Journal of Sound and Vibration 387 (2017) 207–225

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Effect of group walking traffic on dynamic properties of pedestrian structures

E. Shahabpoor [a,*], A. Pavic [b], V. Racic [a,c], S. Zivanovic [d]

[a] *INSIGNEO Institute for In-silico Medicine, The University of Sheffield, Department of Civil & Structural Engineering, Sir Frederick Mappin Building, Mappin Street, Sheffield S1 3JD, UK*
[b] *University of Exeter, College of Engineering, Mathematics and Physical Sciences, Vibration Engineering Section, North Park Road, Exeter EX4 4QF, UK*
[c] *Politecnico di Milano, Department of Civil and Environmental Engineering, Piazza di Leonardo Da Vinci 32, Milano 20133, Italy*
[d] *University of Warwick, School of Engineering, Coventry CV4 7AL, UK*

ARTICLE INFO

Article history:
Received 23 February 2015
Received in revised form
5 October 2016
Accepted 11 October 2016
Handling Editor: Dr. J. Macdonald
Available online 20 October 2016

Keywords:
Vibration serviceability
Human-structure dynamic interaction
Walking
Crowd
Footbridge
Floor

ABSTRACT

The increasing number of reported vibration serviceability problems in newly built pedestrian structures, such as footbridges and floors, under walking load has attracted considerable attention in the civil engineering community over the past two decades. The key design challenges are: the inter- and intra-subject variability of walking people, the unknown mechanisms of their interaction with the vibrating walking surfaces and the synchronisation between individuals in a group. Ignoring all or some of these factors makes the current design methods an inconsistent approximation of reality. This often leads to considerable over- or under-estimation of the structural response, yielding an unreliable assessment of vibration performance.

Changes to the dynamic properties of an empty structure due to the presence of *stationary* people have been studied extensively over the past two decades. The understanding of the similar effect of walking people on laterally swaying bridges has improved tremendously in the past decade, due to considerable research prompted by the Millennium Bridge problem. However, there is currently a gap in knowledge about how *moving* pedestrians affect the dynamic properties of vertically vibrating structures. The key reason for this gap is the scarcity of credible experimental data pertinent to moving pedestrians on vertically vibrating structures, especially for multi-pedestrian traffic.

This paper addresses this problem by studying the dynamic properties of the combined human-structure system, i.e. occupied structure damping ratio, natural frequency and modal mass. This was achieved using a comprehensive set of frequency response function records, measured on a full-scale test structure, which was occupied by various numbers of moving pedestrians under different walking scenarios. Contrary to expectations, it was found that the natural frequency of the joint moving human-structure system was *higher* than that of the empty structure, while it was *lower* when the same people were standing still. The damping ratio of the joint human-structure system was considerably higher than that of the empty structure for both the walking and standing people – in agreement with previous reports for stationary people - and was more prominent for larger groups. Interestingly, it was found that the walking human-structure system has more damping compared with the equivalent standing human-structure

* Corresponding author.
E-mail address: e.shahabpoor@sheffield.ac.uk (E. Shahabpoor).

http://dx.doi.org/10.1016/j.jsv.2016.10.017

system. The properties of a single degree of freedom mass-spring-damper system representing a moving crowd needed to replicate these observations have been identified.

1. Introduction

Over the past two decades, there has been a growing interest in vibration serviceability assessments of civil engineering structures, such as footbridges and floors due to people walking on them. The current trend towards more slender design and longer spans has made structures more sensitive to pedestrian-induced dynamic loading and consequently more susceptible to vibration problems, giving a serious cause for concern about the safety and comfort of the occupants. Moreover, such problems emphasise the need for more accurate and inclusive design methods which will take into account all aspects of human-structure dynamic interaction (HSI) [1–6].

Most of the current design guidelines, such as ISO 10137 standard [7] and UK National Annex to Eurocode 1 (BSI) [8], either ignore (ISO) or do not adequately treat (BSI) the main aspects of HSI [9]: (1) the effect of people on the dynamic properties of the structure, and (2) the changing of pedestrians' gait due to structural vibrations. The latter aspect occurs because the human body is a very sensitive vibration receiver, characterised by the innate ability to adapt quickly to almost any type and level of vibration which normally occurs in nature [10]. It has been experimentally observed that people change their pacing frequency and step width to adapt to clearly perceptible lateral vibrations of the supporting ground, which may lead to the so called 'lock-in' effect, as observed on the Millennium Bridge during its opening day in 2000 [11–13]. However, similar studies on the effect of the vertical vibrations on pedestrian gait are very rare and limited to individuals [14].

A number of studies, mostly based on full-scale measurements, have found that a passive human (sitting and standing) has a significant effect on the dynamic properties of a structure and, therefore, cannot be ignored. Typical findings include a considerable reduction in vibration response and slight changes in the natural frequency and damping of the structure [15]. This effect has been successfully modelled analytically by describing a group of passive people as an SDOF mass-spring-damper system attached to the empty structure [16].

Motivated by these findings, the present study was designed to collect a uniquely extensive experimental data,[1] which is needed for the analytical parameter identification of a mass-spring-damper (MSD) model of multiple *walking* people. Section 2 of this paper describes the modal parameters of the empty (unoccupied) structure used as a test bed for the walking people. Sections 3 and 4 study experimentally measured modal parameters of the structure when occupied by different numbers of standing and walking people, respectively. A comparison of the effects of standing and walking people on the modal parameters of the empty structure is presented in Section 5. In Section 6, a simple two-degrees-of-freedom mass-spring-damper model is used to simulate the joint human-structure system for both standing and walking scenarios. The analytical results are then critically evaluated against the corresponding experimental results. Finally, the main findings of this research are highlighted in Section 7.

2. Modal properties of the empty test structure

The test structure used in this study was a simply supported, in-situ cast, post-tensioned concrete slab strip constructed in the Light Structures Laboratory at the University of Sheffield (Fig. 1a). It rested on two knife edge supports along its shorter edges (Fig. 1b) and, with a span-to-depth ratio of 40, could be considered to be a representation of both a footbridge and a long-span slender floor. More specifically, the total length of the slab was 11.2 m, including 200 mm overhangs over the supports. Its rectangular cross section had a width of 2.0 m and a depth of 275 mm, and it weighed just over 14 t. The modal tests carried out by Shahabpoor and Pavic [17] showed natural frequencies of 4.44 Hz and 16.78 Hz for the first two vertical modes of the structure (Fig. 2) with a modal mass of 7128 kg for both modes, which is half of the total mass, assuming unity-scaled sinusoidal mode shapes. These two modes were selected for further analysis.

2.1. Non-linear behaviour of the empty structure

Knowledge about the potential non-linear behaviour of the structure plays an important role when judging whether changes in the modal properties of the occupied structure are related to the presence of people or to some form of structural non-linearity [18]. The amplitude-dependent behaviour of the damping ratio and natural frequency of the first mode were measured by Racic et al. [19] by curve-fitting cycle-by-cycle the free vibration decay of the structure at the midspan. The results are shown in Fig. 3.

[1] The work described in this article is carried out in accordance with The Code of Ethics of the World Medical Association (Declaration of Helsinki) for experiments involving humans.

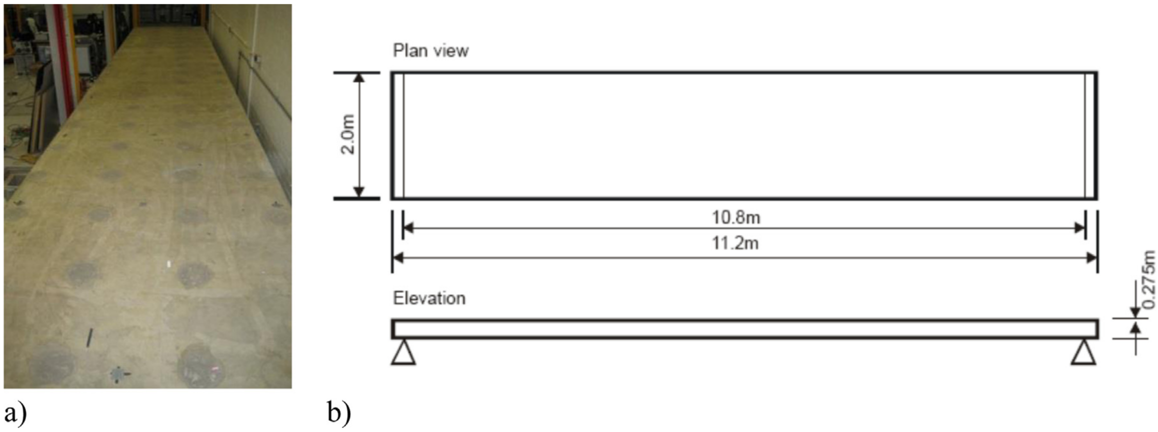


Fig. 1. Test structure (a) photo and (b) plan and elevation view.

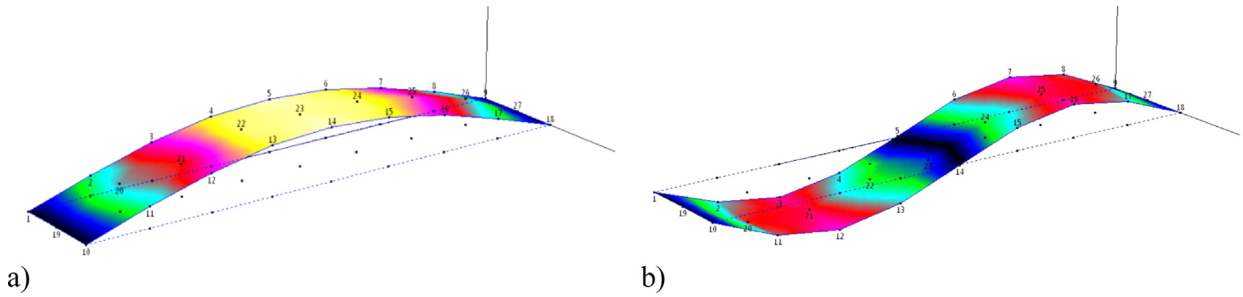


Fig. 2. Experimentally acquired first two vertical mode shapes of the structure: (a) First vertical mode shape @ 4.44 Hz, modal mass = 7128 kg; (b) Second vertical mode shape @ 16.78 Hz, modal mass = 7128 kg.

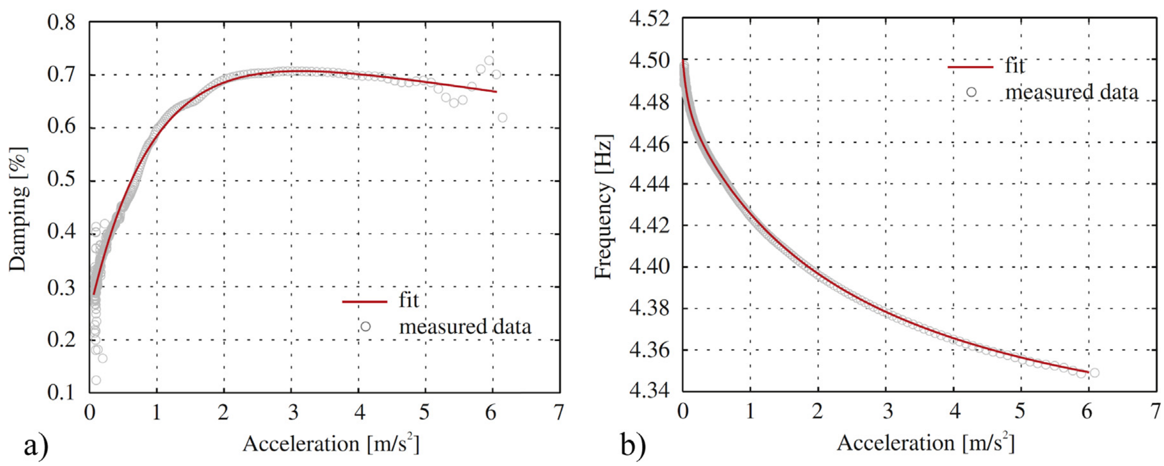


Fig. 3. Nonlinear amplitude-dependant (a) damping ratio of the fundamental mode and (b) natural frequency of structure (after Racic et al. [19]).

