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Uncoupled approaches for walking-induced vertical vibration of a lively footbridge

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ABSTRACT: The recent trend towards the design of flexible footbridges, characterized by a low ratio between permanent and variable load, has made them more sensitive to dynamic forces induced by pedestrians. While walking, the pedestrian moves on the flexible structure adapting his gait to the bridge motion and interacting with the footbridge. At contact points, the pedestrian transmits contact forces to the bridge that, in turn, imposes a set of displacements and velocities to the pedestrian's feet. The pedestrian is here described with an increasing accuracy. Neglecting the interaction with the bridge, the pedestrian is described with a refined force model, that simulates the force pattern transmitted by each foot. The position of the two forces reproduces the sequence of single and double support phases, typical of the human gait. Interaction is accounted for when both systems are described as mechanical systems (having proper mass, stiffness and damping matrices). In this work a newly proposed bipedal pedestrian mass-spring-damper model is adopted, sharing with the force model the same type of locomotion. The mechanical system is excited by an equivalent bio-mechanical force and its equation of motion takes into account the interaction with the bridge. The coupled equations of motion of the bridge-pedestrian system are then derived; with a forced uncoupling of the equations, the two systems can be analyzed separately. The case study concerns a lively footbridge, whose dynamic response is computed with the different modeling approaches.

1 INTRODUCTION

Challenging footbridges, characterized by aesthetics requirements for greater slenderness, result in longer spans, low ratio between permanent and live loads, and low damping (Ingòlfsson et al. 2012). Consequently, their structural systems might exhibit dynamic behavior marked by closely-spaced natural frequencies and/or frequencies very close to the values perceived by human beings, so that the design requires a greater care regarding vibration phenomena (Van Nimmen et al. 2014).

During footbridge vibrations, especially under crowd load, some form of Human-Structure Interaction (HSI) occurs, playing an important role in the structural dynamic response. The correct modeling of pedestrians represents an important issue into the study of HSI. A large research effort in the last 15 years has led to both the development of analytical expressions for the walking force and procedures to determine the footbridge response. Following the simplified models, the increasing sophistication of the proposed expressions did not lead to significant improvements in numerical estimates of the response under pedestrian flows (Jimenez-Alonso et al. 2016). In addition, the dynamic analyses using mathematical force models, both in time and frequency domain, cannot consider the dynamic effects of the mechanical human body on the bridge response (Kim et al. 2007). On the contrary, a mechanical model considers the mass, stiffness and damping of the pedestrian, all of which can interact with the structure (Toso et al. 2016).

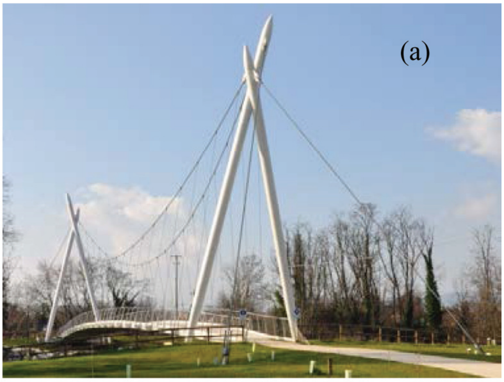
Aim of this work is the study of the dynamic response of a footbridge accounting for the Human-Structure Interaction (HSI) in the vertical direction. Two pedestrian's models, reproducing the human gait in the same way but having increasing accuracy have been proposed: a dynamic travelling force and a Mass-Spring-Damper (MSD) system. Only the latter can be adopted if HSI is to be investigated. Numerical analyses have been carried out with an ad-hoc numerical code, based on an uncoupled analytical formulation recently proposed (Lai 2016). The modeling and analysis approach is inherently uncoupled when forces are considered. For the case of the MSD model, uncoupling is based on a previous study on the vehicle-bridge dynamic interaction (Feriani & Mulas 2008) and involves the derivation of the bridge-pedestrian coupled equations of motion. Due to the pedestrian motion, the structural matrices of the coupled system are time-dependent and should be modified whenever the pedestrian's foot position on the bridge changes. The proposed alternative solution strategy is based on the forced uncoupling of the system, based on the key assumption that the contact points between pedestrian and bridge deck are massless. The uncoupled formulation considers the footbridge and the pedestrian as two separate sub-systems (the former subjected to contact forces transmitted by the pedestrian, the latter excited by the bridge motion), reducing the computational burden associated to the time-varying properties of the matrices of the coupled system.

