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Procedia Engineering 199 (2017) 3362–3367

Procedia Engineering

www.elsevier.com/locate/procedia

X International Conference on Structural Dynamics, EURODYN 2017

Dynamic assessment of the axial force in the tie-rods of the Milan Cathedral

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Abstract

The Milan Cathedral, constructed over a period of more than 400 years, is one of the few Gothic cathedrals where permanent metallic tie-rods are installed under all naves to support a portion of the lateral thrust exerted by vaults and arches. After the recent failure of one tie-rod, an extensive research program was carried out to characterize the metallic material and to evaluate the axial forces and the tensile stresses of the tie-rods through dynamic testing and system identification. After a description of the methodology adopted in the dynamic assessment of tie-rods in the Milan Cathedral, the paper presents the main results of the investigation, in terms of fundamental frequencies and axial forces in the tie-rods.

Peer-review under responsibility of the organizing committee of EURODYN 2017.

Keywords: Axial force, Experimental modal analysis, Frequency Response Function, Historic tie-rods, Identification, Wrought iron.

1. Introduction

The Milan Cathedral (Fig. 1a) [1], partly designed in Gothic style and erected between 1396 and 1805, is one of the largest masonry monuments ever built. The church construction started from the half-octagonal apse and East choir, and proceeded with the transept, the main dome, the *tiburio* (i.e., the prismatic structure with octagonal base, which was built around the main dome) and the main spire; subsequently, the five-nave structure over eight bays was built and finalized with a neo-Gothic façade. The overall dimensions of the Latin cross-shaped plan are about 66 m × 158 m, with the lateral naves and the central nave (Fig. 2a) spanning 9.6 m and 19.2 m, respectively.

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1877-7058
Peer-review under responsibility of the organizing committee of EURODYN 2017.
10.1016/j.proeng.2017.09.442

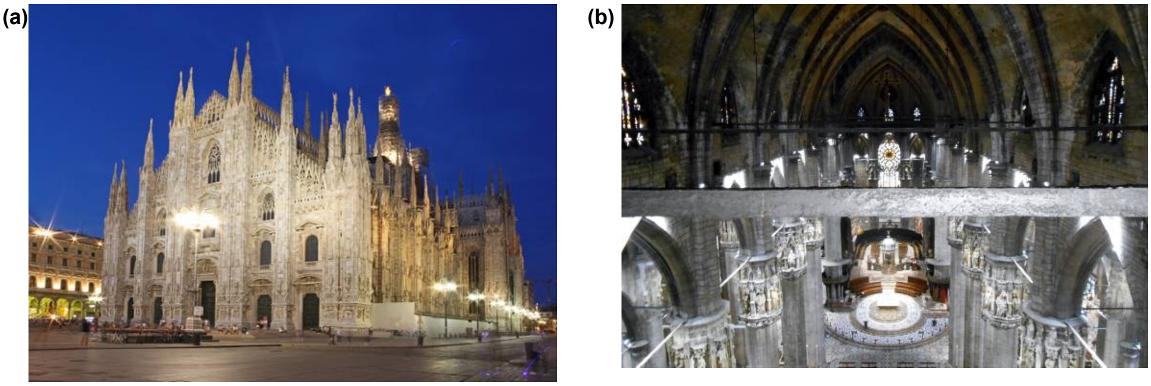

Fig. 1. Milan Cathedral: (a) External view; (b) Inner partial view of arches, vaults and metallic tie-rods.

