. 25: Build-up property for Delta pattern.

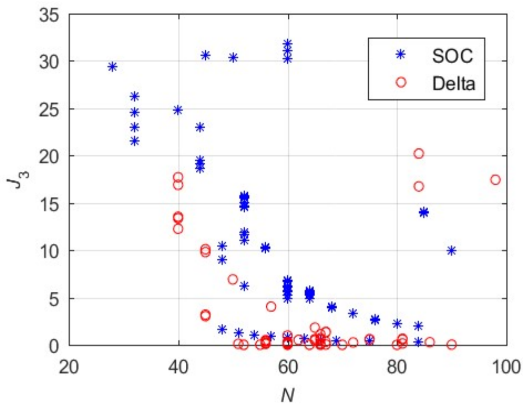


Fig. 26: Comparison of robustness property for SOC and Delta pattern.

Comparison for SOC and Delta Pattern

Comparing the optimisation results of SOC and Delta pattern, several conclusions are drawn from Fig. 26 and Fig. 27:

(1) The optimal Delta constellations have the advantage in terms of robustness, benefited by the nature of symmetrical geometry.

(2) The optimal SOC constellations perform better with respect to collision avoidance and build-up. Both of those properties are greatly influenced by the number of orbital planes and SOC pattern always has less orbital plane than Delta pattern.

(3) For the optimal Delta constellations, the numbers of satellites are larger than 40 and mostly of them are between 60 and 80. While for the optimal SOC constellations, there exists the geometry with the number of satellites less than 40.

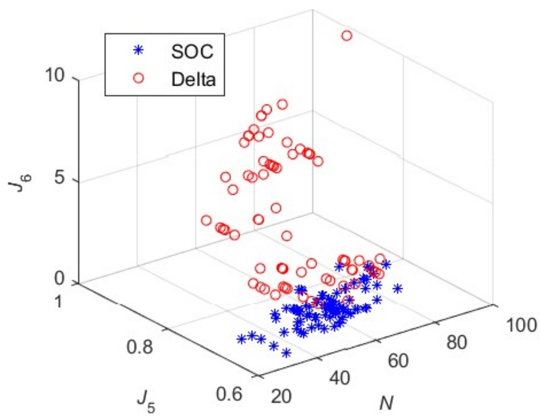


Fig. 27: Comparison of collision avoidance and build-up properties for SOC and Delta pattern.

Several choices in terms of different number of satellites for SOC and Delta pattern are listed in Table 6 and Table 7 respectively. These could be the used for designing the remote sensing constellation.

Table 6: Geometrical information of optimal SOC constellations best suitable for remote sensing.

Parameter	Geometry 1	Geometry 2	Geometry 3
N	28	60	90
P	4	3	5
S	7	20	18
I	82.68 deg	79.03 deg	81.42 deg
θ_{min}	30.93 deg	31.03 deg	18.84 deg
$\Delta\Omega_{s1}$	49.16 deg	62.10 deg	37.50 deg
$\Delta\Omega_{o1}$	32.55 deg	55.80 deg	29.99 deg
$\Delta\varphi_{intra1}$	51.43 deg	18 deg	20 deg
$\Delta\varphi_{inter1}$	19.04 deg	4.07 deg	4.20 deg
H	1935.8km	1948.4km	863.3 km

Table 7: Geometrical information of optimal Delta constellations best suitable for remote sensing.

Parameter	Geometry 1	Geometry 2	Geometry 3
N	60	84	90
P	12	7	5
S	5	12	18
F	3	2	1
i	67.12 deg	88.04 deg	83.63 deg
θ_{min}	28.50 deg	19.63 deg	21.19 deg
$\Delta\Omega$	30 deg	51.43 deg	72 deg
H	1647.8 km	847.6 km	964.3 km

V. CONCLUSION

This paper presented a general study of two classical constellation patterns with circular orbits for continuous global coverage. The constellation geometries for both patterns were introduced and the geometrical parameters determining the constellation geometry were identified. Several significant constellation properties (coverage, launchability, robustness, stationkeeping, build-up, collision avoidance and end-of-life disposal) dominating the constellation design were assessed with a parametrical approach, and the comparison of optimal coverage geometry for both patterns was conducted to analyse the geometrical characteristics of these two patterns. A series of fitness functions were developed based on the assessments in order to quantitatively evaluate the properties. Through a multi-objective optimisation, the optimal geometries that best suitable for the remote sensing mission in the region of LEO and for single continuous global coverage were obtained. The optimisation results showed that the SOC pattern would be favoured in terms of the collision avoidance and build-up properties while the Delta pattern would be favoured for the robustness property. Moreover, only the SOC pattern was feasible for the constellation of less than 40 satellites.

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