Adopting the different approaches, the response of a suspension lively footbridge is investigated for the cases of a single pedestrian and six pedestrians with different trajectories.

2 THE SUSPENSION FOOTBRIDGE

2.1 *Bridge description*

The Seriate footbridge in Figure 1a, 63.75 m long, connects two cycle routes in the Serio Park (near to Milan, Italy). The width of the timber deck on a steel grid (Figure 1b) ranges between 2.5 m at the entrance and 5 m at midspan. The slightly curved longitudinal steel girders have a rise of 1.3m. The transverse beams of the steel grid (Figure 1b), spaced 1 m apart, fall into two categories: the main girders, connected to the hangers, with a tapered section and the secondary ones, with an IPE 120 cross-section. The stringers are a pair of IPE 330 beams at the edges, and a central beam with a circular section (ϕ = 298.5 mm), deemed to stabilize the deck on the horizontal plane.



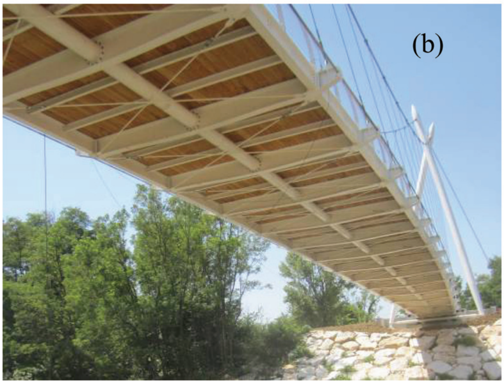


Figure 1. Seriate Footbridge: (a) overall view; (b) footbridge deck, detail of longitudinal and transverse beam.

A series of X-braces (Fig. 1b) connects the main transverse girders and provides stiffness in the horizontal plane. The timber deck (Fig. 1b) has only a minor structural role, providing the walking surface. The ends of the transverse main girders are crossed by stabilizing cables, whose sliding in the longitudinal direction is allowed by the interposition of a polymeric layer between the two contact surfaces.

The suspension system is composed of:

- steel main pylons, slightly inclined with respect to the vertical plane, and arranged in pairs creating an A-shaped portal. The portals support the suspension system and the backstays cables as shown in Figure 1a;
- 2 main suspension cables, ϕ = 60 mm, supporting the longitudinal girders through 42 vertical hangers, of diameter 16 mm;
- backstays cables, ϕ = 60 mm, connecting the pylons top to the ground;
- 2 stabilizing cables of opposed curvatures ϕ = 40 mm.

The suspension system is not symmetric neither about the vertical plane crossing the longitudinal bridge axis nor about the vertical plane crossing the midspan. All the cables were pre-tensioned during construction.

2.2 *FE model*

The bridge model, shown in Figure 2, is based on the as-built design data and has been implemented within the ANSYS framework. The girders of the steel grid, the pylons and the hangers are modeled with “BEAM 188”, a Timoshenko beam element, with 6 Degrees Of Freedom (DOFs) at each node. Cables and braces are modeled with “LINK 180”, a spar element transmitting axial force only, with 3 DOFs at each node. Timber planks and handrails are modeled as lumped masses on steel grid; their weight is included in the dead load. The boundary conditions of the footbridge model, as the constraint conditions between adjacent elements, are inferred from the technical drawings.

