

Determining the quantum expectation value by measuring a single photon

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One of the most intriguing features of quantum mechanics is that variables might not have definite values. A complete quantum description provides only probabilities for obtaining various eigenvalues of a quantum variable. The eigenvalues and the corresponding probabilities specify the expectation value of a physical observable, which is known to be a statistical property of an ensemble of quantum systems. In contrast to this paradigm, here we demonstrate a method for measuring the expectation value of a physical variable on a single particle, namely, the polarization of a single protected photon. This realization of quantum protective measurements could find applications in the foundations of quantum mechanics and quantum-enhanced measurements.

Despite its unprecedented success in accurately predicting experimental results, there is no consensus about the foundational concepts of quantum mechanics. The reality of the wavefunction is still hotly debated¹⁻⁴. In stark contrast to classical physics, quantum observables lack definite values. A complete description of a quantum system predicts only the spectrum and probabilities for the measurement outcomes of a physical observable. Given the quantum state of the system $|\Psi\rangle$, which, according to standard quantum mechanics, comprises its complete description, to each observable A we can associate a definite number: $\langle\Psi|A|\Psi\rangle = \sum p_i a_i$ (p_i being the probability to obtain the (eigen)value a_i as the result of the measurement of A). The meaning of this number is statistical: to find the expectation value of A one needs to measure an ensemble of identically prepared systems.

Single measurements yielding the expectation value of a physical variable seem to be against the spirit of quantum mechanics. However, it has been suggested that, in certain special situations, one can find the expectation value of an observable by performing only a single measurement. This is the method of protective measurement (PM), originally proposed as an argument supporting the reality of the quantum wavefunction⁵.

However, this is a highly controversial issue⁶⁻¹²; namely, does the procedure allow observing the state or only the protection mechanism? Nevertheless, the idea of PM triggered and helped in studying various foundational topics beyond exploring the meaning of the wavefunction, such as Bohmian trajectories¹³, stationary basis determination¹² and analysis of measurement optimization for minimizing the state disturbance¹⁴. The concept of protection was also extended to measurement of a two-state vector¹⁵.

Protection can be realized⁵ both actively or passively: here we employ a variant of an active protection technique based on the Zeno effect¹⁶. Since in our Zeno protection method we strongly project on a particular state, our experiment corresponds to a protocol where Bob wants to measure the expectation value of an observable

on a quantum state unknown to him, in presence of a protection mechanism designed for such state. The state, together with its protection mechanism, is provided by Alice, who needs to know the state to set up the protection. Thus, Bob actually measures the photon in conjunction with the ‘protection apparatus’.

In spite of the rich and diverse analysis of the theory behind PM, it has not yet been realized experimentally. Indeed, although weak measurements (WMs)¹⁷⁻²⁰ and the Zeno effect²¹⁻²⁶ have been largely considered in experiments for several physical systems, no experiment joining them in a PM has been realized yet.

Our main result is the extraction of the expectation value of the photon polarization by means of a measurement performed on a single protected photon (see Fig. 1) that survived

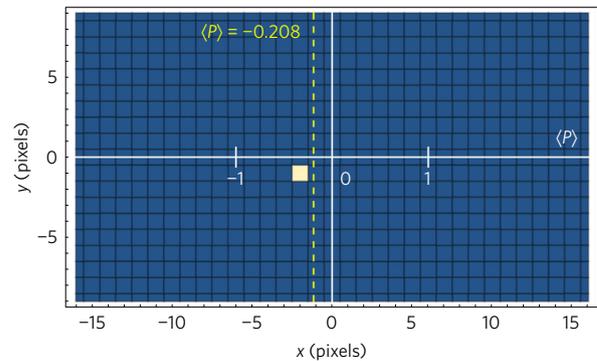


Figure 1 | Estimation of the polarization expectation value $\langle P \rangle$ by means of a single protective measurement. The x coordinate of the pixel which detected the single photon tells us—without the need of any statistics—the expectation value of the polarization operator, $\langle P \rangle = -0.3(3)$, where the uncertainty is estimated from the width of the photon counts distribution presented in the paper, the theoretical value being (for $\theta = 17\pi/60$) $\langle P \rangle = -0.208$.