From a visual inspection of the graphs in Fig. 3, it is apparent that the structure shows some amplitude-dependent non-linear behaviour. It can be seen that, when the acceleration amplitude increases to 2 m/s^2, the damping ratio increases from 0.3 percent to 0.7 percent and remains almost constant between 2–6 m/s^2. The natural frequency, on the other hand, reduces from 4.50 Hz to 4.35 Hz, as the acceleration response amplitude increases to 6 m/s^2.

2.2. Modal testing

The tests presented in this paper were carried out in two test campaigns, referred to as test Series A and B, one year apart, with a nominally identical hardware setup. In each test series, the FRF-based modal testing of the empty structure was carried out using 18 Honeywell QA 750 accelerometers (nominal sensitivity 1.2 mA/g) [20] in two rows along the longer edges of the slab. An APS electro-dynamic shaker model 400 [21] was used to excite the structure. Chirp excitation with frequency ranges of either 3.5–5.5 Hz or 15–18 Hz was used to excite the structure, targeting the first and the second vertical modes of vibrations, respectively. The moving masses of the shaker were not used, as the shaker was operated in a grounded mode, with its armature connected via a rod to the soffit of the bridge to excite it directly. This also helped to ensure the safety and uninterrupted flow of the pedestrians, who did not need to negotiate a shaker as an obstacle on the surface over which they were walking. The shaker was placed at the midspan of the structure to predominantly excite the first mode, and at the quarter span to predominantly excite the second mode of vibration. The shaker force was measured directly using a uniaxial ENTRAN ELAF load cell [22] installed between the shaker and the rod. In each test, the driving point accelerance FRF (where the force and acceleration response are measured, at the same location and in the same vertical direction) was used in the subsequent modal identification.

The previously mentioned modal mass of 7128 kg was used in the curve-fitting of the measured FRFs for both modes. Unity-scaled sinusoidal mode shapes were assumed for both vertical modes, to reduce the redundancy in the curve-fitting process of more or less noisy FRFs. The identified modal properties of the *empty* structure (modal mass m_{es}, damping c_{es}, stiffness k_{es}, frequency f_{es} (Eq. (1)) and damping ratio ζ_{es} (Eq. (2)) are presented in Table 1, and they obey well-known relationships:

$$f_{es} = 1/(2\pi)\sqrt{k_{es}/m_{es}} \tag{1}$$

$$\zeta_{es} = \mathrm{ces}/(2\sqrt{k_{es}m_{es}}) \tag{2}$$

The maximum $a_{es,max}$ and total RMS $a_{es,rms}$ of the acceleration response during modal testing at the anti-node of each mode are also presented for response comparison. It can be seen that the modal properties of the structure have been quite stable over the one year period. The slight difference between the Series A and B results was due to the inevitable small differences in equipment positioning, environmental conditions in the lab during both series of tests, and the ageing of the concrete.

3. Tests with standing people

Although the main focus of the present study was on the effect of walking people on the modal properties of the structure, a limited number of tests involving standing people was carried out for comparison of the results relevant to the first mode of vibration only. Details of these tests are presented in this section.

3.1. Experimental setup

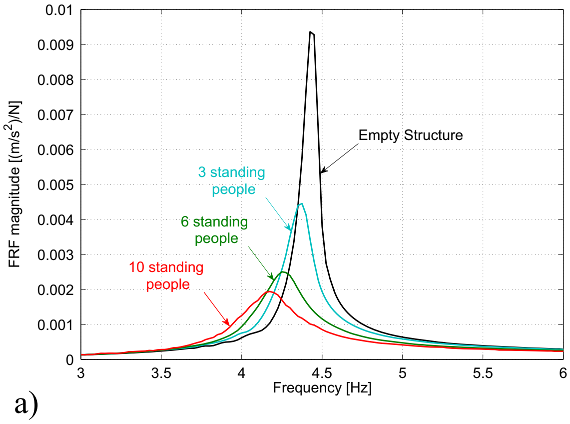
FRF-based modal testing of the human-structure system was carried out using the same accelerometer and shaker layout as in the tests with the empty structure (Section 2.2). Since the human body is a dynamic system, its location on the slab can considerably influence the FRF measurements. For the first vibration mode, a person standing at the midspan (i.e. the anti-node of the first mode) has the greatest interaction with the structure, while a person standing on the supports (i.e. the node of the first mode) makes no difference to the FRF measured.

Three tests were carried out with standing people, where groups of three, six and 10 people were standing together as close as possible to the midspan (so that their location could be assumed at midspan). A chirp force signal with a 3.5–5.5 Hz frequency range was used to excite the occupied slab around the frequency of its first vertical mode. To minimise the adverse effects of signal noise in each test, five FRF data blocks, each lasting 64 s, were recorded and averaged in the frequency domain. In each data block, the excitation lasted for the first 51.2 s, while the remaining 12.8 s allowed the response signal to die out before the acquisition of the next data block started.

The overlaid FRF moduli and phases in Fig. 4a and b show that the damping of the coupled system increases considerably, while the natural frequency decreases as the number of people on the structure increases. To assess the effects of non-linearity on the modal properties of the structure, the results of the empty structure test were compared with the test with 10 people standing on the structure. The acceleration RMS decreased from 0.37 m/s^2 for the empty structure test, to 0.15 m/s^2 for the 10-people test (Table 2). Looking at Fig. 3, this would yield an approximate 0.1 percent *decrease* in damping

Table 1
Results of experimental modal analysis of the empty structure (es).

Mode#	FRF based						
	f_{es}(Hz)	ζ_{es}(%)	m_{es} (kg)	c_{es} (N s/m)	k_{es} (N/m)	$a_{es,max}$ (m/s^2)	$a_{es,rms}$(m/s^2)
1 (Series A)	4.44	0.6	7128	2386	5547×10^3	1.88	0.37
1 (Series B)	4.44	0.7	7128	2784	5547×10^3	2.61	0.48
2 (Series A)	16.87	0.4	7128	6044	$80{,}086\times10^3$	2.51	0.48
2 (Series B)	16.77	0.4	7128	6009	$79{,}140\times10^3$	3.21	0.59



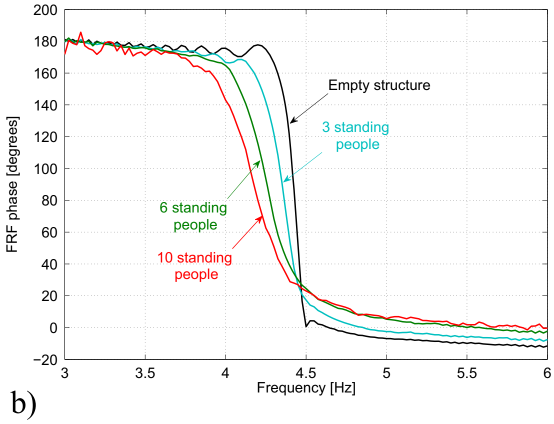


Fig. 4. Experimental FRF (a) magnitude and (b) phase graphs of the occupied structure with different number of standing people.

Table 2
Occupied structure experimental modal properties with different number of standing people.

Number of standing people	Modal properties of occupied structure					Structural response	
	f_{os}(Hz)	ζ_{os} (%)	m_{os}(kg)	c_{os} (N s/m)	k_{os} (N/m)	$a_{os,max}$ (m/s^2)	$a_{os,rms}$(m/s^2)
0	4.440	0.60	7128	2386	5547×10^3	1.88	0.37
3	4.363	1.35	7968	5898	5988×10^3	1.33	0.24
6	4.259	2.30	8808	10,842	6307×10^3	0.89	0.17
10	4.175	2.60	9928	13,543	6832×10^3	0.71	0.15

ratio and a 0.01 Hz *increase* in the modal frequency due to amplitude-dependant nonlinearity. However, a 2 percent *increase* in the modal damping ratio and a 0.27 Hz *decrease* in modal frequency were observed (Table 2). Therefore, the observed behaviour was completely opposite to what would be expected if the amplitude dependency was governing it. Bearing this in mind, it can be concluded that the observed behaviour was due to the HSI (i.e. the effect of passive people) on the modal properties of the slab, which was much more prominent than the effect of the structural nonlinearity. In fact, due to their opposite effect, the amplitude-dependent non-linearity has seemingly reduced the effects of the HSI. Based on this conclusion, in the next section, the changes in the damping ratio and natural frequency of the occupied structure observed in Fig. 4 have been attributed to the presence of the standing people only.

3.2. Parameter identification

If the coupled human-structure dynamic system was to be modelled as a 2DOF oscillator, two modes of vibration (i.e. two peaks in the FRF plot) would be expected in the experimental data [16]. However, the measured driving point accelerance FRFs in Fig. 4 feature only one apparent peak. This could be because the human mode was highly damped or it fell outside the frequency range displayed in the figures. Also, Matsumoto and Griffin [23] suggest that, for a crowd of people, due to the difference between the dynamic behaviour of people, the FRF of the “global crowd” is expected to be characterised by a broad and low peak.

The natural frequency f_{os} and damping ratio ζ_{os} of the occupied structure were initially estimated for each test using peak-picking and half-power bandwidth methods [24]. The modal mass of the occupied structure was estimated for each test, using the mass of the empty structure and the mass of each person, scaled by the squared mode shape amplitude, corresponding to the location of the person. A range of f_{os}, ζ_{os} and m_{os} was defined around initial values and used in the identification process. A set of f_{os}, ζ_{os} and m_{os} parameters that resulted in the best fit according to the least square error criterion was identified and was adopted for further analysis. For example, for six people standing, Fig. 5 shows that the fits of both amplitude and phase FRFs match their experimental counterparts well. This gives confidence that the methodology implemented here was robust, and could be used for identification of modal properties.

Table 2 and Fig. 6 summarise the results and show how the modal properties of the occupied structure change with an increasing number of standing people. The results are in line with those reported elsewhere for groups of *stationary* people [16]. In the next section, the focus of the study shifts to the influence of multiple *walking* people on the dynamic properties of the slab.

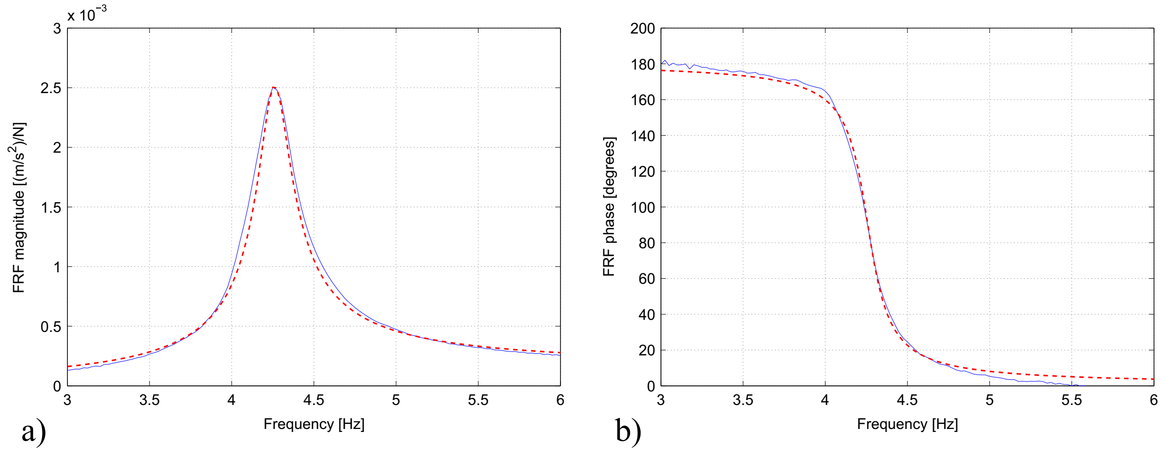


Fig. 5. Analytical (dashed line) and experimental (solid line) FRF (a) magnitudes and (b) phases for the test with six standing people.