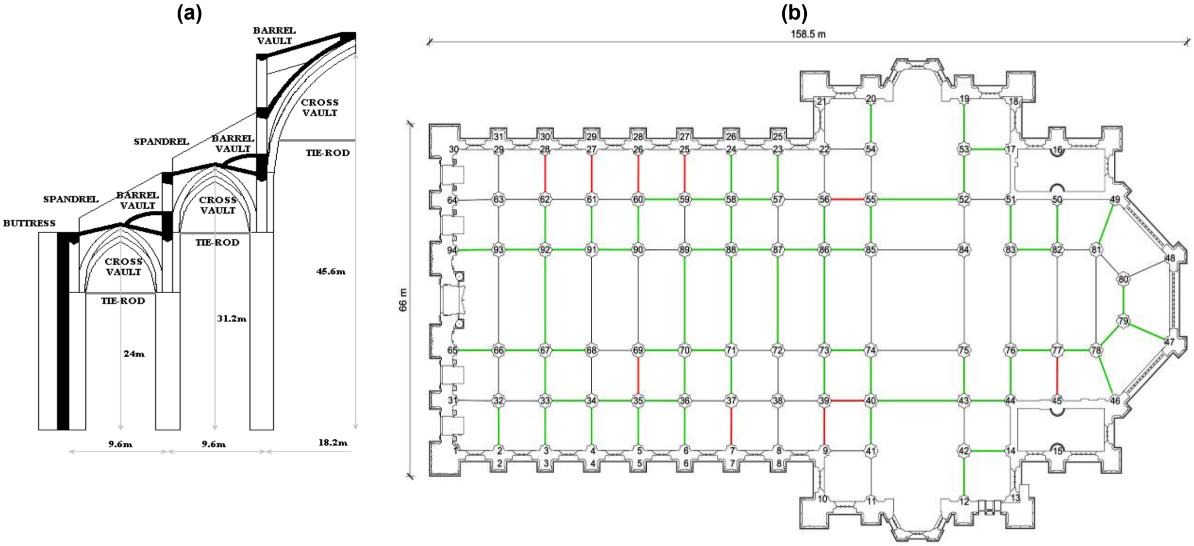

Fig. 2. Milan Cathedral: (a) Structural system exemplified on half transversal cross-section; (b) Plan of the church and schematic of the metallic tie-rods (with green and red colors referring to the structural elements assessed in 2012-2014).

When compared to other Gothic cathedrals, the structural architecture of the Milan Cathedral is characterized by: (a) a system of double masonry vaults, which are denoted to as cross vaults and barrel vaults in Fig. 2a; (b) the structural system resisting the lateral thrusts, which includes massive masonry walls (spandrels), built in the lateral naves over the arches, and metallic ties placed under each arch (Figs. 1b and 2). Historical documents testify that the tension bars in the Milan Cathedral were installed on the top of the piers during the construction, to connect the new parts with already built parts and the buttressing walls in each bay. Even if metallic or wooden ties were used as auxiliary devices to facilitate the construction in many European cathedrals, in most cases they were removed once the building was completed [2].

A total of 118 metallic tie-rods is present in the Milan Cathedral (Fig. 2b) and most of them are the original elements dating back to the age of construction. Only 4 ties, connecting the piers under the *tiburio* [1], and 3 other ties were substituted across the years.

The paper presents the main results of the investigation performed to evaluate the axial forces in the iron ties of the Milan Cathedral. This project followed the failure and substitution of tie-rod 58-88 [3, 4] and began with lab tests on several specimens sampled from the original (substituted) tension bar 58-88: the tests were aimed at providing the mechanical characteristics of the historic iron in terms of Young's modulus, tensile strength and mass density [4].

Subsequently, 112 tie-rods of the cathedral were investigated − in 2012 and 2014 − using the following methodology [4, 5]: (a) visual inspection and geometric characterization of each metallic element on site; (b) forced vibration test and experimental modal analysis [6]; (c) identification of the tensile force and boundary conditions, based on the behaviour (i.e., natural frequency and mode shape) of the fundamental mode [7]. Furthermore, a validation of the structural parameters characterizing the most loaded ties was carried out through a numerical (FE) model. The model of each tie is based on the available information on geometry and materials, as well as on the identified axial force and boundary conditions [5]; the validation involves the comparison between the experimental frequencies of the lower 6 modes and the corresponding numerical outcomes.

2. Mechanical characteristics of the wrought iron of the tie-rods in Milan Cathedral

After the broken tie-rod 58-88 was removed from the Cathedral and replaced with a modern steel member, it was brought to the Laboratory of Politecnico di Milano. Subsequently, the iron rod was divided into two parts, which were used for simulating in the laboratory the scheduled dynamic tests on site and the mechanical characterization of the material, respectively.

Direct observation of the structural element confirmed that the tie-rods in Milan Cathedral are made of hand-produced wrought iron: each bar was shaped by repeating the process of heating and hammering the hot metal on an anvil, which had significant influence on the mechanical properties of resulting materials.

Direct tensile tests were performed on 3 series of samples according to the European standard EN ISO 6892-1 [8]: (a) series "n" included 2 unmachined test specimens with nearly square (18.3×18.7 mm^2) and rectangular cross section (16.6×25.9 mm^2); (b) series "c" included 3 machined test samples with circular cross section ($\varnothing = 14.95$ mm); (c) series "r" included 3 machined test pieces, with rectangular cross section (5.95×20.10 mm^2), extracted from the region of the bar neighboring the cross-section where failure occurred.

Stress-strain diagrams are shown in Fig. 3 for the first series of samples. The three series of samples provided average values of the Young's modulus and tensile strength of 205 GPa and 275 MPa, respectively; furthermore, the mass density of the wrought iron resulted in 7792 kg/m^3.