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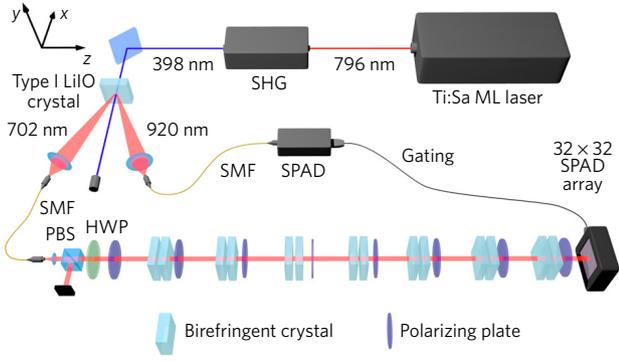


Figure 2 | Experimental setup. Heralded single photons are produced by type-I parametric down-conversion in a LiIO₃ crystal, then properly filtered, fibre coupled and addressed to the open-air path where the experiment takes place. After being prepared in the polarization state $|\psi_\theta\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle$, they pass through a birefringent material, shifting them in the transverse direction x (according to their polarization). The weak interaction is obtained by means of $K = 7$ birefringent units, each unit composed of a first crystal separating the beam by 1.66 pixels (less than the beam width) and a second crystal used to compensate the phase and time shift induced by the first crystal: only the action of all units together allows separating orthogonal polarizations. The protection of the quantum state, implementing the quantum Zeno scheme, is realized by inserting a thin-film polarizer after each birefringent unit, projecting the photons onto the same polarization as the initial state $|\psi_\theta\rangle$. At the end of the optical path, the photons are detected by a spatial-resolving single-photon detector prototype—that is, a two-dimensional array of 32×32 ‘smart pixels’. ML, mode-locked; SHG, second-harmonic generator; SMF, single-mode fibre; SPAD, single-photon avalanche diode; PBS, polarizing beam splitter; HWP, half-wave plate.

the Zeno-type protection scheme. The polarization operator is defined by

$$P = |H\rangle\langle H| - |V\rangle\langle V| \quad (1)$$

where H (V) is the horizontal (vertical) polarization. Because of the presence of active protection in our experiment, the single click of a multi-pixel camera tells us that the expectation value of the polarization operator of the single protected photon is $\langle P \rangle = -0.3 \pm 0.3$ (see Fig. 1), in agreement with the theoretical predictions ($\langle P \rangle = -0.208$).

In our experiment (see Fig. 2), heralded single photons²⁷, prepared in the polarization state $|\psi_\theta\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle$, pass through a birefringent material, shifting them in the transverse direction x (according to their polarization). The spatial mode is close to Gaussian with $\sigma = 4.1$ pixels (σ being the source of uncertainty associated with the estimation of $\langle P \rangle$ presented in Fig. 1). The WM interaction is obtained exploiting by $K = 7$ birefringent units, while the state protection is implemented via the quantum Zeno scheme—that is, by inserting a thin-film polarizer after each birefringent unit, realizing a state filtering equivalent to the one made at the preparation stage. Finally, the photons are detected by a spatially resolving single-photon detector prototype²⁸. Without protection, the photons end up in one of the two regions corresponding to the vertical and horizontal polarizations, centred around $x = \pm a$ (see Fig. 3a).