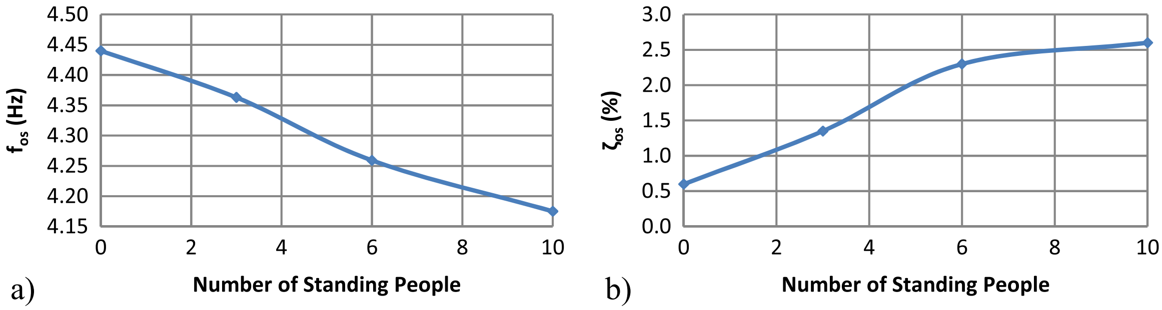


Fig. 6. Modal properties of the occupied structure for different number of people standing at midspan.

4. Tests with walking people

In comparison with stationary occupants, constantly changing positions of moving pedestrians on the structure makes the human-structure system strictly speaking ‘time-varying’ and, therefore, expected to generate ‘noisier’ and less stable FRF data. To reduce the effect of this noise, the FRFs were averaged using 15 data blocks each lasting 64 s. This is three times more averages than what was used in the tests with stationary people in Section 3. In addition, to study the effect of different locations of moving pedestrians on the modal properties of the occupied structure, two walking load scenarios were tested here. In Scenario 1 (S1), pedestrians were walking *along the structure*, as illustrated in Fig. 7a. In Scenario 2 (S2), participants were walking in a ‘tight circle’ around a point at 1/2 (Fig. 7b), then 3/8 and finally 1/4 of the span. S1 represents a realistic walking load case, while S2 was designed to study the effects of the varying locations of the people walking on a spot, as opposed to standing on the spot.

4.1. Experimental setup

In total, 112 test subjects in groups of 2–15 participated in 23 tests to provide statistically reliable FRFs, given the inter-subject variability of participants. Thirteen tests were focused on mode one, and 10 tests were focused on the second vibration mode. Pedestrians were asked to walk as they would normally do, and their pacing was not controlled by any external stimuli, such as metronome beats. They were free to speed up, slow down and pass others if necessary, while maintaining their usual self-selected ‘normal’ walking style.

The accelerometer layout in all walking tests was identical to the tests with standing people and the empty structure (Fig. 7). The shaker was connected to the bottom of the slab at either the midspan or in the quarter span, depending on which mode of vibration was targeted in a particular test. FRF-based modal testing, with pedestrians on the structure and targeting the first mode of vibration, was carried out under the same chirp shaker excitation used in the tests with standing people. The only difference was that the walking tests lasted longer, since it was necessary to collect more data blocks to obtain stable average FRFs. Chirp excitation with the frequency range 15–18 Hz was used to excite the pedestrian-structure system, targeting the second vertical mode of vibration.

Individual walking force time histories could not have been measured reliably due to the lack of adequate technology [6,25]. However, as the normal pacing frequency of the test subjects (measured prior to the FRF tests) were between 1.60 Hz and 1.85 Hz, the frequency ranges of their 1st and 2nd walking force harmonics (1.60–1.85 Hz and 3.2–3.7 Hz, respectively)

Fig. 7. A typical walking path and accelerometer (square) and shaker (triangle; midspan: mode 1 tests; quarter span: mode 2 tests) placement layout of walking tests: (a) Scenario 1 (S1): Walking along the structure; (b) Scenario 2 (S2): Walking in tight circle.

were below the frequency range of the modes of interest of the structure ($f_{es,1} = 4.44$ Hz and $f_{es,2} = 16.8$ Hz). As for the 3rd and 4th harmonics, their spectrum is characterised by low amplitude and wider energy spread compared with their 1st and 2nd harmonics counterparts. Therefore, the walking force spectrums were expected to be relatively flat around the modes of interest of the structure, especially as the number of people increased.

This assumption made it possible to consider the walking excitation as uncorrelated random extraneous excitation (having a flat spectrum around the two natural frequencies of interest), which can be averaged out. Several strategies were used to minimise the effects of the walking force excitation in the measured output. A chirp signal was used (instead of random) to excite the structure as much as possible and build up the resonance to get the maximum response amplitude. The shaker was connected directly to the structure to maximise the energy transferred to the structure. These helped to achieve a maximal signal (shaker force) to noise (walking forces) ratio, which subsequently resulted in RMS vibration amplitudes up to 12 times higher for tests, due to the simultaneous action of the shaker and the walking people compared to the tests with people alone (when the shaker was switched off) [26]. The duration of the tests were prolonged by up to 15 min to enable sufficient averaging of the FRFs. This considerably reduced the effects of uncorrelated noise due to walking, and smoothed and settled the FRF curves. Finally, the H1 estimator was used to reduce the effects of the uncorrelated extraneous excitation, due to non-measured walking forces.

In each test, a set of 18 FRFs corresponding to the 18 TPs was collected. The recordings at these points were used to identify the mode shape of the structure in each test. The response corresponding to test points on both edges of the structure with the same distance from the supports (e.g. the pair TP5 and TP14) was averaged in the time domain to eliminate contribution from torsional modes.

The experimentally measured FRFs for modes 1 and 2 of the occupied structure for S1 (walking along the structure) are presented in Fig. 8. A common trend can be observed in this figure: as the number of walking people on the structure increases, the damping ratio of the system also increases. Furthermore, contrary to what may be expected, considering the additional mass of the people present on the structure, the natural frequency of the occupied structure *increases* as the number of walking people on the structure increases.

4.2. Parameter identification

To identify the modal properties of the structure occupied with walking pedestrians, the same identification process was used as in the case of standing people (Section 3.2). The structure was assumed to behave linearly. Moreover, the modal properties of the occupied structure were considered to be constant values, representing their average values over time.

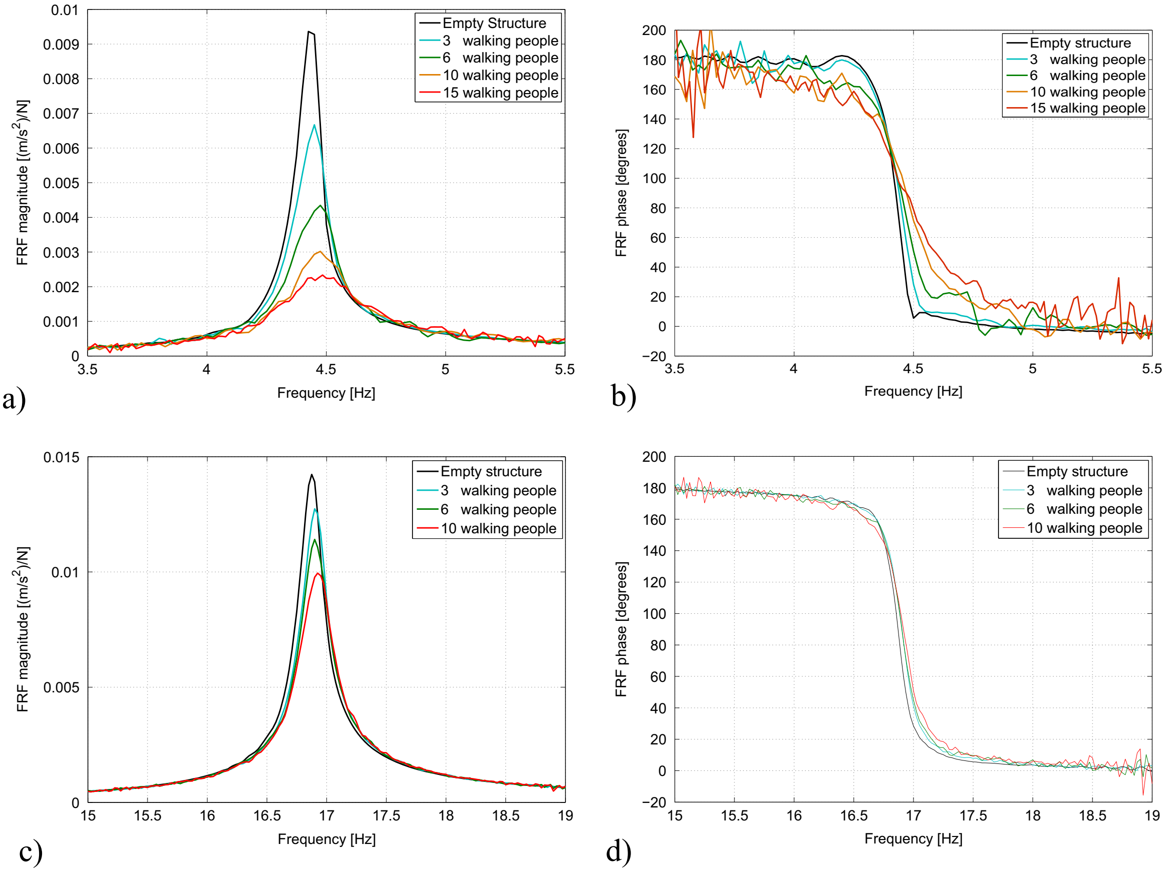


Fig. 8. Experimental FRF magnitude and phase curves of the 1st and 2nd vertical modes of the structure with different number of people walking along the structure (Scenario 1): (a) Mode 1– FRF magnitude curves; (b) Mode 1– FRF phase curves; (c) Mode 2– FRF magnitude curves; (d) Mode 2– FRF phase curves.

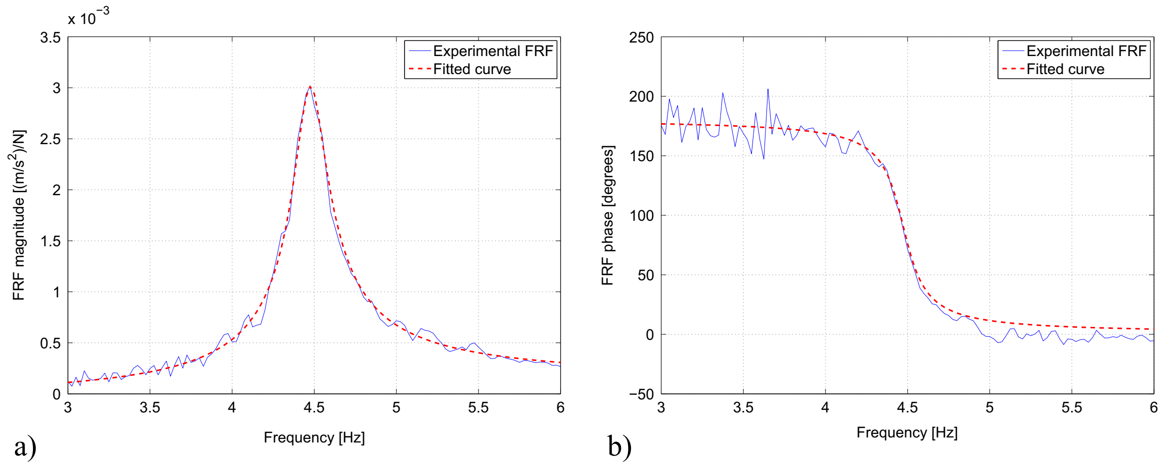


Fig. 9. The driving point experimental FRF (a) amplitude and (b) phase curves for accelerance and their analytical fit – Mode 1 - 10 pedestrians walking along the slab (Scenario S1).

