

14th CIRP Conference on Computer Aided Tolerancing (CAT)

## Robust design of a fixture configuration in the presence of form deviations

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### Abstract

During machining, the tool path is defined with respect to the workpiece reference frame. The workpiece's boundary surfaces have form deviations, and the geometry and the position of the locators are imperfect. The resulting misalignment produces geometrical errors in the features machined on the workpiece.

The main purpose of this work is to investigate how the geometric errors of a machined surface are related to the main sources of the locator errors and to the form deviations of the workpiece. A mathematical framework is presented for an analysis of the relationship among the manufacturing errors, the part form deviations, and the locator errors.

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Peer-review under responsibility of the organizing committee of the 14th CIRP Conference on Computer Aided Tolerancing

*Keywords:* fixture configuration, statistical positioning, six-point locating principle, form deviations.

### 1. Introduction

The accuracy of a manufacturing or inspection operation is mainly determined by the efficiency of the fixturing method. In general, the machined feature may have geometric errors in terms of its form and position in relation to the workpiece datum reference frame. A misalignment error between the workpiece datum reference frame and the machine tool reference frame is known as localization error [1] or datum establishment error [2]. A localization error is essentially caused by a deviation in the position of the contact point between a locator and the workpiece surface from its nominal specification. In this paper, such a theoretical point of contact is referred to as a fixel point or fixel, and its positioning deviation from its nominal position is called fixel error. Within the framework of rigid body analysis, fixel errors have a direct effect on the localization error, as defined by the kinematics between the workpiece feature surfaces and the fixels through their contact constraint relationships [3].

The localization error is highly dependent on the configuration of the locators in terms of their positions relative to the workpiece. A proper design of the locator configuration (or locator layout) may have a significant impact on reducing the localization error. This is often referred to as fixture layout optimization [4].

A main purpose of this work is to present an analysis of the relationships among the manufacturing errors, the workpiece form error, and the fixel errors.

There are several formal methods for fixture analysis based on the classical screw theory [5,6] or geometric perturbation techniques [3]. In the nineties, many studies were devoted to modelling the part deviation due to the fixture [7]. Söderberg calculated a stability index to evaluate the goodness of the locating scheme [8].

The small displacement torsor concept has been used to model the part deviation due to geometric variation of the part-holder [9]. Conventional and computer-aided fixture design procedures have been described in traditional design manuals [10] and recent literature [11,12], especially for designing modular fixtures [13].

A number of methods for localization error analysis and reduction have been reported. A mathematical representation of the localization error was given in [14] using the concept of a displacements screw vector. Optimization techniques were suggested to minimize the magnitude of the localization error vector or the geometric variation of a critical feature [14,15]. An analysis is described by Chouduri and De Meter [2] to relate the locator shape errors to the worst case geometric errors in machined features. Geometric deviations of the workpiece datum surfaces were also analysed by Chouduri and De Meter [2] for positional, profile, and angular

manufacturing tolerance cases. Their effects on machined features, such as those produced by drilling and milling, were illustrated. A second-order analysis of the localization error is presented by Carlson [16]. The computational difficulties of the fixture layout design have been studied, with the objective of reducing the overall measure of the localization error for general three dimensional (3D) workpieces, such as turbine air foils [1,4]. A more recent paper shows a robust fixture layout approach as a multi-objective problem that is solved by means of Genetic Algorithms [17]. A further work presents an analysis describing the impact of localization source errors on the potential datum-related geometric errors of machined features [18]. A genetic algorithm method has been used to find the optimal locating layout within the specified tolerance range for a hole-making process in [19].

The assembly problems among workpiece and fixels may be solved by means of tolerance analysis tools too. Researches on models for tolerance analysis are very extensive, they are summarized in [20-23]. However, actually those model do not deal rightly with form deviations. Some of the models have been developed in Computer Aided Tolerancing (CAT) software packages that are commercially available [24-25]. Commercial CATs are not completely true to the Geometric Dimensioning and Tolerancing (GD&T) standards and need improvement after a better mathematical understanding of geometric variations.

This paper goes beyond the state of the art because it considers the fixel errors together with the form deviation on the profile in contact with the fixels. The form deviation is taken into account in the form of a discrete skin model shape [26].