The cables geometric stiffness and the variation of configuration associated to dead loads have been accounted for in a preliminary non-linear static

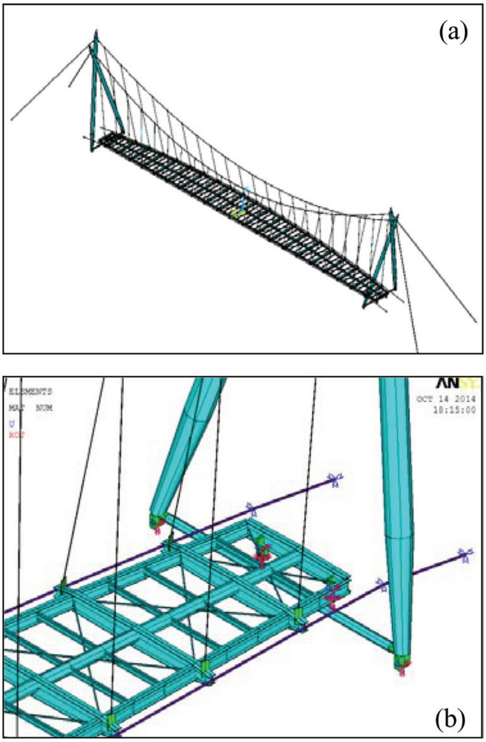


Figure 2. FE model of Seriate Footbridge: (a) overall view; (b) detail of steel grid.

Table 1. Modal properties of the footbridge.

Mode #	Type	f_E [Hz]	f_N [Hz]	ε(%)	MAC
1	V-F	1.025	1.079	5.22	0.995
2	V-F	1.475	1.565	6.07	0.994
3	V-T	1.924	1.997	3.77	0.908
4	V-TR	1.953	2.109	7.99	0.842
5	V-F	2.168	2.311	6.58	0.984
6	V-T	2.754	2.635	–4.33	0.975
7	V-F	2.861	2.827	–1.17	0.996
8	V-T	3.691	3.645	–1.24	0.957
9	V-F	4.121	4.076	–1.10	0.988
10	V-T	4.385	4.409	0.55	0.982

analysis. Once the model correctly reproduces the design value of tension in cables and the deformed geometry, the modal analysis has been performed. The modal properties of the footbridge, for the first experimentally identified 10 modes, are listed in Table 1, where f_E and f_N denote the experimental and numerical value of the frequency, respectively, and ε is the percentage error between the two. The second column lists the type of mode shape (V-F: vertical-flexural; V-T: vertical-torsional and V-TR: vertical-transversal). The value of MAC index, a measure of the correlation between the mode shapes, is shown in the last column. The agreement between experimental and numerical value is fully satisfactory. Further details on the derivation of the FE model are found in Lai et al. 2015.

3 MODELING OF PEDESTRIAN

3.1 *Pedestrian movement*

The pedestrian movement along the bridge deck represents a crucial aspect in the study of the response of a footbridge subjected to walking people. Both the classic single pedestrian loading model (Bachmann et al. 1995) and more sophisticated force models (Zivanovic et al. 2007) are modelled as a continuous walking force, based on the assumption that pedestrian remains always in contact with the bridge deck. This approach models the feet movement as the motion of a roller and it is not realistic for two reasons. First, when a human being walks or runs, he covers only a discrete series of points in space. Secondly, the pedestrian's position does not change at each time step and is characterized by time intervals in which, alternatively and in a sequence, the human transmits either two forces (double support phase, DSP) or only one (single support phase, SSP). Furthermore, during the DSP, the two forces are placed at two different positions and their mutual distance is equal to the step length.

Figure 3 depicts the human gait cycle, highlighting the alternation of SSP and DSP. The former begins at the instant when the leading foot hits the ground and the trailing foot is off the ground, moving through the air towards the next position. The latter starts when the trailing foot hits the ground becoming the leading foot and both feet are in contact with the bridge deck.