Fig. 9 presents an example of a satisfactory match between the measured and fitted driving point accelerance FRFs, corresponding to the fundamental mode of the structure while 10 people were walking, according to S1.

Fig. 10 shows the recorded driving point accelerance FRF curves for groups of six (Tests 1.6 and 1.7) and 10 (Tests 1.8 and 1.9) walking people (Scenario 1). The tests were repeated, with the same group size, but different participants. The figure

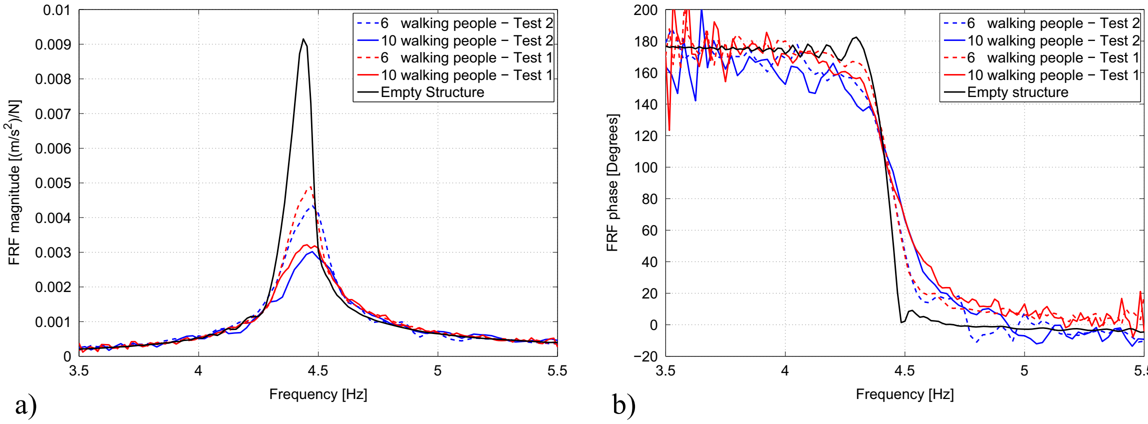


Fig. 10. The driving point accelerance FRF (a) magnitude and (b) phase graphs captured for six and 10 walking people, repeated twice with different participants – Mode 1, Scenario 1.

Table 3
Identified modal properties of the first mode of the occupied structure for different number of pedestrians and walking scenarios.

Test no.	Test Series	Location of pedestrians	No. of Pedestrians	Modal properties of the occupied structure– First mode					Structural response	
				f_{os} (Hz)	ζ_{os}	m_{os} (kg)	c_{os} (N s/m)	k_{os} (N/m)	$a_{os,max}$ (m/s^2)	$a_{os,rms}$ (m/s^2)
Empty structure properties										
1.1	A	–	0	4.440	0.0060	7128	2386	5547×10^3	1.88	0.37
1.2	B	–	0	4.440	0.0070	7128	2784	5547×10^3	2.61	0.49
Scenario 1: Pedestrians are walking along the footbridge										
1.3	B	All over	2	4.443	0.0100	7165	4000	5583×10^3	2.44	0.41
1.4	A	All over	3	4.445	0.0110	7183	4413	5603×10^3	1.75	0.30
1.5	B	All over	4	4.450	0.0128	7201	5154	5630×10^3	2.18	0.36
1.6	B	All over	6	4.465	0.0155	7238	6294	5696×10^3	1.88	0.33
1.7	A	All over	6	4.465	0.0165	7238	6701	5696×10^3	1.49	0.25
1.8	A	All over	10	4.475	0.0230	7311	9456	$5{,}780\times10^3$	1.13	0.21
1.9	B	All over	10	4.476	0.0210	7311	8635	5782×10^3	1.59	0.29
1.10	B	All over	15	4.485	0.0291	7402	12,140	5878×10^3	1.13	0.25
Scenario 2: Pedestrians are walking in a tight circle										
1.11	A	Midspan	3	4.455	0.0200	7214	8077	5652×10^3	1.32	0.25
1.12	A	Midspan	6	4.480	0.0290	7300	11,918	5784×10^3	1.09	0.20
1.13	A	Midspan	10	4.500	0.0340	7415	14256	5928×10^3	0.87	0.19
1.14	A	3/8 –span	6	4.465	0.0250	7287	10,222	5735×10^3	0.99	0.20
1.15	A	Quarter span	6	4.460	0.0205	7250	8329	5693×10^3	1.10	0.22

shows that FRF curves of different tests with the same number of people follow the same trend of change in the natural frequency and damping ratio. The higher the number of pedestrians, the lower and more shifted towards higher natural frequency the FRF peaks are. This demonstrates that individual differences in human body mechanics for different participants do not affect the general trend of changing the FRF shape for a different number of people. The conclusions based on these trends appear to be valid for an arbitrary group of people.

4.2.1. Results for the first vibration mode

The identified modal properties for the tests focused on the first mode of the structure are summarised in Table 3 and their trends are illustrated in Fig. 11. Both the natural frequency of the occupied structure f_{os} and its damping ratio ζ_{os} increase as the number of walking people increases. Modal mass m_{os}, stiffness k_{os} and damping c_{os} of the occupied structure also increase for more people on the structure. Moreover, considering the fact that the natural frequency is directly proportional to $\sqrt{k/m}$, it appears that modal stiffness increases faster than its modal mass counterpart to make the observed increase in the natural frequency possible.

The similarity of the trends of the changes in the modal frequency and damping ratio observed in S2 and S1 again confirms the validity of the observed trends in the results. On the other hand, higher values of all modal properties in S2

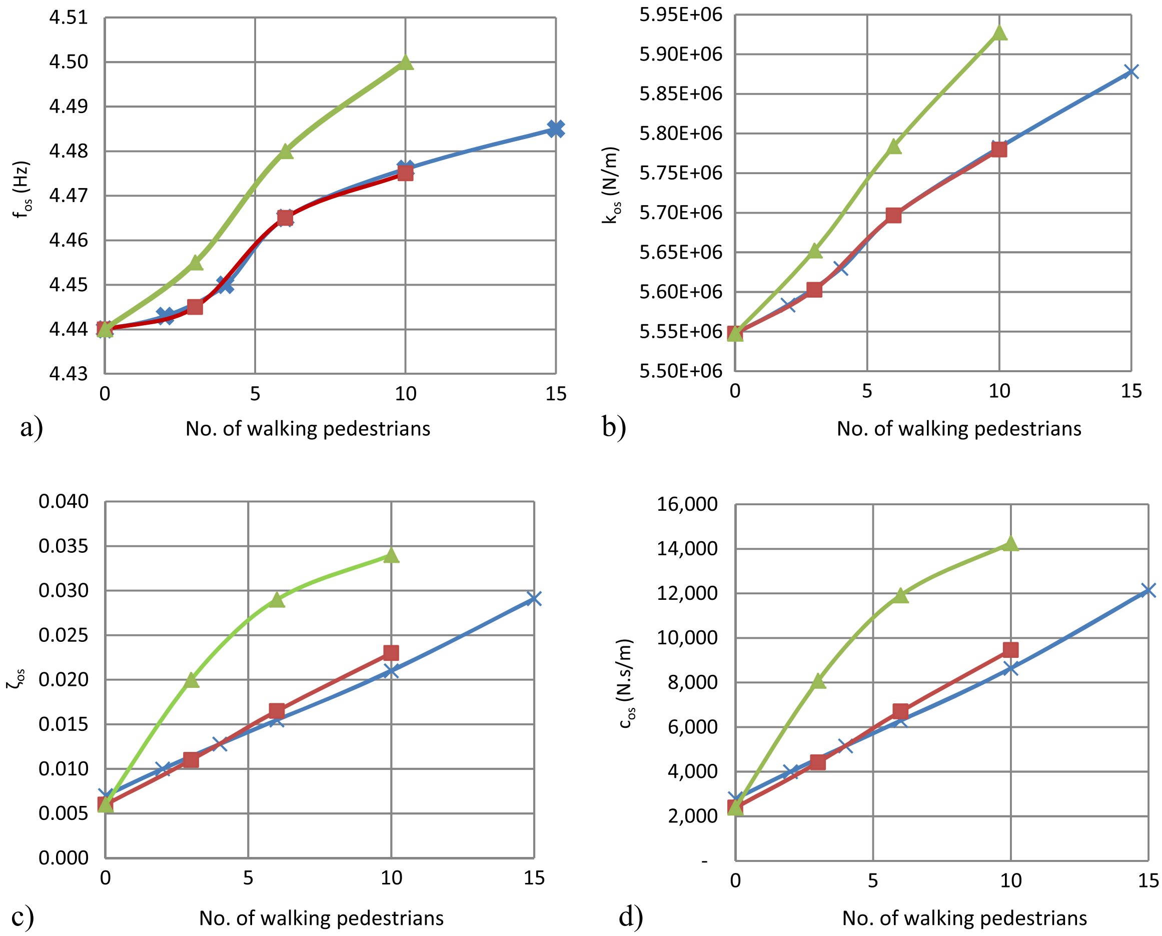


Fig. 11. Mode 1 - Trends of occupied structure (a) modal frequency f_{os} (b) stiffness k_{os} , (c), damping ratio ζ_{os} and (d) damping c_{os} versus number of walking pedestrians – (Red/square: Scenario 1 - Series A; Blue/cross: Scenario 1 - Series B; Green/triangle: Scenario 2). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

compared with S1 with the same participants confirm that human body location relative to a mode shape amplitude plays a significant role in the level of the interaction with the structure. HSI is apparently greatest when walking happens close to the anti-node of the structural mode (e.g. midspan for the first mode).

Comparing the results of this set of tests with the changes observed in Section 2.1 due to the amplitude-dependent behaviour of the structure, it can be concluded that the changes in the modal properties of the structure under walking people were much more prominent than the effect of the non-linearity of structure. For instance, comparing Tests 1.1 and 1.13, in Table 3; with the highest change in RMS response (and therefore maximum expected effects of non-linearity) there was a 0.06 Hz increase in the fundamental frequency of the occupied structure f_{os}, while the corresponding increase of the natural frequency of the empty structure due to its non-linear nature could be expected to be less than 0.01 Hz (according to Fig. 3 based on RMS response). A significantly more prominent difference can be seen in the damping ratio, where ζ_{os} was increased by 2.8 percent in the occupied structure, while the corresponding change due to amplitude-dependant non-linearity was a 0.05 V decrease (rather than increase). In the rest of the tests, the effects of amplitude-dependant non-linearity were less pronounced. Therefore, the observed changes in Table 3 are overwhelmingly due to the presence of walking humans on the structure.

4.2.2. Results for the second vibration mode

The identified modal parameters of the second mode of the occupied structure and their trends are presented in Table 4 and Fig. 12, respectively. The same trend can be observed in all modal properties of the occupied structure. Similar to the results for Mode 1, f_{os}, ζ_{os}, m_{os}, k_{os} and c_{os} for Mode 2 all increase as the number of walking people on it increases.

4.2.3. Location effects

In modal analysis, the contribution of a physical force to the force exciting a specific mode is called modal force, and is calculated by scaling the physical force with the corresponding mode shape ordinate at its point of application. In the case of

Table 4
Identified modal properties of the second mode of the occupied structure for different number of pedestrians and walking scenarios.