Then, the expectation value can be statistically found by the counts ratio:

$$\langle P \rangle = \frac{N_H - N_V}{N} \quad (2)$$

In contrast, with PM the photons end up in a region centred at $x = a\langle P \rangle$ (see Fig. 3b). A large ensemble of measurements allows

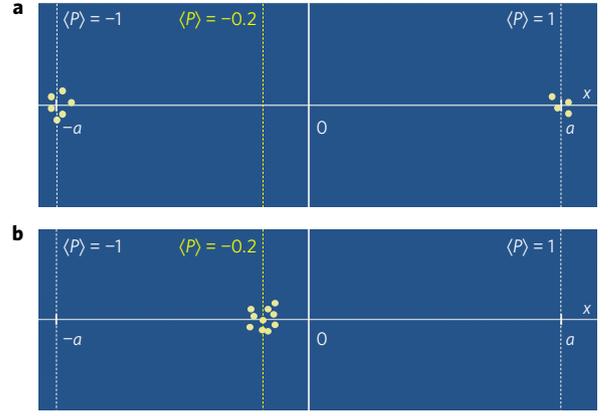


Figure 3 | Illustrative drawing showing the measurement of unprotected and protected photons. **a**, Unprotected photons: six photons fall close to $x = -a$ (corresponding to $P = -1$), while four photons fall near $x = a$ (corresponding to $P = 1$), giving the expectation value $\langle P \rangle = -0.2$. **b**, Protected photons: all the photons accumulate close to $x = -0.2a$, the $\langle P \rangle = -0.2$ position; this indicates that, with PM, we can estimate the expected value of our observable even with a single photon.

the centre to be found with arbitrarily good precision, but even one single-photon detection provides information about $\langle P \rangle$, albeit with a finite precision defined by the distribution width.

In Fig. 4a–d we show the results obtained by collecting heralded single photons for a measurement time of 1,200 s. Figure 4a,c shows, respectively, a histogram and a contour plot of the photon counts distribution observed in the unprotected case for the input state $|\psi_{17\pi/60}\rangle = 0.629|H\rangle + 0.777|V\rangle$. As in a standard Stern–Gerlach experiment, we observed photons only in two regions corresponding to the eigenvalues of P . The polarization expectation value $\langle P \rangle$ evaluated using (2) from this distribution (dark counts subtracted) is $\langle P_{17\pi/60} \rangle = -0.21(4)$, in agreement with theoretical expectations, $\langle P_{17\pi/60} \rangle = -0.208$. Figure 4b,d shows, respectively, a histogram and a contour plot of the photon counts distribution obtained in the protected case for the same polarization state. Instead of two distributions around $x = \pm a$, here we find a single distribution of photon detections centred very close to $x = \langle P \rangle a$. The measured expectation value is $\langle P_{17\pi/60} \rangle = -0.19(2)$ (dark counts subtracted). This result demonstrates that we have been able to realize and exploit the PM concept, providing the estimation of the polarization operator, $\langle P \rangle$, by detecting a single photon.

This is further confirmed in Fig. 4e,f, presenting typical photon detection maps for the input state $|\psi_{17\pi/60}\rangle$ obtained from a small number of detected photons. Specifically, Fig. 4e,f corresponds respectively to the unprotected ($N = 14$ detection events) and protected ($N = 17$ detection events) case; the circles drawn in the two panels represent the width of the distributions reported in Fig. 4a–d. As expected, counts are clearly concentrated inside the circles (despite the non-negligible dark counts level of our non-ideal SPAD array, likely responsible for the detection events outside the circles), demonstrating the validity of PM even when just few detections are considered. The first detected photons in the runs are signified with white pixels. We see that, although the white pixel of Fig. 4f provides a good estimate of $\langle P \rangle$, we cannot learn much from the white pixel of Fig. 4e.