In previous papers, a statistical method for estimating the position deviation of a hole due to the inaccuracy of all the six locators of the 3-2-1 locating scheme was developed for 2D plates and 3D parts [27, 28]. A further step was to consider the random error of the locator positions and the volumetric error of the machine tool adopted for the operation [29]. The following describes a conceptual demonstrator to investigate how fixel errors and part form deviations affect machining operations quality. In §2, the theoretical approach is introduced; in §3, a variational model is used to solve the case study; and in §4, the results obtained by the theoretical approach are compared with those due to the variational model, and the differences are discussed.

## 2. Methodology for the simulation of the drilling accuracy

To illustrate the proposed conceptual demonstrator, the case study of a drilled hole on a plate was considered. The case study is shown in Fig. 1. The basic dimensions and a level of straightness specify the form deviation of a plate side. Two locators on the primary datum and one on the secondary datum determine the position of the workpiece. Each locator has coordinates related to the machine tool reference frame, as represented by the following three terms of values:

$$P_1(x_1, y_1) \quad P_2(x_2, y_2) \quad P_3(x_3, y_3) \quad (1)$$

The proposed approach considers the uncertainty source in the positioning error of the machined hole due to the error in the positioning of the locators and the form deviations of the workpiece side that goes into contact with two locators. The final aim of the model is to define the location deviation of the centre of the drilled hole. The model input includes the nominal locator configuration, the nominal hole location (supposed coincident with the drill tip) and the characteristics of typical errors that can affect these nominal parameters.

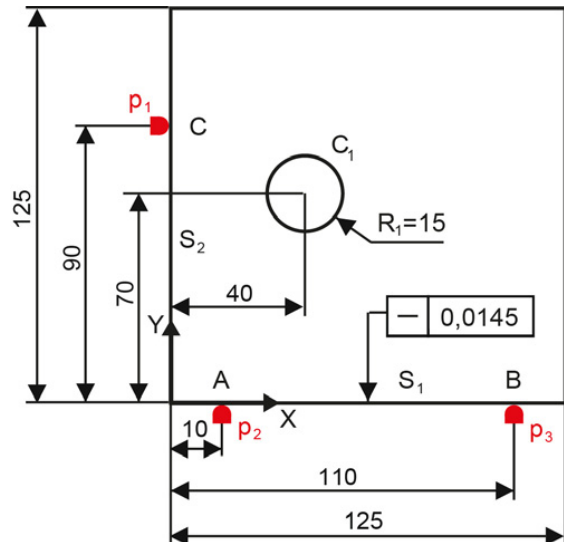


Fig. 1. Locator configuration schema

### 2.1. Effect of the error of the locators

The positions of the three locators are completely defined by their six coordinates. It is assumed that each of these coordinates is affected by an error behaving independently, according to a Gaussian  $N(0, \sigma^2)$  distribution.

The actual locator coordinate will then identify the workpiece reference frame. In particular, the  $x'$  axis is the straight line passing through the actual position of locators  $P_2$  and  $P_3$ , whereas the  $y'$  axis is straightforwardly computed as being perpendicular to the  $x'$  axis and passing through the actual position of locator  $P_1$ . The origin of the reference frame can be obtained as intersection of the two axes. The formulas for computing the axis-direction vectors and the origin coordinates from the actual locators coordinates are omitted here; for reference, see the work by Armillotta et al. [28].

The axis-direction vectors and origin coordinates define an homogeneous transformation matrix  ${}^0R_p$  [30], which enables conversion of the drill tip coordinate expressed in the machine tool reference frame  $P_0$  to the same coordinates expressed in the workpiece reference frame  $P'_0$  through the formula:

$$P'_0 = {}^0R_p^{-1} P_0 \quad (2)$$

The matrix  ${}^0\mathbf{R}_p$  depends only by the coordinates  $x_1, y_2$  and  $y_3$  of the locators, whereas the other coordinates may be considered nominal, as demonstrated in [28].

2.2. Effect of the part form error

To simulate the straightness deviation of the workpiece side, a discrete skin model shape was considered. To describe the skin model shape, a statistical Autoregressive-moving-average model with exogenous inputs (ARMAX) model was used [30]. The ARMAX model combines an harmonic model that represents the systematic pattern with the autoregressive one that corresponds to the random component:

$$Y_t = \sqrt{\frac{2}{N}} \sum_{i=2}^3 \left[ b_{2i-1} \cdot \cos\left(\frac{i \cdot t \cdot 2 \cdot \pi}{N}\right) + b_{2i} \cdot \sin\left(\frac{i \cdot t \cdot 2 \cdot \pi}{N}\right) \right] + \frac{1}{1 - a_1 B - a_2 B^2} \cdot \varepsilon_t \quad (3)$$