It is worth noting that the tie-rods of the Milan Cathedral were made and placed over a period of almost 200 years; therefore, elements made in different centuries were probably made of wrought iron with different material characteristics (see e.g. [9]). Nevertheless, the above average Young's modulus and mass density were assumed as representative of all ties in the dynamic assessment of the axial forces.

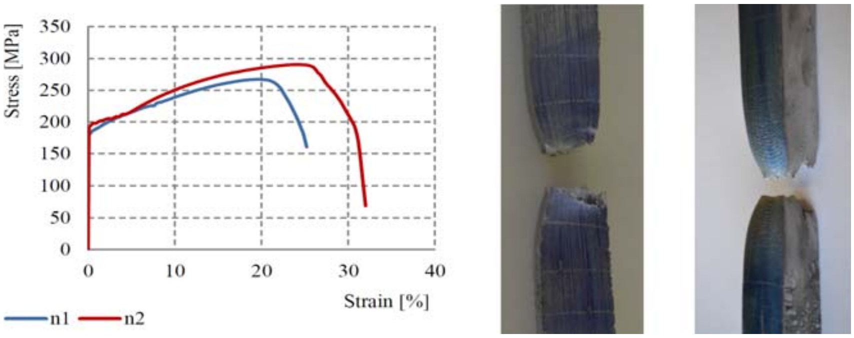

Fig. 3. Tensile stress-strain diagrams and sample failures for wrought iron specimens extracted from one tie-rod of the Milan Cathedral.

3. Dynamic assessment of the axial force in the tie-rods of the Milan Cathedral: methodology

Between 2012 and 2014, almost all the tie-rods of the Cathedral underwent visual inspection, geometric characterization and impact hammer tests to evaluate the dynamic characteristics and the relevant axial force. As previously mentioned, 112 elements (out of 118) were investigated and the only tie-rods missing were the ones connecting the piers under the *tiburio* (as those elements, in modern steel, were continuously monitored using vibrating wire rod extensometers) and the ones placed over the major altar.

The non-destructive survey was carried out making use of a movable platform employed in scheduled weeks for the maintenance interventions planned in view of EXPO 2015.

After a visual inspection including the geometric characterization, each tie-rod was investigated adopting the methodology summarized in Fig. 4.

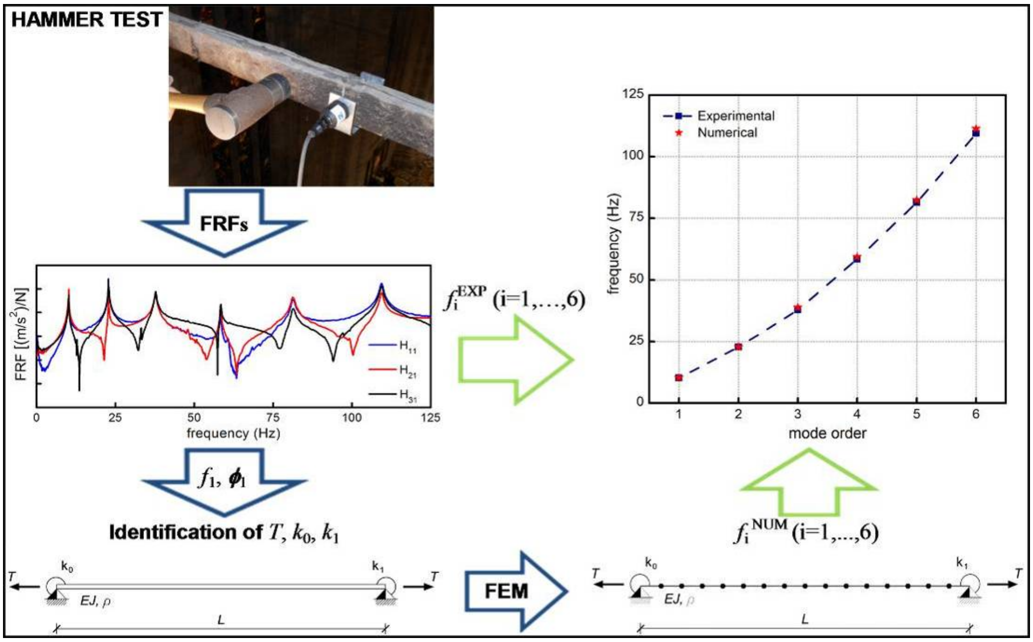

Fig. 4. Flow chart of the methodology adopted to evaluate the axial force in the metallic tie-rods of the Milan Cathedral.

