Mean-Square Prefiltered Generalized Delayed Decision Feedback Sequence Detection

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ABSTRACT

The Generalized Delayed Decision Feedback Sequence Detector (GDDFSD) is a scheme for detecting uncoded data corrupted by ISI and noise. The GDDFSD is a variant of the DDFSD, the variant being that in the GDDFSD multiple survivors are allowed for each state. In the paper, it is proposed to adopt as a front-end the mean-square whitened matched filter in place of the classical whitened matched filter. Simulation results show that our proposed design gives substantial benefits when a severe frequency selective channel is considered.

I. INTRODUCTION

The concern of the present paper is a suboptimal technique for signal equalization. The receiver that guarantees minimum Bit Error Rate (BER) is the maximum a posteriori probability receiver. However, one often renounces to this receiver, because it is too complex. A simpler receiver is obtained if the probability of sequence error is considered. This approach leads to Maximum Likelihood Sequence Detection (MLSD) [1]. Unfortunately, even MLSD is often too complex. Actually, the MLSD receiver is realized by a Viterbi algorithm with a number of states that is exponential in the channel memory. Hence, when dealing with channels with long memory, one is forced to consider suboptimal receivers. A popular technique for complexity reduction is the Delayed Decision Feedback Sequence Detection (DDFSD) proposed in [2]. The DDFSD is based on a Viterbi algorithm where the channel memory is truncated. The performance loss due to memory truncation is mitigated by a per survivor processing, where the past history of each survivor is used in a decision feedback scheme. The Whitened Matched Filter (WMF) of [1] was adopted as a front-end for the DDFSD in [2]. Recently, the benefits offered by the Mean Square Whitened Matched Filter (MSWMF) have been pointed out in [3, 4, 5]. In the present paper, it is proposed to adopt the MSWMF in a more general scheme, called Generalized DDFSD (GDDFSD), where multiple survivors are considered for each state [6]. Simulation results show that the MSWMF-GDDFSD outperforms the WMF-GDDFSD when a severe frequency selective channel is considered.

II. SYSTEM MODEL

We consider the model of a binary uncoded data sequence transmitted over a baseband linear channel corrupted by Additive White Gaussian Noise (AWGN). The block diagram of the system is reported in figure 1. In figure 1, $\tilde{a}_k \in \{+1, -1\}$ is the bit transmitted at time k and $w(t)$ is AWGN with two-sided power spectral density σ^2. Let $r(z) = \sum_{i=-\nu}^{\nu} r_i z^{-i}$ be the z-transform of the impulse response of the system from the source to the output of the sampler (z^{-1} indicates the unit delay). The trellis for MLSD has 2^ν states, while in the DDFSD the trellis has 2^μ states, $\mu \le \nu$, and to each state a Decision Feedback Equalizer (DFE) with $\nu - \mu$ taps is attached. With reference to figure 1, the branch metric at time k in the reduced trellis of the DDFSD is

$$b_k(a_{k-\mu}, \ldots, a_k) = (x_k - \sum_{j=0}^{\mu} d_j a_{k-j} - \sum_{j=\mu+1}^{\nu} d_j \hat{a}_{k-j}(a_{k-\mu}, \ldots, a_{k-1}))^2, \qquad (1)$$

where $\hat{a}_{k-j}(a_{k-\mu}, \ldots, a_{k-1})$ is the estimate of the bit transmitted at time $k-j$ which is present in the survivor that at time $k-1$ visits the state $(a_{k-\mu}, \ldots, a_{k-1})$. According to [6], the GDDFSD is obtained by allowing M DFEs for each state. Hence in the GDDFSD there are 2^μ states and $2M$ transi-

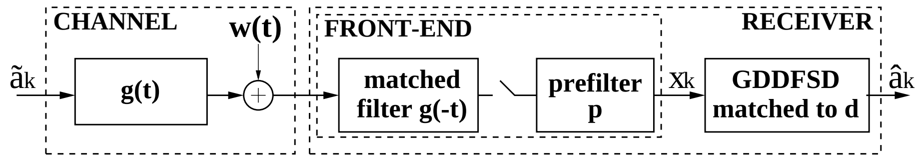

Figure 1: Channel and receiver block diagram

tions diverging from and merging in each state. At each step in the trellis, the metrics of the $2M$ transitions that merge in each state are sorted, and the sequences associated to the M lower metrics are selected as survivors. The prefilter and $d(z)$ are hereafter presented for the WMF and for the MSWMF.