where  $t=1,2, \dots, N$  represent the index of data points in the sampled profile,  $B$  is the backshift operator, and  $N$  is the number of equally spaced points measured on that profile. Thus, the signature model in Eq. (3) is a linear combination of two harmonic terms plus a second-order autoregressive model of the noise. Each term of the first part of Eq. (3) represents the  $i$ -th harmonic ( $i=2$  and  $3$ ), characterized by  $i$  undulations per revolution, by an amplitude equal to  $\sqrt{2/N(b_{2i-1}^2 + b_{2i}^2)}$  and by a phase- equal to  $\tan^{-1}(b_{2i} / b_{2i-1})$ . The constant term  $\sqrt{2/N}$  is introduced to normalize the harmonic predictors. The parameters' vector in Eq. (3) forms a stochastic vector that has a multivariate normal distribution with the mean vector and the variance-covariance matrix, as reported in Table 1. The  $\mathbf{b}_i$  vectors that describe the systemic pattern left on the manufactured surface were defined by studying in detail the manufacturing process in [30]. The term  $\varepsilon_t$  in Eq. (1) was modelled as Gaussian white noise with a standard deviation equal to  $0,374 \mu\text{m}$ . Fig. 2 shows an example of the ARMAX profile.

The geometrical model implements an algorithm similar to Iterative Closest Point (ICP) to put into contact the plate side with the fixels. The model is constituted by four steps (see Fig. 3); it was developed in the Matlab® environment.

The first step generates a profile of 60.000 evenly distributed points  $\mathbf{S}_1$  by means of eq. (3). Then, the first point of the profile  $\mathbf{S}_1$  is randomly chosen among the first 10.000 points, and then, from the first point, the final profile  $\mathbf{S}_1$  of 50.000 points is generated.

The second step calculates the least square substitute geometry line  $\mathbf{S}_1$  profile. This straight line is rotated of  $90^\circ$  anticlockwise and it is called profile  $\mathbf{S}_2$ . It is brought into contact with profile  $\mathbf{S}_1$  at the origin.

The third step finds the two points of the profile  $\mathbf{S}_1$  nearest to the fixels  $\mathbf{p}_2$  and  $\mathbf{p}_3$  and it calculates the homogeneous transformation matrix, which brings those two points (A and B) into contact with locators  $\mathbf{p}_2$  and  $\mathbf{p}_3$  respectively.

The fourth step defines the minimum distance of fixel  $\mathbf{p}_1$  from  $\mathbf{S}_2$  and it translates the workpiece of this value along  $x$ -axis to bring the profile  $\mathbf{S}_2$  into contact with fixel  $\mathbf{p}_1$ , by keeping the side  $\mathbf{S}_1$  in contact with fixels  $\mathbf{p}_2$  and  $\mathbf{p}_3$ . If a solution is found

and the plate arrives to be in contact with the three locators, the algorithms stops. Otherwise it goes back to step 3. The distance of the hole centre from the nominal position is calculated as:

$$\text{location deviation} = \sqrt{\Delta x^2 + \Delta y^2} \quad (4)$$

where  $\Delta x$  and  $\Delta y$  are calculated as the distance of the drilled hole from the nominal position along the  $x$  and  $y$  axes, respectively.

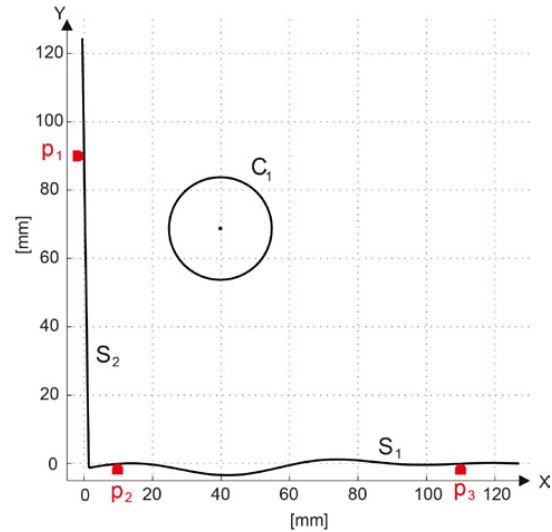


Fig. 2. ARMAX profile (amplified 500 times)