3.2 *Pedestrian's model*

The level of adequacy of the structural analysis of the dynamic interaction between footbridge and pedestrians depends on the modelling of the pedestrian. Different models of pedestrian have been implemented to carry out analyses of increas-

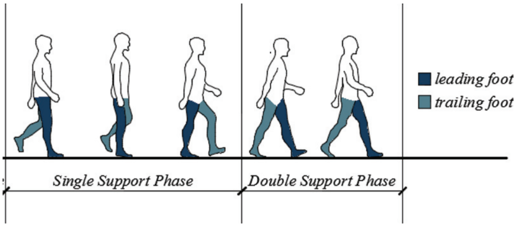


Figure 3. Human gait cycle.

ing accuracy. First, the human being is described as a couple of vertical dynamic forces, each following the single foot force model in Figure 4 (Li et al. 2010). Each foot moves over a discrete series of points along the bridge deck (see Section 3.1). The single foot force model allows the simulation of the gait cycle in Figure 3, since the two feet can be considered individually without neglecting the relation existing between them. Following Figure 4, the SSP and DSP alternate themselves according to the law:

$$\begin{cases} t < T_s & SSP \\ T_s \le t \le T_e & DSP \\ t > T_e & SSP \end{cases} \tag{1}$$

where T_s is the period related to the step frequency, T_e is the period of the single foot force and Δt is the overlapping period when both feet are in contact. Each foot remains in the same position during all the period T_e and moves forward only when the period ends. Therefore, within a numerical time integration, the force position does not change at each time step and occupies only a discrete numbers of points along its path, spaced apart at a distance equal to the step length.

Modelling the human being as a dynamic force takes into account the variation of both intensity and position of the forces transmitted by the pedestrian feet, but neglects the interaction between bridge and pedestrians and the subsequent variation of the bridge dynamic properties. Second, the pedestrian is modelled as a SDOF mass-spring-damper mechanical system. The MSD system has two spring-damper legs (Fig. 5), capable to reproduce the correct human gait (SSP and DSP in sequence). The legs parameters follows the data provided by Kim & Park (2011). The mass, lumped at the center of mass of the body, oscillates in the vertical direction only. A fictitious bio-mechanical force F_b excites the MSD system. By solving an inverse problem, in this work F_b has been derived as the force that, applied on the system moving on a rigid surface, produces a ground reaction force (transmitted by each leg) matching the single foot force model in Figure 3.

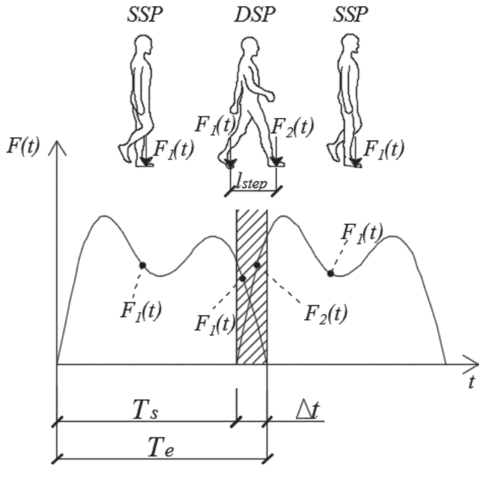


Figure 4. Single foot force: F_2 is the force transmitted by the leading leg during DSP, F_1 denotes the remaining situations.

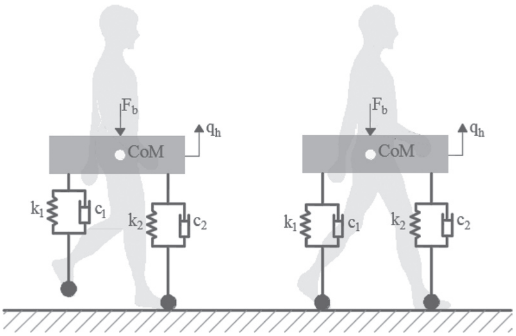


Figure 5. MSD model: left, only one leg is in contact, right, both legs are in contact.

4 NUMERICAL ANALYSIS OF THE FOOTBRIDGE

The analytical procedure for the uncoupled analysis of the pedestrian-footbridge interaction, including the correct human locomotion, has been implemented in a numerical code, named INTER 2.0 (Lai 2016), able to consider the pedestrian as either a dynamic force or a mechanical system. In the numerical implementation, the bridge is modelled using the FE method, and its 3D geometry is considered correctly without any approximation.