Each tie-rod was instrumented with 3 piezoelectric accelerometers (500 mV/g nominal sensitivity) installed at 1/4, 1/2 and 3/4 of the total length and the excitation was provided by an instrumented hammer; the transversal responses and the input force were recorded with sampling frequency of 1600 Hz. 5 hammer blows were applied in correspondence of each instrumented cross sections (Fig. 4); the time interval between two subsequent blows was set approximately equal to 60 s, so that data records of about 900 s were acquired.

Frequency Response Functions (FRFs) were estimated from the time histories recorded during the hammer tests in order to extract the main dynamic characteristics of the tested tension bars (Fig. 4). The inertance FRFs $H_{ik}(f)$ − at section x_i for an excitation force F_k applied at section x_k − were computed using the H_2-estimator [6] (i.e., through the ratio $G_{xi,xi}(f)/G_{xi,Fk}(f)$ between the auto-spectrum of response and the cross-spectrum of response and excitation). In the practice, the time-histories associated to the different blowing positions were grouped together so that the different columns of the FRF matrix were computed with a frequency resolution of 0.0488 Hz; subsequently, one column was used to extract the modal parameters, whereas an accuracy assessment was carried out based on the estimate of the other terms.

As suggested by Figs. 4 and 5a, at least 6 vibration modes were usually identifiable from the FRFs; of course, due to the occurrence of "spatial aliasing", only the mode shapes of the lower 3 modes can be correctly represented, whereas the low number of installed sensors prevents to obtain the amount of information needed to describe the articulated mode shapes associated to the higher modes.

Once the experimental modal analysis has been performed, the procedure proposed by Tullini and Laudiero [7] was adopted to identify the axial force T in each tension bar. According to [7], the reference model assumed for the transverse vibration of a pre-tensioned tie-rod is the classic Euler-Bernoulli beam of length L, uniform bending stiffness EJ and linear mass density ρ. As shown in Fig. 4, it is also assumed that the beam is elastically restrained at the opposite ends by rotational springs of stiffness k_0 and k_1. If the geometric and mechanical parameters (L, EJ and ρ) are known and some parameters of the fundamental mode (i.e., the natural frequency and the modal deflections at 1/4, 1/2 and 3/4 of the total length) have been experimentally evaluated, it can be shown [7] that the 3 unknowns (T, k_0 and k_1) can be identified by iteratively solving an inverse problem in closed form.

In the authors' opinion, the method proposed in [7] exhibits several advantages, including quite easy experimental procedures and good accuracy in the estimate of the axial force T. In addition – once T, k_0 and k_1 have been identified – the experimental data related to the higher modes should be used to validate an analytical or numerical (FE) model of the tie-rod at study. This validation task (Fig. 4), generally not implemented in the dynamic assessment of tie-rods and proposed in the present study, consists in the comparison between the experimental and numerical (or analytical) modal frequencies and could provide additional information on the accuracy of the (assumed and identified) parameters of the model, as well as on the overall structural condition of the investigated element.

4. Dynamic assessment of the axial force in the tie-rods of the Milan Cathedral: results and discussion

The hammer tests performed, in 2012-2014, on 112 tie-rods highlighted that:

1. 75 tie-rods exhibit a dynamic behavior which is fully corresponding to the expectations. In more details, the FRFs are characterized by well-defined and sharp modal peaks and do not show significant singularities or anomalies. For the structural elements of this group, highlighted in green and red in Fig. 2b, complete experimental modal analysis and estimation of the axial force was carried out;
2. the FRFs of 37 tie-rods are characterized by clear anomalies, which are conceivably associated to the presence of heavy lamps directly supported by the tension bars. It should be noticed that a few of those elements highlighted the splitting of the first natural frequency [10]: although the frequency splitting is occasionally reported in the literature as associated to the response of cracked reinforced concrete beams [11], in the present case, its dependence on other phenomena (such as the presence of added masses) cannot be excluded. Of course, no results are shown herein for the ties of this second group. On the other hand, it is worth mentioning that recently (Summer-Autumn 2016), the experimental survey of those tie-rods was repeated after the removal of the lamps and both hammer and ambient vibration tests were performed. For some elements, the ambient vibration response was continuously acquired for 24 hours (or more) and the inner and outer temperature was measured as well.

Fig. 5 refers to tie-rod 45-77 and exemplify the typical results obtained for the 75 ties of the first group: (a) the FRFs (Fig. 5a) are regular and highlight well-defined and consistent modal peaks; (b) after the identification of the axial force and boundary conditions (T, k_0 and k_1), the natural frequencies of the resulting (optimal) numerical model exhibit an excellent agreement with the corresponding experimental frequencies. It is worth mentioning that, for some elements, the frequency discrepancy – although satisfactory – tends to increase with the increased mode order. In those cases, sensitivity analysis and manual tuning provided the evidence that the discrepancy was mainly dependent on the assumed value of the Young's modulus; after trial-and-error tuning of this parameter (involving also new estimates of T, k_0 and k_1, according to [7]), the match between experimental and numerical results turned out to be very similar to the one exemplified in Fig. 5b.