Table 1. ARMAX model parameters

	$\mathbf{b}_3$	$\mathbf{b}_4$	$\mathbf{b}_5$	$\mathbf{b}_6$	$\mathbf{a}_1$	$\mathbf{a}_2$
(a)	-0,0341	0,0313	0,0080	-0,0322	0,3714	0,2723
(b)	0,0004	-0,0002	0,0001	0	0,0001	0,0003
	-0,0002	0,0004	0,0001	0	0,0001	-0,0002
	0,0001	0,0001	0,0002	0	0,0001	0
	0	0	0	0,0003	0,0003	0,0003
	0,0001	0,0001	0,0001	0,0003	0,0072	0,0012
	0,0003	-0,0002	0	0,0003	0,0012	0,0036

3. Variational model to calculate the position deviation

The variational model has been considered in the literature [31]. Considering the assembly of Fig. 1, in the model,  $\mathbf{S}_1$  and  $\mathbf{S}_2$  are the sides of the plate in contact with the locators,  $\mathbf{C}_1$  is the hole, and  $\mathbf{p}_1, \mathbf{p}_2$  and  $\mathbf{p}_3$  are the locators. A straightness tolerance of  $0,0145 \text{ mm}$  is applied to  $\mathbf{S}_1$  side, and a location tolerance of  $0,012 \text{ mm}$  (6 times  $0,002 \text{ mm}$ ) is applied to  $\mathbf{p}_2$  and  $\mathbf{p}_3$  along the  $y$ -axis and to  $\mathbf{p}_1$  along  $x$ -axis. Two joints of vertex-line type are between the side  $\mathbf{S}_1$  with  $\mathbf{p}_2$  and  $\mathbf{p}_3$  at points A and B, and one joint of vertex line type is between

the side  $S_2$  with  $p_1$  at point C. Fig. 4 shows the assembly graph, which highlights the functional requirement, i.e., the shift of hole location along the  $x$  and  $y$  axes ( $\Delta x$  and  $\Delta y$ , respectively). A datum reference frame (DRF) was assigned to each part and to the entire assembly (see Fig. 5); all the global DRFs have the  $x$ -axis horizontal. The DRF of the plate, on the lower left corner, is also considered to be the global DRF of the assembly. Fig. 5 shows the model parameters.

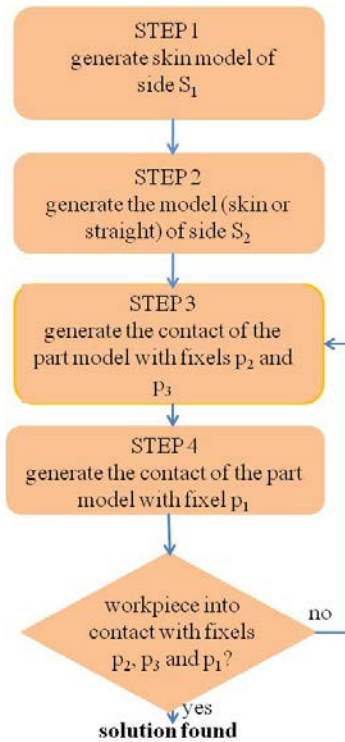


Fig. 3. Part positioning algorithm with respects to the fixels

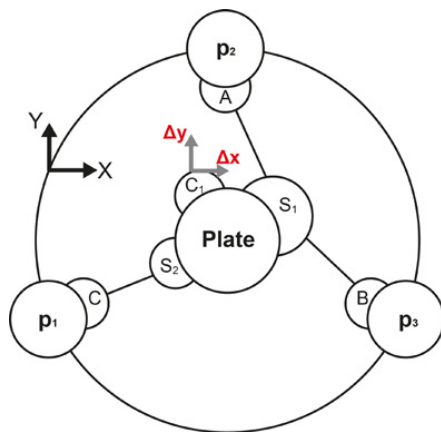


Fig. 4. Assembly graph

Once the DRFs are located, the model parameters can be assigned, enabling evaluation of the equations of the features in the global DRF of the assembly:

$$S_1: -r_{z01}X + Y + (62,5r_{z01} - t_{y01}) = 0 \tag{5}$$

$$S_2: X + r_{z04}Y - (62,5r_{z04} + t_{y04}) = 0 \tag{6}$$

$$p_1: [d_{x1}, 90] \tag{7}$$

$$p_2: [10, d_{y2}] \tag{8}$$

$$p_3: [110, d_{y3}] \tag{9}$$

where  $r_{ji}$  are the rotation parameters of the generic features  $S_i$  measured in their Datum Reference Frame (DRF in Fig. 5),  $t_{yi}$  are the translation parameters of the generic features  $S_i$  in their DRF, and  $d_{x1}$ ,  $d_{y2}$  and  $d_{y3}$  are the model parameters due to the location tolerance of the locators  $p_1$ ,  $p_2$  and  $p_3$ , respectively.