The pedestrians can freely walk following a rectilinear trajectory, parallel to the bridge axis, with a transverse x_P coordinate inside the deck grid chosen by the analyst. In general, the pedestrian position does not coincide with a bridge mesh node and therefore the human-induced force will load a contact point that does not belong to the mesh. To solve this problem, a proper set of shape functions is introduced, with a twofold aim. Moving forces are transformed into equivalent nodal loads on properly selected mesh nodes, and the contact point displacement and velocity are computed from analogous quantities at surrounding mesh nodes. As shown in

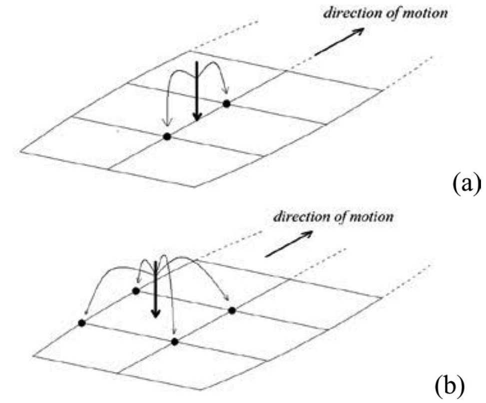


Figure 6. Loaded nodes: (a) pedestrian moving along a line of nodes; (b) pedestrian in a generic position.

Figure 6a, if the pedestrian trajectory coincides with a line of nodes, the selected nodes will be those placed immediately before and after the force position. Otherwise, the selected nodes will be the four nodes of the mesh surrounding the pedestrian's position (Fig. 6b).

5 NUMERICAL RESULTS

The relevant response parameters are the time histories of vertical accelerations and vertical displacements, evaluated in seven transverse sections of the deck for three points, as shown in Figure 7.

First, the analyses have been performed adopting only a single pedestrian described with the two pedestrian models investigated. Second, the bridge response, in terms of displacements and accelerations, produced by two different groups of six MSD pedestrians, is computed.

5.1 *Single pedestrian*

In the first and simplest case study investigated in this work, a single pedestrian, with a step frequency and a weight of 1.6 Hz and 700 N, respectively, walks at a constant speed (1,3 m/s) along the longitudinal bridge axis. Bridge crossing takes about 50s. Figure 8 shows that the displacement time histories at midspan node 11, for each pedestrian's model, have a common pattern and similar maximum values (those due to the MSD system being slightly lower), reached at the same instant of time. The displacements produced by both the dynamic force and the MSD model show oscillations, at the step frequency, about the average value. This pattern is tied to the periodicity of both the couple of single foot forces and the force itself, reproducing the alternation of SSP and DSP.

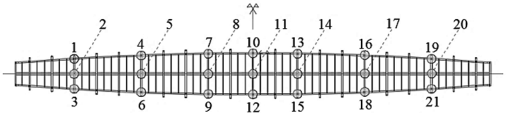


Figure 7. Bridge nodes considered in the analysis.

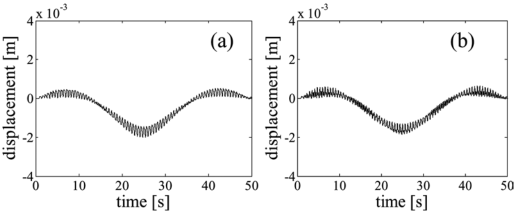


Figure 8. Displacement time history at node 11: (a) dynamic travelling force, (b) MSD model.

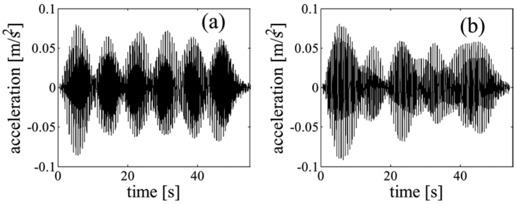


Figure 9. Acceleration time history at node 6: (a) dynamic travelling force, (b) MSD model.

On the contrary, the comparison in terms of accelerations shows that larger values are found for the MSD system than those experienced with the dynamic force. For the accelerations of nodes on the same section, the MSD results are more scattered (general trend) than those obtained with the force model. Furthermore, in general the MSD model produces accelerations larger in the central nodes than in the lateral ones since the former are directly loaded by the pedestrian's walk.