Test No.	Test series	Location of pedestrians	No. of pedestrians	Modal properties of the occupied structure - Second mode					structural response	
				f_{os} (Hz)	ζ_{os}	m_{os} (kg)	c_{os} (N s/m)	k_{os} (N/m)	$a_{os,max}$ (m/s^2)	$a_{os,rms}$ (m/s^2)
Empty structure properties										
2.1	A	–	0	16.870	0.0040	7128	6,044	$80,086\times10^3$	2.51	0.48
2.2	B	–	0	16.770	0.0040	7128	6,009	$79,140\times10^3$	3.21	0.59
Scenario 1: Pedestrians are walking along the structure										
2.3	A	All over	3	16.900	0.0055	7128	8326	$80,372\times10^3$	2.41	0.45
2.4	B	All over	6	16.813	0.0053	7128	7982	$79,548\times10^3$	2.90	0.56
2.5	A	All over	6	16.910	0.0065	7128	9846	$80,468\times10^3$	2.29	0.42
2.6	B	All over	8	16.819	0.0061	7128	9190	$79,605\times10^3$	2.56	0.51
2.7	B	All over	10	16.822	0.0064	7128	9644	$79,634\times10^3$	2.52	0.52
2.8	A	All over	10	16.935	0.0075	7128	11,377	$80,708\times10^3$	2.14	0.40
2.9	B	All over	15	16.825	0.0079	7128	11,907	$79,665\times10^3$	2.24	0.47
Scenario 2: Pedestrians are walking around a tight circle										
2.10	A	Quarter span	3	16.913	0.0061	7128	9241	$80,496\times10^3$	2.23	0.49
2.11	A	Quarter span	6	16.925	0.0082	7128	12,432	$80,611\times10^3$	1.94	0.35
2.12	A	Quarter span	10	16.975	0.0099	7,128	15,054	$81,091\times10^3$	1.69	0.37

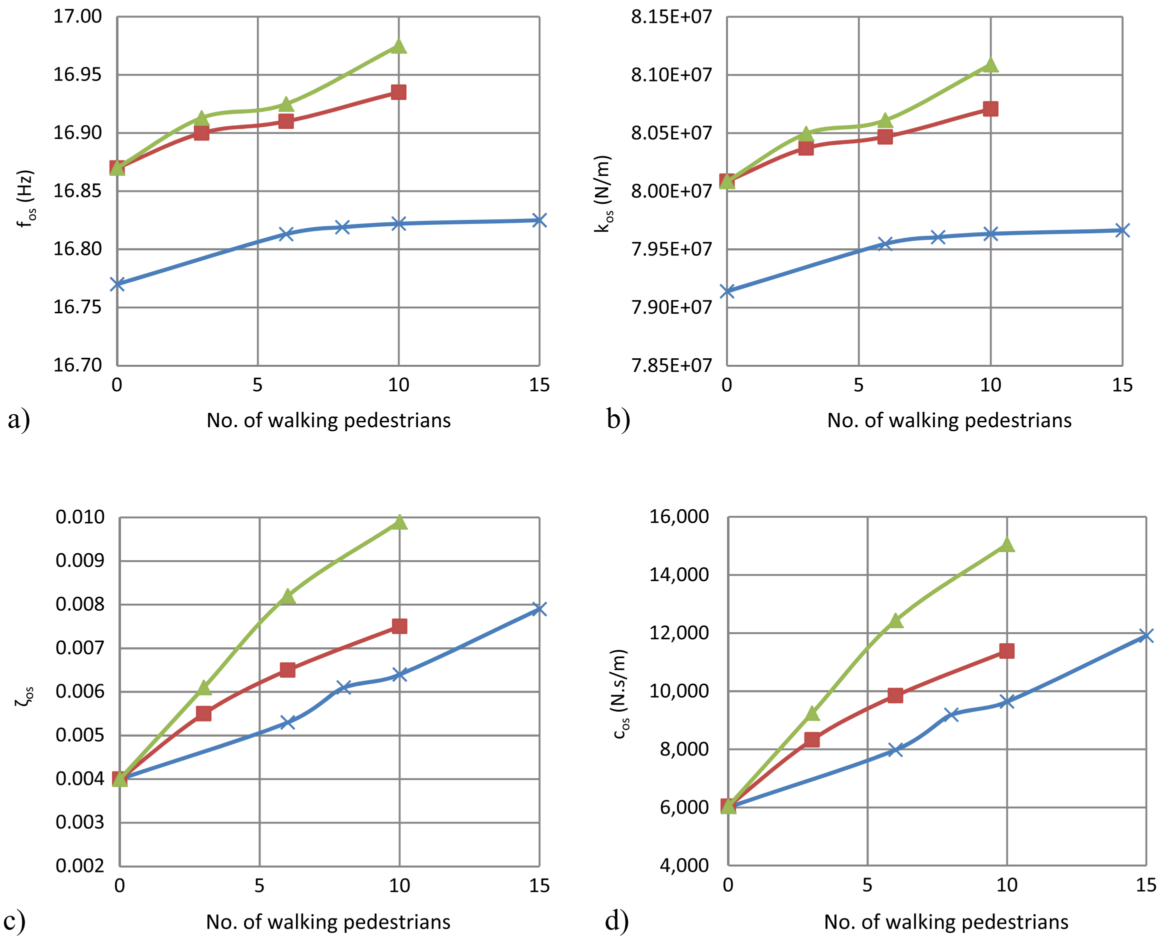


Fig. 12. Mode 2 - Trends of occupied structure (a) modal frequency f_{os}, (b) stiffness k_{os}, (c) damping ratio ζ_{os} and (d) damping c_{os} against number of walking pedestrians – (Red/square: Scenario 1 - Series A; Blue/cross: Scenario 1 - Series B; Green/triangle: Scenario 2). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

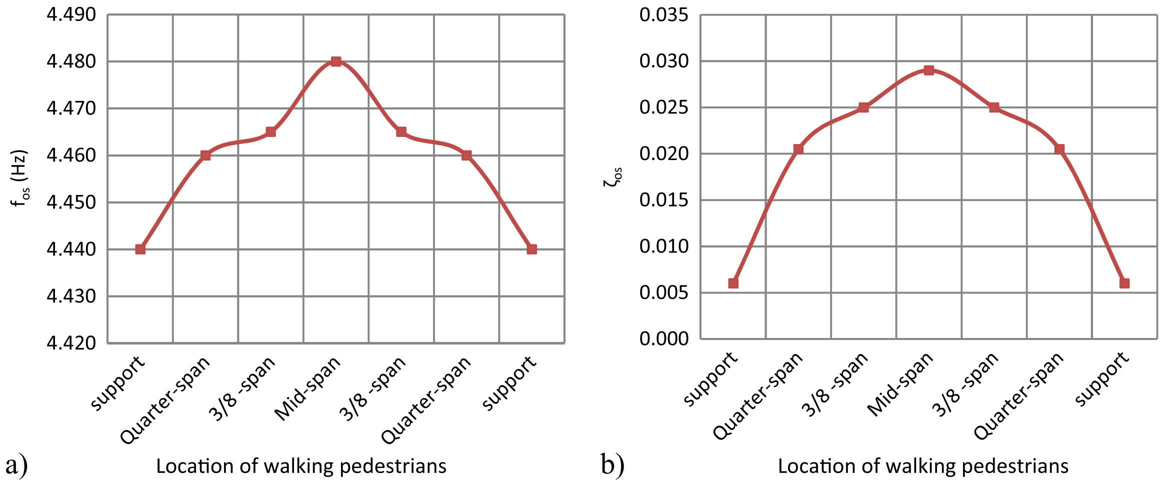


Fig. 13. (a) Natural frequency f_{os} and (b) damping ratio ζ_{os} of occupied structure vs. the location of six walking pedestrians on the structure – Mode 1, Scenario2. For presentation purposes, results corresponding to walking at both supports are shown by assuming that walking on them does not change dynamic properties.

the test footbridge in this paper, the first vertical mode shape can be approximated with a half sinusoid (Fig. 2a). This means that, for an arbitrary physical force applied on the structure, the modal force in Mode 1 is maximum when the physical force is applied at the midspan and decreases to its minimum (zero) when the physical load is applied at the supports' location.

To investigate how the location of people on the structure influences the HSI, f_{os} and ζ_{os} are compared in Fig. 13 for a group of six people walking in a tight circle (i.e. S2) at different locations on the footbridge. During the experiment, the influence of walking humans was greatest when they were at midspan (anti-node of the first mode), while it was naturally negligible when they were located at supports (nodes of the first mode – compare with the empty structure properties presented in Table 1). Such observations are in line with the modal analysis concept of the modal force, and confirm the mechanical behaviour of the human body system.

5. Comparison of effects of standing and walking people

The observed trends in the modal parameters of the occupied structure for nominally identical groups of standing and walking people are compared in this section. The FRF magnitude and phase plots for groups of three, six and 10 walking and standing people are shown in Fig. 14. The same test subjects participated in each pair of the walking/standing tests. Only the results of walking 'around a tight circle' at the midspan (i.e. S2) are compared with the corresponding standing tests, in order to compare like with like as far as location is concerned.

Changes in the natural frequency f_{os} and damping ratio ζ_{os} of the first mode occupied structure, with regard to the change in the number of walking/standing people, are presented in Fig. 15. For standing people, the FRFs shifted towards lower frequencies, while for walking people they shifted towards higher frequencies. In both walking and standing scenarios, the damping ratio of the occupied structure increased considerably. However, ζ_{os} for walking was consistently higher compared to its standing counterpart. This is a new observation, to the best knowledge of the authors. A straightforward explanation behind this phenomenon is that a human body simply adds more damping to the structure when walking than in the standing posture at nominally the same location. Also, it could be attributed to the component of the walking force proportional to the velocity of the vibrating structure, which acts as an 'active damper' – the effect already reported in case of the HSI in the lateral direction. These observations require further research, and their further investigation is beyond the scope of this paper.

6. Analytical verification

The experimental results presented in previous sections show a clear and significant change in the modal properties of the test structure when interacting with walking or standing people. To validate the findings, aggregated dynamic effects of standing and walking groups of people (referred to as *crowd* hereafter) were simulated using a conventional single degree of freedom (SDOF) mass-spring-damper (MSD) model. The aim was to check if such a simple dynamic system can simulate accurately enough the interaction of the crowd with a particular vertical vibration mode of the empty structure.

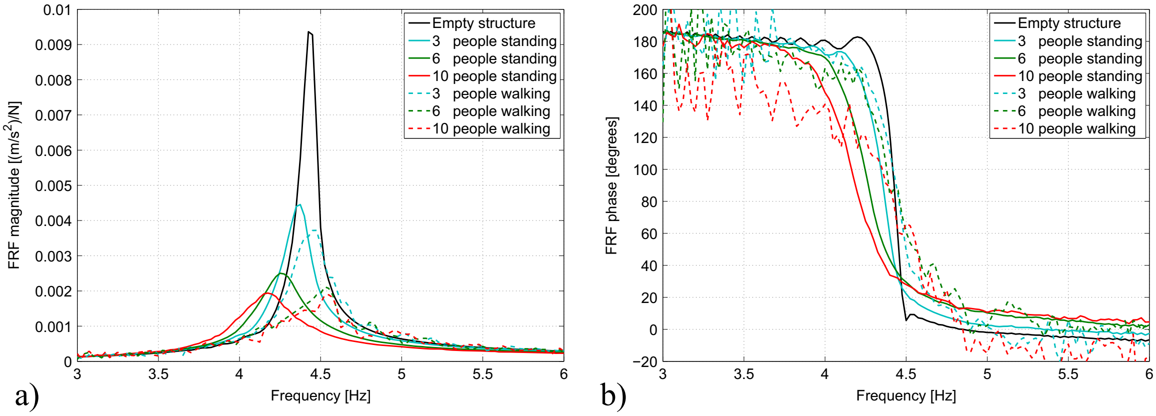


Fig. 14. Mode 1 - FRF (a) magnitude and (b) phase plots for groups of three, six and 10 walking/standing people at the midspan.