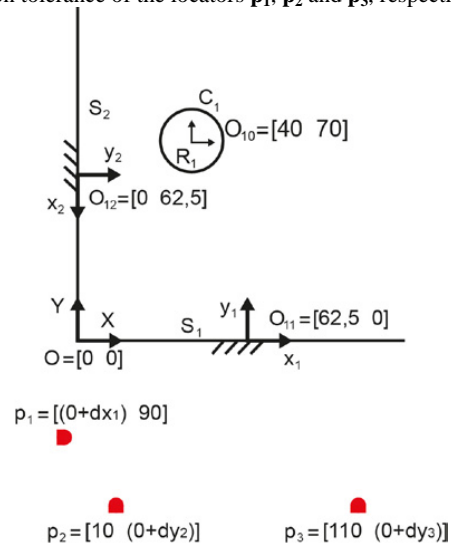


Fig. 5. DRFs and the model parameters

Once all of the features are expressed in the same global DRF of the assembly, the assembly is created by imposing the assembly conditions. The analytical equation to impose a joint of vertex-line type is:

$$n_{ix}t_x + n_{iy}t_y + (v_{ix}n_{iy} - v_{iy}n_{ix})\theta + v_{ix}n_{ix} + v_{iy}n_{iy} + c_i - d_i = 0 \tag{10}$$

where  $t_x$ ,  $t_y$  and  $\theta$  are the roto-translation components of the plate (i.e., the two functional requirements  $\Delta x$  and  $\Delta y$  and the rotation  $\theta$ ),  $v_{ix}$  and  $v_{iy}$  represent the  $x$  and  $y$  coordinates, respectively, of the  $i^{th}$  locators,  $n_{ix}$  and  $n_{iy}$  are the coefficients associated with  $X$  and  $Y$ , respectively, of the  $i^{th}$  line equation (sides of the plate).

There are three joints of vertex-line kind:

$$\mathbf{S}_1\text{-}\mathbf{P}_2: -r_{z01} \cdot \Delta x + \Delta y + (62,5 \cdot r_{z01} - t_{y01}) + [10 + r_{z01} \cdot d_{y2}] \cdot \theta - r_{z01} \cdot 10 + d_{y2} - d_A = 0 \quad (11)$$

$$\mathbf{S}_1\text{-}\mathbf{P}_3: -r_{z01} \cdot \Delta x + \Delta y + (62,5 \cdot r_{z01} - t_{y01}) + [110 + r_{z01} \cdot d_{y3}] \cdot \theta - r_{z01} \cdot 110 + d_{y3} - d_B = 0 \quad (12)$$

$$\mathbf{S}_2\text{-}\mathbf{P}_1: \Delta x + r_{z02} \cdot \Delta y - (62,5 \cdot r_{z02} + t_{y02}) + [r_{z02} \cdot d_{x1} - 90] \cdot \theta + r_{z02} \cdot 90 + d_{x1} = 0 \quad (13)$$

where  $d_A$  and  $d_B$  are the model parameters due to the straightness tolerance of the profile  $\mathbf{S}_1$ . The parameters  $r_{zj}$  and  $t_{yj}$  are equal to zero because there are no orientation and location tolerances on profiles  $\mathbf{S}_1$  and  $\mathbf{S}_2$ . As a result, the system of three equations can be simplified and solved as follows:

$$\Delta y = \frac{(d_B - d_{y3} + 110/10 [d_{y2} - d_A])}{\left(1 - \frac{110}{10}\right)} \quad (14)$$

$$\theta = \frac{(d_A - d_{y1} - d_{y2})}{10} \quad (15)$$

$$\Delta x = 90 \cdot \theta - d_{x1} \quad (16)$$

where  $d_A$  and  $d_B$  are distributed according to a Gaussian  $N(0, 0,0145/6=0,0024)$  distribution;  $d_{x1}$ ,  $d_{y2}$  and  $d_{y3}$  are distributed according to a Gaussian  $N(0, 0,002)$  distribution.

#### 4. Case study results

The model proposed so far was considered to identify the expected quality due to the locator configuration and the form tolerance applied to the workpiece.