Figure 9 shows the time-histories of acceleration at node 6 with the two models. When the node is reached by the pedestrian, the acceleration attains the maximum value. A beat phenomenon is well apparent with both models. The MSD system, as the dynamic force, might excite the footbridge at study with the first, second and third harmonic of the load (biomechanical force in the case of MSD model).

5.2 *Six pedestrians*

The six people are placed in a single line, parallel to the bridge axis, with a mutual distance of 2 m. Two pedestrian configurations are analyzed. In the first one, the pedestrians are walking on the left side of the deck, with a rectilinear trajectory having an eccentricity of 1 m from the longitudinal axis. In the second configuration, the six pedestrians are walking along the bridge longitudinal axis.

The six MSD pedestrians are considered uncorrelated and unrestricted: they have their own step frequency ($f_1 = 1.843$ Hz, $f_2 = 2.057$ Hz, $f_3 = 1.675$ Hz, $f_4 = 1.776$ Hz, $f_5 = 1.634$ Hz, $f_6 = 1.799$ Hz) drawn from a normal distribution, and no synchronization has been taken into account. The pedestrians have a common weight and height of 700 N and 1.70 m, respectively. Each pedestrian crosses the bridge in about 49 s and the last pedestrian exits the bridge about 8 s after the first. Aim of the analyses is to investigate how transits, with the same spatial distribution but different eccentricity, affect the bridge response.

The comparison in terms of time histories of displacements (Fig. 10) highlights how eccentric transits induce a torsional behavior, which could have been amplified by the configuration of the bridge suspension system, anti-symmetric with respect to both the bridge longitudinal axis and the mid-span.

Figure 10a shows the comparison among the displacement histories of nodes 4, 5 and 6 (section at ¼ span) for the case of people walking along the left side. The torsional behavior becomes larger when the pedestrians cross the section at study. Obviously, nodes closer to the pedestrians' trajectory, i.e. node 4 and 5 respectively, experience displacements larger than those of the node on the opposite side.

The pedestrians walking along the longitudinal axis do not induce a torsional effect. In fact, in Figure 10b, the displacement histories are overlapped during the whole time history.

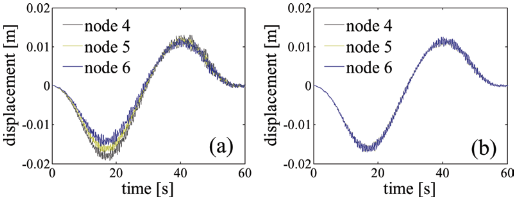


Figure 10. Displacement time history at ¼ span nodes: (a) left eccentricity, (b) no eccentricity.

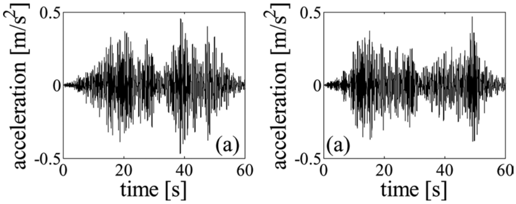


Figure 11. Acceleration time history at node 4: (a) left eccentricity, (b) no eccentricity.

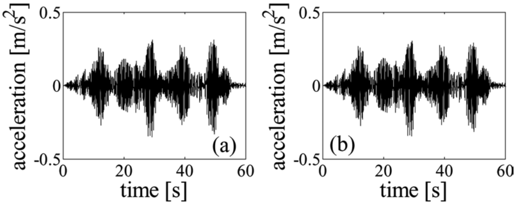


Figure 12. Acceleration time history at node 11 (mid-span): (a) left eccentricity, (b) no eccentricity.

The acceleration time histories show a general trend. The eccentric transit produces accelerations larger than those induced by a non-eccentric load. In addition, when nodes on the same cross-section are analyzed, the response in terms of acceleration has the same pattern of the displacement response. The lateral nodes experience the largest acceleration with the eccentric configuration, whereas a central trajectory does not produce a considerable difference among the nodes of the same cross-section.