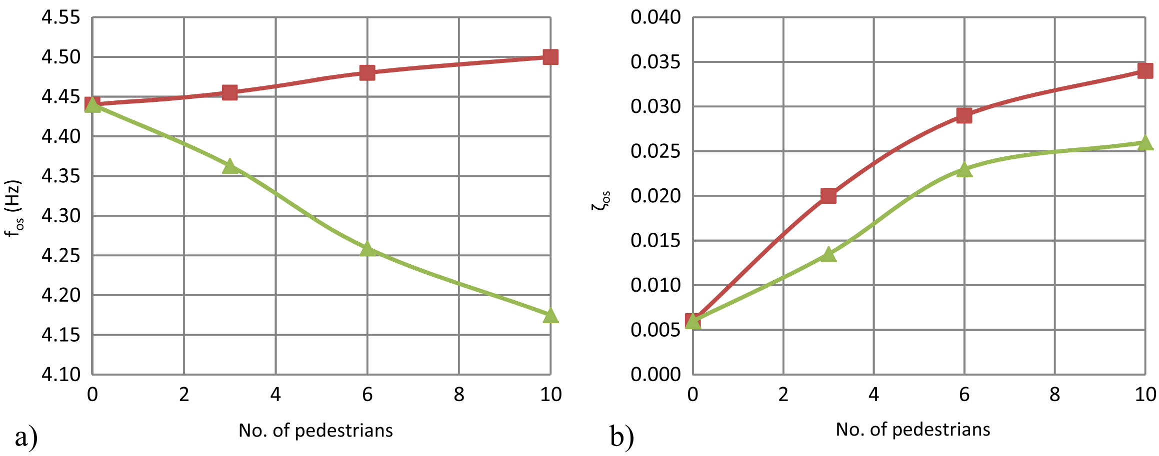


Fig. 15. Mode 1 – (a) Natural frequency f_{os} and (b) damping ratio ζ_{os} for varying number of standing and walking people on the structure (Red/square: Circular walking at midspan; Green/triangle: standing at midspan). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

6.1. 2DOF crowd-structure model

Here, only the first mode of the empty structure was considered and was conceptualised using an SDOF oscillator with the corresponding modal properties (m_s, k_s and c_s). Assuming that the structure was linear, the mode superposition principle applies [27]. Therefore, considering only one structural mode at a time does not affect the generality of the results.

Another SDOF model was used to simulate the crowd (m_{sc}, k_{sc} and c_{sc}) standing-still as close as possible to the anti-node of the first structural mode (i.e. the midspan). Similarly, an SDOF model was used to simulate the walking crowd (m_{wc}, k_{wc} and c_{wc}). The walking crowd parameters were considered time-invariant under the assumption that the pedestrian flow was in the steady state regime, i.e. individuals in a group do not significantly change their gait during the test and their locations on the structure are evenly distributed.

The walking crowd SDOF model was further assumed to be 'stationary' to avoid the complexity of modelling the 'time-varying system' caused by the change of location of people as they walk on the structure. This assumption was based on the experimental observation that, during the walking tests, after 3–4 averages, FRFs settled (did not change noticeably with further averaging) and represented the averaged effects of the walking crowd on the structure over time. This is conceptually equal to approximating the moving crowd with an equivalent stationary crowd that are walking on a series of treadmills located at the midspan of the structure (Fig. 16a) and create the same dynamic effects on the structure as the corresponding moving crowd.

By connecting the SDOF crowd and SDOF structure models in the vertical direction, as illustrated in Fig. 16b, a 2DOF crowd-structure (CS) mechanical system is created. Assuming a 'stationary' crowd, both standing and walking crowd-structure (denoted generally by m_c, k_c and c_c) coupled systems can be represented as a simple conventional 2DOF system, the dynamic response of which can be calculated by solving its equations of motion [27]:

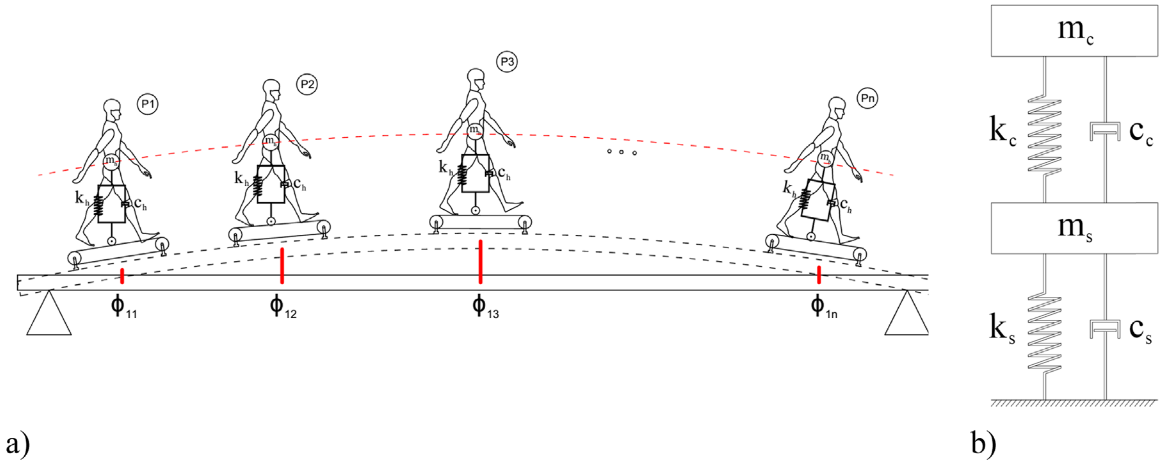


Fig. 16. Conceptual 2DOF model of a coupled crowd-structure system. Crowd model parameters are shown generally by m_c, k_c and c_c.

$$\begin{bmatrix} m_s & 0 \\ 0 & m_c \end{bmatrix}\begin{Bmatrix} \ddot{x}_s(t) \\ \ddot{x}_c(t) \end{Bmatrix}+\begin{bmatrix} c_s+c_c & -\ c_c \\ -\ c_c & c_c \end{bmatrix}\begin{Bmatrix} \dot{x}_s(t) \\ \dot{x}_c(t) \end{Bmatrix}+\begin{bmatrix} k_s+k_c & -\ k_c \\ -\ k_c & k_c \end{bmatrix}\begin{Bmatrix} x_s(t) \\ x_c(t) \end{Bmatrix}=\begin{Bmatrix} f_s(t) \\ f_c(t) \end{Bmatrix} \tag{3}$$

Here, m_s, c_s and k_s are the mass, damping and stiffness of the empty structure and m_c, c_c and k_c are those of the crowd (standing or walking) model. Moreover,$\ddot{x}_s(t)$, $\dot{x}_s(t)$ and $x_s(t)$ are respectively the vertical acceleration, velocity and displacement response of structure in the coupled 2DOF system. Similarly,$\ddot{x}_c(t)$, $\dot{x}_c(t)$ and $x_c(t)$ represent the vertical acceleration, velocity and displacement of the crowd DOF, respectively. Finally, $f_s(t)$ and $f_c(t)$ are the externally applied forces at the structure and crowd degrees of freedom, respectively. To extract modal properties from this system, a condition of free vibration is assumed:

$$f_s(t)=0 \tag{4}$$

$$f_c(t)=0 \tag{5}$$

To find possible combinations of parameters m_c, c_c and k_c, a modal analysis formulation for systems with a non-proportional damping matrix was used here. A new coordinate vector '**y**' containing displacement and velocity vectors was defined first:

$$\mathbf{y}(t) = \begin{Bmatrix} x(t) \\ \dot{x}(t) \end{Bmatrix} \tag{6}$$

Then Eq. (3) is re-written into the following form for modal analysis [28]:

$$\begin{bmatrix} \mathbf{C} & \mathbf{M} \\ \mathbf{M} & 0 \end{bmatrix}\dot{\mathbf{y}}(t) + \begin{bmatrix} \mathbf{K} & 0 \\ 0 & -\mathbf{M} \end{bmatrix}\mathbf{y}(t) = \{0\} \tag{7}$$

Where **M**, **C** and **K** represent mass, damping and stiffness matrices from Eq. (3), respectively. Eq. (7) leads to a standard eigenvalue problem and can be solved for eigenvectors and eigenvalues accordingly [27,28].

6.2. Modelling specification

For each test, the described 2DOF crowd-structure model was used to simulate the observed changes in the dynamic properties of the structure when exposed to a standing/walking crowd. The modal parameters of the first mode of the empty structure were adopted from Table 1. The mass of the crowd model m_c was calculated up front for each test, to reduce the number of variables and improve the convergence and quality of the linear system identification procedure needed to make use of real, more or less noisy and non-linear FRFs measured.

Based on an analogy used to model both active (bouncing) and passive (standing) people as MSD systems on grandstands [29], the effects of people *distributed* along the structure (Fig. 16a) were simulated using a lumped SDOF model (Fig. 16b) attached at the midspan i.e. at the antinode of the beam's fundamental mode of vibration. Expressions can be developed for a modal force caused by all individual SDOF oscillators and for a modal force caused by the 'lumped' SDOF system under base excitation due to Mode 1 vibration. By equating these two modal forces, and assuming identical individual SDOF oscillators without loss of generality, the following equations linking properties of individual human SDOF systems ($m_{h,i}$, $c_{h,i}$ and $k_{h,i}$) with their 'equivalent' lumped SDOF (m_c, c_c and k_c) (Fig. 16b) can be developed:

$$m_c = \sum m_{h,i} \varnothing_i^2;\ i = 1, \ldots, number\ \ of\ \ pedestrians, \tag{8}$$

$$c_c = \sum c_{h,i} \varnothing_i^2;\ i = 1, \ldots, number\ \ of\ \ pedestrians, \tag{9}$$

and

$$k_c = \sum k_{h,i} \varnothing_i^2;\ i = 1, \ldots, number\ \ of\ \ pedestrians, \tag{10}$$

where $\varnothing_i$ is the structure (unity-normalized) mode shape ordinate corresponding to the location of pedestrian '*i*' modelled as an SDOF MSD with properties $m_{h,i}$, $c_{h,i}$ and $k_{h,i}$. This ensures that the effect of the *location* of walking people on the structure is taken into account in the properties of the 'equivalent' crowd SDOF model. Therefore, m_c was taken as the sum of known physical masses of each person weighted by the squared mode shape amplitude (assumed to be sinusoidal unity-scaled) at their location on the structure (Tables 5 and 6).

Based on this analogy, in Scenario 1 (walking along the structure), m_c was calculated as equal to half of the total actual mass of the crowd, assuming sinusoidal mode shape and uniform distribution of the walking pedestrians along the length of the test structure (Table 5). Such approach, to assume 50 percent of the mass of the uniformly distributed people, when each spectator is modelled as an SDOF, is commonly used for both active and passive humans on grandstands [29].

Similarly, in Scenario 2, as pedestrians are walking around a tight circle at *midspan* (the anti-node of the mode one), the mode shape amplitudes at the location of all pedestrians on the structure were assumed unity and, therefore, m_c was calculated to be equal to the total physical mass of the whole crowd i.e. 100 percent of the pedestrians' mass was taken (Table 5).

The analogy presented above is applicable to the stiffness (Eq. (10)) and damping coefficient (Eq. (9)) of the crowd as well, but it is automatically addressed by identifying the natural frequency and damping ratio of the crowd, and then deriving the contributory stiffness and damping coefficient using relationships such as Eqs. (1) and (2).

For each experiment, the 2DOF model of the CS system was used to find the occupied structure modal properties for

Table 5
Walking crowd model properties obtained from simulation of 2DOF crowd-structure model.