A. Whitened Matched Filter

In the time discrete white Gaussian model of [1], $d(z)$ is obtained from the spectral factorization

$$r(z) = d(z)d(z^{-1}) \tag{2}$$

by taking for $d_{WM}(z)$ that $d(z)$ that is causal and minimum phase. The autocorrelation $r(z)$ is factorizable if its Fourier transform

$$S(f) = r(e^{j2\pi f}), \tag{3}$$

is nonnull over any measurable interval [7]. When this condition is satisfied, the roots of $d_{WM}(z)$ are on or inside the unit circle. Note that the case where some of the roots of $d_{WM}(z)$ are on the unit circle, that is when $S(f)$ is null in some non measurable interval, is a limiting case. When the roots of $d_{WM}(z)$ are inside the unit circle, the prefilter is the noise whitening filter

$$p_{WM}(z) = d_{WM}^{-1}(z^{-1}). \tag{4}$$

When the roots of $d_{WM}(z)$ are on the unit circle, the noise whitening filter does not exist, because $d_{WM}(z^{-1})$ is not invertible. However, the existence of the WMF is still guaranteed [1].

B. Mean Square Whitened Matched Filter

In the MSWMF, $d(z)$ is determined from the spectral factorization

$$d(z)d(z^{-1}) = r(z) + \sigma^2, \tag{5}$$

by taking for $d_{MS}(z)$ that $d(z)$ that is causal and minimum phase. Note that, for $\sigma^2 > 0$, factorizability of $r(z) + \sigma^2$ is guaranteed. Therefore, in contrast to the WMF, here the case where $S(f)$ (3) is null in some interval is not a limiting case. Let $e(z) = x(z) - \tilde{a}(z)d_{MS}(z)$ be the error sequence. In [7] it is shown that the prefilter that minimizes the mean square error is

$$p_{MS}(z) = \frac{d_{MS}(z)}{r(z) + \sigma^2} = d_{MS}^{-1}(z^{-1}). \tag{6}$$

When $p_{MS}(z)$ is used as a prefilter, the error sequence turns out to be white [7]. For this reason, the front-end filter takes the name of mean-square whitened matched filter. It has been proved in [3] that when $\mu = \nu$ the Viterbi detector based on the MSWMF performs MLSD.

III. EXPERIMENTAL RESULTS

To obtain substantial difference between the MSWMF-GDDFSD and the WMF-GDDFSD, a severely distorted channel should be considered. The channels studied in [8] are actually severe, in the sense that they give the lower minimum distance for a fixed duration of the impulse response. We focus on the channel with $\nu = 6$. The z-transform of the impulse response at the output of the WMF, that is $r(z)p_{WM}(z) = d_{WM}(z)$, is $d_{WM}(z) = 0.176 + 0.316z^{-1} + 0.476z^{-2} + 0.532z^{-3} + 0.476z^{-4} + 0.316z^{-5} + 0.176z^{-6}$. The shape of the impulse response, depicted in figure 2 together with the spectrum $r\left(e^{j\omega}\right)$, resembles a bell, a shape that is often found in channels from the real world. Note that this channel has three pairs of roots on the unit circle, that is three spectral nulls. It is intuitive that the effect of the spectral nulls is more severe for the WMF, where the spectral nulls are treated as a lim-