Indeed, using a single photon is what makes PMs special. However, Zeno protection ensures survival of the photon only for the ideal case of noiseless devices in the infinite protection operations limit $K \rightarrow \infty$. For our, non-ideal case, one could argue that our experiment concerns a single post-selected photon (that is, that survived all protection stages) and, allowing post-selection, one

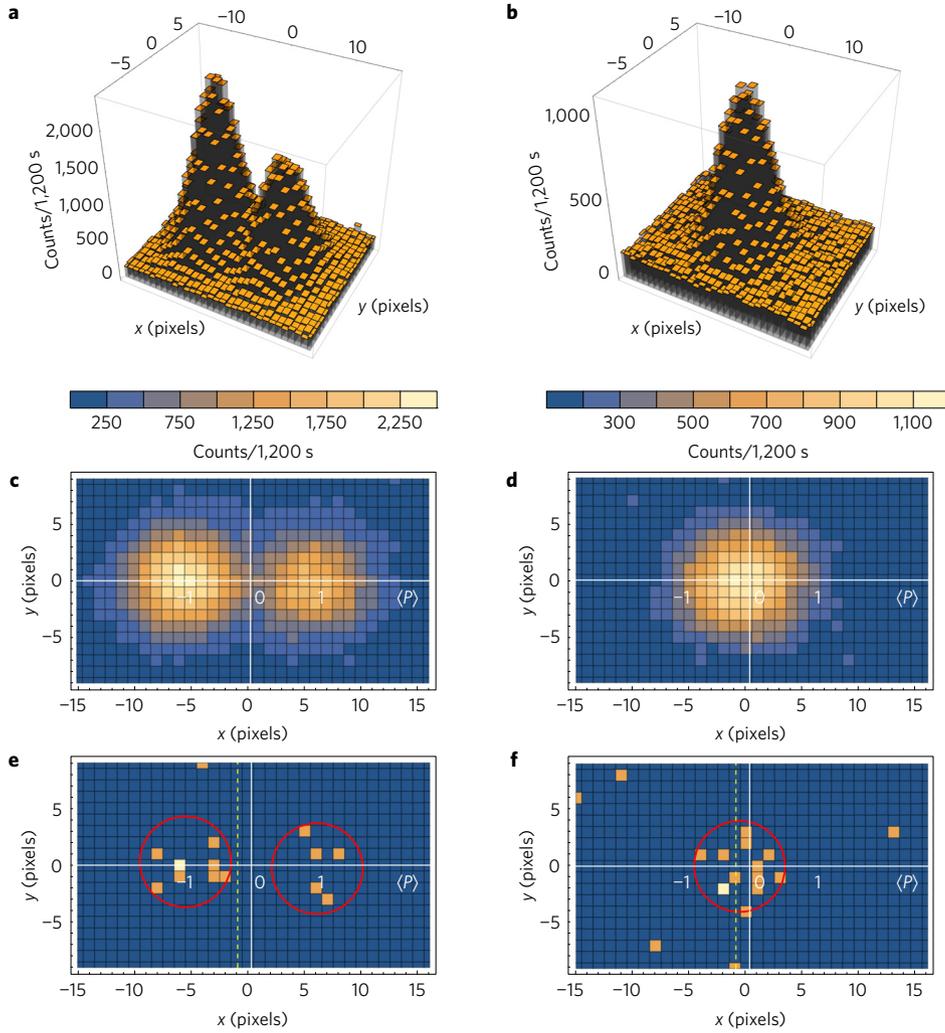


Figure 4 | Results obtained for the input state $|\psi_{17\pi/60}\rangle = 0.629|H\rangle + 0.777|V\rangle$. **a,c**, Histogram and contour plot of the photon counts distribution, respectively, obtained for the unprotected state. **b,d**, Histogram and contour plot of the photon counts distribution, respectively, obtained for the protected state. **e**, Experiment with 14 single events (the first one in white), without protection. The yellow dashed line indicates the $x = a\langle P_{17\pi/60} \rangle$ value. **f**, Experiment with 17 single events (the first one in white), with protection: as expected, all the photons accumulate around the $x = a\langle P_{17\pi/60} \rangle$ position (yellow dashed line).

can perform a measurement yielding the expectation value in the case of both weak and strong interaction.