Test No.	No of pedestrians	Occupied structure – experimental					Walking crowd model – analytical				
		f_{os} Hz	ζ_{os} %	m_{os} kg	c_{os} N.s/m	k_{os} N/m	f_{wc} Hz	ζ_{wc} %	m_{wc} kg	c_{wc} N.s/m	k_{wc} N/m
Scenario 1: Walking along the footbridge – Series B											
1.2	0	4.440	0.70	7128	2784	5547×10^3	–	–	–	–	–
1.3	2	4.443	1.00	7165	4000	5583×10^3	2.406	36	70	762	15,997
1.5	4	4.450	1.30	7201	5154	5629×10^3	2.552	30	140	1347	35,996
1.6	6	4.465	1.60	7238	6294	5696×10^3	2.645	24	210	1675	58,000
1.9	10	4.476	2.10	7311	8635	5782×10^3	2.770	22	350	2680	106,020
1.10	15	4.485	2.90	7402	12,140	5878×10^3	2.800	21	525	3879	162,493
Scenario 1: Walking along the footbridge – Series A											
1.1	0	4.440	0.60	7128	2386	5547×10^3	–	–	–	–	–
1.4	3	4.445	1.10	7183	4413	5603×10^3	2.504	32	105	1057	25,991
1.7	6	4.465	1.65	7238	6701	5696×10^3	2.778	28	210	2053	63,980
1.8	10	4.475	2.30	7311	9456	5780×10^3	2.900	24	350	3061	116,205
Scenario 2: Walking around a tight circle at midspan – Series A											
1.11	3	4.450	2.11	7214	8077	5652×10^3	2.906	30	210	2301	70,012
1.12	6	4.480	2.90	7300	11918	5784×10^3	2.950	26	420	4048	144,296
1.13	10	4.500	3.40	7415	14256	5928×10^3	2.962	22	560	4586	193,963

Table 6
Standing-still crowd model properties obtained from simulation of 2DOF crowd-structure model – standing at midspan.

No of people	Occupied structure – experimental					Standing crowd model – analytical				
	f_{os} Hz	ζ_{os} %	m_{os} kg	c_{os} N s/m	k_{os} N/m	f_{sc} Hz	ζ_{sc} %	m_{sc} kg	c_{sc} N s/m	k_{sc} N/m
0	4.440	0.60	7128	2386	5547×10^3	–	–	–	–	–
3	4.363	1.35	7968	5898	5988×10^3	5.436	57	210	8177	244,984
6	4.259	2.30	8808	10,842	6307×10^3	5.267	45	420	12,509	459,977
10	4.175	2.60	9928	13,543	6832×10^3	5.171	43	630	17,603	665,042

different combinations of crowd parameters k_c and c_c, with 1000 N/m and 10 Ns/m increments, respectively. The ranges of k_c and c_c values used in the simulations (10 kN/m < k_c < 80 kN/m and 0.2 kNs/m < c_c < 15 kN s/m) were selected to be wide enough to cover the ranges found in the relevant literature, which was m ostly in the field of biomechanics [30–34]. As the 2DOF model of the CS system has two modes of vibration, the *dominant* mode of vibration of the CS system was chosen to represent the modal properties of the occupied structure. The dominant mode of vibration was defined as the mode which dominates the response at the structure DOF. A pair of k_c and c_c parameters corresponding to the best FRF match (according to the least square error) between the measured and the analytical FRF curves of the occupied structure were chosen to represent the crowd model for that experiment.

For consistency, and to allow comparison, in all simulations the mode shape ordinate corresponding to the structure DOF of the dominant mode was scaled to unity. Such scaling ensures that the modal properties of the crowd-structure system were found with the same scaling as the empty structure and, therefore, they were comparable.

6.3. Simulation results

The experimentally measured dynamic properties of the occupied structure and the corresponding crowd model properties found from the simulations are summarised for walking and standing tests in Tables 5 and 6, respectively. To make the comparison easier, the simulation results in Table 5 are presented in the order of increasing number of pedestrians. The trends of the walking crowd model properties observed in simulations are plotted in Fig. 17.

It can be seen in Fig. 17 that, when the number of people increased, the natural frequency of the walking crowd model f_{wc} increased too. This is also valid, but much less pronounced, for the S2 'walking around a tight circle' (green trace), in which the effect of people's location is minimal. The same trend can be seen for the walking crowd model stiffness k_{wc}. Considering that the natural frequency of an SDOF is proportional to $\sqrt{k/m}$, and knowing that the mass of a walking crowd model m_{wc} increases as the number of people in the crowd increases (as calculated and shown in Table 5), it appears that k_{wc} increases faster than m_{wc}, allowing f_{wc} to increase. An explanation for this could be the progressively faster stiffening of the body as the speed of walking in more crowded situations reduces. Although the damping of the walking crowd c_{wc} increased as the number of people in the crowd increased, the damping ratio ζ_{wc} decreased, as it was dependant on the square root of a

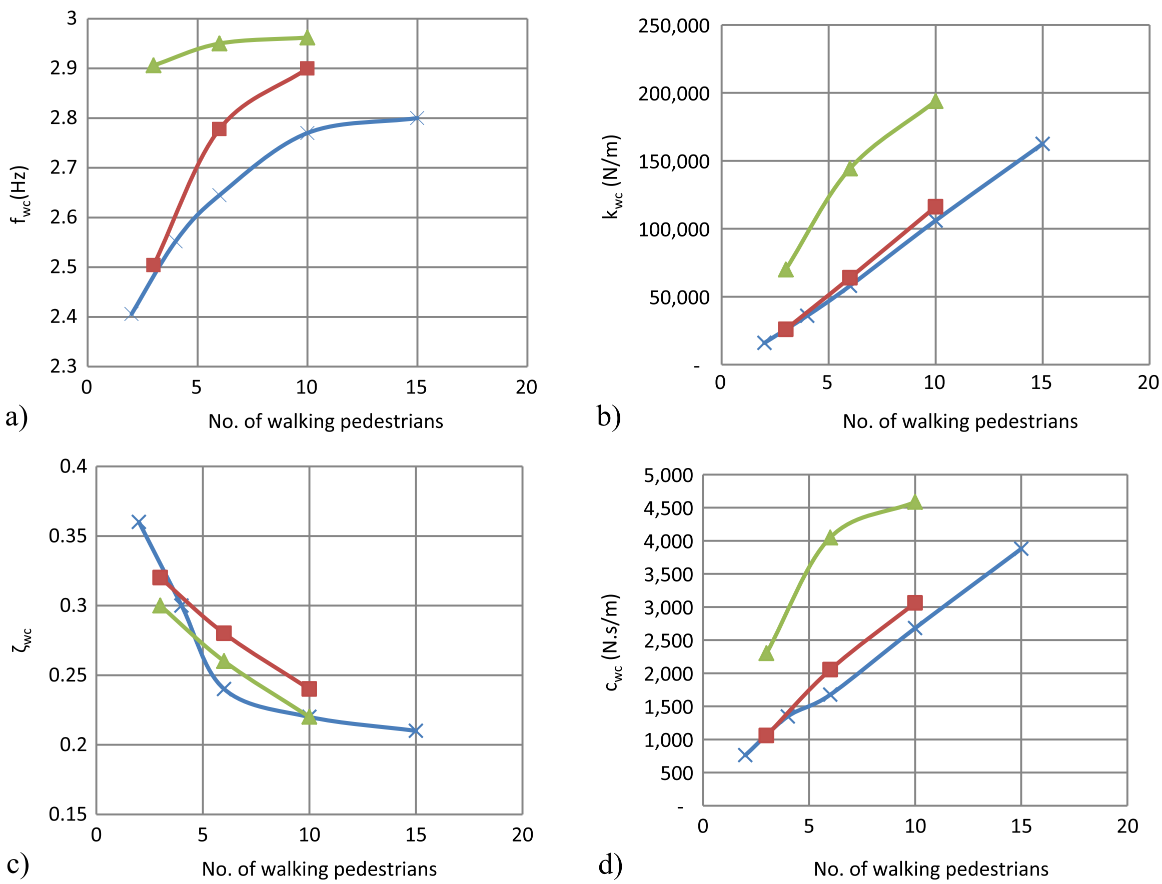


Fig. 17. Trends of walking crowd SDOF model (a) natural frequency f_{cw}, (b) stiffness k_{cw}, (c) damping ratio ζ_{cw} and (d) damping c_{cw} against number of walking pedestrians (Red/square: S1-Series A; Blue/cross: S1-Series B; Green/triangle: S2).

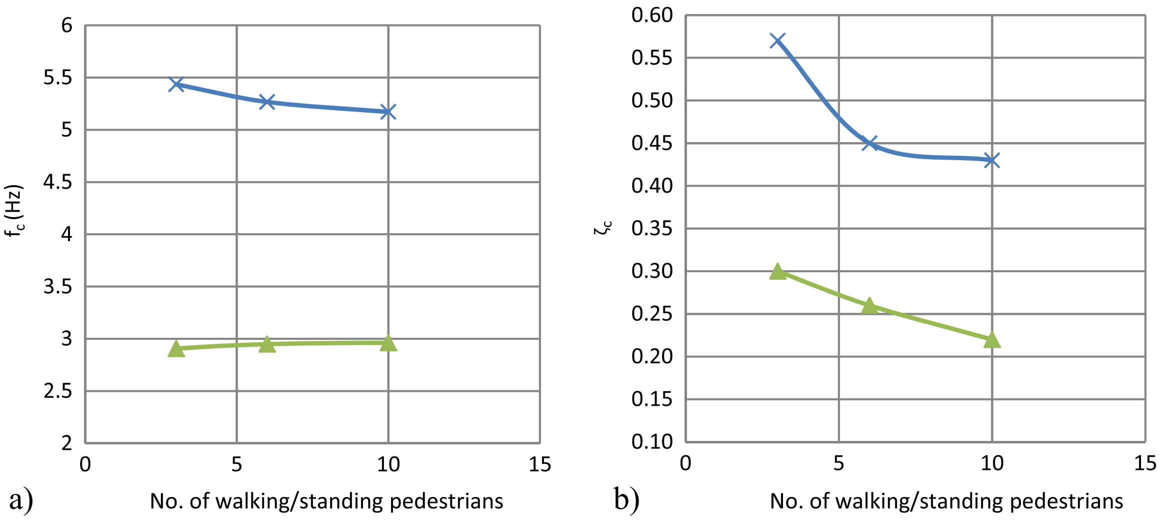


Fig. 18. Comparison of changes in (a) natural frequency f_c and (b) damping ratio ζ_c of walking (green/triangle) and standing (Blue/cross) crowd models against the number of pedestrians.

product of the walking crowd SDOF mass and stiffness, similar, to what is shown in (Eq. (2)) for an SDOF of an empty structure.

In all simulations, changes to the parameters for both walking scenarios show the same trend, which increases the confidence in the results obtained.

Fig. 18 compares the trend in the natural frequency f_c and damping ratio ζ_c of standing-still (blue) and walking (green) crowd models for a different number of people on the structure. Results of walking 'around a tight circle' at midspan tests are compared with standing still at midspan tests. Increasing the number of walking people on the structure slightly increased the natural frequency of the crowd model f_c, while increasing the number of standing people decreased f_c. In both walking and standing-still scenarios, the damping ratio of the crowd model (ζ_{wc} and ζ_{sc}) decreased as the number of people on the structure increased, although their damping coefficients (c_{wc} and c_{sc}) also increased. Finally, the difference between the natural frequencies of the standing still (above 5 Hz) and walking crowd (just below 3 Hz) is remarkable.