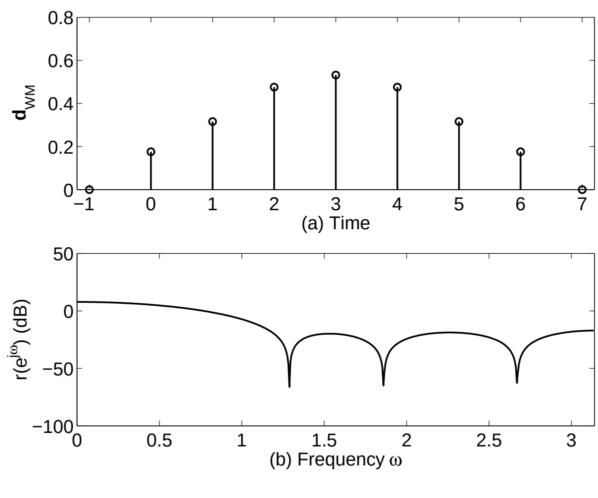

Figure 2: (a) Discrete time channel. (b) Spectrum.

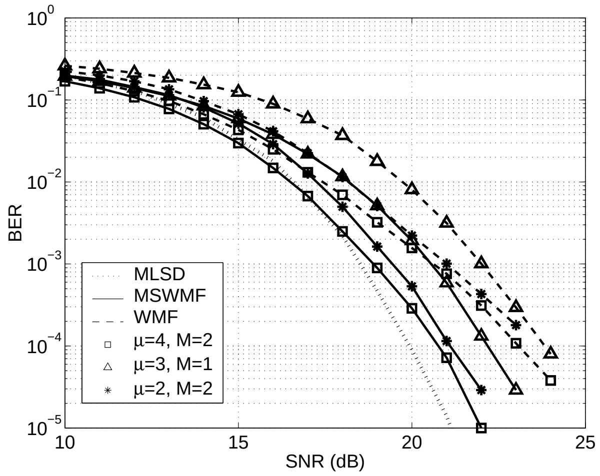

Figure 3: Performance of MLSD and of the GDDFSD with the WMF and the MSWMF. M is the number of survivors per state, and 2^{μ} is the number of states

iting case. The error rate is evaluated by computer simulation. In the simulations, the z-transform of the sequence at the output of the WMF is $x_{WM}(z) = d_{WM}(z)\tilde{a}(z) + w(z)$, where the variance of the white noise is σ^2. For the MSWMF, the polynomial $x_{MS}(z)$ is $x_{MS}(z) = x_{WM}(z)p_{WM}^{-1}(z)p_{MS}(z) = x_{WM}(z)d_{WM}(z^{-1})d_{MS}^{-1}(z^{-1})$, the existence of $d_{MS}^{-1}(z^{-1})$ being guaranteed for $\sigma > 0$. The product $d_{WM}(z^{-1})d_{MS}^{-1}(z^{-1})$ is truncated to 91 terms. The BER is measured by a random sequence of $2 \cdot 10^6$ data. Figure 3 reports the BER of MLSD, MSWMF-GDDFSD, and WMF-GDDFSD, versus Signal to Noise Ratio (SNR), $SNR = r_0/\sigma^2$. From the figure, it is apparent that the MSWMF outperforms the WMF. Examining the results reported in figure 3 for the MSWMF, one observes that the MSWMF-GDDFSD with $\mu = 2$, $M = 2$ outperforms the MSWMF-GDDFSD with $\mu = 3$, $M = 1$ (that is the pure DDFSD with 8 states), and that its performance is close to the performance achieved with $\mu = 4$, $M = 2$. This observation suggests that, when severe complexity reduction is necessary, a well-balanced design of μ and M may offer the best performance.

IV. CONCLUSIONS

The MSWMF is widely known and studied in the theory of the DFE [7], and has been recently adopted as a front-end for the DDFSD in [3, 4, 5]. In the present paper, the MSWMF has been proposed as a front end also for the GDDFSD. The results show that the MSWMF-GDDFSD outperforms the WMF-GDDFSD when a severe frequency selective channel is considered. The results also suggest that, in the design of the GDDFSD, a studied balancement between the number of states and the number of survivors may offer the best performance when severe complexity reduction is necessary.

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