To discuss quantitatively the performance of PM, we compare it with the straightforward alternative, a projective measurement exploiting, for example, a polarizing beam splitter (note that, when estimating P , the simple projective measurement saturates the Quantum Cramér–Rao bound²⁹). For this purpose we plot in Fig. 5 the ratio $R = (u_{\text{PBS}}(P))/u(P)$ between the uncertainties on $\langle P \rangle$ in the two cases. We consider in both cases the same initial number of photons, taking into account the photons lost in PM (see Supplementary Information). We consider two different scenarios: $K = 7$ (yellow surface) and $K = 100$ (blue surface) interaction–protection stages. In both cases, PM is almost always advantageous ($R > 1$) with respect to the projective measurement, going below the $R = 1$ plane (in magenta) only for extremely weak interactions. In our experiment, with $\xi \sim 0.4$ and just $K = 7$, a 10% advantage is already present for most of the possible states, even if the maximum for R corresponds to $\xi \sim 1$. For $K = 100$, instead, the reasonably weak interaction $\xi \sim 0.4$ grants the maximum of the advantage ($R > 8.5$ almost everywhere), while for stronger interaction the advantage is reduced to $R < 4$. The advantage of PM stems from the very high survival probability of the protected photons (see Supplementary Information). We also point out that our

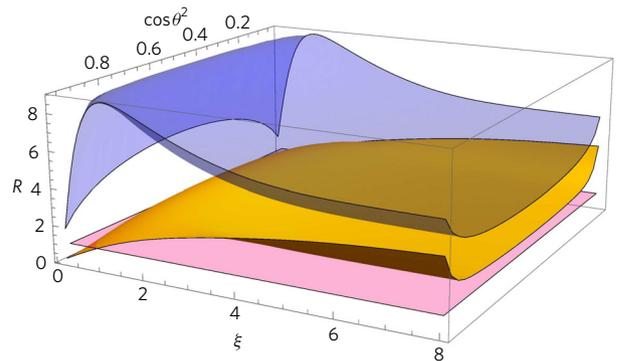


Figure 5 | Comparison between the uncertainty on P with the PM approach ($u(P)$) and the one given by projective measurement ($u_{\text{PBS}}(P)$). Yellow surface: ratio $R = (u_{\text{PBS}}(P))/u(P)$ for a PM scheme with $K = 7$ interaction–protection stages (as in our experiment), plotted versus the interaction strength ξ and the H -polarization component $(\cos\theta)^2$ of the single-photon state $|\psi\rangle$. Blue surface: ratio R for a PM with $K = 100$ stages. Magenta surface: $R = 1$ bound, discriminating the part where PM approach is advantageous (above) and disadvantageous (below) with respect to the projective measurement.

experiment is the first realization of a ‘robust’ WM³⁰ at the single-photon level.

This is the first experimental realization of PMs⁵. Our results demonstrate that a single-event detection can provide reliable information regarding a certain property of a quantum system—the expectation value of the polarization operator—supposed to be only statistical, belonging to an ensemble of identically prepared quantum systems. In doing this, PMs require that prior information on the preparation stage is exploited in realizing the protection. Although our results may not resolve the controversy regarding the meaning of PMs, they are of interest for all approaches. Proponents of the quantum state ontic interpretation should be excited to see this first single-particle measurement (they will argue that the necessity of protection is not surprising: every measurement obeys the Heisenberg uncertainty principle). At the same time, proponents of the minimalist approach, where only measurement outcomes exist, should also be interested to see a property of various preparation/protection methods of the quantum state directly inferred from a single-photon detection as the pointer shift.

Furthermore, we demonstrate that PMs outperform the traditional quantum measurements and could find several significant applications, for example, when testing an unknown state preparation–protection procedure, given that both the state preparation and protection exploit the same projective measurement system (or equivalently a set of identical projective measurements, as in our case).

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