Figure 11 and Figure 12 shows the time histories of the vertical acceleration at node 4 and 11, respectively. The bridge responses at mid-span have a common pattern. The time histories of lateral nodes are slightly different.

6 CONCLUSIONS

This paper, aiming to analyze the response of a footbridge accounting for the HSI, summarizes in a qualitative way, the main findings of a research devoted to this topic. In more detail, both the procedure of analysis and the derivation of a MSD system representing a pedestrian, have been sketched. The case study is a lively footbridge, whose dynamic behavior has been extensively analyzed both experimentally and numerically. In this work, the results of the numerical analyses aiming to evaluating the effect on the bridge response of a few parameters (pedestrian's model adopted and spatial distribution) can be listed as it follows:

- The displacement time histories induced by a single pedestrian, described with the two models, have similar values and common pattern.
- The dynamic travelling force induces accelerations having the same order of magnitude of those produced by a MSD model, but lower.
- Acceleration time histories show a beat phenomenon.
- Eccentric transits produce larger accelerations than those induced by a group of pedestrian with

no eccentricity. Moreover, as it was expected, the eccentric transit induces a torsional behaviour in the bridge response.

Because of the promising results, the research is currently under way, to refine both the MSD system and the uncoupled numerical procedure.

REFERENCES

Bachmann, H., Pretlove A.J. & Rainer, H. 1995. Dynamic forces from rhythmical human body motions. *Vibration Problems in Structures: Pratical Guidelines*, Birkhäuser, Appendix G.

Feriani, A., Mulas, M.G. & Lucchini, G. 2008. Vehicle-bridge interaction analysis: an uncoupled approach. In *Proceedings of ISMA* 2008, Leuven, Belgium.

Ingólfsson, E.T., Georgakis, C.T. & Jönsson, J. 2012. Pedestrian-induced lateral vibrations of footbridges: A literature review", *Engineering Structures*, 45, 21–52.

Jimenez-Alonso, J.F., Sáez, A., Caetano, E. & Magalhãe, F. 2016. Vertical Crowd-Structure Interaction Model to Analyze the Change of the Modal Properties of a Footbridge. *J of Bridge Engineering*, C4015004.

Kim, S.H., Mha, H.S., Cho, K.I., Won, J.H. & Kim, M.A. 2007. Study on dynamic behaviour of footbridge under pedestrian excitation generated by human body model. In: *Proceedings of the 8th Pacific Structural Steel Conference—Steel Structures in Natural Hazards* PSSC, March, New Zealand.

Kim, S. & Park, S. 2011. Leg stiffness increases with speed to modulate gait frequency and propulsion energy. *J of Biomechanics*, 44, 1253–1258.

Lai, E., Gentile, C. & Mulas, M.G. 2015. Vibration testing and FE modelling of a lively footbridge. In *6th International Operational Modal Analysis Conference* (IOMAC'15), Gijón, Spain, May 12–14.

Lai, E. 2016. *Pedestrian-footbridge dynamic interaction: uncoupled analysis using a MSD model*. PhD. Thesis. Politecnico di Milano, Department of Civil and Environmental Engineering.

Li, Q., Fan, J., Nie, J., Li, Q. & Chen, Y. 2010. Crowd-induced random vibration of footbridge and vibration control using multiple tuned mass dampers. *J of Sound and Vibration*, 329, 4068–4092.

Toso, M.A., Gomes, H.M., De Silva, F.T, Pimentel & R.L. 2016. Experimentally fitted biodynamic models for pedestrian-structure interaction in walking situations. *J of Mechanical Systems and Signal Processing*, 72–72, 590–606.

Van Nimmen, K., Lombaert, G., De Roeck, G. & Van den Broeck, P. 2014. Vibration serviceability of footbridges: Evaluation of the current codes of practice. *Engineering Structures*, 59, 448–461.

Živanović, S., Pavic, A. & Ingolfsson, E.T. 2010. Modelling spatially unrestricted pedestrian traffic on footbridges. *J of Structural Engineering* ASCE, 136(10), 1296–1308.