These observations are in line with the findings of Shahabpoor et al. [35]. Based on an analytical study of a 2DOF MSD model of a crowd-structure system, they suggest that, when the natural frequency of the crowd model f_c is less than the natural frequency of the empty structure $f_c < f_s$ (similar to the walking people tests in this study), the natural frequency of the occupied structure f_{os} increases. Similarly, when $f_c > f_s$ (similar to the standing-still people tests in this study), the natural frequency of the occupied structure f_{os} decreases. In both cases, the damping ratio ζ_{os} of the occupied structure increases. These trends are also in line with other experimental observations reported in the literature [36–45], concerning the effects of walking pedestrians on vibrating structures in the vertical direction. Most recently, in 2015, Zhang et al. [44] and Van Nimmen et al. [45] also observed a considerable increase in damping ratio and a slight change in natural frequency of the structure occupied by walking people. This agreement with other reported trends and the fact that modelling human/crowd using an SDOF MSD linear model (as the simplest approximation) can coherently reproduce all the experimentally observed trends provide confidence in the validity of the results, specifically the trends of changes observed in the dynamic properties of the occupied structure.

It needs to be mentioned that the magnitude of the crowd model dynamic parameters reported here are dependent on the number of pedestrians, walking regime (walking speed, etc.) and pedestrians biomechanics. An extensive set of experiments with large population of pedestrians, different group sizes and walking regimes would be required to build the statistical descriptors of human/crowd dynamic parameters.

7. Conclusions

Extensive FRF-based modal identification tests were carried out on a full-scale prototype footbridge, with a total of over 150 human participants, walking or standing on it in groups of 2–15. Analysis and subsequent modelling of this experimental data led to the following conclusions:

- Both human biomechanics (including mass, stiffness and damping of a walking human) and dynamic (modal) properties of the structure affect the HSI.
- Given that real structures are usually considerably heavier, stiffer and less damped than the human body, a 2DOF model to simulate walking crowd-structure interaction in the vertical direction was able to consistently replicate the measured

FRF data around relevant resonances of the vertical bending modes of vibration.

- Multi-person walking traffic effects in the vertical direction were possible to simulate using a simple SDOF MSD model of the walking crowd and its mass, stiffness and damping properties were identified for different number of walking people.
- If the natural frequency of the SDOF walking crowd model f_c is less than the natural frequency of the empty structure $f_c < f_s$ both the natural frequency f_{os} and damping ratio ζ_{os} of the occupied structure increase.
- Walking people can increase the damping of the occupied structure tested, more than standing people.
- The results of tests focused on Mode 2 of the prototype structure show that crowd-structure interactions can affect modes with natural frequencies much higher than the fundamental frequency of the walking crowd MSD model.
- The effect of a crowd on the modal parameters of the occupied structure becomes stronger as the size of the crowd increases.

Acknowledgements

The authors acknowledge the financial support which came from the UK Engineering and Physical Sciences Research Council (EPSRC) for Platform Grant EP/G061130/2 (Dynamic Performance of Large Civil Engineering Structures: An Integrated Approach to Management, Design and Assessment), responsive mode grants EP/I03839X/1 (Pedestrian Interaction with Lively Low-Frequency Structures), EP/I029567/1 (Synchronization in Dynamic Loading Due to Multiple Pedestrians Occupants of Vibration-Sensitive Structures) and EP/K03877X/1 (Modelling Complex and Partially Identified Engineering Problems - Application to the Individualised Multiscale Simulation of the Musculoskeletal System).

References

[1] S. Zivanovic, I.M. Diaz, A.Pavi, Influence of walking and standing crowds on structural dynamic properties, Proceedings of the IMAC-XXVII, 2009.
[2] S. Zivanovic, A. Pavic, E.T. Ingolfsson, Modelling spatially unrestricted pedestrian traffic on footbridges, *ASCE Journal of Structural Engineering* 136 (10) (2010) 1296–1308.
[3] M.Kasperski, Actual problems with stand structures due to spectator-induced vibration, Proceedings of Structural Dynamics-EURODYN '96, 1996, pp. 455–461.
[4] W.M. Reid, J.F. Dickie, J.R. Wright, Stadium structures: are they excited, The Structural, *Engineer* 75 (1997) 383–388.
[5] SCOSS, Structural Safety, 13th Report of the Standing Committee on Structural Safety, 2000–2001, pp. 23-24.
[6] V. Racic, A. Pavic, J.M.W. Brownjohn, Experimental identification and analytical modelling of human walking forces: literature review, *Journal of Sound and Vibration* 326 (2009) 1–49.
[7] ISO, Bases for design of structures – Serviceability of buildings and walkways against vibration, ISO 10137:2007. International Organization for standardization, 2007.
[8] BSI, UK National Annex to Eurocode 1: Actions on Structures - Part 2: Traffic Loads on Bridges, NA to BS EN 1991-2:2003, British Standards Institution, 2008.
[9] R. Sachse, A. Pavic, P. Reynolds, Human-structure dynamic interaction in civil engineering dynamics: a literature review, *The Shock and Vibration Digest* 35 (1) (2003) 3–18.
[10] M.J. Griffin, *Handbook of Human Vibration*, Academic Press, London, 1996.
[11] S.H. Strogatz, D.M. Abrams, A. McRobie, B. Eckhardt, E. Ott, Crowd synchrony on the Millennium Bridge, *Nature* 438 (2005) 43–44.
[12] A. Fitzpatrick, P. Dallard, S. le Bourva, A. Low, R. Ridsill-Smith, M. Willford, *Linking London: the Millennium Bridge*, The Royal Academy of Engineering, London, 2001.
[13] A. McRobie, G. Morgenthal, J. Lasenby, M. Ringer, Section model tests on human–structure lock-in, *Bridge Engineering* 156 (2003) 71–79.
[14] M. Bocian, J.H.G. Macdonald, J.F. Burn, Biomechanically inspired modelling of pedestrian-induced vertical self-excited forces, *Journal of Bridge Engineering* 18 (2013).
[15] B.R. Ellis, T. Ji, Floor vibration induced by dance-type loads: verification, *The Structural Engineer* 72 (3) (1994) 45–50.
[16] R. Sachse, *The Influence of Human Occupants on the Dynamic Properties of Slender Structures (PhD Thesis)*, University of Sheffield, Sheffield, UK, 2002.
[17] E. Shahabpoor, A. Pavic, Comparative evaluation of current pedestrian traffic models on structures, Conference Proceedings of the Society for Experimental Mechanics Series. 26, 2012, pp. 41–52.
[18] M.S. Blanter, I.S. Golovin, H. Neuhäuser, H.R. Sinning, *Internal Friction in Metallic Materials: A Handbook*, *Springer Series in Materials Science*, 90, Springer-Verlag Berlin, Heidelberg, 2007 XVII, ISBN 978-3-540-68758-0.
[19] V. Racic, J.M.W. Brownjohn, A. Pavic, Reproduction and application of human bouncing and jumping forces from visual marker data, *Journal of Sound and Vibration* 329 (2010) 3397–3416.
[20] *QA 750 User Manual*, Honeywell International Inc., Washington, USA, 2009.
[21] *APS-400 User Manual*, Acoustic Power Inc., Carlsbad, USA, 2009.
[22] ELAF ENTRAN load cell data sheet. [online] Available at: ⟨http://www.meas-spec.com/downloads/ELAF.pdf⟩ (accessed 17.07.15).
[23] Y. Matsumoto, M.J. Griffin, dynamic response of the standing human body exposed to vertical vibration: influence of posture and vibration magnitude, *Journal of Sound and Vibration* 212 (1) (1998) 85–107.
[24] J. He, Z.F. Fu, *Modal Analysis*, Butterworth-Heinemann, 2001 ISBN:9780750650793.
[25] V. Racic, A. Pavic, J.M.W. Brownjohn, Modern facilities for experimental measurement of dynamic loads induced by humans: a literature review, *Shock and Vibration* 20 (1) (2013) 53–67.
[26] E.Shahabpoor, Dynamic interaction of walking humans with pedestrian structures in vertical direction: experimentally based probabilistic modelling. [Ph.D. thesis], 2014.
[27] R.W. Clough, J. Penzien, *Dynamics of Structures*, 2nd ed. McGraw-Hill, New York. ISBN: 0-07-011394-7, 1993.
[28] C. Min, H. Park, S. Park, Direct identification of non-proportional modal damping matrix for lumped mass system using modal parameters, *Journal of Mechanical Science and Technology* 26 (4) (2012) 993–1002.
[29] C.A. Jones, A. Pavic, P. Reynolds, R.E. Harrison, Verification of equivalent mass–spring–damper models for crowd–structure vibration response prediction, *Canadian Journal of Civil Engineering* 38 (2011) 1122–1135, http://dx.doi.org/10.1139/l11-076.
[30] L.Zhang, D.Xu, M.Makhsous, F.Lin, Stiffness and viscous damping of the human leg, Proceedings of the 24th Annual Meeting of the American Society of Biomechanics, 2000.
[31] S. Rapoport, J. Mizrahi, E. Kimmel, O. Verbitsky, E. Isakov, Constant and variable stiffness and damping of the leg joints in human hopping, *Journal of*

Biomechanical Engineering, ASME 125 (2003) 507–514.
[32] G.Bertos, D.Childress, S.Gard, The vertical mechanical impedance of the locomotor system during human walking with applications in rehabilitation, in: Proceeding of the IEEE Ninth International Conference on Rehabilitation Robotics, 2005.
[33] C. Lee, C. Farley, Determinants of the center of mass trajectory in human walking and running, *The Journal of Experimental Biology* 201 (1998) 2935–2944.
[34] H.Geyer, A.Seyfarth, R.Blickhan, Compliant leg behaviour explains basic dynamics of walking and running, in: Proceeding of the Royal Society Biological Sciences, 273 (1603), 1998, pp. 2861–2867.
[35] E.Shahabpoor, A.Pavić, V.Racic, Using MSD Model to Simulate Human-structure interaction during walking. Conference Proceedings of the Society for Experimental Mechanics Series, 2013.
[36] N.W.M. Bishop, M. Willford, R. Pumphrey, *Multi-person excitation of modern slender staircases*, Engineering for Crowd Safety, London, UK, 1993, 399–408.
[37] J.M.W. Brownjohn, P. Fok, M. Roche, P. Omenzetter, Long span steel pedestrian bridge at Singapore Changi Airport—part 2: crowd loading tests and vibration mitigation measures, *Structural Engineer* 82 (16) (2004) 28–34.
[38] E.Shahabpoor, A.Pavić, V.Racic, Using MSD Model to Simulate Human-Structure Interaction during Walking. Conference Proceedings of the Society for Experimental Mechanics Series, 2013.
[39] A.Ebrahimpour, R.L.Sack, Design Live Loads for Crowds in Motion. Conference proceeding of Building an International Community of Structural Engineers, ASCE, 1, 1996, pp. 420–427.
[40] P. Fanning, P. Healy, A. Pavic, Pedestrian Bridge vibration serviceability: a case study in testing and simulation, *Advances in Structural Engineering* 13 (2010) 5.
[41] R.L. Pimentel, P. Waldron, Validation of the numerical analysis of a pedestrian bridge for vibration serviceability applications. International Conference on Identification in Engineering Systems, Swansea, UK, 1996, pp. 648-657.
[42] R.L. Pimentel, *Vibration performance of pedestrian bridges due to human-induced loads (PhD thesis)*, University of Sheffield, Sheffield, UK, 1997.
[43] M. Willford, Dynamic actions and reactions of pedestrians. In proceedings of the International Conference on the Design and Dynamic Behaviour of Footbridges, Paris, France, 2002, pp. 66–73.
[44] M. Zhang, C.T. Georgakis, W. Qu, J.Chen, SMD model parameters of pedestrians for vertical human-structure Interaction, in: proceeding of the 33rd conference of the society of experimental mechanics IMAC 2015, DOI http://dx.doi.org/10.1007/978-3-319-15248-6_33.
[45] M. Zhang, C.T.Georgakis, W.Qu, J.Chen, SMD model parameters of pedestrians for vertical human-structure Interaction. In the proceeding of the 33rd conference of the society of experimental mechanics IMAC 2015, DOI http://dx.doi.org/10.1007/978-3-319-15248-6_33.