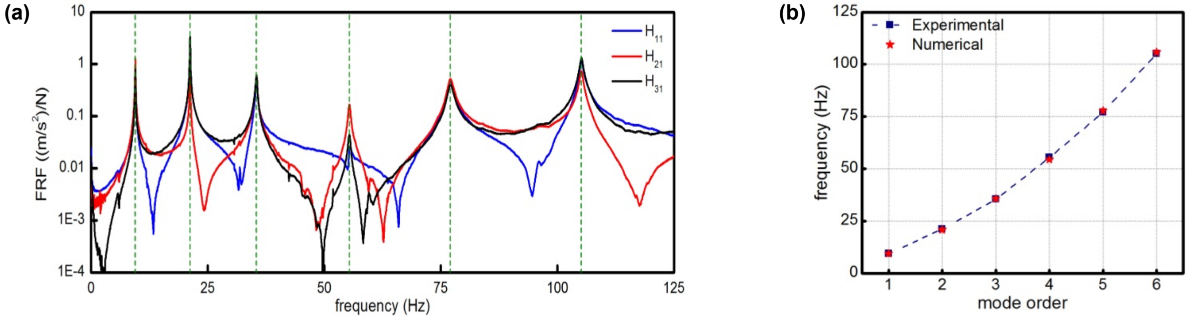

Fig. 5. Tie-rod 45-77: (a) Frequency Response Functions (FRF); (b) Experimental vs. numerical (FE) natural frequencies.

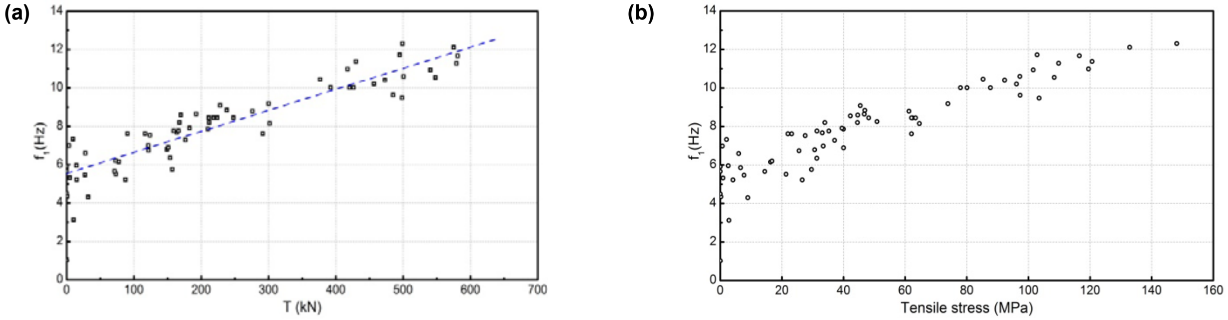

Fig. 6. Fundamental frequency vs. (a) axial force and (b) tensile stress, for the tie-rods with length *L* ≤ 7.66m.

As shown in Fig. 2b, a very large amount of the completely assessed tie-rods is characterized by very similar length. Hence, an almost complete summary of the results can be obtained by considering the elements with *L* ≤ 7.66 m (70 tie-rods out of 75) and representing in the same plot the fundamental frequency vs. the identified axial force (Fig. 6a) and vs. the tensile stress (Fig. 6b). In particular, Fig. 6b demonstrates that 10 tie-rods (highlighted in red in Fig. 2b) exhibit a tensile stress which is larger than 100 MPa, with the stress of only element (tie-rod 28-62) being quite close to the estimated yield stress of the iron material.

5. Conclusions

Taking advantage of recent maintenance interventions performed inside the Milan Cathedral in 2012-2014, 112 tie-rods were tested by performing a classic hammer test in order to estimate the FRFs and the actual tensile load. The adopted assessment methodology turned out to be fully successful for 75 tie-rods and the results indicated that a large portion of the investigated tie-rods is characterized by a tensile stress lower than 100 MPa.

Acknowledgements

The authors gratefully acknowledge the support of the “Veneranda Fabbrica del Duomo” and the co-operation of B. Mörling Visconti Castiglione and F. Aquilano. Sincere thanks are due to M. Cucchi and C. Tiraboschi (Politecnico di Milano), who assisted the authors in conducting all the tests.

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