Some of the model parameters are kept constant: the nominal size of the plate (125 mm x 125 mm); the standard deviation of the random errors in locator positioning ( $\sigma=0,002$  mm); the nominal location of the hole ( $\mathbf{P}_0=[40 \ 70]^T$ ); the straightness value of 0,0145 mm. In contrast, the coordinates  $x_j$ ,  $y_2$  and  $y_3$  of the locators are left free to change.

Monte Carlo simulation was performed by implementing 1000 and 10.000 runs for both the proposed method and the variational one. The obtained values of  $\Delta x$  and  $\Delta y$  are shown in Fig. 6 and 7, respectively.

Moreover, the mean value and the standard deviation of the obtained results are evaluated about  $\Delta x$  and  $\Delta y$ . The mean value of  $\Delta x$  and  $\Delta y$  is equal to 0 for the two models. The values of the standard deviation are reported in Table 2. The standard deviations of the estimated location deviation due to the proposed model are found to be significantly lower than those due to the variational model:

$$\Delta_{location} = \frac{0,0028 - 0,0056}{0,0056} \cdot 100 = -50\% \quad (17)$$

This result occurs because the proposed approach combines the form deviation with the locators' position deviations, both in a negative (i.e., the crests of the profile range into contact with locators) or positive way (i.e., the valleys of the profile range into contact with locators) randomly. In contrast, the variational model always adds the two deviations.

#### 5. Conclusions

This work proposes a conceptual demonstrator for robust design of fixture configuration considering the random error of the positions of the locators (due to the locator mounting on the machine table, the contact on irregular surface of the workpiece, etc.) and the form deviation on the surface in contact with the locators.

Table 2. Standard deviation of the results of the case study

Functional requirement [mm]	Runs	Variational model	Proposed model
$\Delta x$	1000	0,0044	0,0021
	10000	0,0044	0,0021
$\Delta y$	1000	0,0034	0,0019
	10000	0,0034	0,0019
location deviation $= \sqrt{\Delta x^2 + \Delta y^2}$	1000	0,0056	0,0028
	10000	0,0056	0,0028

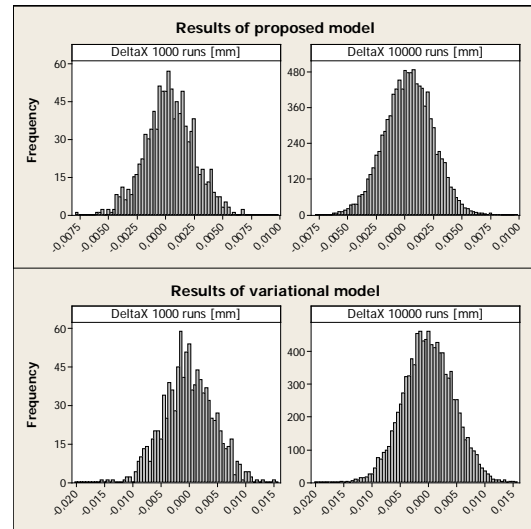


Fig. 6. Histograms of the  $\Delta x$  results

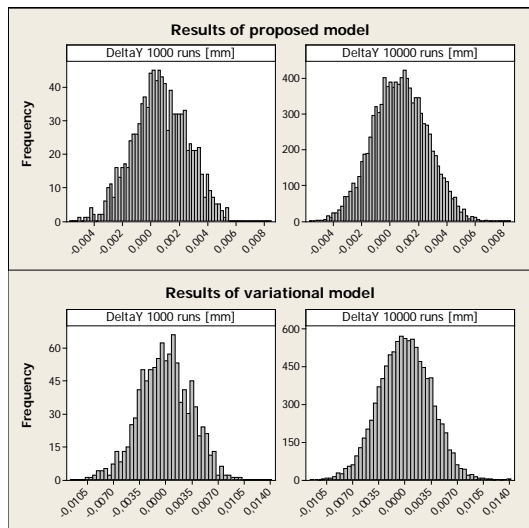


Fig. 7. Histograms of the  $\Delta y$  results

The proposed demonstrator was compared with a model of the literature for tolerance analysis. The comparison revealed how the proposed demonstrator allows the combination of the crests and valleys of the workpiece profile, on which is applied a form tolerance, with the locators' deviations, as actually occurs. Therefore, the proposed model can significantly reduce the estimated range of hole drilled location error. The model of the literature for tolerance analysis tends to overestimate the location error of the drilled hole because it always adds the two deviations of locators' position and workpiece straightness.

Small changes in the parameters of the adopted ARMAX model that involve studying different processes or different operative conditions of the turning process to verify if it is possible to represent them with the adopted ARMAX model are a matter for further studies.

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